Online Appendix to the research article:

Stock market alphas help predict macroeconomic innovations.

Abstract

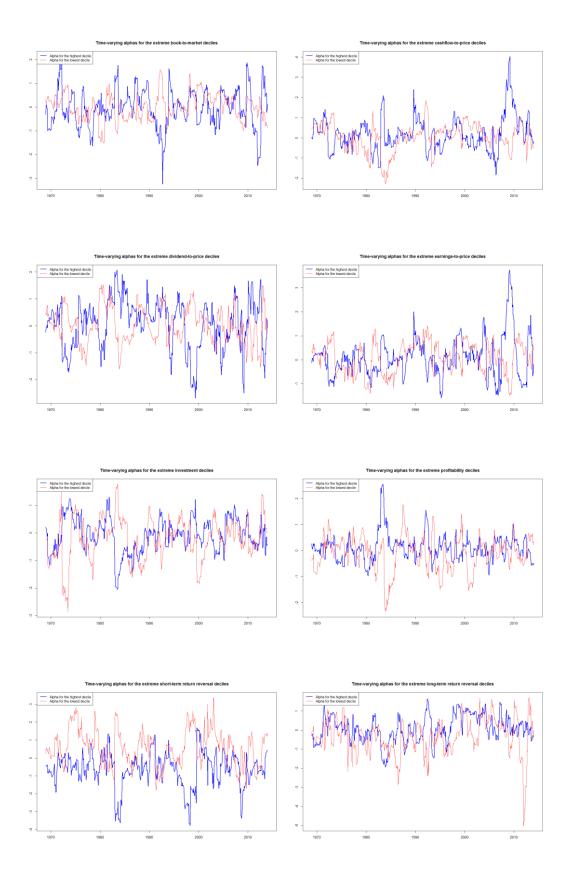
We extract dynamic conditional factor premiums from the Fama-French factor model and find that most anomalies disappear after one accounts for time variation in these premiums. Vector autoregression evidence shows that mutual causation between dynamic conditional alphas and macroeconomic surprises serves as a core qualifying condition for fundamental factor selection. This economic insight is an incremental step toward drawing a distinction between rational risk and behavioral mispricing models. To the extent that dynamic conditional alphas can reveal the marginal investor's fundamental news and expectations about the cross-section of average asset returns, our economic insight helps enrich macroeconomic asset return prediction.

Appendix 1: Dynamic conditional alpha visualization for the extreme deciles

This appendix shows the time-series charts of the dynamic alphas for the extreme deciles based on the stock portfolio tilts such as size, value, momentum, profitability, investment, short-run return reversal, and long-run return reversal. These dynamic factor premiums exhibit wide variation for the top and bottom deciles. Some large comovements tend to influence the central tendency of the alpha spread between the extreme deciles. Each alpha spread often switches its sign and becomes econometrically insignificant. In addition to Gibbons, Ross, and Shanken's (1989) AGRS F-test, the AGMM C-test and AGMM Q-test of dynamic multifactor mean-variance efficiency (MMVE) suggests that the average alpha spreads are not different from zero. The distance between the squared Sharpe ratios for each individual stock portfolio and the MMVE tangency portfolio is not large enough for one to reject the null hypothesis of a correct asset pricing model specification. This inference accords with the spirit of the intertemporal context of Merton (1973), Campbell (1993), and Fama (1996). Investors care about not only their terminal wealth but also several behavioral considerations such as human capital, labor income, consumption, and hedging investment opportunities that covary with the conditional expectations of their terminal wealth. In this light, the Fama-French (2015) factors serve as valid and relevant state variables that reflect these comovements in response to the typical investor's demand for hedging instruments. To the extent that the dynamic factor premiums on each Fama-French (2015) state variable is econometrically significant across the entire data span (i.e. each dynamic multifactor beta consistently differs from zero), the resultant dynamic alpha exhibits too much variability for the pricing error to be significant enough for the econometrician to reject the null hypothesis that the dynamic multifactor model is correctly specified.



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Appendix 2: ARMA-GARCH representation for each dynamic factor premium

We show that each dynamic conditional factor premium can be modeled as a typical financial time-series. We apply both ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional-mean-and-variance models to fit each factor premium that the econometrician extracts from a "dynamic" variant of the Fama-French (2015) factor model. Although it is possible to identify a more precise time-series representation for each factor premium, our goal here is more straight-forward. In fact, our primary goal is to use the standard toolkit in time-series econometrics to establish the empirical fact that each dynamic alpha or beta spread exhibits the major properties of most financial time series. Each factor premium embeds autoregressive mean reversion in the conditional mean specification of ARMA(1,1), and volatility clusters and asymmetries in the conditional volatility specification of EGARCH(1,1,1) or GJR-GARCH(1,1,1) (cf Engle (1982); Bollerslev (1986); Nelson (1991); Glosten et al (1993)). One can readily fit an ARMA-EGARCH or ARMA-GJR-GARCH model to characterize the dynamic evolution of each conditional alpha or beta spread over time. This characterization entails both reasonable and flexible assumptions about the true conditional mean and variance processes for each dynamic conditional factor premium.

This time-series analysis also differs from several earlier studies that exclusively focus on the single-beta CAPM (cf. Adrian and Franzoni (2009); Ang and Chen (2007); Lewellen and Nagel (2006)). The recursive multivariate filter helps extract dynamic conditional alphas and betas from the Fama-French (2015) multi-factor model, and then the econometrician can apply **Eq(14)-Eq(17)** to model each dynamic conditional alpha or beta spread as a financial time series. While it is reasonable to identify the "best" ARMA-GARCH representation for each conditional alpha or beta spread, we establish the empirical fact that each dynamic conditional alpha or beta spread exhibits the most prevalent properties of a typical financial time series. In turn, this empirical fact defies the conventional wisdom of pure point estimates of factor premiums in most static time-series ordinary least-squares regressions.

Table A2.1 presents the ARMA-GARCH results for each dynamic alpha spread between the extreme deciles. While there is substantive evidence in support of autoregressive dynamic alpha spreads across all of the portfolio tilts (*t*-ratios>33), only the size portfolio tilt demonstrates some trace of a moving average alpha spread (*t*-ratio>2.6). Across the EGARCH and GJR-GARCH panels, there is strong evidence in support of both volatility clusters and asymmetries across all of the portfolio tilts (|*t*-ratios|>2). In this light, it is reasonable to infer that each dynamic alpha spread exhibits the common properties of a typical financial time series. Subsequent analysis can shine new light on both the economic content and predictive power of each dynamic alpha spread. To the extent that stock market information serves as a leading indicator of economic activity, each alpha spread can convey material information about economic growth, market valuation, financial stress, cyclical variation, or forecast combination. This conjecture calls for some further empirical confirmation in the spirit of several recent studies (Liew and Vassalou, 2000; Vassalou, 2003; Vassalou and Xing, 2004; Petkova, 2006; Hahn and Lee, 2006).

Only the momentum and short-term reversal portfolio tilts produce significant average alpha spreads (|t-ratios|>2.9). The respective intercepts are 1.028 and -1.122. These average conditional alpha spreads are reasonably close to the corresponding average alpha spreads for momentum and short-term return reversal of 0.989 and -1.180 in Table 3. Although these average alpha spreads seem to persist in the extreme deciles (Fama and French, 2008), it is key to recall the more formal Sharpe ratio test evidence that the average alphas do not jointly differ from zero across all the momentum and short-term reversal deciles. This logic leads the econometrician to infer that the average alpha spreads are consistent between Table 3 and Table A2.1.

Table A2.2 encapsulates the ARMA-GARCH results for each dynamic MRP beta spread between the top and bottom deciles. This conditional beta spread represents the relative sensitivity of the excess return on each stock portfolio tilt to changes in the market risk premium. All of the AR(1) coefficients are highly significant across the board (*t*-ratios>9), but only the short-term reversal portfolio tilt carries a significant MA(1) coefficient (*t*-ratio>2). Thus, the conditional mean specification is largely autoregressive in nature. In regard to the conditional variance specification, the GARCH effect is significant for the momentum, book-to-market, cashflow-to-price, profitability, and short-term reversal portfolio tilts. Among these tilts, only the MRP beta spreads for momentum and book-to-market exhibit significant volatility asymmetries in both the EGARCH and GJR-GARCH models. In comparison, the cashflow-to-price, profitability, and short-term reversal tilts exhibit significant volatility asymmetries only in the EGARCH model. Similar to the case for the dynamic alpha spreads, the MRP beta spreads can be further synthesized to offer new insights into a better prediction of economic or financial variables. The time variation in each MRP beta spread reflects shifts in the response of the excess return on a given portfolio strategy to changes in the market risk premium. As a result, this variation indicates changes in the investor's exposure to systematic risk after the econometrician controls for the other Fama-French (2015) state variables. A deeper analysis of MRP beta spread gyrations shows promise beyond the asset pricing literature.

Table A2.3 presents the ARMA-GARCH results for each dynamic SMB beta spread between the extreme deciles. The dynamic SMB beta spread reflects the relative sensitivity of the excess return on each stock portfolio tilt to changes in the return spread between the small and big stock portfolios. With respect to the conditional mean specification, the AR(1) coefficients are significant across the board (*t*-ratios>20), whereas, the MA(1) coefficients are insignificant at any reasonable confidence level. The conditional mean specification is first-order autoregressive. With respect to the conditional variance specification, the GARCH effect is prevalent across the EGARCH and GJR-

GARCH models for the size, momentum, book-to-market, profitability, and short-term reversal portfolio tilts (*|t*-ratios|>2.1). With the EGARCH model, the GARCH effect of volatility clusters is also evident for the cashflow-toprice, investment, and long-term reversal portfolio tilts (*|t*-ratios|>2.1). Moreover, the presence of volatility asymmetries is real for the size, momentum, book-to-market, cashflow-to-price, investment, profitability, short-term reversal, and long-run reversal portfolio tilts (*|t*-ratios|>2.1). In sum, the SMB beta spread exhibits large variability over time. It is reasonable to infer that the SMB beta spread gyrates sufficiently to capture shifts in the relative response of the excess return on a given portfolio strategy to changes in the return spread between the small and big stock portfolios.

Table A2.4 summarizes the ARMA-GARCH results for each HML beta spread between the extreme deciles. The HML beta spread throws light on the sensitivity of the excess return on a given portfolio strategy to changes in the return spread between the high and low book-to-market stock portfolios. With respect to the conditional mean specification, the AR(1) coefficients are significant (*t*-ratios>25.8). Moreover, the MA(1) coefficients for momentum and earnings-to-price are significant (*t*-ratios>2) in the ARMA-EGARCH model. Thus, there is substantial serial correlation in the conditional mean specification of the dynamic HML beta spread. With respect to the conditional variance specification, the EGARCH model suggests significant volatility clusters for the dividend-to-price, investment, and short-term reversal portfolio tilts while the GJR-GARCH model suggests significant volatility clusters for the size, momentum, cashflow-to-price, and investment portfolio tilts. In this light, the GJR-GARCH seems to better pick up the GARCH effect for the HML beta spread. Among these portfolio tilts, volatility asymmetries prevail in the GJR-GARCH model for both momentum and investment tilts. All this evidence supports the view that the dynamic HML beta spread exhibits the common properties of a typical financial time series.

The above results are informative in the sense that each HML beta spread exhibits much variability over time for the original Fama-French HML factor to be economically meaningful in explaining the variation in stock returns. In conjunction with the evidence of significant mean HML betas in Table A5.3, the ARMA-GARCH results support the use of HML as a relevant state variable that helps better span the investor's mean-variance space. Thus, HML conveys non-trivial information about at least some of the variation in excess returns for a variety of stock portfolio tilts. This inference is inconsistent with the recent claim of Fama and French (2015) and Hou, Xue, and Zhang (2014) that HML becomes redundant after the econometrician incorporates RMW and CMA into the multifactor asset pricing model. The economic content of HML and even SMB relates to whether these state variables serve as proxies for financial distress risk (Griffin and Lemmon, 2002; Vassalou and Xing, 2004), macroeconomic innovations (Liew and Vassalou, 2000; Vassalou, 2003; Petkova, 2006; Hahn and Lee, 2006), or some other behavioral considerations (Campbell, Hilscher, and Szilagyi, 2008).

Table A2.5 presents the ARMA-GARCH results for each dynamic RMW beta spread between the extreme deciles. The RMW beta spread reflects the relative sensitivity of the excess return on a given portfolio to changes in the return spread between the robust and weak stock portfolios in terms of their profitability. With respect to the conditional mean specification, the AR(1) coefficients are highly significant across the board (t-ratios>34). Also, the ARMA-EGARCH model yields significant MA(1) coefficients for the book-to-market and profitability portfolio tilts (t-ratios>2), whilst the alternative ARMA-GJR-GARCH model yields significant MA(1) coefficients for the investment and profitability tilts (t-ratios>2.9). Thus, there is substantial serial correlation in the conditional mean specification of each dynamic RMW beta spread. With respect to the conditional variance specification, the GJR-GARCH model seems to better pick up the GARCH effect than the EGARCH model. Specifically, the GJR-GARCH model suggests a significant GARCH effect for the size, momentum, book-to-market, cashflow-to-price, dividendto-price, earnings-to-price, and profitability portfolio tilts (t-ratios>1.8), whereas, the EGARCH model picks up a large GARCH effect only for the size, book-to-market, cashflow-to-price, and investment portfolio tilts (t-ratios>1.7). In addition, the presence of volatility asymmetries prevails in the GJR-GARCH model for the size, book-to-market, cashflow-to-price, investment, and profitability portfolio tilts (|t-ratios|>2.3). In light of the above evidence, the RMW beta spread exhibits much variability over time. Similar to the case for the other state variables, RMW carries informative dynamic beta spreads that are analogous to a typical financial time series. To the extent that RMW captures the return spread between profitable stocks and less profitable stocks, this state variable adds value to the explanatory power of a dynamic variant of the Fama-French (2015) multifactor model.

Table A2.6 summarizes the ARMA-GARCH results for each CMA beta spread between the top and bottom deciles. The CMA beta spread describes the sensitivity of the excess return on a given stock portfolio to changes in the return spread that reflects differences in a firm's capital investment or asset growth. With respect to the conditional mean specification, the AR(1) coefficients are significant across the board (*t*-ratios>36). Some of the MA(1) coefficients are also significant for the size, cashflow-to-price, dividend-to-price, investment, profitability, and short-term reversal portfolio tilts (*t*-ratios>1.6). Similar to the case for the other Fama-French beta spreads, the CMA beta spread exhibits serial correlation in its conditional mean. With respect to the conditional variance specification, the EGARCH model appears to better pick up the prevalence of volatility clusters than the GJR-GARCH model. Specifically, the EGARCH model finds a substantial GARCH effect for the size, momentum, book-to-market, cashflow-to-price, dividend-to-price, profitability, and short-term reversal portfolio tilts (*|t*-ratios|>2), while the GJR-GARCH model does so only for the book-to-market, dividend-to-price, and short-term reversal portfolio tilts (*t*-ratios>2). However, the evidence of volatility asymmetries is less conclusive. In sum, CMA seems to be a useful state variable that yields wide time-series heterogeneity in its beta spread between the extreme deciles.

Table A2.1: ARMA-GARCH representation of each dynamic conditional alpha spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}}\varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f\ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 9 summarizes the quantitative estimates of the main parameters for each dynamic alpha in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters a, b, c, d, e, f, and g correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding t-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each t-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	,1)-EGAR	CH(1,1,1)				1	ARMA(1,1)-GJR-GA	RCH(1,1,	1)	
Stock portfolio sort	а	b	с	d	e	f	g	a	b	с	d	e	f	g
Size														
Coefficient	0.084	0.887	0.114	0.008	0.035	1.000	0.042	0.087	0.889	0.116	0.000	0.033	0.984	-0.037
t-ratio	0.9	48.6	2.7	2.9	10.1	41597	3.1	0.8	45.5	2.6	3.3	3.6	170.4	-2.5
Momentum														
Coefficient	1.028	0.918	0.041	-0.016	0.078	0.959	0.057	0.987	0.915	0.031	0.013	0.071	0.951	-0.076
t-ratio	2.9	54.6	0.8	-1.0	3.6	50.4	2.3	2.9	49.0	0.6	1.9	3.4	66.7	-3.4
Book-to-market														
Coefficient	-0.195	0.906	0.047	-1.430	0.187	0.170	0.283	-0.173	0.914	0.075	0.003	0.000	1.000	-0.029
t-ratio	-1.0	46.0	0.8	-5.2	2.5	1.1	2.5	-6.2	65.3	4.7	180.6	0.0	380425	-205.7
Cashflow-to-price														
Coefficient	-0.047	0.909	0.012	-0.017	-0.007	0.980	0.068	0.021	0.912	0.022	0.004	0.000	0.979	0.012
t-ratio	-0.2	60.8	0.3	-3.1	-0.4	341	14.3	0.1	47.3	0.5	6.0	0.0	217.4	1.8
Dividend-to-price														
Coefficient	-0.142	0.932	0.002	0.010	0.037	1.000	0.059	-0.239	0.922	0.047	0.007	0.063	0.958	-0.082
t-ratio	-0.6	50.8	0.0	3.3	2.4	2309	10.1	-3.0	56.9	1.3	12.8	208.6	1764	-203.5
Earnings-to-price														
Coefficient	-0.082	0.904	0.031	-0.083	0.068	0.944	0.048	-0.095	0.897	0.043	0.013	0.071	0.910	-0.086
t-ratio	-0.5	46.0	0.7	-11.7	3.8	255	3.4	-0.5	45.5	1.0	3.0	3.4	36.5	-3.7
Investment														
Coefficient	0.056	0.938	0.062	-1.074	-0.403	0.465	0.127	0.051	0.907	0.063	0.062	0.000	0.409	0.351
t-ratio	0.3	59.2	1.7	-5.1	-6.5	4.6	2.0	0.3	51.2	1.2	4.4	0.0	3.7	4.1
Profitability														
Coefficient	0.304	0.928	0.024	-0.227	0.017	0.898	0.034	0.329	0.931	0.032	0.001	0.000	1.000	-0.011
t-ratio	1.9	53.6	0.5	-2.9	0.7	24.2	3.6	2.2	55.4	0.8	1.9	0.0	294824	-2.0
Short-term reversal														
Coefficient	-1.122	0.858	0.079	0.001	0.003	0.980	0.098	-1.076	0.901	0.066	0.004	0.000	1.000	-0.015
t-ratio	-4.8	41.1	1.5	0.1	0.2	98.8	4.1	-2.6	49.9	1.6	7.6	0.0	214006	-9.0
Long-term reversal														
Coefficient	0.378	0.854	0.065	-0.092	0.093	0.928	0.070	0.380	0.860	0.072	0.023	0.062	0.881	-0.065
t-ratio	2.6	38.2	1.4	-2.3	4.3	39.0	2.2	2.4	33.6	1.5	3.3	2.4	27.6	-2.0

 Table A2.1: ARMA-GARCH representation of each dynamic conditional alpha spread

Table A2.2: ARMA-GARCH representation of each dynamic conditional MRP beta spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}} \varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f \ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 10 shows the quantitative estimates of the key parameters for each dynamic MRP beta in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters a, b, c, d, e, f, and g correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding t-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each t-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	1)-EGAR	CH(1,1,1)					A	ARMA(1,1)	-GJR-GA	RCH(1,1,1	l)	
Stock portfolio sort	а	b	с	d	e	f	g	-	а	b	с	d	e	f	g
Size															
Coefficient	0.048	0.700	0.012	-0.017	0.023	0.993	0.071		0.047	0.697	0.010	0.000	0.035	0.962	-0.006
t-ratio	1.8	18.8	0.2	-1.0	1.5	176.9	3.8		1.6	15.7	0.2	1.2	2.4	88.2	-0.3
Momentum															
Coefficient	-0.083	0.699	0.021	-0.563	-0.130	0.462	0.454		-0.065	0.701	0.024	0.225	0.178	0.090	0.271
t-ratio	-1.0	12.1	0.2	-2.8	-2.3	2.6	5.1		-0.8	17.4	0.3	4.3	2.5	0.6	1.9
Book-to-market															
Coefficient	0.213	0.708	-0.021	-0.125	0.079	0.941	0.062		0.216	0.711	-0.035	0.005	0.066	0.933	-0.099
t-ratio	6.1	18.1	-0.4	-9.3	4.9	144.6	2.8		4.6	16.0	-0.5	2.8	3.6	32.5	-11.7
Cashflow-to-price															
Coefficient	0.108	0.682	-0.023	-0.336	-0.064	0.822	0.350		0.102	0.670	-0.016	0.040	0.243	0.465	0.119
t-ratio	3.0	22.3	-1.2	-2.2	-1.6	11.0	3.9		2.4	14.6	-0.2	3.7	2.4	4.0	1.1
Dividend-to-price															
Coefficient	-0.254	0.699	-0.010	-0.020	0.002	0.982	0.174		-0.249	0.700	-0.012	0.003	0.072	0.913	0.011
t-ratio	-6.5	23.1	-0.8	-0.8	0.1	68.2	4.1		-4.7	15.8	-0.2	1.5	3.1	37.2	0.3
Earnings-to-price															
Coefficient	0.177	0.713	-0.018	-0.392	-0.072	0.800	0.157		0.169	0.730	-0.024	0.000	0.005	0.988	0.011
t-ratio	2.4	9.0	-0.1	-1.6	-1.8	6.7	2.4		3.0	15.3	-0.4	0.6	0.6	455.4	0.6
Investment															
Coefficient	-0.025	0.665	0.011	-0.278	-0.012	0.885	0.114		-0.021	0.678	-0.006	0.000	0.000	0.991	0.015
t-ratio	-0.7	14.1	0.2	-1.3	-0.4	9.9	2.0		-0.6	14.7	-0.1	0.5	0.0	837.4	2.4
Profitability															
Coefficient	-0.285	0.733	-0.021	-0.138	0.012	0.937	0.184		-0.275	0.735	-0.011	0.006	0.088	0.857	-0.015
t-ratio	-7.2	16.8	-0.3	-2.6	0.4	43.0	4.6		-6.0	16.5	-0.2	2.9	2.4	29.9	-0.3
Short-term reversal															
Coefficient	-0.199	0.661	0.134	-0.044	-0.012	0.956	0.153		-0.185	0.662	0.121	0.015	0.067	0.887	-0.003
t-ratio	-2.6	15.0	2.3	-1.7	-0.5	43.5	3.9		-2.5	14.4	2.0	1.7	2.6	20.2	-0.1
Long-term reversal															_
Coefficient	-0.186	0.637	0.109	-0.272	-0.003	0.833	0.205		-0.169	0.642	0.130	0.001	0.008	1.000	-0.024
t-ratio	-4.0	14.5	1.6	-1.9	-0.1	10.0	3.2		-1.8	9.2	1.3	3.8	0.9	213718	-0.9

Table A2.2: ARMA-GARCH representation of each dynamic conditional MRP beta spread

Table A2.3: ARMA-GARCH representation of each dynamic conditional SMB beta spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}}\varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f\ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 11 shows the quantitative estimates of the key parameters for each dynamic SMB beta in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters a, b, c, d, e, f, and g correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding t-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each t-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	,1)-EGAR	CH(1,1,1)				1	ARMA(1,1)-GJR-GA	RCH(1,1,1	1)	
Stock portfolio sort	а	b	с	d	e	f	g	a	b	с	d	e	f	g
Size														
Coefficient	-1.419	0.844	0.007	-0.050	0.050	0.979	0.061	-1.436	0.845	0.003	0.001	0.042	0.965	-0.046
t-ratio	-24.5	38.0	0.8	-12.4	3.8	4260	12.0	-22.6	30.0	0.0	3.4	4.4	124.6	-3.1
Momentum														
Coefficient	0.034	0.845	0.014	-0.043	0.082	0.939	0.104	0.002	0.840	0.011	0.023	0.099	0.900	-0.094
t-ratio	0.2	31.7	0.3	-1.6	3.0	31.1	2.8	0.0	29.9	0.2	1.7	2.8	20.6	-2.6
Book-to-market														
Coefficient	0.403	0.828	0.066	-0.031	0.038	0.986	-0.024	0.409	0.797	0.080	0.001	0.007	0.996	-0.026
t-ratio	4.4	27.7	1.3	-15.7	3.9	5170	-5.2	5.2	27.8	1.6	2.9	3.4	553289	-4.4
Cashflow-to-price														
Coefficient	0.109	0.797	0.034	-0.329	-0.077	0.827	0.166	0.086	0.788	0.047	0.023	0.041	0.751	0.100
t-ratio	1.7	24.4	0.7	-2.2	-2.1	11.0	2.9	1.1	22.9	0.8	2.5	1.4	9.1	1.8
Dividend-to-price														
Coefficient	-0.317	0.790	0.033	-0.071	-0.043	0.947	0.162	-0.308	0.784	0.052	0.011	0.023	0.875	0.098
t-ratio	-4.5	21.0	0.5	-1.9	-1.5	42.2	4.4	-3.6	21.9	0.8	2.4	1.1	23.6	2.4
Earnings-to-price														
Coefficient	0.168	0.813	0.058	-0.265	0.002	0.858	0.066	0.165	0.824	0.055	0.000	0.000	0.994	0.011
t-ratio	1.8	25.7	1.1	-1.0	0.0	5.9	1.3	1.7	28.7	1.1	0.6	0.0	1046	2.0
Investment														
Coefficient	-0.092	0.871	-0.003	-0.196	0.053	0.918	-0.053	-0.081	0.838	0.012	0.001	0.000	1.000	-0.018
t-ratio	-1.0	30.1	-0.1	-55.1	2.9	18417	-1.8	-1.0	31.7	0.2	25.0	0.0	952657	-25.1
Profitability														
Coefficient	-0.482	0.750	0.084	-0.175	-0.002	0.918	0.132	-0.462	0.754	0.092	0.008	0.045	0.871	0.012
t-ratio	-7.7	20.7	1.3	-2.4	-0.1	29.8	3.1	-8.0	20.6	1.5	2.7	2.1	23.3	0.4
Short-term reversal														
Coefficient	-0.138	0.813	0.035	-0.044	0.077	0.948	0.103	-0.137	0.812	0.026	0.022	0.118	0.880	-0.106
t-ratio	-1.0	25.5	0.6	-1.9	3.0	42.3	3.5	-1.0	25.6	0.5	2.5	3.4	28.9	-2.6
Long-term reversal														_
Coefficient	-0.577	0.860	0.025	-0.036	-0.042	0.969	0.097	-0.567	0.857	0.049	0.005	0.010	0.946	0.044
t-ratio	-5.1	34.6	0.6	-1.4	-2.1	62.1	2.9	-4.3	32.4	0.9	2.0	0.6	49.0	2.1

Table A2.3: ARMA-GARCH representation of each dynamic conditional SMB beta spread

Table A2.4: ARMA-GARCH representation of each dynamic conditional HML beta spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}} \varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f \ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 12 shows the quantitative estimates of the key parameters for each dynamic HML beta in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters a, b, c, d, e, f, and g correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding t-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each t-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	,1)-EGAR	CH(1,1,1)				I	ARMA(1,1)	-GJR-GA	RCH(1,1,	1)	
Stock portfolio sort	а	b	с	d	e	f	g	а	b	с	d	e	f	g
Size														
Coefficient	-0.593	0.808	0.026	-0.013	0.005	0.992	0.070	-0.591	0.841	-0.009	0.001	0.040	0.965	-0.028
t-ratio	-12.9	39.0	1.0	-4.4	0.4	1732	28.0	-9.4	25.8	-0.2	2.3	2.6	126.5	-1.3
Momentum														
Coefficient	-0.452	0.920	0.060	0.006	0.021	0.988	0.087	-0.409	0.929	0.064	0.009	0.057	0.961	-0.065
t-ratio	-2.1	53.8	4.5	0.7	1.0	77.8	2.9	-1.1	51.1	1.4	1.6	3.3	54.3	-2.9
Book-to-market														
Coefficient	1.933	0.877	-0.009	-1.518	-0.127	0.259	0.385	1.972	0.877	-0.005	0.004	0.000	0.947	0.047
t-ratio	17.6	39.8	-0.1	-4.1	-2.2	1.49	4.4	17.3	37.9	-0.1	1.8	0.0	42	2.7
Cashflow-to-price														
Coefficient	1.642	0.860	0.059	-0.316	0.065	0.817	0.261	1.620	0.852	0.084	0.035	0.194	0.646	-0.096
t-ratio	15.0	33.4	1.0	-2.3	1.3	10.8	3.5	14.4	32.7	1.4	3.6	2.2	7.8	-1.1
Dividend-to-price														
Coefficient	1.587	0.867	0.063	0.006	-0.037	1.000	0.026	1.633	0.863	0.068	0.001	0.009	0.979	0.022
t-ratio	11.8	46.4	1.4	3.2	-3.7	400458	4.9	11.7	35.4	1.4	1.7	1.2	233.5	1.8
Earnings-to-price														
Coefficient	1.739	0.854	0.057	0.005	0.000	1.000	0.031	1.742	0.856	0.037	0.000	0.004	0.992	0.006
t-ratio	14.7	35.4	2.0	6.7	0.0	105492	97.8	14.4	32.7	0.7	0.7	0.4	981	0.5
Investment														
Coefficient	-0.686	0.888	-0.010	-0.603	0.128	0.716	0.235	-0.725	0.894	0.024	0.001	0.000	1.000	-0.025
t-ratio	-6.5	41.2	-0.2	-3.1	3.2	8.28	3.7	-7.1	49.3	0.5	52.7	4.7	9274919	-41.7
Profitability														
Coefficient	-0.873	0.854	0.076	-0.468	-0.052	0.792	0.075	-0.874	0.851	0.085	0.021	0.017	0.757	0.042
t-ratio	-9.0	33.1	1.6	-1.3	-1.3	5.0	1.4	-9.2	32.9	1.7	1.5	0.7	5.0	0.8
Short-term reversal														
Coefficient	-0.087	0.840	0.012	-0.030	-0.059	0.967	0.068	-0.055	0.844	0.015	0.009	0.000	0.944	0.060
t-ratio	-0.8	40.1	0.7	-7.1	-4.5	260.6	9.3	-0.4	33.1	0.3	1.9	0.0	50.6	3.3
Long-term reversal														_
Coefficient	-1.089	0.907	0.018	0.004	-0.026	1.000	0.022	-1.031	0.907	0.018	0.000	0.004	0.992	0.005
t-ratio	-5.8	46.0	0.4	2.8	-1.7	194322	5.99	-5.4	42.4	0.4	0.6	0.4	947.4	0.3

Table A2.4: ARMA-GARCH representation of each dynamic conditional HML beta spread

Table A2.5: ARMA-GARCH representation of each dynamic conditional RMW beta spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}}\varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f\ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 13 shows the quantitative estimates of the key parameters for each dynamic RMW beta in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters *a*, *b*, *c*, *d*, *e*, *f*, and *g* correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding *t*-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each *t*-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	1)-EGAR	CH(1,1,1)				1	ARMA(1,1)	-GJR-GA	RCH(1,1,1	1)	
Stock portfolio sort	а	b	с	d	e	f	g	a	b	с	d	e	f	g
Size														
Coefficient	0.581	0.907	0.049	-0.027	0.071	0.989	0.017	0.579	0.892	0.052	0.001	0.041	0.977	-0.053
t-ratio	5.6	48.9	1.0	-9.8	5.5	2102663	16.0	5.8	45.2	1.1	11.3	7.9	121.6	-4.2
Momentum														
Coefficient	0.275	0.904	0.020	-0.023	-0.023	0.943	0.139	0.267	0.897	0.035	0.039	0.045	0.858	0.039
t-ratio	0.9	43.3	0.4	-1.0	-0.8	30.0	3.5	1.0	40.4	0.6	1.9	1.8	16.1	1.1
Book-to-market														
Coefficient	-1.161	0.889	0.067	-0.130	0.080	0.917	0.205	-1.184	0.898	0.044	0.010	0.189	0.838	-0.147
t-ratio	-6.5	48.3	2.2	-2.8	2.8	39	4.5	-8.1	41.4	0.8	3.2	3.5	28	-3.0
Cashflow-to-price														
Coefficient	-0.322	0.901	-0.014	-0.080	0.042	0.949	0.090	-0.331	0.904	-0.020	0.005	0.047	0.945	-0.045
t-ratio	-0.9	39.9	-0.3	-1.7	1.7	39.1	2.6	-2.0	44.7	-0.4	2.2	2.6	50.6	-2.3
Dividend-to-price														
Coefficient	-1.083	0.886	0.055	-0.019	0.018	0.962	0.225	-1.117	0.889	0.073	0.010	0.128	0.875	-0.052
t-ratio	-7.2	51.9	1.9	-0.8	0.8	58.0	4.8	-6.7	40.3	1.4	2.4	3.8	29.2	-1.5
Earnings-to-price														
Coefficient	-0.341	0.871	-0.049	-0.364	-0.006	0.757	0.360	-0.303	0.877	-0.019	0.001	0.014	0.988	-0.006
t-ratio	-3.7	38.3	-0.7	-3.0	-0.2	10.5	4.5	-2.1	34.5	-0.4	0.8	3.0	409.3	-0.7
Investment														
Coefficient	0.597	0.910	0.063	-0.224	-0.039	0.893	-0.008	0.622	0.899	0.086	0.001	0.000	1.000	-0.008
t-ratio	3.7	45.7	1.4	-10.6	-1.8	105	-0.4	4.2	45.6	2.9	4.1	0.0	49347	-10.8
Profitability														
Coefficient	1.460	0.891	0.105	-0.301	-0.003	0.858	0.151	1.505	0.900	0.093	0.001	0.005	1.000	-0.031
t-ratio	10.8	39.2	2.0	-2.1	-0.1	13.3	2.7	15.1	58.8	3.0	33.7	6.2	705608	-314.5
Short-term reversal														
Coefficient	0.217	0.897	0.035	-0.741	-0.102	0.117	0.390	0.127	0.894	0.030	0.331	0.173	0.000	0.211
t-ratio	0.8	44.5	0.7	-3.6	-1.7	0.5	4.5	0.5	45.8	0.6	12.7	1.9	0.0	1.4
Long-term reversal														
Coefficient	0.334	0.915	0.026	-0.162	-0.008	0.881	0.144	0.404	0.917	0.000	0.021	0.023	0.866	0.041
t-ratio	1.4	48.2	0.5	-1.9	-0.2	15.7	2.6	1.8	44.6	0.0	1.9	0.9	14.1	1.2

Table A2.5: ARMA-GARCH representation of each dynamic conditional RMW beta spread

Table A2.6: ARMA-GARCH representation of each dynamic conditional CMA beta spread

The econometrician demonstrates that each dynamic factor premium can be modeled as a typical financial time-series. The econometrician can use the canonical ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) models to fit each dynamic factor premiums that one extracts from the dynamic multifactor variant of the Fama-French (2015) asset pricing model. The conditional mean specification is ARMA(1,1) while the conditional variance specification can take the form of EGARCH(1,1,1) or GJR-GARCH(1,1,1). For simple and intuitive exposition, the econometrician describes the ARMA(1,1) conditional mean specification and EGARCH(1,1,1) and GJR-GARCH(1,1,1) conditional variance specifications below (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993):

$$m_{t} = a + bm_{t-1} + cw_{t-1} + w_{t} \qquad w_{t} = \sqrt{h_{t}}\varepsilon_{t}$$

$$h_{t} = \exp\left\{d + e\left(\frac{w_{t}}{\sqrt{h_{t-1}}}\right) + f\ln h_{t-1} + g\left(\left|\frac{w_{t}}{\sqrt{h_{t-1}}}\right| - \left|\frac{E(w_{t})}{\sqrt{h_{t-1}}}\right|\right)\right\}$$

$$h_{t} = d + ew_{t-1}^{2} + fh_{t-1} + gD_{t-1}w_{t-1}^{2}$$

where m_t is the dynamic alpha or beta spread; w_t is the residual error; h_t is the conditional variance process; ε_t is a Gaussian white noise; D_t is a binary variable with a numerical value of unity if w_t is negative or zero if w_t is positive; a, b, c, d, e, f, and g are the parameters for quasi-maximum likelihood estimation. While the ARMA model serves as the conditional mean specification to capture any serial correlation in the dynamic alpha or beta spread between the extreme deciles, the EGARCH or GJR-GARCH model fits the conditional variance specification to encapsulate any volatility clusters and asymmetries in the above time-series under study.

Table 14 shows the quantitative estimates of the key parameters for each dynamic CMA beta in the ARMA(1,1)-EGARCH(1,1,1) and ARMA(1,1)-GJR-GARCH(1,1,1) conditional mean-variance specifications. The main parameters a, b, c, d, e, f, and g correspond to the key coefficients in the above time-series representation. This table reports each coefficient estimate and its corresponding t-statistic across the pervasive asset pricing anomalies such as size, value, momentum, investment, profitability, short-term return reversal, and long-term return reversal. Each t-test statistic helps examine statistical significance. On this basis, the econometrician can assess the presence or absence of the common properties of a typical financial time-series (cf. autoregressive mean reversion in the conditional mean specification).

Asset pricing puzzle			ARMA(1	,1)-EGAR	CH(1,1,1)				A	ARMA(1,1)-GJR-GA	RCH(1,1,1)	
Stock portfolio sort	а	b	с	d	e	f	g	a	b	с	d	e	f	g
Size														
Coefficient	0.044	0.898	0.114	-0.009	-0.053	0.992	0.057	0.078	0.890	0.097	0.001	0.006	0.960	0.051
t-ratio	0.4	46.0	2.2	-2.7	-4.1	28362	7.8	0.8	41.3	1.9	2.2	0.8	98.3	2.8
Momentum														
Coefficient	0.038	0.922	0.060	-0.010	-0.047	0.977	0.001	0.109	0.926	0.067	0.002	0.000	0.995	0.008
t-ratio	0.1	59.5	1.5	-3.5	-3.6	78097	0.3	0.3	53.4	1.5	1.4	0.0	1076.1	2.0
Book-to-market														
Coefficient	0.649	0.866	0.031	-0.697	0.096	0.641	0.192	0.644	0.870	0.026	0.065	0.169	0.434	-0.150
t-ratio	6.2	39.5	0.9	-2.3	2.3	4	2.5	5.5	36.9	0.5	3.1	2.5	3	-2.2
Cashflow-to-price														
Coefficient	-0.465	0.959	0.213	-0.074	-0.047	0.972	-0.140	0.034	0.953	0.144	0.003	0.000	1.000	-0.025
t-ratio	-1264	2157	1387	-2225	-1067	2563	-2295	2.3	68.4	3.7	25.0	0.0	20049	-25.9
Dividend-to-price														
Coefficient	1.901	0.944	0.135	-0.025	0.159	0.984	-0.090	0.804	0.934	0.090	0.001	0.024	1.000	-0.055
t-ratio	1737	3885	3314	-1576	98105	3586	-2159	9.0	4752.9	7.0	9.3	66.6	26788	-224.4
Earnings-to-price														
Coefficient	-0.293	0.884	0.068	-0.332	0.085	0.755	0.190	-0.218	0.927	0.054	0.003	0.000	1.000	-0.025
t-ratio	-3.2	47.9	4.8	-1.1	1.5	3.6	2.0	-1.6	64.0	1.5	24.1	0.7	48349	-35.9
Investment														
Coefficient	-1.341	0.880	0.147	-0.277	-0.028	0.872	0.128	-1.346	0.901	0.146	0.084	0.203	0.075	-0.184
t-ratio	-13.1	47.5	6.8	-2.0	-1.0	15	2.6	-9.4	41.0	2.9	4.5	2.3	0	-1.9
Profitability														
Coefficient	0.140	0.919	0.056	-0.083	0.000	0.973	-0.132	-0.093	0.925	0.069	0.001	0.000	1.000	-0.012
t-ratio	2287.7	2531.7	16.0	-3552	-3.2	3506	-1720	-0.4	50.7	1.6	3.4	0.0	25955	-3.4
Short-term reversal														
Coefficient	0.177	0.923	0.065	-0.976	-0.426	-0.105	0.016	0.166	0.893	0.079	0.001	0.005	1.000	-0.012
t-ratio	0.5	56.8	1.6	-7.3	-6.2	-0.8	0.2	0.6	46.5	1.8	1.6	9.3	3122	-7.4
Long-term reversal														
Coefficient	-1.119	0.927	0.023	-0.115	-0.034	0.905	0.092	-0.984	0.928	0.024	0.040	0.000	0.818	0.041
t-ratio	-12.8	47.0	0.5	-1.4	-0.9	15.1	2.4	-3.9	49.8	0.5	2.1	0.0	9.7	1.3

Table A2.6: ARMA-GARCH representation of each dynamic conditional CMA beta spread

Appendix 3: Macroeconomic variable definitions and their data sources

This appendix describes the main macroeconomic variable definitions and their public data sources in our vector autoregression analysis of Granger mutual causation between fundamental macro surprises and dynamic conditional factor premiums. To the extent that macroeconomic innovations manifest in the form of dynamic conditional factor premiums, this causation reveals the marginal investor's fundamental news and rational expectations about the cross-section of average returns.

We specify 15 major monthly time-series in a standard macroeconometric vector autoregressive system. There are 12 macroeconomic variables, 2 financial uncertainty time-series, and 2 investor sentiment proxies. For the Baker-Wurgler investor sentiment index, we use the original first principal component as a better empirical proxy. The resultant dataset spans the 285-month sample period from April 1990 to December 2013. These macroeconomic time-series include changes in the national economic activity index, Treasury bill rate, unemployment rate, term spread, default spread, prime bank loan rate, aggregate equity market dividend yield, and percent changes in U.S. industrial production, non-farm payroll, house price index, consumer price index, exchange rate, financial stress index, economic policy uncertainty, and investor sentiment.

Macroeconomic variable and name definition	Source
Chicago Fed's national economic activity index change from historical trend	Chicago Fed
St. Louis Fed Treasury 3-month secondary-market bill rate change	St. Louis Fed
St. Louis Fed unemployment rate change (total unemployment/labor force participation)	St. Louis Fed
Term spread between the 10-year Treasury and 3-month Treasury constant maturity rates	St. Louis Fed
Default spread between Moody's Baa-corporate-bond and 10-year Treasury bill rates	St. Louis Fed
Prime bank loan rate change for Top 25 U.S. commercial banks in terms of total assets	St. Louis Fed
S&P 500 dividend yield from Professor Robert Shiller's book on irrational exuberance	Shiller
St. Louis Fed national industrial production index change with the base year in 2007	St. Louis Fed
Bureau of Labor Statistics non-farm payroll (in thousands of persons) percent change	Bureau of Labor Statistics
Freddie Mac U.S. metropolitan-area residential house price index percent change	Freddie Mac
Bureau of Labor Statistics consumer price index (for urban consumers) percent change	Bureau of Labor Statistics
Federal Reserve U.S. trade-weighted average composite dollar index percent change	Federal Reserve Board
Baker-Bloom-Davis U.S. economic policy uncertainty index percent change	Baker, Bloom, and Davis (2012)
Kansas City Fed financial stress index change from 11 key financial market variables	Kansas City Fed
Baker-Wurgler investor sentiment percent change (from the first principal component)	Baker and Wurgler (2006)

Appendix 4: Conceptual nexus between our study and several recent contributions

In this appendix, we discuss the conceptual similarities and differences between the current study and several recent contributions. This discussion clarifies the core themes of our study in comparison to several concurrent ideas in the recent asset pricing literature. For instance, Harvey, Liu, and Zhu (2015) introduce a multiple testing framework (e.g. Harvey and Liu (2014a, 2014b, 2014c, 2014d)) and provide a unique variety of historical significance cut-offs from the first empirical tests in the 1960s to the present. This recent strand of asset pricing literature suggests that financial economists should lift the test hurdle from a *t*-ratio of 2.0 to a *t*-ratio of 3.0 for most cross-sectional tests. Specifically, Harvey, Liu, and Zhu (2015) find that this higher hurdle reduces the number of cross-sectional anomalies from 316 to only 2 (value and momentum) (cf. Asness et al (2013); Fama and French (2016); Hou, Xue, and Zhag (2017)). In addition, Harvey, Liu, and Zhu (2015) contend that a theoretically-derived factor should have a lower hurdle than an empirically-discovered factor. In accordance with the central thesis of Harvey, Liu, and Zhu (2015), a factor can be important in some economic environments but unimportant in some other environments.

While our econometric innovation complements Harvey, Liu, and Zhu's (2015) multiple testing analysis, the current study serves as a time-series equivalent to their cross-sectional adjustment for empirical asset pricing tests. Back-of-the-envelope calculations suggest that the typical stock portfolio's Sharpe ratio has to increase by at least 3.1 to 8.2 times for the dynamic alphas to be jointly significant at the conventional confidence level. The critical values for the χ^2 -test with 525 degrees of freedom are 603.31, 579.4, and 566.9 at the respective 99%, 95%, and 90% confidence levels. Table 3 suggests that the highest *C*-test or *Q*-test statistic is 59.29 while the lowest *C*-test or *Q*-test statistic is 8.97. Therefore, the smallest Sharpe ratio multiplier can be calculated as $(566.932/59.29)^{1/2}$ =3.092 while the largest Sharpe ratio multiplier is $(603.31/8.97)^{1/2}$ =8.201. As a consequence, the econometrician has to specify a higher test hurdle for each anomaly. Across the deciles, most dynamic conditional alphas need to be larger on average with much less variability for the resultant Sharpe ratio to increase by at least 3 to 8 times. The equivalent Sharpe ratio would be in the approximate range of 1.15 to 2.4 (cf. Kozak, Nagel, and Santosh (2017)). In other words, our dynamic analysis of conditional factor premiums proposes raising the bar for the econometric time-series asset pricing test. This recommendation echoes the cross-sectional counterpart of Harvey, Liu, and Zhu (2015).

McLean and Pontiff (2016) analyze the out-of-sample and post-publication stock return predictability of about 100 firm characteristics that prior academic papers demonstrate to explain the cross-sectional stock return heterogeneity. The long-short portfolio return for the average predictor declines by 26% out-of-sample. Moreover, the long-short stock portfolio return for the average predictor shrinks by 58% post-publication. While there is sufficient evidence to reject the null hypothesis that stock return predictability does not change post-publication, there is also sufficient evidence to reject the null hypothesis that stock return predictability completely vanishes. McLean and Pontiff (2016) interpret the results as sufficient evidence in support of the behavioral mispricing conjecture that investors learn from anomalous stock returns while the evidence accords with the comovement models of Lee, Shleifer, and Thaler (1991) and Barberis, Shleifer, and Wurgler (2005). As academic research draws public attention to many useful predictors, stock return predictability gradually dissipates over time.

Our evidence contradicts McLean and Pontiff's (2016) empirical study. Their study does not apply a dynamic variant of the Fama-French (2015) multifactor model. Neither do their regressions with several dummy variables incorporate state variables or factors that mimic intertemporal changes in the typical investor's hedging demand for investment opportunities. Our current study proposes the use of a recursive multivariate filter to extract key dynamic conditional factor premiums from the Fama-French (2015) multifactor model. The vast majority of dynamic conditional alphas turn out to be insignificant at the conventional confidence level. Moreover, most dynamic conditional alphas are not jointly different from zero and therefore would need to increase at least 3 to 8 times for the econometrician to reject the hypothesis that our chosen dynamic version of the Fama-French (2015) factor model is a correct specification. Our work shines skeptical light on McLean and Pontiff's (2016) behavioral mispricing interpretation of conditional factor premiums in the dynamic context. Should investors learn from a diverse set of asset pricing anomalies so that these anomalies decay over time, McLean and Pontiff (2016) cannot explain why the anomalous returns persist for a prolonged period of time in the first place. In contrast, our analysis suggests that the pervasive asset pricing puzzles can be readily reconciled within a dynamic conditional factor model. Regardless of whether investors can learn from unique and viable stock portfolio strategies, our dynamic analysis helps demystify at least some ubiquitous anomalies to the extent that mutual causation between macroeconomic innovations and dynamic conditional factor premiums reflects the rational investor's fundamental information about the mysterious cross-section of average returns..

Berk and Green (2004) derive a canonical model of how the financial market for mutual fund investment equilibrates in a way that accords with the empirical facts. In highly competitive financial markets, all mutual funds must have enough assets under management such that these funds face diminishing returns to scale. When new information arrives and convinces investors that a particular fund represents a positive net-present-value investment opportunity, investors react to this opportunity by injecting more capital into that mutual fund. This process continues until enough new capital gets invested to eliminate the opportunity. As a result, mutual fund flows reflect past fund performance. Investors chase past fund performance because this performance conveys rich information about whether the mutual fund manager has skill and expertise in stock selection. By competing to take advantage of this information, investors phase out the opportunity to predict future mutual fund performance. Berk and Van Binsbergen (2016) and Barber, Huang, and Odean (2016) independently report that mutual fund flows reveal investor preferences for their use of asset pricing models. Their evidence is consistent with the view that the single-beta CAPM is the clear "victor" in the empirical horserace against the multifactor Fama-French-Carhart and dynamic equilibrium models because the alpha or abnormal fund return seems most heavily discounted by the CAPM. This joint evidence has implications for the broader proposition that both the multifactor and dynamic equilibrium asset pricing models may not represent true progress toward a better model of the nexus between risk and return. Investor preferences appear to be more closely aligned with the CAPM despite the fact that the model has been found to perform poorly relative to the other models in explaining the cross-sectional variation in stock returns. This issue remains an important puzzle in the asset pricing literature. Kozak, Nagel, and Santosh's (2017, 2018) recent studies shed skeptical light on whether these empirical horseraces can reflect investor beliefs, behaviors, and preferences in a clear dichotomy between the rational risk paradigm and the behavioral mispricing counterpart.

Our current study provides a midle refutation of Berk and Van Binsbergen's (2016) and Barber, Huang, and Odean's (2016) joint inference that the CAPM outperforms the multifactor models in their separate tests on mutual fund flows data. There are several major differences between our work and these recent studies. First, Berk and Van Binsbergen (2016) and Barber, Huang, and Odean (2016) do not consider the Fama-French (2015) multifactor model. Just as the founders of a firm have incentives to employ proprietary technologies, state-of-the-art work mechanisms, or efficient means of production, financial economists should make proper use of innovative econometric methods and models that help resolve the pervasive anomalies. The exclusion of both asset investment growth and operating profitability variables RMB and CMA is likely to introduce a key omitted-variables bias. Second, Berk and Van Binsbergen (2016) and Barber, Huang, and Odean (2016) cannot account for the time variation in dynamic conditional factor premiums. Their independent studies rest upon the assumption that the multifactor premiums are invariant over time. The static analysis does not take into account the key impact of measurement noise that might be present in each factor premium. To the extent that conditional factor premiums tend to change over time, this measurement noise does not vanish but can persist even in a large long-term dataset. Given this rationale, the emergence of anomalous returns or significant alphas can arise from the fact that the conventional static multifactor model cannot properly control for time variation in dynamic conditional factor premiums. Finally, Berk and Van Binsbergen (2016) and Barber, Huang, and Odean (2016) both find that a large fraction of mutual fund flows remains unknown. Although the use of mutual fund flows or quantities is valid and innovative, this empirical work represents a major departure from most prior asset pricing literature that focuses on prices, returns, or factor premiums. In contrast to the unconventional use of stock quantities rather than stock prices, our current study helps reconcile many ubiquitous asset pricing anomalies with the dynamic conditional factor model. Consequently, our new approach poses a conceptual challenge to the behavioral mispricing interpretation of anomalous stock returns that the prior factor models cannot explain in practice.

More recent studies contribute to the ongoing debate on whether the Fama-French (2015) multi-factor asset pricing model wins the horse race against most alternative static counterparts. For instance, Fama and French (2016) apply their factor model to dissect many asset pricing anomalies such as market beta, net share issuance, idiosyncratic volatility, accrual, and momentum. Each of these variables sorts stocks into deciles that result in abnormal returns or pricing errors. Fama and French (2016) report that the list of anomalies shrinks when the econometrician applies their static factor model. Positive exposures to the state variables for investment and profitability RMW_t and CMA_t capture the high average returns on profitable firms that invest conservatively with low market beta, share buyback, and low volatility. Conversely, negative exposures to RMW_t and CMA_t help explain the low average stock returns on unprofitable firms that invest aggressively with high market beta, high share buyback, and high volatility. Thus, each stock's fundamental factors such as size, value, investment, and profitability help better explain the cross-section of average stock returns.

In addition to Fama and French's (2016) recent attempt to dissect many anomalies with their factor model, Hou, Xue, and Zhang (2016) apply an alternative *q*-theoretic factor model to examine an extensive database with 430 anomalies. In comparison to the Fama-French five-factor model, the *q*-theoretic factor model yields smaller average static alphas. Yet, at least 161 to 216 anomalies persist with significant alphas. Although these separate empirical contributions of Fama and French (2016) and Hou, Xue, and Zhang (2016) seem to tell the same economic intuition that fundamental characteristics such as asset investment and operating profitability explain much of the cross-sectional variation in average returns, it is difficult to draw a clear distinction between both rational risk and behavioral mispricing models because numerous anomalies remain statistically significant in the general form of non-trivial static alphas (Kozak, Nagel, and Santosh, 2017, 2018).

Contrary to Fama and French's (2016) and Hou, Xue, and Zhang's (2016) fixation on the explanatory power of their static multifactor models, our current study designs a rigorous conditional specification test to differentiate the new and useful dynamic conditional model from the static baseline model. Not only does our dynamic conditional model outperform the static counterpart in driving the long-term average pricing errors to zero, but the dynamic conditional factor model also passes the conditional specification test that rejects the static counterpart across 104 of 110 deciles for the major portfolio tilts. In direct response to Cochrane's critique (2005: 168), the test evidence corroborates the central economic story that most static asset pricing anomalies evaporate after the econometrician properly accounts for time variation in conditional factor premiums. Our analysis helps demystify the inexorable puzzle that significant asset-pricing errors persist in the static cross-section of average returns.

Appendix 5: Fama-French beta coefficients and t-statistics on multiple factors

Table A5.1 shows the average dynamic conditional MRP betas across the deciles for each anomaly. While there is no monotonic relation between average MRP beta and decile rank, the vast majority of these average MRP betas hover around unity. Also, the average MRP betas are all significantly greater than nil (p-values>0.001). This key evidence affirms the close empirical nexus between the excess returns on both the market portfolio and each portfolio tilt.

Also, the average conditional MRP beta spread between the extreme deciles is significant for all the portfolio strategies except the momentum and investment tilts. One has to interpret this result with caution because the lack of a monotonic trend in the average conditional MRP betas does not necessarily suggest the absence of a positive relation between risk and average return. It would be important to consider the complete set of results within the broader context of dynamic portfolio efficiency. Along with most other dynamic conditional factor betas, the conditional MRP beta varies sufficiently to reflect time variation in the sensitivity of each given portfolio to changes in the market risk premium. This time variation may arise due to shifts in the marginal investor's information set or changes in the macroeconomic environment. In light of this logic, we defer the assessment of a risk-reward nexus to an in-depth subsequent analysis.

Table A5.2 presents the average conditional SMB betas across the deciles for each anomaly. SMB helps capture the predictable variation in excess returns on the size deciles. The average conditional SMB beta monotonically declines from 1.096 for the smallest size decile to -0.28 for the largest size decile (p-values<0.08). Ceteris paribus small stocks carry higher conditional SMB factor premiums than large stocks with an average conditional SMB beta spread of -1.376 (p-value<0.01). Moreover, the average conditional SMB beta spread is significant for the value, profitability, and long-term reversal tilts (p-values<0.06). Yet, the average SMB beta spread is insignificant for the momentum, asset growth, and short-run reversal tilts (p-values>0.18). This evidence suggests that SMB helps explain the variation in excess returns on some but not all of the portfolio tilts. The average SMB beta series are lower than the MRP counterparts by an order of magnitude. Thus, the conditional SMB beta exhibits non-negligible heterogeneity over time as a complement to the conditional MRP beta and other factor betas.

Table A5.3 encapsulates the average conditional HML betas on the deciles for each anomaly. The long-term average conditional HML betas are predominantly significant across the deciles for the size, value, investment, profitability, and long-run return reversal tilts. In particular, the average conditional HML beta monotonically decreases from the top decile to the bottom decile for all of the value tilts. For example, the average conditional HML beta declines monotonically from 1.118 for the top book-to-market decile to -0.899 for the bottom decile. For the cashflow-to-price, dividend-to-price, and earnings-to-price value tilts, the mean HML beta monotonically shrinks from 1.001, 0.983, and 1.028 respectively for the top decile to -0.68, -0.46, and -0.698 for the bottom decile. This evidence contradicts the recent studies of Fama and French (2015) and Hou, Xue, and Zhang (2014) who empirically show that HML becomes a redundant factor in U.S. data after the econometrician includes the investment and profitability return spreads in the factor model. Whether HML serves as an empirical proxy for financial distress risk remains open to controversy (Fama and French, 1995, 1996; Griffin and Lemmon, 2002; Vassalou and Xing, 2004; Petkova, 2006; Campbell, Hilscher, and Szilagyi, 2008; Fama and French, 2016). Notwithstanding this controversy, the current study shines new light on the explanatory role of HML in a dynamic conditional context. This rare resurrection suggests that one might want to revisit the economic content of HML and even SMB in the Fama-French (2015) factor model.

Yet, HML cannot readily contain the time variation in the excess returns on the short-term reversal and momentum portfolio tilts. Analogous to SMB, HML helps explain the variation in the excess returns on some but not all of the portfolio tilts. Thus, SMB and HML only serve as imperfect states variables that enter the marginal investor's intertemporal information set. The joint economic content of HML and SMB pertains to whether these factors serve as proxies for financial distress risk (Griffin and Lemmon, 2002; Vassalou and Xing, 2004), macroeconomic innovations in the term and default spreads (Liew and Vassalou, 2000; Vassalou, 2003; Petkova, 2006; Hahn and Lee, 2006), or some behavioral mispricing considerations (Campbell, Hilscher, and Szilagyi, 2008).

Table A5.4 presents the average dynamic conditional RMW betas across the deciles for each portfolio tilt. RMW explains the variation in the excess returns for the size, momentum, book-to-market, dividend-to-price, investment growth, operating profitability, and long-term return reversal tilts (p-values<0.006). However, the average conditional RMW betas are insignificant for the cash-flow-to-price, earnings-to-price, and short-run return reversal tilts. In stark contrast to the separate cases of SMB and HML, the average RMW beta does not monotonically increase from the bottom decile to the top decile for the profitability tilt. It is thus difficult to rationalize whether this result represents a statistical aberration or a rotten apple in the barrel of the hefty profitability premium.

Similar to the informative cases of SMB and HML, the absolute values of dynamic RMW betas turn out to be smaller by a full order of magnitude than the dynamic MRP beta near unity. On this basis, it is fair to infer that each of the state variables (SMB, HML, and RMW) provides at best a partial view of time variation in the excess returns for multiple tilts that are known to produce anomalous returns in static regression analysis. Subsequent analysis should shed some fresh light on the economic content of each of these factors in a dynamic conditional context.

Table A5.5 summarizes the average conditional CMA betas across the deciles for each anomaly. CMA explains the variation in excess returns on the top-and-bottom deciles for the size, value, momentum, and long-run return reversal tilts and most of the investment deciles. For the former intermediate deciles, however, CMA lacks explanatory power. This evidence shows that CMA complements the other state variables only marginally in the dynamic conditional factor model. Unlike SMB and HML, CMA does not carry average conditional factor betas that demonstrate a monotonic trend across the asset growth deciles. Nevertheless, the average conditional CMA betas are significantly positive for the bottom asset growth deciles and become negative for the top asset growth deciles. This evidence accords with q-theoretic economic intuition: stocks with high asset growth experience lower subsequent average returns. For the extreme asset growth deciles, the conditional CMA factor beta spread of -1.432 is significant at any confidence level. CMA complements the other Fama-French (2015) factors in capturing time variation in excess returns for a variety of portfolio tilts.

It is important to further explain the q-theoretic prediction of a negative empirical relation between corporate investment and average return performance. Recent literature focuses on the q-theory that connects a given firm's sequential investment decisions to its market risk exposure and subsequent average return (Berk, Green, and Naik, 1999; Gomes, Kogan, and Zhang, 2003; Carlson et al, 2004, 2006; Zhang, 2005; Cooper, 2006; Anderson and Garcia-Feijoo, 2006; Liu, Whited, and Zhang, 2009; Li, Livdan, and Zhang, 2009). When the firm invests in M&A and capital stock, this investment transforms risky real options into less risky assets that yield steady streams of future cash flows. This transformation continues until the firm exhausts its positive net-present-value investment projects. As a result, the firm reduces its exposure to systematic risk during this investment transformation while most rational investors require a lower average stock return. So the q-theory predicts a negative empirical nexus between corporate investment and subsequent average stock return performance. Anderson and Garcia-Feijoo (2006) confirm this negative nexus.

Overall, the Fama-French factors serve as useful state variables for the typical investor who cares about his or her intertemporal payoffs (Merton, 1973; Campbell, 1993; Fama, 1996; Fama and French, 2004). Each of these state variables carries significant long-run average conditional betas across the deciles for a variety of portfolio tilts that are known to generate abnormal return spreads in a static regression analysis. In fact, the collective wisdom of the Fama-French factors suggests that the dynamic conditional recursion exhausts nearly all time variation in the excess returns on most deciles. As a consequence, the vast majority of average conditional alphas turn out to be insignificant at the conventional confidence level. More formal hypothesis tests further affirm that these alphas exhibit substantial variability around nil. Therefore, there is insufficient evidence for the econometrician to infer that dynamic conditional factor model. The resultant tangency portfolio is multifactor mean-variance efficient in a dynamic sense that this portfolio achieves the highest excess returns for a unique set of return variances and covariances. As one swallow does not make a summer, the transient emergence of mispricing opportunities does not necessarily indicate an econometrically persistent trend. Dynamic conditional alphas thus converge toward zero, and transitory price misalignment vanishes on the conditional mean-variance efficient frontier.

Table A5.1: Average dynamic MRP betas, MRP beta spreads, and Newey-West t-tests

Over the 50-year period from January 1964 to December 2013, the econometrician applies the recursive multivariate Filter to extract dynamic factor premiums from the Fama-French (2015) fivefactor asset pricing model. At each time increment, the econometrician takes into account the Fama-French (2015) factors such as the excess return on the market portfolio (MRP), the return spread between the top 30% small and bottom 30% big stocks (SMB), the return spread between the top 30% high book-to-market and bottom 30% low book-to-market stocks (HML), the return spread between the top 30% robust and bottom 30% weak stocks in terms of their relative profitability (RMW), and the return spread between the top 30% conservative investment and bottom 30% aggressive investment stocks (CMA) to explain the variation in the excess return on each stock decile for size, momentum, value (cf. book-to-market, cashflow-to-price, dividend-to-price, and earnings-to-price), investment, profitability, short-term return reversal, and long-term return reversal. The econometrician presents the mathematical time-series representation below:

$$R_{kt} - R_{ft} = \alpha_t + \beta_{mt} \left(R_{mt} - R_{ft} \right) + \beta_{st} SMB_t + \beta_{ht} HML_t + \beta_{rt} RMW_t + \beta_{ct} CMA_t + \varepsilon_t$$

Table A5.1 sums up the long-run mean MRP beta for each stock decile sorted on size, value, momentum, investment, profitability, short-run return reversal, and long-run return reversal. The first 10 columns summarize each long-term average MRP beta and its corresponding *p*-value for the null hypothesis of zero dynamic MRP beta. The next column encapsulates the long-term average MRP beta spread for the long-short trading strategy that entails both a long position in the top decile and a short position in the bottom decile in the 50-year period from January 1964 to December 2013. For each hypothesis test, the econometrician uses the Newey-West (1987) method with quadratic spectral kernel estimation to correct the standard errors to safeguard against any serial correlation and heteroskedasticity.

Portfolio	Low	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	High	Spread
Size											
Beta (test statistic)	0.892	1.057	1.073	1.065	1.051	1.044	1.077	1.082	1.030	0.957	0.065
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.038
Momentum											
Beta (test statistic)	1.225	1.128	0.997	0.977	0.921	0.983	0.996	0.993	1.009	1.124	-0.101
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.268
Book-to-market											
Beta (test statistic)	0.891	0.991	1.035	1.077	1.029	1.025	1.017	1.015	1.073	1.124	0.234
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Cashflow-to-price											
Beta (test statistic)	0.966	0.926	0.949	1.029	0.996	1.014	1.007	1.077	1.069	1.091	0.125
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009
Dividend-to-price											
Beta (test statistic)	1.022	0.959	0.992	0.996	1.014	1.019	1.032	1.000	0.925	0.806	-0.215
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Earnings-to-price											
Beta (test statistic)	0.954	0.971	0.949	0.935	0.990	1.020	0.970	1.016	1.087	1.130	0.176
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Investment											
Beta (test statistic)	1.122	1.040	1.054	0.954	0.987	0.952	0.962	0.967	0.989	1.103	-0.019
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.595
Profitability											
Beta (test statistic)	1.193	1.008	1.033	0.972	0.990	1.029	1.046	1.018	0.989	0.891	-0.302
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Short-term reversal											
Beta (test statistic)	1.199	1.101	1.054	1.057	1.010	0.971	0.936	0.963	0.983	0.985	-0.214
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005
Long-term reversal											
Beta (test statistic)	1.250	1.091	1.061	1.029	1.015	0.946	0.965	0.907	0.983	1.065	-0.185
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002

 Table A5.1: Average dynamic MRP betas, MRP beta spreads, and Newey-West t-tests

Table A5.2: Average dynamic SMB betas, SMB beta spreads, and Newey-West t-tests

Over the 50-year period from January 1964 to December 2013, the econometrician applies the recursive multivariate Filter to extract dynamic factor premiums from the Fama-French (2015) fivefactor asset pricing model. At each time increment, the econometrician takes into account the Fama-French (2015) factors such as the excess return on the market portfolio (MRP), the return spread between the top 30% small and bottom 30% big stocks (SMB), the return spread between the top 30% high book-to-market and bottom 30% low book-to-market stocks (HML), the return spread between the top 30% robust and bottom 30% weak stocks in terms of their relative profitability (RMW), and the return spread between the top 30% conservative investment and bottom 30% aggressive investment stocks (CMA) to explain the variation in the excess return on each stock decile for size, momentum, value (cf. book-to-market, cashflow-to-price, dividend-to-price, and earnings-to-price), investment, profitability, short-term return reversal, and long-term return reversal. The econometrician presents the mathematical time-series representation below:

$$R_{kt} - R_{ft} = \alpha_t + \beta_{mt} \left(R_{mt} - R_{ft} \right) + \beta_{st} SMB_t + \beta_{ht} HML_t + \beta_{rt} RMW_t + \beta_{ct} CMA_t + \varepsilon_t$$

Table A5.2 sums up the long-run mean SMB beta for each stock decile sorted on size, value, momentum, investment, profitability, short-run return reversal, and long-run return reversal. The first 10 columns summarize each long-term average SMB beta and its corresponding *p*-value for the null hypothesis of zero dynamic SMB beta. The next column encapsulates the long-term average SMB beta spread for the long-short trading strategy that entails both a long position in the top decile and a short position in the bottom decile in the 50-year period from January 1964 to December 2013. For each hypothesis test, the econometrician uses the Newey-West (1987) method with quadratic spectral kernel estimation to correct the standard errors to safeguard against any serial correlation and heteroskedasticity.

Portfolio	Low	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	High	Spread
Size											
Beta (test statistic)	1.096	1.044	0.919	0.856	0.703	0.513	0.350	0.184	0.046	-0.280	-1.376
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.079	0.000	0.000
Momentum											
Beta (test statistic)	0.380	0.079	-0.022	0.017	0.016	-0.070	-0.108	0.046	0.053	0.411	0.031
p-value	0.000	0.231	0.655	0.681	0.659	0.046	0.006	0.249	0.395	0.000	0.843
Book-to-market											
Beta (test statistic)	-0.031	-0.061	-0.001	-0.036	-0.073	0.041	0.004	0.100	0.141	0.362	0.393
p-value	0.343	0.088	0.964	0.322	0.051	0.257	0.902	0.002	0.001	0.000	0.000
Cashflow-to-price											
Beta (test statistic)	0.113	-0.033	-0.124	-0.095	-0.071	-0.053	0.001	-0.058	0.068	0.206	0.093
p-value	0.025	0.342	0.001	0.021	0.075	0.111	0.978	0.229	0.064	0.000	0.223
Dividend-to-price											
Beta (test statistic)	0.140	0.011	-0.058	-0.047	-0.042	-0.154	-0.201	-0.131	-0.210	-0.139	-0.279
p-value	0.004	0.771	0.122	0.284	0.342	0.000	0.000	0.004	0.000	0.010	0.000
Earnings-to-price											
Beta (test statistic)	0.118	-0.030	-0.056	-0.185	-0.064	-0.072	-0.052	0.058	0.035	0.268	0.150
p-value	0.012	0.389	0.079	0.000	0.058	0.062	0.101	0.166	0.382	0.000	0.059
Investment											
Beta (test statistic)	0.333	0.213	-0.027	-0.054	-0.056	-0.114	-0.109	-0.083	0.033	0.247	-0.085
p-value	0.000	0.000	0.411	0.055	0.081	0.000	0.007	0.010	0.281	0.000	0.283
Profitability											
Beta (test statistic)	0.455	0.084	0.042	0.049	-0.088	-0.045	-0.074	-0.042	-0.082	0.003	-0.451
p-value	0.000	0.002	0.336	0.264	0.018	0.252	0.030	0.201	0.005	0.920	0.000
Short-term reversal											
Beta (test statistic)	0.434	0.159	0.046	-0.034	-0.099	-0.041	-0.094	-0.087	-0.015	0.279	-0.155
p-value	0.000	0.003	0.320	0.332	0.006	0.224	0.020	0.014	0.737	0.000	0.188
Long-term reversal											
Beta (test statistic)	0.697	0.225	0.090	0.038	-0.010	-0.110	-0.068	-0.187	-0.065	0.128	-0.570
p-value	0.000	0.000	0.064	0.416	0.783	0.001	0.051	0.000	0.109	0.011	0.000

 Table A5.2: Average dynamic SMB betas, SMB beta spreads, and Newey-West t-tests

Table A5.3: Average dynamic HML betas, HML beta spreads, and Newey-West t-tests

Over the 50-year period from January 1964 to December 2013, the econometrician applies the recursive multivariate Filter to extract dynamic factor premiums from the Fama-French (2015) fivefactor asset pricing model. At each time increment, the econometrician takes into account the Fama-French (2015) factors such as the excess return on the market portfolio (MRP), the return spread between the top 30% small and bottom 30% big stocks (SMB), the return spread between the top 30% high book-to-market and bottom 30% low book-to-market stocks (HML), the return spread between the top 30% robust and bottom 30% weak stocks in terms of their relative profitability (RMW), and the return spread between the top 30% conservative investment and bottom 30% aggressive investment stocks (CMA) to explain the variation in the excess return on each stock decile for size, momentum, value (cf. book-to-market, cashflow-to-price, dividend-to-price, and earnings-to-price), investment, profitability, short-term return reversal, and long-term return reversal. The econometrician presents the mathematical time-series representation below:

$$R_{kt} - R_{ft} = \alpha_t + \beta_{mt} \left(R_{mt} - R_{ft} \right) + \beta_{st} SMB_t + \beta_{ht} HML_t + \beta_{rt} RMW_t + \beta_{ct} CMA_t + \varepsilon_t$$

Table A5.3 sums up the long-run mean HML beta for each stock decile sorted on size, value, momentum, investment, profitability, short-run return reversal, and long-run return reversal. The first 10 columns summarize each long-term average HML beta and its corresponding *p*-value for the null hypothesis of zero dynamic HML beta. The next column encapsulates the long-run average HML beta spread for the long-short trading strategy that entails both a long position in the top decile and a short position in the bottom decile in the 50-year period from January 1964 to December 2013. For each hypothesis test, the econometrician uses the Newey-West (1987) method with quadratic spectral kernel estimation to correct the standard errors to safeguard against any serial correlation and heteroskedasticity.

Portfolio	Low	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	High	Spread
Size											
Beta (test statistic)	0.476	0.288	0.181	0.206	0.119	0.186	0.156	0.204	0.232	-0.147	-0.624
p-value	0.000	0.000	0.000	0.000	0.002	0.000	0.004	0.000	0.000	0.000	0.000
Momentum											
Beta (test statistic)	0.216	0.075	0.256	0.175	0.136	0.153	0.195	0.109	-0.010	-0.199	-0.415
p-value	0.234	0.519	0.006	0.016	0.059	0.007	0.002	0.084	0.903	0.068	0.109
Book-to-market											
Beta (test statistic)	-0.899	-0.475	-0.092	0.174	0.374	0.610	0.833	0.896	0.870	1.118	2.016
p-value	0.000	0.000	0.150	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Cashflow-to-price											
Beta (test statistic)	-0.686	-0.586	-0.229	0.014	0.089	0.250	0.303	0.411	0.743	1.001	1.687
p-value	0.000	0.000	0.007	0.793	0.269	0.000	0.001	0.000	0.000	0.000	0.000
Dividend-to-price											
Beta (test statistic)	-0.462	-0.394	-0.309	-0.081	-0.110	0.172	0.382	0.515	0.631	0.983	1.445
p-value	0.000	0.000	0.000	0.284	0.129	0.019	0.000	0.000	0.000	0.000	0.000
Earnings-to-price											
Beta (test statistic)	-0.698	-0.506	-0.365	-0.176	-0.036	0.306	0.444	0.536	0.818	1.028	1.726
p-value	0.000	0.000	0.000	0.014	0.687	0.000	0.000	0.000	0.000	0.000	0.000
Investment											
Beta (test statistic)	0.249	0.221	0.267	0.018	0.272	0.126	0.074	-0.127	-0.373	-0.412	-0.661
p-value	0.001	0.000	0.000	0.764	0.000	0.030	0.131	0.031	0.000	0.000	0.000
Profitability											
Beta (test statistic)	0.314	0.452	0.514	0.366	0.227	0.035	0.079	-0.156	-0.213	-0.533	-0.847
p-value	0.000	0.000	0.000	0.000	0.000	0.508	0.092	0.016	0.000	0.000	0.000
Short-term reversal											
Beta (test statistic)	-0.001	-0.037	0.027	0.185	0.077	0.002	0.095	-0.013	0.000	-0.116	-0.116
p-value	0.994	0.535	0.696	0.005	0.092	0.964	0.130	0.818	0.994	0.150	0.419
Long-term reversal											
Beta (test statistic)	0.399	0.418	0.145	0.359	0.313	0.209	0.191	-0.006	-0.176	-0.601	-1.000
<u>p-value</u>	0.007	0.000	0.141	0.000	0.000	0.000	0.004	0.913	0.004	0.000	0.000

 Table A5.3: Average dynamic HML betas, HML beta spreads, and Newey-West t-tests

Table A5.4: Average dynamic RMW betas, RMW beta spreads, and Newey-West t-tests

Over the 50-year period from January 1964 to December 2013, the econometrician applies the recursive multivariate Filter to extract dynamic factor premiums from the Fama-French (2015) fivefactor asset pricing model. At each time increment, the econometrician takes into account the Fama-French (2015) factors such as the excess return on the market portfolio (MRP), the return spread between the top 30% small and bottom 30% big stocks (SMB), the return spread between the top 30% high book-to-market and bottom 30% low book-to-market stocks (HML), the return spread between the top 30% robust and bottom 30% weak stocks in terms of their relative profitability (RMW), and the return spread between the top 30% conservative investment and bottom 30% aggressive investment stocks (CMA) to explain the variation in the excess return on each stock decile for size, momentum, value (cf. book-to-market, cashflow-to-price, dividend-to-price, and earnings-to-price), investment, profitability, short-term return reversal, and long-term return reversal. The econometrician presents the mathematical time-series representation below:

$$R_{kt} - R_{ft} = \alpha_t + \beta_{mt} \left(R_{mt} - R_{ft} \right) + \beta_{st} SMB_t + \beta_{ht} HML_t + \beta_{rt} RMW_t + \beta_{ct} CMA_t + \varepsilon_t$$

Table A5.4 shows the long-run mean RMW beta for each stock decile sorted on size, value, momentum, investment, profitability, short-run return reversal, and long-run return reversal. The first 10 columns summarize each long-term average RMW beta and its corresponding *p*-value for the null hypothesis of zero dynamic RMW beta. The next column presents the long-run average RMW beta spread for the long-short trading strategy that entails both a long position in the top decile and a short position in the bottom decile in the 50-year period from January 1964 to December 2013. For each hypothesis test, the econometrician uses the Newey-West (1987) method with quadratic spectral kernel estimation to correct the standard errors to safeguard against any serial correlation and heteroskedasticity.

Portfolio	Low	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	High	Spread
Size											
Beta (test statistic)	-0.461	-0.289	-0.323	-0.149	-0.104	-0.270	-0.169	-0.307	-0.281	0.203	0.664
p-value	0.000	0.000	0.000	0.044	0.011	0.000	0.007	0.000	0.000	0.000	0.000
Momentum											
Beta (test statistic)	-0.479	-0.218	-0.291	0.036	-0.066	-0.100	0.000	-0.002	0.082	0.093	0.572
p-value	0.003	0.027	0.000	0.589	0.384	0.163	0.996	0.982	0.216	0.336	0.006
Book-to-market											
Beta (test statistic)	0.498	0.166	0.242	-0.142	-0.282	-0.230	-0.305	-0.532	-0.568	-0.590	-1.088
p-value	0.000	0.009	0.000	0.102	0.003	0.001	0.000	0.000	0.000	0.000	0.000
Cashflow-to-price											
Beta (test statistic)	0.152	0.195	0.117	-0.053	-0.122	-0.198	-0.198	0.246	0.194	-0.030	-0.183
p-value	0.066	0.003	0.093	0.467	0.105	0.006	0.007	0.009	0.016	0.734	0.153
Dividend-to-price											
Beta (test statistic)	0.373	0.197	0.046	0.125	0.174	0.030	-0.168	-0.013	-0.186	-0.737	-1.111
p-value	0.000	0.006	0.528	0.264	0.032	0.652	0.090	0.885	0.088	0.000	0.000
Earnings-to-price											
Beta (test statistic)	0.017	0.230	0.150	-0.022	-0.083	-0.110	-0.071	0.044	0.048	-0.103	-0.119
p-value	0.850	0.002	0.046	0.713	0.296	0.199	0.417	0.596	0.540	0.363	0.414
Investment											
Beta (test statistic)	-0.442	-0.029	-0.103	-0.092	-0.013	-0.036	0.062	0.167	0.166	0.184	0.626
p-value	0.000	0.687	0.142	0.137	0.863	0.524	0.373	0.003	0.015	0.016	0.000
Profitability											
Beta (test statistic)	-1.006	-1.016	-0.776	-0.587	-0.378	0.299	0.101	0.395	0.464	0.562	1.567
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.197	0.000	0.000	0.000	0.000
Short-term reversal											
Beta (test statistic)	-0.394	-0.138	-0.142	-0.031	-0.005	0.028	-0.021	0.045	0.142	-0.047	0.347
p-value	0.000	0.229	0.145	0.594	0.948	0.708	0.792	0.538	0.047	0.620	0.051
Long-term reversal											
Beta (test statistic)	-0.345	-0.584	-0.186	-0.283	-0.116	0.013	0.108	0.118	0.136	0.299	0.644
p-value	0.022	0.000	0.059	0.001	0.167	0.853	0.171	0.235	0.039	0.000	0.000

 Table A5.4: Average dynamic RMW betas, RMW beta spreads, and Newey-West t-tests

Table A5.5: Average dynamic CMA betas, CMA beta spreads, and Newey-West t-tests

Over the 50-year period from January 1964 to December 2013, the econometrician applies the recursive multivariate Filter to extract dynamic factor premiums from the Fama-French (2015) fivefactor asset pricing model. At each time increment, the econometrician takes into account the Fama-French (2015) factors such as the excess return on the market portfolio (MRP), the return spread between the top 30% small and bottom 30% big stocks (SMB), the return spread between the top 30% high book-to-market and bottom 30% low book-to-market stocks (HML), the return spread between the top 30% robust and bottom 30% weak stocks in terms of their relative profitability (RMW), and the return spread between the top 30% conservative investment and bottom 30% aggressive investment stocks (CMA) to explain the variation in the excess return on each stock decile for size, momentum, value (cf. book-to-market, cashflow-to-price, dividend-to-price, and earnings-to-price), investment, profitability, short-term return reversal, and long-term return reversal. The econometrician presents the mathematical time-series representation below:

$$R_{kt} - R_{ft} = \alpha_t + \beta_{mt} \left(R_{mt} - R_{ft} \right) + \beta_{st} SMB_t + \beta_{ht} HML_t + \beta_{rt} RMW_t + \beta_{ct} CMA_t + \varepsilon_t$$

Table A5.5 shows the long-run mean CMA beta for each stock decile sorted on size, value, momentum, investment, profitability, short-run return reversal, and long-run return reversal. The first 10 columns summarize each long-term average CMA beta and its corresponding *p*-value for the null hypothesis of zero dynamic CMA beta. The next column presents the long-run average CMA beta spread for the long-short trading strategy that entails both a long position in the top decile and a short position in the bottom decile in the 50-year period from January 1964 to December 2013. For each hypothesis test, the econometrician uses the Newey-West (1987) method with quadratic spectral kernel estimation to correct the standard errors to safeguard against any serial correlation and heteroskedasticity.

Portfolio	Low	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	High	Spread
Size											
Beta (test statistic)	0.039	0.047	-0.046	-0.146	-0.173	-0.203	-0.099	-0.140	-0.168	0.107	0.069
p-value	0.689	0.332	0.377	0.004	0.011	0.035	0.277	0.061	0.013	0.005	0.444
Momentum											
Beta (test statistic)	-0.608	-0.188	-0.044	-0.039	-0.130	0.118	0.076	0.002	0.190	-0.249	0.359
p-value	0.004	0.222	0.769	0.710	0.140	0.112	0.386	0.980	0.014	0.108	0.255
Book-to-market											
Beta (test statistic)	-0.318	0.061	0.116	0.038	0.220	0.072	0.168	-0.027	0.145	0.333	0.651
p-value	0.000	0.278	0.108	0.667	0.003	0.297	0.018	0.753	0.015	0.002	0.000
Cashflow-to-price											
Beta (test statistic)	-0.362	-0.134	0.011	-0.035	0.188	0.240	0.266	0.408	0.024	-0.415	-0.052
p-value	0.000	0.056	0.875	0.671	0.034	0.011	0.009	0.000	0.846	0.017	0.763
Dividend-to-price											
Beta (test statistic)	-0.444	-0.167	-0.083	-0.029	0.142	0.088	0.217	0.409	0.335	0.274	0.718
p-value	0.000	0.125	0.210	0.758	0.075	0.359	0.004	0.000	0.010	0.112	0.001
Earnings-to-price											
Beta (test statistic)	-0.384	-0.094	0.041	0.213	0.119	0.208	0.260	0.077	-0.033	-0.650	-0.266
p-value	0.000	0.064	0.521	0.004	0.123	0.007	0.002	0.451	0.620	0.000	0.109
Investment											
Beta (test statistic)	0.622	0.639	0.669	0.671	0.297	-0.094	-0.202	-0.230	-0.594	-0.809	-1.432
p-value	0.000	0.000	0.000	0.000	0.000	0.203	0.013	0.001	0.000	0.000	0.000
Profitability											
Beta (test statistic)	0.136	-0.014	-0.029	0.010	0.038	-0.004	0.055	0.042	0.018	-0.071	-0.207
p-value	0.109	0.830	0.747	0.895	0.574	0.955	0.429	0.447	0.753	0.337	0.116
Short-term reversal											
Beta (test statistic)	-0.262	-0.188	0.032	-0.005	-0.007	-0.026	-0.091	0.130	-0.152	-0.282	-0.021
p-value	0.136	0.080	0.765	0.956	0.911	0.662	0.234	0.221	0.078	0.016	0.933
Long-term reversal											
Beta (test statistic)	0.477	0.295	0.467	0.138	0.077	0.185	-0.002	-0.212	-0.224	-0.480	-0.957
<u>p-value</u>	0.001	0.020	0.000	0.181	0.401	0.030	0.980	0.028	0.016	0.001	0.000

 Table A5.5: Average dynamic CMA betas, CMA beta spreads, and Newey-West t-tests