Online appendix

A Computation

In this section, I briefly describe how to solve the model numerically. First, I discretize the state space is $\#_d \times \#_n$ points, where $\#_d = 60$ is the number of points in the grid for productivity and $\#_n = 50$ is the number of points in the grid for employment.[39]

Solving the value function

The problem in (2) is solved by value function iteration. For each point in the state space, (d, n), I find the optimal employment choice, n', using the Golden Search algorithm. This algorithm does not ensure finding a global maximum when the objective function is not well-behaved. To make sure I pick the optimal employment choice, I use the algorithm to solve for the optimal employment choice conditional on $n' > n$ and $n' < n$ separately, and then compare the two solutions with $n' = n$. Given the optimal choice of n', I compute the distribution of next period's productivity using equation (8). I repeat this algorithm until the value function converges.

Solving the innovation problem

The exponential term in equation (8) can easily go to infinity, depending on the maximum real number the computer can manage. To avoid this computational problem, one can redefine the value function and define equation (8) as:

$$\pi(d_i|d,n) = \frac{\eta(d_i|d)\exp\left(\tilde{V}(d_i,n)/\kappa_I\right)}{\sum_{j=1}^{D}\eta(d_j|d)\exp\left(\tilde{V}(d_j,n)/\kappa_I\right)} \tag{13}$$

where $\tilde{V}(d,n) = V(d,n) - \mathbb{C}$ and $\mathbb{C} = \max\{V(\cdot,n)\}$. Note that this normalization does not alter the value of $\pi(d'|d,n)$, but ensures that the exponential term is never larger than one. Using this normalization, the cost of innovation becomes:

$$\begin{aligned}
\mathcal{D}(\pi||\eta) &= \kappa_I \left[\sum_{i=1}^{D} \pi(d_i|d,n) \log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)} \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) \tilde{V}(d_i,n) dx \;-\; \kappa_I \log\left[\sum_{i=1}^{D} \eta(d_i|d) \exp\left(\tilde{V}(d_i,n)/\kappa_I \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) dx \;-\; \mathbb{C} \;+\; \kappa_I \frac{1}{\kappa_I}\mathbb{C} \;-\; \kappa_I \log\left[\sum_{i=1}^{D} \eta(d_i|d) \exp\left(V(d_i,n)/\kappa_I \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) dx \;-\; \kappa_I \log\left[\sum_{i=1}^{D} \eta(d_i|d) \exp\left(V(d_i,n)/\kappa_I \right) \right]
\end{aligned}$$

and the value function at the innovation stage:

$$\mathcal{I}^I(d,n) \;=\; \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) - \mathcal{D}(\pi||\eta) \;=\; \kappa_I \log\left[\sum_{i=1}^{D} \eta(d_i|d) \exp\left(V(d_i,n)/\kappa_I \right) \right]$$

which equals the expression derived in section 3.3.

Note that this result makes the model particularly tractable. In fact, it allows to account for the effect of firing costs on the distribution of productivity without adding model complexity. Overall, the problem of a firm with state (d,n) is:

$$\begin{aligned}
V(d,n) \;&=\; \max_{n'} \;\; \Pi(d,n,n') \;+\; \beta(1-\delta)\hat{V}(d,n') \;+\; \beta\delta V_E(n') \\
\text{s.t.} \quad & \hat{V}(d,n) \;=\; \lambda \mathcal{I}^I(d,n) + (1-\lambda)\mathcal{I}^N(d,n) \\
& \mathcal{I}^I(d,n) \;=\; \kappa_I \log\left[\sum_{i=1}^{D} \eta(d_i|d) \exp\left(V(d_i,n)/\kappa_I \right) \right] \\
& \mathcal{I}^N(d,n) \;=\; \sum_{i=1}^{D} \eta(d_i|d) V(d_i,n) \\
& \lambda \;=\; \frac{\bar{\lambda} \exp\left(\mathcal{I}^I(d,n)/\kappa_I \right)}{\bar{\lambda} \exp\left(\mathcal{I}^I(d,n)/\kappa_I \right) + (1-\bar{\lambda}) \exp\left(\mathcal{I}^N(d,n)/\kappa_I \right)}
\end{aligned}$$

Note that the computational cost of solving this problem is similar to the one required to solve a standard firm dynamics model.

B Additional figures and tables

Figure B.1: Firm growth and growth volatility by firm size

Notes: Dots represent size-specific average and standard deviation of employment growth rates, and the dark line is a quadratic fit. *Source*: *Central de Balances* dataset, 2005-2007.

Figure B.2: Firm growth and growth volatility across sectors

Notes: Dots represent sector-specific average and standard deviation of employment and revenues growth rates, and the dark line is a linear fit. *Source*: *Central de Balances* dataset, 2005-2007.

Figure B.3: Distribution of productivity and employment. Baseline economy

Figure B.4: Expenses in innovation and firm size

Notes: To obtain these numbers I compute the average innovation expenses across different productivity level for each given level of employment.

Figure B.5: Innovation choices, by firm size

Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π or η, and then average across productivity for each value of n. The probability of innovation is also averaged across productivity for every value of n.

Figure B.6: Aggregate effects of firing costs. General *vs.* Partial equilibrium

Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The light line represents the partial equilibrium results, where the wage rate is not adjusted. The dark line represents the general equilibrium results that emerge from adjusting the wage rate.

Figure B.7: Innovation choices. Experiment, $\kappa_F = 0.4$ *vs.* $\kappa_F = 0$

Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.8: Innovation choices. Experiment, $\kappa_F = 1$ *vs.* $\kappa_F = 0$

Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.9: Firing rate by productivity and size in the baseline economy

Notes: The firing rate is defined as the share of initial workers that are fired at the beginning of the period. Consequently, I exclude from the figure firms that enter the period with no workers. Areas in red represent regions of the state space (combinations of productivity and size) in which firms fire a larger share of their workers, while areas in blue represent firms that do not fire any worker.

Figure B.10: Innovation choices. Experiment, $\kappa_F = 0.2$ *vs.* $\kappa_F = 0$

Notes: This figure plots the ratio of the differential productivity growth rate and the percentage change in innovation volatility, both presented in figure 5.

Figure B.11: Aggregate effects of firing costs. Exogenous *vs.* Endogenous innovation

Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The dark line represents the results when innovation is endogenous, and thus, firms' innovation choices react to changes in the firing cost. The light line represents the results when innovation is exogenous so that innovation choices are unaffected by changes in the firing cost.

Table B.1: Sensitivity Analysis – More results(% fall relative to frictionless economy)

		Aggregate productivity	Average productivity	Innovation expenses	Aggregate output	Aggregate employment	Job Destruction	Job Creation
Benchmark		3.71	1.41	2.68	1.91	1.84	52.6	30.9
+10%	σ_z^0	3.68	1.42	2.70	1.92	1.84	52.6	30.8
	μ	3.84	1.36	2.70	1.94	1.90	51.8	31.0
	σ_z	3.70	1.62	2.71	2.00	1.84	51.4	30.3
	κ_0	3.73	1.27	2.65	1.88	1.88	51.8	30.9
	κ_1	3.79	1.25	2.65	1.89	1.89	52.4	31.3
	$\bar{\lambda}$	3.57	1.42	2.59	1.88	1.78	53.4	30.9
-10%	σ_z^0	3.73	1.40	2.66	1.91	1.84	52.7	29.5
	μ_z	3.51	1.44	2.63	1.87	1.76	53.5	28.8
	σ_z	3.44	0.99	2.33	1.69	1.75	53.8	32.0
	κ_0	3.57	1.50	2.62	1.90	1.77	53.6	28.2
	κ_1	3.32	1.56	2.58	1.88	1.71	53.5	28.1
	$\bar{\lambda}$	3.81	1.39	2.77	1.94	1.9	51.88	30.3

Table B.2: Aggregate effects of firing cost.(% fall relative to frictionless economy)

	Endogenous Inn.			Exogenous Inn.			AR(1) process		
Firing cost, κ_F	0.20	0.40	1.00	0.20	0.40	1.00	0.20	0.40	1.00
Wage rate	1.77	3.16	6.3	1.35	2.49	5.08	1.28	2.27	4.78
Aggregate productivity	3.71	5.99	10.94	2.38	3.86	7.1	1.64	2.98	5.48
Average productivity	1.41	2.25	4.47	0	0	0	0	0	0
Productivity growth	1.84	2.97	5.29	0	0	0	0	0	0
Innovation expenses	2.68	4.23	8.04	0	0	0	0	0	0
Output	1.91	3.33	6.57	1.14	2.09	4.28	0.95	1.7	3.57
Employment	1.84	3.17	6.1	1.5	2.61	5.2	1.26	2.1	4.32
Profits	3.05	5.07	10.23	2.27	3.9	8.07	2.06	3.67	7.66
Consumption	1.77	3.16	6.3	1.35	2.49	5.08	1.28	2.27	4.78
Firings	52.64	68.8	86.24	51.98	67.97	84.19	51.93	68.19	84.22
Hirings	30.89	40.37	50.61	30.5	39.89	49.4	28.78	37.79	46.67