Mortgage Debt and Time-Varying Monetary Policy Transmission

DAVID FINCK∗ JÖRG SCHMIDT† PETER TILLMANN‡

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APPENDIX

A. Data and Sources

Table (1) reports the four variables which are used in our baseline model. All data series are taken from the FRED data base of the Federal Reserve Bank of St. Louis.

Table 1: DATA & DESCRIPTION

<table>
<thead>
<tr>
<th>unemployment</th>
<th>civilian unemployment rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>prices</td>
<td>implicit price deflator</td>
</tr>
<tr>
<td>short-rate</td>
<td>effective federal funds rate</td>
</tr>
<tr>
<td>mortgages</td>
<td>households and NPO; home mortgages</td>
</tr>
</tbody>
</table>

As the unemployment rate we take the civilian unemployment rate (UNRATE), i.e. the number of people 16 and older actively searching for a job as a percentage of the total labor force. We calculate the inflation rate as the year-on-year growth rate of the implicit price deflator (GDPDEF). As the policy rate, we choose the effective Federal funds rate (EFFR). For the time when the effective federal funds rate is at the Zero Lower Bound, we follow common practice and amend this series by the Wu and Xia (2016) shadow short rate. Lastly, mortgage debt is taken as the cyclical component of home mortgages to households and nonprofit organizations (HHMS-DODNS). This is obtained by Baxter and King (1999) filtering the original

∗University of Giessen, Germany. E-mail: david.finck@wirtschaft.uni-giessen.de.
†University of Giessen, Germany. E-mail: joerg.h.schmidt@gmail.com
‡University of Giessen, Germany. E-mail: peter.tillmann@wirtschaft.uni-giessen.de.
series. Following the observation of Alpanda and Zubairy (2019) that debt cycles are about twice as long as the business cycle, we choose a band-length of eight and let frequencies between four and 64 quarters pass.

B. Convergence Diagnostics

This appendix assesses the convergence of our MCMC algorithm in the baseline case presented in section (3) in the main paper. We apply different metrics in order to judge how well our chain mixes. Remember that we use 20,000 iterations and discard the first 10,000. It stands out that choosing different burn-in periods delivers exactly the same results.

It is common practice to observe the inefficiency factors for convergence analysis. The inefficiency factor is the inverse of the relative numerical efficiency measure of Geweke (1992) and defined by $1 + 2\sum_{j=1}^{\infty} \rho_j$, where $\rho_j$ is the autocorrelation of $j^{th}$ order for the underlying parameter. Inefficiency factors of around 20 are regarded as satisfactory. Table (2) reports the distribution of inefficiency factors of our entire parameter space. Except for the hyperparameters, the inefficiency factors are on average far below 20. Not taking single outliers too serious as our parameter space is large, we conclude that our chain mixes quite fast.

Table 2: Distribution of Inefficiency Factors

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>10th Percentile</th>
<th>90th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>36.98</td>
<td>33.32</td>
<td>22.07</td>
<td>96.15</td>
<td>23.57</td>
<td>62.36</td>
</tr>
<tr>
<td>$B$</td>
<td>4.05</td>
<td>3.04</td>
<td>1.29</td>
<td>15.88</td>
<td>2.06</td>
<td>7.14</td>
</tr>
<tr>
<td>$A$</td>
<td>3.13</td>
<td>2.09</td>
<td>1.00</td>
<td>14.96</td>
<td>1.42</td>
<td>6.79</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>4.46</td>
<td>3.05</td>
<td>1.05</td>
<td>34.41</td>
<td>1.61</td>
<td>8.20</td>
</tr>
</tbody>
</table>

Notes: The elements of $V$ are the elements of the covariance matrix of the model’s innovations. The elements of $B$, $A$ and $\Sigma$ are the time-varying coefficients, the time-varying volatilities of structural shocks and the time-varying simultaneous relations among our endogenous variables.

As a second check, we look at the Raftery and Lewis (1992) diagnostic. They use a two-state Markov chain assumption to construct a univariate diagnostic which is aimed to report the recommended number of iterations needed for a given level of precision in posterior samples. Table (3) reports

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1We follow the most common values (see Raftery and Lewis, 1992; Primiceri, 2005) and set our quantiles of the marginal posteriors to 2.5% and 97.5%, the minimum probability needed to achieve our stationary posterior distribution of 95% and the desires accuracy of 2.5%, such that our reporting based on a 95% interval result in the true posterior values with a probability lying between 92.5%–97.5%.
the corresponding distribution of the recommended minimum number of draws.
Overall, for both the hyperparameters in \( \mathbf{V} \) as well as the parameters in \( \mathbf{B}, \mathbf{A} \) and \( \mathbf{\Sigma} \), we have far more than the required number of draws, conditional on our desired accuracy goal.

### Table 3: Distribution of Raftery-Lewis Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>10(^{th}) Percentile</th>
<th>90(^{th}) Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{V} )</td>
<td>1330</td>
<td>1115</td>
<td>813</td>
<td>4340</td>
<td>882</td>
<td>2353</td>
</tr>
<tr>
<td>( \mathbf{B} )</td>
<td>382</td>
<td>182</td>
<td>157</td>
<td>2340</td>
<td>161</td>
<td>1141</td>
</tr>
<tr>
<td>( \mathbf{A} )</td>
<td>245</td>
<td>164</td>
<td>153</td>
<td>1611</td>
<td>157</td>
<td>342</td>
</tr>
<tr>
<td>( \mathbf{\Sigma} )</td>
<td>457</td>
<td>182</td>
<td>157</td>
<td>2640</td>
<td>161</td>
<td>1141</td>
</tr>
</tbody>
</table>

Notes: The elements of \( \mathbf{V} \) are the elements of the covariance matrix of the model’s innovations. The elements of \( \mathbf{B}, \mathbf{A} \) and \( \mathbf{\Sigma} \) are the time-varying coefficients, the time-varying volatilities of structural shocks and the time-varying simultaneous relations among our endogenous variables.

As a last check, we also apply the Geweke (1992) convergence diagnostic test, which can be sketched as follows: for each single parameter, the idea is to compare the mean of the first \( n_0 \) draws of the chain to the mean of the \( n_1 \) draws by dropping the corresponding draws in between.\(^2\) It turns out that for 97.67\% of the hyperparameters in \( \mathbf{V} \), for 99.99\% of the parameters in \( \mathbf{B} \), for 99.92\% of the parameters in \( \mathbf{A} \) and for 100\% of the parameters \( \mathbf{\Sigma} \), the chains converges to the target distribution within the first 2000 draws, suggesting that the influence of the priors decays fast.

To sum up, the convergence diagnostics seem satisfactory and justify our prior choice, considering the large parameter space of our model.

### C. Details on the DSGE Model

We use the DSGE model from Alpanda and Zubairy (2017). For our purposes, the key element of the model is the description of the mortgage market, which we present here in order to facilitate the interpretation of our results. The rest of the model can best be found in Alpanda and Zubairy (2017).

\(^2\)The statistics are calculated as

\[
G = \frac{(\bar{x}_0 - \bar{x}_1)}{\sqrt{\hat{\sigma}^2_0/n_0 + \hat{\sigma}^2_1/n_1}},
\]

where \( x_i = \frac{1}{n_j}\sum_{m=j}^{m+n_j} x^i \) with \( x^i \) being the \( i^{th} \) draw. \( \hat{\sigma}^2_j/n_j \) is the standard error of \( \bar{x}_j \) for \( j = 0,1 \), where \( \hat{\sigma}^2_j \) is computed using a Parzen window. We follow common practice and choose \( n_0 \) as the first 10\% and \( n_1 \) as the last 50\% of our Markov chain. Note, however, that \( G \) is of course below 0.05 if the whole chain is stationary, saying that the means of the first \( n_0 \) and the last \( n_1 \) values are quite similar.
The law of motion of the stock of mortgage debt is

\[
\frac{D_t}{P_t} = (1 - \kappa) \frac{D_{t-1}}{P_{t-1}} + \frac{L_t}{P_t}.
\]  

(1)

Here, \(D_t\) is the stock of nominal mortgage debt, \(P_t\) is the price level, \(L_t\) are new mortgages and \(\kappa\) is the amortization rate, which is assumed to be constant.

The fixed interest rate on new mortgages is \(R^F_t\). Each period, a fraction \(\Phi\) of the stock of mortgage debt is refinanced at the rate of new mortgages, \(R^F_t\). The effective interest rate on the stock of mortgages \(R^M_t\) is the weighted average of the previous effective rate, \(R^M_{t-1}\), and the rate on new mortgages. The weights are given by the share of old loans which are not refinanced and the sum of new loans and refinanced existing loans, respectively. Hence, the expression for the effective rate is

\[
R^M_t = \frac{(1 - \Phi) \left(1 - \frac{L_t}{D_t}\right) R^M_{t-1} + \frac{L_t}{D_t} + \Phi \left(1 - \frac{L_t}{D_t}\right)}{\frac{L_t}{D_t} + \Phi \left(1 - \frac{L_t}{D_t}\right)} R^F_t
\]

The calibration of \(\Phi\) proposed by Alpanda and Zubairy (2017) contains information on the ARM share, among other things. In short, the ratio of repayments resulting from refinance originations to the stock of mortgages in Alpanda and Zubairy (2017) relies on data in Greenspan and Kennedy (2005) and Greenspan and Kennedy (2008) and averages around 4.35% quarterly, which corresponds to a duration of 31.16 quarters (7.79 years) using the half life formula.\(^3\) Alpanda and Zubairy (2017) then calculate an average duration assuming a 10% share for adjustable rate mortgages:

\[
\Phi = 1 - \exp\left(\frac{\ln 0.25}{4 \times (0.9 \times 7.79 + 0.1 \times 1)}\right).
\]

This results in a duration of 28.43 quarters (7.11 years) which is converted back to a \(\Phi = 0.0475\) used in the paper.\(^4\)

In our simulation exercise, we assume different ARM (FRM) shares ranging from 5% to 80% and calibrate the ratio of repayments resulting from

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\(^3\)That is, the duration is calculated as \(31.16 = 2 \times \frac{\ln 0.5}{\ln(1 - 0.0435)}\).  
\(^4\)We are thankful to Sarah Zubairy for insightful comments and for providing us data and details on the calibration of \(\Phi\).
refinance originations to the stock of mortgages $\Phi$ according to

$$\Phi = 1 - \exp\left(\frac{\ln 0.25}{4 \times (\text{FRM share} \times 7.79 + \text{ARM share} \times 1)}\right)$$

For each $\Phi$ under consideration, we then re-simulate the model, keeping the entire calibration of all remaining parameters unchanged, i.e. identical to the calibration as in Alpanda and Zubairy (2017).

Note that for a larger ARM share, $\Phi$ is closer to one, such that there is a closer connection between the effective interest rate and the rate on new mortgages.

D. Explanatory Power

Table (4) compares the empirical fit of the TVP-VAR model and the constant parameter VAR model, respectively. We use the $R^2$ as a metric of explanatory power. In both cases, i.e. for the restricted and the unrestricted short-rate equation, the TVP-VAR has a superior empirical fit for unemployment, inflation and mortgages. Only for the short-rate, the fit of the constant parameter VAR is superior.

<table>
<thead>
<tr>
<th></th>
<th>Restricted Policy Rule</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unemployment</td>
<td>Inflation</td>
<td>Mortgages</td>
<td>Short-Rate</td>
</tr>
<tr>
<td>TVP-VAR</td>
<td>0.962</td>
<td>0.984</td>
<td>0.978</td>
<td>0.900</td>
</tr>
<tr>
<td>Constant Parameter VAR</td>
<td>0.953</td>
<td>0.977</td>
<td>0.950</td>
<td>0.911</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Unrestricted Policy Rule</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unemployment</td>
<td>Inflation</td>
<td>Mortgages</td>
<td>Short-Rate</td>
</tr>
<tr>
<td>TVP-VAR</td>
<td>0.962</td>
<td>0.984</td>
<td>0.978</td>
<td>0.906</td>
</tr>
<tr>
<td>Constant Parameter VAR</td>
<td>0.953</td>
<td>0.977</td>
<td>0.950</td>
<td>0.913</td>
</tr>
</tbody>
</table>

Notes: The upper part of the table refers to the model where the coefficients of the short-rate equation to changes in mortgage debt are restricted, whereas the bottom part of the table refers to a specification where all parameters in the short-rate equation remain unrestricted.
E. Additional Results on Counterfactual Simulations

Figure 1: COUNTERFACTUAL HISTORICAL SIMULATION

Notes: Counterfactual historical simulation for unemployment, inflation and mortgage debt. The parameters for the short-rate equation (monetary policy rule) are drawn from the posterior mean distribution of the Bernanke tenure.
F. Additional Results for TVP-VAR Model

Figures (2) and (3) report the full three-dimensional profiles of the time-varying impulse responses.

Figure (4) shows the posterior mean as well as the 16th and 84th percentiles of the standard deviation of the structural shocks.\(^5\) Remember that stochastic volatility is meant to capture possible heteroscedasticity in shocks. In this sense, the conditional variance of the shocks is a possible driver of time variation in the linear structure of our model. Overall, our results indicate time-variation in the volatility of shocks. Thus, some of the variation in the model’s linear structure comes from the variance-covariance matrix in addition to the VAR coefficients. Besides the high volatility of structural shocks from the short rate equation around 1980, it is particularly noticeable that the volatility of structural shocks from the inflation equation and from the mortgages equation is much more persistent.

To investigate whether the time-variation in our paper is a feature of the data or driven by our priors, we follow the formal test proposed by Cogley and Sargent (2005) and compare the trace of the prior for the variance-covariance matrix of \(\Sigma_\beta\) with the trace of the posterior distribution of \(\Sigma_\beta\). Following Cogley and Sargent (2005), we would point to time-variation in the parameters stemming from the data rather than the priors if the trace of the prior is below the 16th percentile of the trace of posterior distribution.

<table>
<thead>
<tr>
<th></th>
<th>16th</th>
<th>50th</th>
<th>84th</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.053</td>
<td>0.054</td>
<td>0.056</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Notes: The first three columns report the 16th, the 50th and the 84th percentiles of the trace of the posterior of \(\Sigma_\beta\), respectively. The fourth column reports the trace of the prior for \(\Sigma_\beta\).

These results suggest that the data rather than the priors are the source of the time-variation in the coefficients.

\(^5\)Note that our MCMC algorithm obtains draws from \(h_t = [h_{1,t}, \cdots, h_{k,t}]’\) with \(h_{i,t} = \log \sigma_{i,t}^2\) for \(i = 1, \cdots, k\). Therefore, the volatility of structural shocks is obtained by calculating \(\sigma_{i,t} = \sqrt{\exp(h_{i,t})}\).
Figure 2: Mean Responses to a Monetary Policy Shock

Notes: Mean responses to a 25bp monetary policy shock.
Figure 3: Mean Responses to a Monetary Policy Shock

Notes: Mean responses to a 25bp monetary policy shock.
Figure 4: **Stochastic Volatility**

Notes: Posterior mean and 16th and 84th percentiles of the standard deviation of structural shocks.
G. Additional Results for Rolling-Window VAR Model

The main text reports the results from a rolling-window VAR with two lags. Here, we show the results from a rolling-window VAR with the optimally chosen lag order.

For each estimation within our rolling window procedure, we calculate the log likelihood as

\[
\text{loglik} = -\frac{kT}{2} (1 + \ln 2\pi) - \frac{T}{2} \ln \det \hat{\Sigma},
\]

where \(k\) is the number of endogenous variables, \(T\) is the sample size and \(\det \hat{\Sigma}\) is the determinant of the estimated variance-covariance matrix. Note that also for the TVP-VAR, the marginal (or log) likelihood would in general be a natural candidate to determine the optimal lag length. However, several articles criticized that the harmonic mean method (as was typically used in the literature for TVP-VARs) could be biased.\(^6\)

Figure 5: Optimal Lag Order for the Rolling-Window VAR Model

\[\text{Notes: Distribution of preferred lag length by different information criteria.}\]

We use the log likelihood to calculate the values of standard information criteria. Figure (5) shows the share of each lag length as preferred by the different information criteria. As the Bayesian Information Criterion penalizes the complexity of the model the most among all information criteria, we choose seven lags as an alternative lag order.

For the alternative lag order, our results remain qualitatively unchanged, see Figure (6). If anything, the shape of the impulse responses is not as smooth as with a lag length of two.

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\(^6\)We are grateful to Jouchi Nakajima and thank him for several helpful and insightful comments on this topic.
Figure 6: Response of Mortgage Debt from a Rolling-Window VAR

Notes: Mean responses (red-solid) with 16th and 84th percentiles to a 25bp monetary policy shock obtained from a rolling-window VAR model with four lags.
Figure 7: Response of Real Mortgage Debt from a Rolling-Window VAR in (log) Levels

Notes: Mean responses (red-solid) with 16th and 84th percentiles to a 25bp monetary policy shock obtained from a rolling-window VAR model in (log) levels.
REFERENCES


