Supplementary Material

This supplementary material contains appendices for the article "Tax Rules to Prevent Expectations-Driven Liquidity Traps."

A Proofs

This Appendix provides the details of the proofs for the propositions in the main article.

A.1 Proof of proposition 1

A system is locally determinate if the eigenvalues of the matrix A lie within the unit circle. Let us denote the eigenvalues of the matrix A as λ_1 and λ_2. Following Bullard and Mitra (2002), the conditions can be expressed as

$$|\lambda_1\lambda_2| = |\det(A)| < 1, \tag{A.1}$$

$$|\lambda_1 + \lambda_2| = |\mathrm{trace}(A)| < 1 + \det(A). \tag{A.2}$$

The first inequality (A.1) can be modified as

$$\begin{aligned}
&|\det(A)| < 1 \\
\Leftrightarrow &\Bigg| \frac{1+\zeta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \times \frac{\beta + \frac{\kappa}{\sigma}(1-\xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} - \\
&\frac{\kappa}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \times \frac{\frac{1}{\sigma}(1+\zeta)(1-\xi\sigma) - \frac{\beta}{\sigma}(\phi_\pi - \xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \Bigg| < 1 \\
\Leftrightarrow &\left| \frac{\beta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \right| < 1 \\
\Leftrightarrow &\left| \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa - \Lambda} \right| < 1.
\end{aligned} \tag{A.3}$$

Therefore, Λ must satisfy the following conditions:

$$\Lambda < -\beta + 1 + \frac{\phi_\pi}{\sigma}\kappa, \tag{A.4}$$

or

$$\Lambda > \beta + 1 + \frac{\phi_\pi}{\sigma}\kappa. \tag{A.5}$$

The second inequality (A.2) can be modified as

$$\begin{aligned}
&|\mathrm{trace}(A)| < 1 + \det(A) \\
\Leftrightarrow & \left| \frac{1+\zeta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} + \frac{\beta + \frac{\kappa}{\sigma}(1-\xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \right| \\
& < \frac{\beta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} + 1 \\
\Leftrightarrow & \left| \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} \right| < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1.
\end{aligned} \tag{A.6}$$

First, assuming $\Lambda < -\beta + 1 + \frac{\phi_\pi}{\sigma}\kappa$, we obtain the following relation:

$$\begin{aligned}
& \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1 \\
\Leftrightarrow \;\; & \phi_\pi > 1.
\end{aligned} \tag{A.7}$$

This is satisfied from our assumption.

Next, assuming $\Lambda > \beta + 1 + \frac{\kappa}{\sigma}\phi_\pi$, we obtain the following relation:

$$\begin{aligned}
& \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1 \\
\Leftrightarrow \;\; & \phi_\pi < 1.
\end{aligned} \tag{A.8}$$

This contradicts our assumption. Therefore, the condition to ensure local determinacy around the targeted steady state is

$$\Lambda < 1 - \beta + \phi_\pi \frac{\kappa}{\sigma} \equiv \Psi^D. \tag{A.9}$$

(End of Proof)

A.2 Proof of proposition 2

The ELT equilibrium exists if and only if the following inequality is satisfied:

$$\hat{\pi}_U = \frac{\frac{\log \beta}{1 - p_U} \frac{\kappa}{\sigma}}{-(1 - \beta p_U) + \frac{p_U}{1 - p_U} \frac{\kappa}{\sigma}} < \frac{\log \beta}{\phi_\pi}$$

$$\Leftrightarrow \frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma} p_U - (1 - \beta p_U)(1 - p_U)} > \frac{1}{\phi_\pi}. \tag{A.10}$$

Since the right-hand side of the inequality is positive, the denominator in the left-hand side must be also positive to satisfy the inequality. Hence the following two inequalities are the necessary and sufficient conditions for the ELT equilibrium to exist:

$$\frac{\kappa}{\sigma} p_U - (1 - \beta p_U)(1 - p_U) > 0, \tag{A.11}$$

and

$$\frac{\kappa \phi_\pi}{\sigma} > \frac{\kappa}{\sigma} p_U - (1 - \beta p_U)(1 - p_U). \tag{A.12}$$

Inequality (A.12) can be modified as

$$\left[p_U - \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)\right]^2 + \frac{\kappa \phi_\pi}{\beta \sigma} + \frac{1}{\beta} - \frac{1}{4}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)^2 > 0. \tag{A.13}$$

Above inequality is satisfied under standard calibration. The solution of (A.11) is

$$\underbrace{\frac{1}{2}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)-\sqrt{\frac{1}{4}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)^2-\frac{1}{\beta}}}_{\equiv \underline{p}} < p_U$$
$$< \underbrace{\frac{1}{2}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)+\sqrt{\frac{1}{4}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)^2-\frac{1}{\beta}}}_{\equiv \bar{p}}. \tag{A.14}$$

Since $\bar{p} > 1$ holds, the necessary and sufficient condition is

$$\underline{p} < p_U < 1. \tag{A.15}$$

(End of Proof)

A.3 Proof of proposition 3

The same steps are taken as in proposition 2. The ELT equilibrium exists if and only if the following inequality is satisfied:

$$\hat{\pi}_U = \frac{\frac{\log\beta}{1-p_U}\frac{\kappa}{\sigma}}{\Lambda-(1-\beta p_U)+\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}} < \frac{\log\beta}{\phi_\pi}$$
$$\Leftrightarrow \frac{\frac{1}{1-p_U}\frac{\kappa}{\sigma}}{\Lambda-(1-\beta p_U)+\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}} > \frac{1}{\phi_\pi}. \tag{A.16}$$

The numerator of the left-hand side is positive. Therefore, inequality (A.16) holds for the following Λ:

$$\underbrace{1-\beta p_U-\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}}_{\equiv \Psi} < \Lambda < \underbrace{1-\beta p_U+\frac{\phi_\pi-p_U}{1-p_U}\frac{\kappa}{\sigma}}_{\equiv \Psi^N}. \tag{A.17}$$

Taking the contraposition, the ELT does not exist if and only if

$$\Psi^N \leq \Lambda, \tag{A.18}$$

or

$$\Lambda \leq \Psi. \tag{A.19}$$

Since $\Psi < \Psi^D < \Psi^N$, the second inequality (A.19) satisfies the determinacy condition (A.9) while the first inequality (A.18) does not.

Note that if we assume the ELT equilibrium exists without any policy intervention, (A.11) indicates:

$$\Psi \equiv (1 - \beta p_U) - \frac{p_U}{1 - p_U}\frac{\kappa}{\sigma} < 0. \tag{A.20}$$

Therefore, the threshold Ψ is negative. (End of Proof)

A.4 Proof of proposition 4

Taking the derivative of Ψ with respect to p_U shows

$$\frac{\partial \Psi}{\partial p_U} = -\beta - \frac{\kappa}{\sigma}\frac{1}{(p_U - 1)^2} < 0.$$

Therefore, the threshold level Ψ is decreasing in transition probability p_U. (End of Proof)

A.5 Proof of proposition 5

Λ is equal to 0 when $\lambda_w = 0$. The ELT equilibrium exists if and only if

$$\hat{\pi}_U = \log\beta\frac{\Phi}{\Upsilon} < \frac{\log\beta}{\phi_\pi} \tag{A.21}$$

$$\Leftrightarrow \frac{\Phi}{\Upsilon} > \frac{1}{\phi_\pi}. \tag{A.22}$$

Note that Φ is positive from our assumption. Since the right-hand side of the inequality (A.22) is positive, Υ must be also positive ($\Upsilon > 0$). Then, the condition can be arranged as

$$\Phi\phi_\pi - \Upsilon > 0. \tag{A.23}$$

(End of Proof)

A.6 Proof of proposition 6

The ELT equilibrium does not exist if and only if the following inequality holds:

$$\hat{\pi}_U \geq \frac{\log\beta}{\phi_\pi}. \tag{A.24}$$

Equation $\hat{\pi}_U = \log\beta(\Phi - \Omega\Lambda)[(1-\Omega)\Lambda + \Upsilon]^{-1}$ can be regarded as a hyperbola taking $\hat{\pi}_U$ in the vertical axis and Λ in the horizontal axis. The equation can be arranged as

$$\hat{\pi}_U + \frac{\Omega}{1-\Omega}\log\beta = \frac{\Omega}{1-\Omega}\log\beta\frac{\Upsilon + \frac{1-\Omega}{\Omega}\Phi}{(1-\Omega)\Lambda + \Upsilon}. \tag{A.25}$$

Following inequality shows that the horizontal asymptote of the hyperbola is higher than the threshold level:

$$\frac{\log\beta}{\phi_\pi} < -\frac{\Omega}{1-\Omega}\log\beta. \tag{A.26}$$

There are two regions of Λ that satisfies (A.24). The first region is

$$\Lambda \geq \frac{\Phi\phi_\pi - \Upsilon}{\Omega(\phi_\pi - 1) + 1}. \tag{A.27}$$

However, above inequality contradicts the determinacy condition given in (A.9). The second region is

$$\Lambda \leq -\frac{\Upsilon}{1-\Omega} \equiv \tilde{\Psi}. \tag{A.28}$$

Above region satisfies the determinacy condition. (End of Proof)

A.7 Proof of proposition 7

An equilibrium exists in the crisis state if and only if the following inequalities are satisfied in each case:

$$\hat{\pi}_L = \frac{\frac{-r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{-(1-\beta p_L^*) - \frac{\phi_\pi - p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} \geq \frac{\log\beta}{\phi_\pi}, \tag{A.29}$$

$$\hat{\pi}_L = \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} < \frac{\log\beta}{\phi_\pi}. \tag{A.30}$$

(i) When the ZLB does not bind
The first inequality can be modified as

$$\frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma}(p_L^* - \phi_\pi) - (1-\beta p_L^*)(1-p_L^*)} \geq -\frac{\log\beta}{\phi_\pi r_L^n}. \tag{A.31}$$

The denominator of the left-hand side in inequality (A.31) is negative under standard calibration:

$$-\left[p_L^* - \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)\right]^2 - \frac{\kappa\phi_\pi}{\beta\sigma} - \frac{1}{\beta} + \frac{1}{4}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)^2 < 0. \tag{A.32}$$

Therefore inequality (A.31) can be modified as

$$\frac{\kappa}{\sigma}(p_L^* - \phi_\pi) - (1-\beta p_L^*)(1-p_L^*) \leq \frac{\kappa}{\sigma}\frac{\phi_\pi}{\log\beta}(-r_L^n). \tag{A.33}$$

The solution to the inequality is

$$p_L^* \leq \underbrace{\frac{1}{2}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big) - \sqrt{\frac{1}{4}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)^2 - \frac{1}{\beta} - \frac{\kappa}{\sigma\beta}\phi_\pi + \frac{\kappa}{\sigma\beta}\frac{\phi_\pi}{\log\beta}r_L^n}}_{\equiv p_\dagger}, \tag{A.34}$$

or

$$\underbrace{\frac{1}{2}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big) + \sqrt{\frac{1}{4}\Big(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\Big)^2 - \frac{1}{\beta} - \frac{\kappa}{\sigma\beta}\phi_\pi + \frac{\kappa}{\sigma\beta}\frac{\phi_\pi}{\log\beta}r_L^n}}_{\equiv p^\dagger} \leq p_L^*. \tag{A.35}$$

Since $0 < p_\dagger$ and $1 < p^\dagger$, the condition is

$$0 < p_L^* \leq p_\dagger. \tag{A.36}$$

(ii) When the ZLB binds

The second inequality can be modified as

$$\frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma}p_L^* - (1-\beta p_L^*)(1-p_L^*)} < \frac{\log\beta}{\phi_\pi(\log\beta - r_L^n)}. \tag{A.37}$$

Inequality (A.37) holds if and only if the following two inequalities are satisfied:

$$\underbrace{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}}_{\equiv -\Psi^F} < 0, \tag{A.38}$$

and

$$\frac{\kappa}{\sigma}p_L^* - (1-\beta p_L^*)(1-p_L^*) > \frac{\kappa}{\sigma}\frac{\phi_\pi}{\log\beta}(-r_L^n) + \frac{\kappa}{\sigma}\phi_\pi. \tag{A.39}$$

Inequalities (A.38) and (A.39) can be solved as

$$p_L^* < \underline{p} \quad \text{or} \quad \bar{p} < p_L^*, \tag{A.40}$$

and

$$p_\dagger < p_L^* < p^\dagger. \tag{A.41}$$

Since $p_\dagger < \underline{p} < 1 < p^\dagger$, the conditions can be summarized to

$$p_\dagger < p_L^* < \underline{p}. \tag{A.42}$$

Combining the two conditions (A.36) and (A.42), we obtain the condition as follows:

$$0 < p_L^* < \underline{p}. \tag{A.43}$$

Note that $p_L^* < \underline{p} < p_U$ implies $\Psi < 0 < \Psi^F$. (End of Proof)

A.8 Proof of proposition 8

Let us denote the equilibrium inflation and output as $\hat{\pi}_L^{NI}$ and $\hat{y}_L^{NI}$ (NI stands for "No Intervention") in the case where the tax rate does not respond to inflation ($\Lambda = 0$). Inflation rate in the crisis state is higher compared to the case without the tax rule if the following inequality holds:

$$\hat{\pi}_L > \hat{\pi}_L^{NI}. \tag{A.44}$$

Since we have restricted our focus to the case where the ZLB binds in the crisis state, we can modify the condition as

$$\frac{\dfrac{\log\beta - r_L^n}{1-p_L^*}\dfrac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) + \dfrac{p_L^*}{1-p_L^*}\dfrac{\kappa}{\sigma}} > \frac{\dfrac{\log\beta - r_L^n}{1-p_L^*}\dfrac{\kappa}{\sigma}}{-(1-\beta p_L^*) + \dfrac{p_L^*}{1-p_L^*}\dfrac{\kappa}{\sigma}}. \tag{A.45}$$

The numerator in both sides are positive while the denominator in the right-hand side ($-\Psi^F$) is negative under the assumption $p_\dagger \leq p_L^* < \underline{p}$ from (A.38). Therefore, above inequality can be solved as

$$\Lambda < 0 \quad \text{or} \quad (1-\beta p_L^*) - \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma} < \Lambda. \tag{A.46}$$

The first condition $\Lambda < 0$ is satisfied when the fiscal authority sets $\Lambda \leq \Psi$ to avoid the ELT equilibrium since $\Psi < 0$. Therefore, as long as the fiscal authority targets to prevent the ELT equilibrium, $\hat{\pi}_L > \hat{\pi}_L^{NI}$ holds. (End of Proof)

B Models with Different Tax Instruments

This Appendix provides a full description of the optimization problem and the conditions to prevent the ELT equilibrium with different tax instruments. For completeness, dividend tax and consumption tax are also included as the fiscal authority's target in addition to the labor income tax. In the following analysis, steady state tax rates are calibrated to $\tau_w = 0.2$, $\tau_c = 0.2$, and $\tau_d = 0.2$ respectively.

B.1 Optimization problem

B.1.1 Household

A representative household maximizes its lifetime utility subject to the budget constraint:

$$U = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Big[\frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{l_{t+s}^{\eta+1} - 1}{\eta + 1} \Big],$$

$$(1 + \tau_{c,t}) c_t + \frac{b_t}{R_t} = (1 - \tau_{w,t}) w_t l_t + \frac{b_{t-1}}{\Pi_t} + (1 - \tau_{d,t}) d_t. \tag{B.1}$$

The Lagrangian can be set up as follows:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \Big\{ \beta^s \Big[\frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{l_{t+s}^{\eta+1} - 1}{\eta + 1} \Big] - \mu_{t+s} \Big[(1 + \tau_{c,t+s}) c_{t+s} \\ + \frac{b_{t+s}}{R_{t+s}} - (1 - \tau_{w,t+s}) w_{t+s} l_{t+s} - \frac{b_{t+s-1}}{\Pi_{t+s}} - (1 - \tau_{d,t+s}) f_{t+s} - \tau_{t+s} \Big] \Big\}. \end{aligned} \tag{B.2}$$

Household takes prices $\{w_t, P_t, R_t\}_{t=0}^{\infty}$ as given. The first order conditions can be derived as

$$\text{w/r} \quad c_{t+s} \ : \ \beta^s c_{t+s}^{-\sigma} - \mu_{t+s} (1 + \tau_{c,t+s}) = 0, \tag{B.3}$$

$$\text{w/r} \quad l_{t+s} \ : \ -\beta^s l_{t+s}^{\eta} + \mu_{t+s} (1 - \tau_{w,t+s}) w_{t+s} = 0, \tag{B.4}$$

$$\text{w/r} \quad b_{t+s} \ : \ -\frac{\mu_{t+s}}{R_{t+s}} + \mathbb{E}_{t+s} \frac{\mu_{t+s+1}}{\Pi_{t+s+1}} = 0. \tag{B.5}$$

Equilibrium conditions are

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.6}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1+\tau_{c,t}}{1-\tau_{w,t}}\frac{1}{w_t}. \tag{B.7}$$

B.1.2 Firms

The optimization problem for the final goods producer is

$$\max_{\{y_t, y_{i,t}\}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di - \lambda\Big[y_t - \Big(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\Big)^{\frac{\theta}{\theta-1}}\Big]. \tag{B.8}$$

First order conditions are

$$\text{w/r} \quad y_t \ : \ P_t = \lambda, \tag{B.9}$$

$$\text{w/r} \quad y_{i,t} \ : \ P_{i,t} = \lambda\Big[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\Big]^{\frac{1}{\theta-1}} y_{i,t}^{-\frac{1}{\theta}}. \tag{B.10}$$

Substituting out the Lagrange multiplier, we obtain

$$y_{i,t} = \Big(\frac{P_{i,t}}{P_{t+s}}\Big)^{-\theta} y_t, \tag{B.11}$$

$$P_t = \Big(\int_0^1 P_{i,t}^{1-\theta} di\Big)^{\frac{1}{1-\theta}}. \tag{B.12}$$

The optimization problem for the intermediate goods producers are

$$\max_{\{y_{i,t+s}, P_{i,t+s}, l_{i,t+s}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{c,t+s}(1-\tau_{d,t+s}) d_{i,t+s}, \tag{B.13}$$

$$\text{s.t.} \quad d_{i,t+s} = \frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1\Big)^2 y_{t+s}, \tag{B.14}$$

$$y_{i,t+s} = l_{i,t+s}, \tag{B.15}$$

$$y_{i,t+s} = \Big(\frac{P_{i,t+s}}{P_{t+s}}\Big)^{-\theta} y_{t+s}. \tag{B.16}$$

where the stochastic discount factor is defined as

$$Q_{c,t+s} \equiv \beta^s \frac{c_{t+s}^{-\sigma}}{1+\tau_{c,t+s}}. \tag{B.17}$$

Note that the consumption tax rate is included in the stochastic discount factor. We can set up the Lagrangian as

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \Big\{ Q_{c,t+s}(1-\tau_{d,t+s}) \Big[\frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2} \Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1 \Big)^2 y_{t+s} \Big]$$
$$- \mu_{t+s}(y_{i,t+s} - l_{i,t+s}) - \phi_{t+s} \Big(y_{i,t+s} - \Big(\frac{P_{i,t+s}}{P_{t+s}} \Big)^{-\theta} y_{t+s} \Big) \Big\}. \tag{B.18}$$

First order conditions of the optimization problem for the firms are

$$\text{w/r } l_{i,t} \;:\; \mu_t - Q_{c,t}(1-\tau_{d,t}) w_t = 0, \tag{B.19}$$

$$\text{w/r } y_{i,t} \;:\; Q_{c,t}(1-\tau_{d,t}) \frac{P_{i,t}}{P_t} - \mu_t - \phi_t = 0, \tag{B.20}$$

$$\text{w/r } P_{i,t} \;:\; Q_{c,t}(1-\tau_{d,t}) \Big[\frac{y_{i,t}}{P_t} - \psi \Big(\frac{P_{i,t}}{P_{i,t-1}} - 1 \Big) \frac{y_t}{P_{i,t-1}} \Big] - \theta \phi_t \Big(\frac{P_{i,t}}{P_t} \Big)^{\theta} \frac{y_t}{P_{i,t}}$$
$$= \mathbb{E}_t \Big[Q_{c,t+1}(1-\tau_{d,t+1}) \psi \Big(\frac{P_{i,t+1}}{P_{i,t}} - 1 \Big) \Big(- \frac{P_{i,t+1}}{P_{i,t}^2} \Big) y_{t+1} \Big]. \tag{B.21}$$

Substituting out the Lagrange multipliers and imposing symmetry across firms, we can derive the Philips curve as

$$\psi(\Pi_t - 1)\Pi_t - \theta w_t + \theta - 1 = \beta \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \frac{1-\tau_{d,t+1}}{1-\tau_{d,t}} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \Big]. \tag{B.22}$$

We can observe that both consumption tax and dividend tax are included in the PC.

B.1.3 Central Bank and Fiscal Authority

The central bank sets the interest rate following the standard Taylor rule. The net nominal interest rate is bounded below by zero:

$$R_t = \max \Big[1, \frac{1}{\beta} \Pi_t^{\phi_\pi} \Big]. \tag{B.23}$$

The government's budget constraint with consumption tax, dividend tax, and labor income tax is

$$\frac{b_t}{R_t} + \tau_{c,t} c_t + \tau_{w,t} w_t l_t + \tau_{d,t} d_t = \frac{b_{t-1}}{\Pi_t} + g_t. \tag{B.24}$$

It is further assumed that the government spending is determined endogenously. Namely, the total amount of tax revenue constrains the amount of goods that the fiscal authority purchases:

$$\tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t = g_t. \tag{B.25}$$

Therefore, the fiscal authority does not issue bonds in the equilibrium ($b_t = 0$).

B.2 Equilibrium conditions

The complete set of equilibrium conditions with eleven variables $\{c_t, y_t, l_t, d_t, g_t, w_t, \Pi_t, R_t, \tau_{w,t}, \tau_{c,t}, \tau_{d,t}\}$ are as follows:

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.26}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1+\tau_{c,t}}{1-\tau_{w,t}}\frac{1}{w_t}, \tag{B.27}$$

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}}(1-\tau_{d,t})\Big[\psi(\Pi_t-1)\Pi_t - \theta w_t + \theta - 1\Big]$$

$$= \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}(1-\tau_{d,t+1})\psi(\Pi_{t+1}-1)\Pi_{t+1}\frac{y_{t+1}}{y_t}\Big], \tag{B.28}$$

$$y_t = l_t, \tag{B.29}$$

$$d_t = y_t - w_t l_t - \frac{\psi}{2}(\Pi_t - 1)^2 y_t, \tag{B.30}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{B.31}$$

$$y_t = c_t + g_t + \frac{\psi}{2}(\Pi_t - 1)^2 y_t, \tag{B.32}$$

$$g_t = \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t, \tag{B.33}$$

$$\tau_{c,t} = \tau_c\Pi_t^{\lambda_c}, \quad \tau_{w,t} = \tau_w\Pi_t^{\lambda_w}, \quad \tau_{d,t} = \tau_d\Pi_t^{\lambda_d}. \tag{B.34}$$

Equilibrium conditions other than the tax rules can be summarized to the following four equations with four variables:

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.35}$$

$$\frac{1-\tau_{d,t}}{1+\tau_{c,t}}\Big[\psi(\Pi_t-1)\Pi_t - \theta\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta}+\theta-1\Big]$$
$$= \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1-\tau_{d,t+1}}{1+\tau_{c,t+1}}\frac{y_{t+1}}{y_t}\psi(\Pi_{t+1}-1)\Pi_{t+1}\Big], \tag{B.36}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{B.37}$$

$$(1-\tau_{d,t})y_t\Big[1-\frac{\psi}{2}(\Pi_t-1)^2\Big] = (1+\tau_{c,t})c_t + (\tau_{w,t}-\tau_{d,t})\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta+1}. \tag{B.38}$$

Steady state values are

$$R_{TSS} = \frac{1}{\beta}, \tag{B.39}$$

$$y_{TSS} = (1+\tau_c)^{\frac{\sigma}{\eta+\sigma}}\Big[1-\tau_d-(\tau_w-\tau_d)\frac{\theta-1}{\theta}\Big]^{-\frac{\sigma}{\eta+\sigma}}\Big(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\Big)^{\frac{1}{\eta+\sigma}}, \tag{B.40}$$

$$c_{TSS} = (1+\tau_c)^{-\frac{\eta}{\eta+\sigma}}\Big[1-\tau_d-(\tau_w-\tau_d)\frac{\theta-1}{\theta}\Big]^{\frac{\eta}{\eta+\sigma}}\Big(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\Big)^{\frac{1}{\eta+\sigma}}. \tag{B.41}$$

Log-linearized equilibrium conditions are

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}(\hat{\tau}_{c,t}-\mathbb{E}_t\hat{\tau}_{c,t+1}), \tag{B.42}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t + \frac{\theta-1}{\psi}\frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{B.43}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{B.44}$$

$$\gamma_y\hat{y}_t = \gamma_c\hat{c}_t + \gamma_{\tau,c}\hat{\tau}_{c,t} + \gamma_{\tau,w}\hat{\tau}_{w,t} + \gamma_{\tau,d}\hat{\tau}_{d,t}, \tag{B.45}$$

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t, \quad \hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \quad \hat{\tau}_{d,t} = \lambda_d\hat{\pi}_t, \tag{B.46}$$

where

$$\gamma_y \equiv 1-\tau_d-(\eta+1)(\tau_w-\tau_d)\frac{\theta-1}{\theta}, \quad \gamma_c \equiv (1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + \sigma(\tau_w-\tau_d)\frac{\theta-1}{\theta},$$
$$\gamma_{\tau,c} \equiv \Big[(1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + (\tau_w-\tau_d)\frac{\theta-1}{\theta}\Big]\frac{\tau_c}{1+\tau_c}, \quad \gamma_{\tau,w} \equiv \frac{\theta-1}{\theta}\frac{\tau_w}{1-\tau_w}, \quad \gamma_{\tau,d} \equiv \frac{1}{\theta}\tau_d.$$

Above equilibrium conditions can be further summarized to following equations:

$$
\begin{aligned}
\hat{y}_t &= \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) + \Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)\hat{\tau}_{c,t} + \frac{\gamma_{\tau,w}}{\gamma_y}\hat{\tau}_{w,t} \\
&\quad + \frac{\gamma_{\tau,d}}{\gamma_y}\hat{\tau}_{d,t} + \Big(\frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y}\Big)\mathbb{E}_t\hat{\tau}_{c,t+1} - \frac{\gamma_{\tau,w}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{w,t+1} - \frac{\gamma_{\tau,d}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{d,t+1}, \qquad \text{(B.47)} \\
\hat{\pi}_t &= \beta\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\theta-1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big)\hat{y}_t + \frac{\theta-1}{\psi}\Big(\frac{\tau_c}{1+\tau_c} - \sigma\frac{\gamma_{\tau,c}}{\gamma_c}\Big)\hat{\tau}_{c,t} \\
&\quad - \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\Big)\hat{\tau}_{w,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\hat{\tau}_{d,t}, \qquad \text{(B.48)} \\
\hat{\tau}_{c,t} &= \lambda_c\hat{\pi}_t, \quad \hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \quad \hat{\tau}_{d,t} = \lambda_d\hat{\pi}_t. \qquad \text{(B.49)}
\end{aligned}
$$

After substitution, the equilibrium conditions simplify to the following EE and PC with two variables $\hat{\pi}_t$ and $\hat{y}_t$:

$$
\hat{y}_t = \xi\hat{\pi}_t + \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) - \xi\mathbb{E}_t\hat{\pi}_{t+1}, \qquad \text{(B.50)}
$$

$$
\hat{\pi}_t = \kappa\hat{y}_t - \zeta\hat{\pi}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \qquad \text{(B.51)}
$$

where

$$
\begin{aligned}
\kappa &\equiv \frac{\theta-1}{\psi}\Big(\eta + \sigma\frac{\gamma_y}{\gamma_c}\Big), \\
\xi &\equiv \Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)\lambda_c + \frac{\gamma_{\tau,w}}{\gamma_y}\lambda_w + \frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d, \\
\zeta &\equiv \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,c}}{\gamma_c} - \frac{\tau_c}{1+\tau_c}\Big)\lambda_c + \frac{\theta-1}{\psi}\Big(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\Big)\lambda_w + \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\lambda_d.
\end{aligned}
$$

Equations (B.50) and (B.51) are identical to equations (34) and (35) in the main article with different definitions for ξ and ζ. Therefore, all propositions hold for models in this Appendix by replacing Λ with

$$
\Lambda \equiv \xi\kappa - \zeta = \frac{\theta-1}{\psi}\Big[\eta\Big(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\Big)\lambda_c + \Big(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\Big)\lambda_w + \eta\frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d\Big].
$$

Note that equations in the main article are particular cases with $\tau_c = 0$, $\gamma_{\tau,c} = 0$, $\tau_d = 0$, and $\gamma_{\tau,d} = 0$.

B.3 Preventing the ELT equilibrium

Proposition 3 in the main article claims that the fiscal authority prevents the ELT equilibrium if the tax response parameters satisfy the following condition:

$$\Lambda \leq 1 - \beta p_U - \frac{\kappa}{\sigma} \frac{p_U}{1 - p_U} \equiv \Psi \tag{B.52}$$

In the following subsubsections, we discuss how the use of different tax instruments affects the equilibrium outcome.

B.3.1 Consumption tax rate adjustment

Changes in the consumption tax rate operate through the demand side, while it can prevent the ELT equilibrium as long as the utility function of the representative household is not a logarithmic utility. Let us set λ_w and λ_d to zero.

(i) When $\sigma = 1$

The utility function takes the form of log, therefore income effect and substitution effect perfectly offset each other. This is reflected in the coefficients:

$$\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma} \frac{\tau_c}{1 + \tau_c} \frac{\gamma_c}{\gamma_y} = 0$$

on λ_c. Therefore, altering λ_c cannot affect the equilibrium, and whether the ELT equilibrium exists or not does not depend on the choice of λ_c.

(ii) When $\sigma \neq 1$

Inequality (B.52) simplifies to

$$\lambda_c \leq -\frac{\psi}{(\theta - 1)\eta} \left(\frac{1}{\sigma} \frac{\tau_c}{1 + \tau_c} \frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y} \right)^{-1} \Psi \equiv \Psi_c < 0 \tag{B.53}$$

Since Ψ_c is negative, the fiscal authority raises the consumption tax rate in response to a decrease in the inflation rate.

B.3.2 Dividend tax rate adjustment

The dividend tax rate operates through the demand side and affects household income. Let us set λ_w and λ_c equal to zero. Then, the condition (B.52) simplifies to

$$\lambda_d \leq \frac{\psi}{(\theta-1)\eta}\frac{\gamma_y}{\gamma_{\tau,d}}\Psi \equiv \Psi_d < 0. \tag{B.54}$$

Since Ψ_d is negative, the fiscal authority raises the dividend tax rate in response to a decline in the inflation rate.

The mechanism through which the demand-side policy affects the equilibrium is as follows. The negative Ψ_d implies that an increase in inflation causes the dividend tax rate to decline and increases household income. When the ZLB does not bind, the increase in consumption induced by this increase in income partially offsets the decline in consumption caused by the increase in real interest rate through intertemporal substitution.

Alternatively, when the ZLB binds, the Taylor rule is inactive, and an increase in inflation decreases the real interest rate, which induces the household to increase current consumption. However, in addition to the increase in consumption according to the household's intertemporal substitution, the increase in income caused by the decline in the dividend tax rate also operates to increase consumption.

Changes in the dividend tax rate also affect the PC. When the inflation rate rises, output increases, driven by the rise in household income. This induces the household to increase its labor supply, which adds further inflationary pressure.

B.3.3 Combining different tax rates

We have confirmed that both supply-side and demand-side policies affect labor supply and consumption in different ways. Although we have examined each tax individually, we can combine different taxes to achieve our goal.

Let us focus on the labor income tax and the dividend tax. Any combination of λ_w and λ_d that satisfies inequality (B.52) can prevent the ELT equilibrium. The condition stated in proposition 3 of the main article can be rearranged as

$$\lambda_d \leq \frac{\gamma_y}{\gamma_{\tau,d}}\Big(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\Big)\lambda_w + \frac{\gamma_y}{\gamma_{\tau,d}}\frac{\psi}{(\theta-1)\eta}\Psi. \tag{B.55}$$

Figure 1 displays the area that satisfies inequality (B.55). Both the edge and the

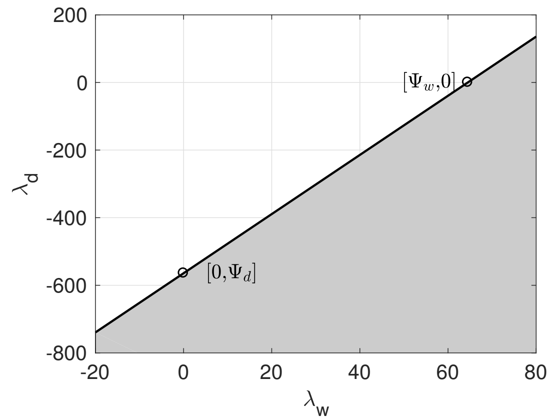


Figure 1: Parameter space where the ELT equilibrium does not exist.

area in gray depict the parameter space where the ELT equilibrium does not exist. Any linear combination $\mu\Psi_w + (1-\mu)\Psi_d$ lies on the edge and therefore satisfies (B.55). For simplicity, let us restrict our focus on the case with $0 \leq \mu \leq 1$.

The mechanism through which the inflation-sensitive tax rule prevents the ELT equilibrium can be summarized as follows. The existence of the ELT equilibrium requires both inflation and output to fall simultaneously. However, the proposed tax rule counteracts the decline in output when the inflation rate declines by inducing the household to increase its labor supply. If the tax rule drives the household to supply more labor, ceteris paribus, firms are operating at too low a marginal revenue product of their labor input, and the monopolistic competitor reacts by raising prices. Therefore, the private agents' belief that a decline in inflation and output occurs without any changes in the fundamentals becomes inconsistent under the fiscal authority's commitment.

B.4 Endogenous government spending

In the benchmark case, the labor income tax rate was the only tax instrument available for the fiscal authority. In such a case, whether government spending g_t increases or decreases according to changes in the inflation rate was determined by λ_w.

However, when the fiscal policy manipulates more than two different tax rates, whether g_t increases or not depends on the combination of the tax response parameters. This subsection investigates how the government spending g_t is affected by the choice of tax response parameters.

B.4.1 Increasing government spending when inflation becomes lower

Let us assume that the fiscal authority adjusts both λ_w and λ_d as its policy instrument and chooses μ that satisfies $\mu\Psi_w + (1-\mu)\Psi_d$. It is not obvious whether government spending increases or decreases in response to a decline in inflation since spending is determined endogenously.

On the one hand, the more the fiscal authority relies on the use of the labor income tax rate (higher μ) to prevent the ELT equilibrium, the more government spending is likely to decline due to the reduction in tax revenue. On the other hand, relying more on the dividend tax rate (lower μ) increases government spending as inflation declines. Therefore, the fiscal authority can control the variation in government spending by combining the labor income tax and the dividend tax.

Log-linearizing the government budget constraint (B.24) around the deterministic steady state and substituting out the rest of the endogenous variables, we obtain the following representation:

$$\hat{g}_t = \Gamma_\pi \hat{\pi}_t + \Gamma_y \hat{y}_t, \tag{B.56}$$

$$\text{where } \Gamma_\pi \equiv \frac{\left(\frac{\tau_w w_{TSS} l_{TSS}}{g_{TSS}} - \frac{1-2w_{TSS}}{1-w_{TSS}}\frac{\tau_w}{1-\tau_w}\right)\lambda_w + \frac{\tau_d d_{TSS}}{g_{TSS}}\lambda_d}{1 - \frac{1-2w_{TSS}}{1-w_{TSS}}\sigma\frac{g_{TSS}}{c_{TSS}}},$$

$$\Gamma_y \equiv \frac{2 + \frac{1-2w_{TSS}}{1-w_{TSS}}\left(\eta - \sigma\frac{y_{TSS}}{c_{TSS}}\right)}{1 - \frac{1-2w_{TSS}}{1-w_{TSS}}\sigma\frac{g_{TSS}}{c_{TSS}}}.$$

Γ_y is positive under standard calibration, which implies that controlling for $\hat{\pi}_t$, $\hat{g}_t$ increases as $\hat{y}_t$ increases. By imposing the restriction $\Gamma_\pi < 0$, the fiscal authority can ensure that $\hat{g}_t$ increases as $\hat{\pi}_t$ declines, controlling for $\hat{y}_t$.

Although one of the main findings of this study is that we can prevent ELT without increasing $\hat{g}_t$, the above restriction may be desirable when the fiscal authority prefers to avoid a decrease in government spending when the economy is experiencing deflation. For the government spending to be decreasing in inflation, fiscal authority is required to put a larger weight on the dividend tax than the labor income tax.

B.4.2 Increasing output in the crisis state

In the main article, we confirmed that fiscal authority cannot improve both output and inflation in the crisis state if it adjusts only one tax instrument. Here we derive the following condition under which both inflation and output improve in the crisis state by allowing the fiscal authority to adjust more than two tax instruments.

Proposition. *Output in the crisis state is higher than where the tax rates do not respond to inflation if the fiscal authority chooses λ_w and λ_d to satisfy the following condition:*

$$\lambda_d < -\frac{\Psi^F\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right) + (1-p_L^*\beta)\left(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\right)}{\Psi^F\sigma\frac{\gamma_{\tau,d}}{\gamma_c} + (1-p_L^*\beta)\eta\frac{\gamma_{\tau,d}}{\gamma_y}}\lambda_w, \tag{B.57}$$

$$\text{where} \quad \Psi^F \equiv (1-\beta p_L^*) - \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}. \tag{B.58}$$

Proof. Output in the crisis state is higher compared to the case where the tax rates do not respond to inflation ($\Lambda = 0$) if the following inequality holds:

$$\hat{y}_L > \hat{y}_L^{NI}. \tag{B.59}$$

Since we have restricted our focus on the case where the ZLB binds in the crisis state ($\Psi^F > 0$), $\Lambda < \Psi^F$ holds from $\Psi < 0 < \Psi^F$. We can modify the condition as

$$\begin{aligned}
\frac{1-p_L^*\beta+\zeta}{\kappa} \times \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} &> \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} \times \frac{1-p_L^*\beta}{\kappa} \\
\Leftrightarrow \frac{1-p_L^*\beta+\zeta}{\Lambda-\Psi^F} &> -\frac{1-p_L^*\beta}{\Psi^F} \\
\Leftrightarrow \zeta\Psi^F &< -\Lambda(1-p_L^*\beta).
\end{aligned} \tag{B.60}$$

The inequality can be arranged as

$$\lambda_d < -\frac{\Psi^F\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right) + (1-p_L^*\beta)\left(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\right)}{\Psi^F\sigma\frac{\gamma_{\tau,d}}{\gamma_c} + (1-p_L^*\beta)\eta\frac{\gamma_{\tau,d}}{\gamma_y}}\lambda_w. \tag{B.61}$$

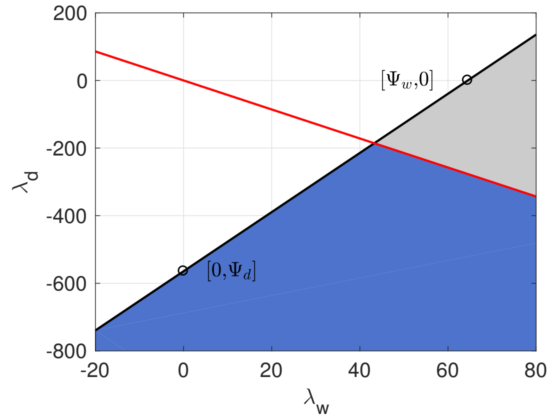


Figure 2: Parameter space where the output is higher in the crisis state with policy intervention.

Note: The blue area shows the parameter space where output is higher in the crisis state compared to where the tax rates do not respond to inflation at all.

Figure 2 depicts the region where the proposed tax rule mitigates the decline in output in the crisis state. The result shows that adjusting both the labor income tax and the dividend tax is desirable when the economy suffers from real interest rate shocks.

C Models with Alternative Fiscal Policies

In the baseline model, we assumed that the government debt outstanding is always set equal to zero. In this Appendix, we investigate two alternative cases: the case with lump-sum transfer and the case with government debt targeting.

C.1 A model with lump-sum transfer

This subsection shows that introducing an inflation-sensitive tax rule prevents the ELT equilibrium when the lump-sum transfer is available. We assume that government spending is set proportional to the output.

Let us consider a case where the lump-sum transfer is used to balance the budget. Household's budget constraint is

$$(1+\tau_{c,t})c_t + \frac{b_t}{R_t} = (1-\tau_{w,t})w_t l_t + \frac{b_{t-1}}{\Pi_t} + (1-\tau_{d,t})d_t - \tau_t. \tag{C.1}$$

Assume that the government spending is set proportional to the output:

$$g_t = s_g y_t, \tag{C.2}$$

$$g_t + \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + \tau_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t. \tag{C.3}$$

Although Ricardian equivalence did not hold in the baseline model of the main article, it holds with the lump-sum transfer and we can set $b_t = 0$ without loss of generality. The government budget simplifies to

$$s_g y_t = \tau_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t. \tag{C.4}$$

In this case, the lump-sum transfer instead of the government spending is determined endogenously. Equilibrium conditions can be summarized as

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{C.5}$$

$$\begin{aligned} &\frac{1-\tau_{d,t}}{1+\tau_{c,t}}\Big[\psi(\Pi_t-1)\Pi_t - \theta\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta}+\theta-1\Big] \\ &\qquad = \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1-\tau_{d,t+1}}{1+\tau_{c,t+1}}\frac{y_{t+1}}{y_t}\psi(\Pi_{t+1}-1)\Pi_{t+1}\Big], \end{aligned} \tag{C.6}$$

$$c_t = \Big(1-s_g-\frac{\psi}{2}(\Pi_t-1)^2\Big)y_t, \tag{C.7}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{C.8}$$

$$\tau_{w,t} = \tau_w\Pi_t^{\lambda_w}, \quad \tau_{c,t} = \tau_c\Pi_t^{\lambda_c}, \quad \tau_{d,t} = \tau_d\Pi_t^{\lambda_d}. \tag{C.9}$$

By log-linearizing these equations around the TSS, we obtain the following equilibrium conditions:

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}(\hat{\tau}_{c,t}-\mathbb{E}_t\hat{\tau}_{c,t+1}), \tag{C.10}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t + \frac{\theta-1}{\psi}\frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{C.11}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{C.12}$$

$$\hat{y}_t = \hat{c}_t, \tag{C.13}$$

$$\hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \tag{C.14}$$

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t. \tag{C.15}$$

Above equations are identical to (B.42) – (B.46) with parameters set to the following values:

$$\gamma_y = 1, \gamma_c = 1, \gamma_{\tau,w} = \gamma_{\tau,c} = \gamma_{\tau,d} = 0. \tag{C.16}$$

Therefore, all propositions established in the main article hold in the model discussed in this Appendix by replacing ξ, ζ and Λ to appropriate values. Note that $\gamma_{\tau,d} = 0$ indicates that the dividend tax does not affect the equilibrium outcome. This is because changes in the dividend tax do not affect the marginal behavior of the representative household as long as the aggregate demand is kept constant by the use of the lump-sum transfer.

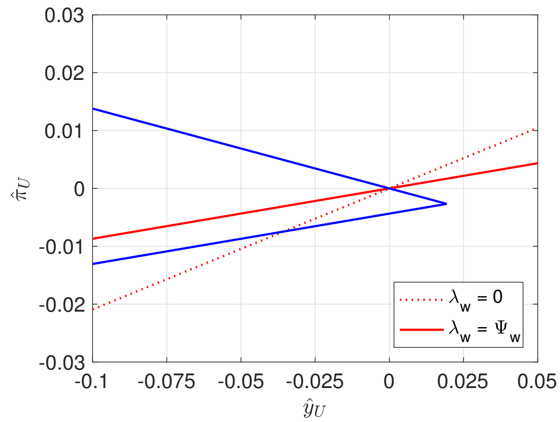


Figure 3: Euler equation and Philips curve with lump-sum transfer.

Figure 3 shows the case where the tax response parameter is set to $\lambda_w = \Psi_w$ and others to zero. We can observe that only the supply side is affected by the introduction of the tax rule since changes in the demand are isolated by the lump-sum transfer.

The case where the government spending is always zero ($g_t = 0$) and the variation in distortionary taxes is fully funded by the lump-sum transfer can be obtained by setting $s_g = 0$. Log-linearized equations (C.10) – (C.15) are not affected by the choice of s_g, therefore the results remain unchanged if we assume balanced government spending.

C.2 A model with endogenous government debt

We assumed that the fiscal authority runs a balanced budget in the baseline model and keeps government debt to zero at all periods. A natural question that arises here is whether the results would be affected if we relax the balanced budget assumption and allow the government debt to vary over time.

To address this question, we assume that the government spending is determined by the following government debt targeting rule:

$$g_t = s_g y_t \left(\frac{b_{t-1}}{b_{target}} \right)^{\phi_b}. \tag{C.17}$$

The parameter s_g determines the ratio of government spending to output. The fiscal authority sets the target level of government debt b_{target} equal to the steady state level of government debt b_{TSS}, which is determined by s_g and τ_w as well as other parameters.

Let us assume that the lump-sum transfer is not available. As shown in the seminal paper of Leeper (1991), the parameter ϕ_b must be appropriately selected for a unique equilibrium to exist. More concretely, the government spending rule must be designed so that government debt does not follow an explosive path.

Let us further assume that $\phi_b < 0$ is satisfied. Then, for a fixed level of output y_t, the fiscal authority reduces government expenditure g_t when the government debt level b_{t-1} is higher than its target b_{target}.

In the rest of the analysis, the parameter is set to $\phi_b = -1$. For simplicity, we assume that the fiscal authority adjusts only the labor income tax rate and the steady state tax rate is set to $\tau_w = 0.2$, $\tau_d = 0$, and $\tau_c = 0$. In this case, equilibrium conditions consist of the following equations:

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t \Big[c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \Big], \tag{C.18}$$

$$\psi(\Pi_t - 1)\Pi_t - \frac{\theta c_t^{\sigma} y_t^{\eta}}{1-\tau_{w,t}} + \theta - 1 = \beta \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \Big], \tag{C.19}$$

$$y_t = c_t + s_g y_t \Big(\frac{b_{t-1}}{b_{target}} \Big)^{\phi_b} + \frac{\psi}{2}(\Pi_t - 1)^2 y_t, \tag{C.20}$$

$$\frac{b_t}{R_t} + \frac{\tau_{w,t}}{1-\tau_{w,t}} c_t^{\sigma} y_t^{\eta+1} = \frac{b_{t-1}}{\Pi_t} + s_g y_t \Big(\frac{b_{t-1}}{b_{target}} \Big)^{\phi_b}, \tag{C.21}$$

$$R_t = \max \Big[1, \frac{1}{\beta} \Pi_t^{\phi_\pi} \Big]. \tag{C.22}$$

Equation (C.20) and (C.21) represent the aggregate resource constraint and the government budget constraint respectively. The steady state values can be calculated as

$$y_{TSS} = \Big[\frac{\theta - 1}{\theta}(1-\tau_w) \Big]^{\frac{1}{\eta+\sigma}} (1 - s_g)^{-\frac{\sigma}{\eta+\sigma}}, \tag{C.23}$$

$$c_{TSS} = \Big[\frac{\theta - 1}{\theta}(1-\tau_w) \Big]^{\frac{1}{\eta+\sigma}} (1 - s_g)^{\frac{\eta}{\eta+\sigma}}. \tag{C.24}$$

The steady state value of the government debt can be derived from the government budget constraint as

$$b_{TSS} = \frac{1}{1-\beta} \Big[\tau_w \frac{\theta - 1}{\theta} - s_g \Big] y_{TSS}. \tag{C.25}$$

The steady state amount of government debt outstanding is positive only if the right-hand side of equation (C.25) is positive. In the remaining analysis, the spending ratio is calibrated to $s_g = 0.16$, which yields the debt-to-output ratio of $b_{TSS}/y_{TSS} = 1.67$.

The log-linearized equilibrium conditions consist of following equations:

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}), \tag{C.26}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{C.27}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{C.28}$$

$$\hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \tag{C.29}$$

$$\hat{c}_t = \hat{y}_t - \frac{s_g}{1-s_g}\phi_b\hat{b}_{t-1}, \tag{C.30}$$

$$\beta\gamma_b\hat{b}_t = \gamma_{y,b}\hat{y}_t - \sigma\frac{\theta-1}{\theta}\tau_w\hat{c}_t - \gamma_b\hat{\pi}_t + (\gamma_b + s_g\phi_b)\hat{b}_{t-1} + \beta\gamma_b\hat{i}_t - \gamma_{\tau,w}\hat{\tau}_t, \tag{C.31}$$

where $\gamma_b \equiv \frac{b_{TSS}}{y_{TSS}}, \quad \gamma_{y,b} \equiv s_g - (\eta+1)\frac{\theta-1}{\theta}\tau_w.$

After substitution, equilibrium conditions simplify to the following EE and PC with three variables $\hat{\pi}_t$, $\hat{y}_t$ and $\hat{b}_{t-1}$:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) + \frac{s_g}{1-s_g}\phi_b(\hat{b}_{t-1} - \hat{b}_t), \tag{C.32}$$

$$\Big(1 + \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\lambda_w\Big)\hat{\pi}_t = (\sigma+\eta)\frac{\theta-1}{\psi}\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} - \sigma\frac{\theta-1}{\psi}\frac{s_g}{1-s_g}\phi_b\hat{b}_{t-1}, \tag{C.33}$$

$$\begin{aligned}\beta\gamma_b\hat{b}_t = \Big(\gamma_{y,b} - \sigma\frac{\theta-1}{\theta}\tau_w\Big)\hat{y}_t + \beta\gamma_b\max[\log\beta, \phi_\pi\hat{\pi}_t] - (\gamma_b + \gamma_{\tau,w}\lambda_w)\hat{\pi}_t \\ +\Big[\sigma\frac{\theta-1}{\theta}\tau_w\frac{s_g}{1-s_g}\phi_b + (\gamma_b + s_g\phi_b)\Big]\hat{b}_{t-1}.\end{aligned} \tag{C.34}$$

Let us focus on the case where the economy fluctuates around the TSS. The ZLB does not bind and monetary policy is active around the TSS. In this case, the equilibrium conditions can be expressed in the following state space representation:

$$B\begin{bmatrix}\hat{b}_t \\ \mathbb{E}_t\hat{y}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1}\end{bmatrix} = C\begin{bmatrix}\hat{b}_{t-1} \\ \hat{y}_t \\ \hat{\pi}_t\end{bmatrix}, \tag{C.35}$$

where

$$B \equiv \begin{bmatrix} -\frac{s_g}{1-s_g}\phi_b & 1 & \frac{1}{\sigma} \\ 0 & 0 & \beta \\ \beta\gamma_b & 0 & 0 \end{bmatrix},$$

$$C \equiv \begin{bmatrix} -\frac{s_g}{1-s_g}\phi_b & 1 & \frac{1}{\sigma}\phi_\pi \\ \sigma\frac{\theta-1}{\psi}\frac{s_g}{1-s_g}\phi_b & -(\sigma+\eta)\frac{\theta-1}{\psi} & 1+\frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\lambda_w \\ \sigma\frac{\theta-1}{\theta}\tau_w\frac{s_g}{1-s_g}\phi_b+(\gamma_b+s_g\phi_b) & \gamma_{y,b}-\sigma\frac{\theta-1}{\theta}\tau_w & \beta\gamma_b\phi_\pi-(\gamma_b+\gamma_{\tau,w}\lambda_w) \end{bmatrix}.$$

Since there are two control variables ($\hat{y}_t$, $\hat{\pi}_t$) and one predetermined variable ($\hat{b}_{t-1}$), one of the eigenvalues of $C^{-1}B$ must lie outside the unit-circle and two of them within the unit-circle.

The solution of the linear system can be represented as

$$\hat{y}_t = a_1\hat{b}_{t-1}, \quad \hat{\pi}_t = a_2\hat{b}_{t-1}, \quad \hat{b}_t = a_3\hat{b}_{t-1}. \tag{C.36}$$

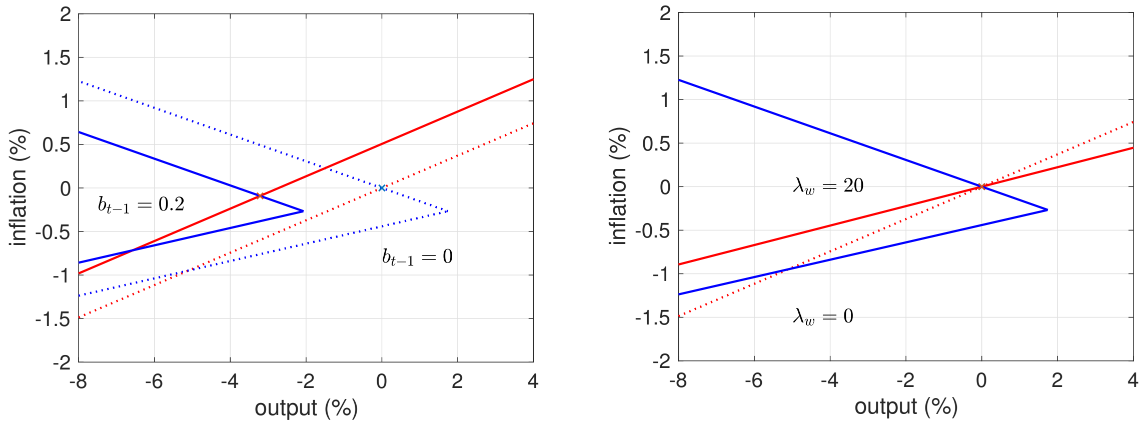
Assuming $\lambda_w = 0$, the solution of the linear system can be obtained numerically[1] as follows:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -0.169 \\ -0.005 \\ 0.910 \end{bmatrix}. \tag{C.37}$$

When there is no uncertainty, we can replace $\mathbb{E}_t\hat{y}_{t+1} = \hat{y}_{t+1} = a_1\hat{b}_t = a_1a_3\hat{b}_{t-1} = a_3\hat{y}_{t-1}$ and $\mathbb{E}_t\hat{\pi}_{t+1} = \hat{\pi}_{t+1} = a_2\hat{b}_t = a_2a_3\hat{b}_{t-1} = a_3\hat{\pi}_{t-1}$, which holds regardless of the value of b_{t-1}.

To investigate how demand and supply are affected by the amount of debt outstanding and the size of the tax response parameter, figure 4 shows the EE and PC under different values of b_{t-1} and λ_w. As shown in the left-hand figure, fiscal authority reduces government spending as b_{t-1} becomes larger, which shifts the EE downwards and the PC upwards. While the equilibrium inflation is little affected, the output is depressed when there is a positive amount of government debt since the fiscal authority

[1]The program `gensys.m` by Sims (2002) is used to compute the solution.



(a) Cases with different level of government debt outstanding.

(b) Cases with $\hat{b}_{t-1} = 0$ and different values of λ_w.

Figure 4: Euler equation and Philips curve with endogenous government debt around the TSS.

cuts government spending. The key feature here is that both EE and PC shift in a parallel manner, and the slope of these two equations are not affected by the level of $\hat{b}_{t-1}$.

The right-hand figure shows that changing λ_w only affects the supply side. This contrasts with the baseline model, where the demand curve was also affected by the level of λ_w. Since government spending is determined by equation (C.17), it is isolated from the tax revenue; a marginal change in the tax revenue does not affect government spending. Therefore, the demand curve remains unchanged. The key observation here is that when the government debt fluctuates over time, the fiscal authority can isolate the demand effects and the supply effects of the fiscal policy.

The above analysis has abstracted from the possibility of switching between the targeted regime and the unintended regime. Simple models that do not include predetermined variables (b_{t-1} in this case) are relatively straightforward to solve even with the regime-switching structure. However, once a predetermined variable enters the model, closed-form solutions are not available, and different algorithms are required to solve the model (see Farmer et al. (2009)). A comprehensive study on ELTs with government debt is left for future work.

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