Sectoral Labor Mobility and Optimal Monetary Policy Online Appendix*

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*The views expressed in this paper are those of the authors and do not necessarily represent those of the International Monetary Fund or IMF policy or Banca d'Italia.

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Appendix

A Symmetric equilibrium

A.1 Baseline model

$$
\begin{aligned}
X_t &= C_t^{1-\alpha} D_t^{\alpha} && \text{(A.1)}\\
U\left(X_t, N_t\right) &= \log\left(X_t\right) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} && \text{(A.2)}\\
U_{C,t} &= \frac{(1-\alpha)}{C_t} && \text{(A.3)}\\
U_{D,t} &= \frac{\alpha}{D_t} && \text{(A.4)}\\
\frac{C_t}{D_t} &= \frac{1-\alpha}{\alpha} Q_t && \text{(A.5)}\\
w_t^C &= -\frac{-\nu \left(\chi^C\right)^{-\dfrac{1}{\lambda}} \left(N_t^C\right)^{\dfrac{1}{\lambda}} N_t^{\varphi-\frac{1}{\lambda}}}{U_{C,t}}, && \text{(A.6)}\\
w_t^D &= -\frac{-\nu \left(1-\chi^C\right)^{-\dfrac{1}{\lambda}} \left(N_t^D\right)^{\dfrac{1}{\lambda}} N_t^{\varphi-\frac{1}{\lambda}}}{U_{C,t}}. && \text{(A.7)}\\
N_t &= \left[\left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^C\right)^{\frac{1+\lambda}{\lambda}} + \left(1-\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^D\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}} && \text{(A.8)}\\
\Lambda_{t,t+1} &\equiv \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{e_{t+1}^B}{e_t^B} && \text{(A.9)}\\
1 &= E_t\left[\Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}^C}\right] && \text{(A.10)}\\
\Pi_t^D &= \Pi_t^C \frac{Q_t}{Q_{t-1}} && \text{(A.11)}\\
Y_t^C &= e_t^A e_t^{A,C} N_t^C && \text{(A.12)}\\
Y_t^D &= e_t^A e_t^{A,D} N_t^D && \text{(A.13)}\\
\epsilon_c MC_t^C &= \left(\epsilon_c - 1\right) + \vartheta_c \left(\Pi_t^C - \Pi^C\right) \Pi_t^C - \\
&- \vartheta_c E_t\left[\Lambda_{t,t+1} \frac{Y_{t+1}^C}{Y_t^C} \left(\Pi_{t+1}^C - \Pi^C\right) \Pi_{t+1}^C\right] && \text{(A.14)}\\
MC_t^C &= \frac{w_t^C}{e_t^A e_t^{A,C}} && \text{(A.15)}
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_d MC_t^D &= (\epsilon_d - 1) + \vartheta_d \left(\Pi_t^D - \Pi^D\right) \Pi_t^D - \\
&- \vartheta_d E_t \left[\Lambda_{t,t+1} \frac{Q_{t+1}}{Q_t} \frac{Y_{t+1}^D}{Y_t^D} \left(\Pi_{t+1}^D - \Pi^D\right) \Pi_{t+1}^D\right] && \text{(A.16)} \\
MC_t^D &= \frac{w_t^D}{e_t^A e_t^{A,D} Q_t} && \text{(A.17)} \\
\widetilde{\Pi}_t &= \left(\Pi_t^C\right)^{1-\tau} \left(\Pi_t^D\right)^{\tau} && \text{(A.18)} \\
\log\left(\frac{R_t}{\bar{R}}\right) &= \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \alpha_\pi \log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right) + \alpha_y \log\left(\frac{Y_t}{Y_t^f}\right) + \\
&+ \alpha_{\Delta y} \left[\log\left(\frac{Y_t}{Y_t^f}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-1}^f}\right)\right], && \text{(A.19)} \\
Y_t^C &= C_t + \frac{\vartheta_c}{2} \left(\Pi_t^C - \Pi^C\right)^2 Y_t^C && \text{(A.20)} \\
Y_t^D &= D_t + \frac{\vartheta_d}{2} \left(\Pi_t^D - \Pi^D\right)^2 Y_t^D && \text{(A.21)} \\
Y_t &= Y_t^C + Q_t Y_t^D && \text{(A.22)}
\end{aligned}
$$

A.2 Two-sector model with durable goods

The symmetric equilibrium changes as follows. Durable goods follow the low of motion

$$
D_{t+1} = (1 - \delta) D_t + I_t^D. \tag{A.23}
$$

Equation (A.5) now reads as

$$
Q_t = \frac{U_{D,t}}{U_{C,t}} + (1 - \delta) E_t \left[\Lambda_{t,t+1} Q_{t+1}\right]. \tag{A.24}
$$

Finally, the market clearing condition (A.21) in sector D becomes

$$
Y_t^D = I_t^D + \frac{\vartheta_d}{2} \left(\Pi_t^D - \Pi^D\right)^2 Y_t^D. \tag{A.25}
$$

A.3 Fully-fledged two-sector model

$$
\begin{aligned}
X_t &= C_t^{1-\alpha} D_t^{\alpha} && \text{(A.26)}\\
C_t &= Z_t - \zeta S_{t-1} && \text{(A.27)}\\
S_t &= \rho_c S_{t-1} + (1-\rho_c) Z_t && \text{(A.28)}\\
U\left(X_t, N_t\right) &= \log\left(X_t\right) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} && \text{(A.29)}\\
U_{C,t} &= \frac{(1-\alpha)}{C_t} && \text{(A.30)}\\
U_{D,t} &= \frac{\alpha}{D_t} && \text{(A.31)}\\
U_{N^C,it} &= -\nu \left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^C\right)^{\frac{1}{\lambda}} \left[\left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^C\right)^{\frac{1+\lambda}{\lambda}} + \left(1-\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^D\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda\varphi-1}{1+\lambda}} && \text{(A.32)}\\
U_{N^D,it} &= -\nu \left(1-\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^D\right)^{\frac{1}{\lambda}} \left[\left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^C\right)^{\frac{1+\lambda}{\lambda}} + \left(1-\chi^C\right)^{-\frac{1}{\lambda}} \left(N_{i,t}^D\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda\varphi-1}{1+\lambda}} && \text{(A.33)}\\
N_t &= \left[\left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^C\right)^{\frac{1+\lambda}{\lambda}} + \left(1-\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^D\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}} && \text{(A.34)}\\
w_t^C &= -\frac{-\nu \left(\chi^C\right)^{-\dfrac{1}{\lambda}} \left(N_t^C\right)^{\dfrac{1}{\lambda}} N_t^{\varphi-\frac{1}{\lambda}}}{U_{C,t}}, && \text{(A.35)}\\
w_t^D &= -\frac{-\nu \left(1-\chi^C\right)^{-\dfrac{1}{\lambda}} \left(N_t^D\right)^{\dfrac{1}{\lambda}} N_t^{\varphi-\frac{1}{\lambda}}}{U_{C,t}}. && \text{(A.36)}\\
\Lambda_{t,t+1} &\equiv \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{e_{t+1}^B}{e_t^B} && \text{(A.37)}\\
0 &= \left[1 - e_t^{w,C}\eta\right] + \frac{e_t^{w,C}\eta}{\tilde{\mu}_t{}^C} - \vartheta_C^w \left(\Pi_t^{w,C} - \Pi^C\right) \Pi_t^{w,C} + \\
&+ E_t \left[\Lambda_{t,t+1} \vartheta_C^w \left(\Pi_{t+1}^{w,C} - \Pi^C\right) \Pi_{t+1}^{w,C} \frac{w_{t+1}^C N_{t+1}^C}{w_t^C N_t^C}\right] && \text{(A.38)}\\
\tilde{\mu}_t{}^C &= -\frac{U_{C,t}}{U_{N,t}^C} w_t^C && \text{(A.39)}\\
0 &= \left[1 - e_t^{w,D}\eta\right] + \frac{e_t^{w,D}\eta}{\tilde{\mu}_t{}^D} - \vartheta_D^w \left(\Pi_t^{w,D} - \Pi^C\right) \Pi_t^{w,D} + \\
&+ E_t \left[\Lambda_{t,t+1} \vartheta_D^w \left(\Pi_{t+1}^{w,D} - \Pi^C\right) \Pi_{t+1}^{w,D} \frac{w_{t+1}^D N_{t+1}^D}{w_t^D N_t^D}\right] && \text{(A.40)}\\
\tilde{\mu}_t{}^D &= -\frac{U_{C,t}}{U_{N,t}^D} w_t^D && \text{(A.41)}\\
Q_t \psi_t &= \frac{U_{D,t}}{U_{Z,t}} + (1-\delta) E_t \left[\Lambda_{t,t+1} Q_{t+1} \psi_{t+1}\right] && \text{(A.42)}
\end{aligned}
$$

$$
\begin{aligned}
1 &= E_t\left\{\Lambda_{t,t+1}\psi_{t+1}\frac{Q_{t+1}}{Q_t}e^I_{t+1}\left[S^{'}\left(\frac{I^D_{t+1}}{I^D_t}\right)\left(\frac{I^D_{t+1}}{I^D_t}\right)^2\right]\right\}+\\
&+ \psi_t e^I_t\left[1-S\left(\frac{I^D_t}{I^D_{t-1}}\right)-S^{'}\left(\frac{I^D_t}{I^D_{t-1}}\right)\frac{I^D_t}{I^D_{t-1}}\right] && \text{(A.43)}\\
S\left(\frac{I^D_t}{I^D_{t-1}}\right) &= \frac{\phi}{2}\left(\frac{I^D_t}{I^D_{t-1}}-1\right)^2 && \text{(A.44)}\\
S^{'}\left(\frac{I^D_t}{I^D_{t-1}}\right) &= \phi\left(\frac{I^D_t}{I^D_{t-1}}-1\right) && \text{(A.45)}\\
1 &= E_t\left[\Lambda_{t,t+1}\frac{R_t}{\Pi^C_{t+1}}\right] && \text{(A.46)}\\
\Pi^D_t &= \Pi^C_t\frac{Q_t}{Q_{t-1}} && \text{(A.47)}\\
Y^C_t &= e^A_t N^C_t && \text{(A.48)}\\
Y^D_t &= e^A_t N^D_t && \text{(A.49)}\\
e^C_t\epsilon_c MC^C_t &= \left(e^C_t\epsilon_c-1\right)+\vartheta_c\left(\Pi^C_t-\Pi^C\right)\Pi^C_t-\\
&- \vartheta_c E_t\left[\Lambda_{t,t+1}\frac{Y^C_{t+1}}{Y^C_t}\left(\Pi^C_{t+1}-\Pi^C\right)\Pi^C_{t+1}\right] && \text{(A.50)}\\
MC^C_t &= \frac{w^C_t}{e^A_t} && \text{(A.51)}\\
e^D_t\epsilon_d MC^D_t &= \left(e^D_t\epsilon_d-1\right)+\vartheta_d\left(\Pi^D_t-\Pi^D\right)\Pi^D_t-\\
&- \vartheta_d E_t\left[\Lambda_{t,t+1}\frac{Q_{t+1}}{Q_t}\frac{Y^D_{t+1}}{Y^D_t}\left(\Pi^D_{t+1}-\Pi^D\right)\Pi^D_{t+1}\right] && \text{(A.52)}\\
MC^D_t &= \frac{w^D_t}{e^A_t Q_t} && \text{(A.53)}\\
\widetilde{\Pi}_t &= \left(\Pi^C_t\right)^{1-\tau}\left(\Pi^D_t\right)^{\tau} && \text{(A.54)}\\
\log\left(\frac{R_t}{\bar{R}}\right) &= \rho_r\log\left(\frac{R_{t-1}}{\bar{R}}\right)+\alpha_\pi\log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right)+\alpha_y\log\left(\frac{Y_t}{Y^f_t}\right)+\\
&+ \alpha_{\Delta y}\left[\log\left(\frac{Y_t}{Y^f_t}\right)-\log\left(\frac{Y_{t-1}}{Y^f_{t-1}}\right)\right], && \text{(A.55)}\\
Y^C_t &= C_t+e^G_t+\frac{\vartheta_c}{2}\left(\Pi^C_t-\Pi^C\right)^2 Y^C_t+\frac{\vartheta^w_c}{2}\left(\frac{w^C_{i,t}}{w^C_{i,t-1}}\Pi^C_t-\Pi^C\right)^2 w^C_t N^C_t && \text{(A.56)}\\
Y^D_t &= I^D_t+\frac{\vartheta_d}{2}\left(\Pi^D_t-\Pi^D\right)^2 Y^D_t+\frac{\vartheta^w_d}{2}\left(\frac{w^D_{i,t}}{w^D_{i,t-1}}\Pi^C_t-\Pi^C\right)^2 w^D_t N^D_t && \text{(A.57)}\\
Y_t &= Y^C_t+Q_tY^D_t && \text{(A.58)}
\end{aligned}
$$

B Ramsey problem

In this section we outline the Ramsey problem in the stylized model of Section 2. For illustrative purposes, we focus on the simpler version of the model, e.g. without durable goods. The symmetric equilibrium of the model is reported in Appendix A.1. The social planner maximizes the present value of households' utility subject to the equilibrium conditions of the model, but does not have to follow an interest rate rule. We now report the Lagrangian function of the optimization problem (B.1), the first-order conditions (B.2) and the steady state procedure (B.3).[1]

B.1 Lagrangian function

$$
\mathcal{L}_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{
\begin{array}{lc}
 & e_t^B \log(X_t) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} - w_r (R_t - R)^2 \\
+\lambda_{1,t} & \left[X_t - C_t^{1-\alpha} D_t^{\alpha}\right] \\
+\lambda_{2,t} & \left[U_{C,t} - \frac{(1-\alpha)}{C_t}\right] \\
+\lambda_{3,t} & \left[U_{D,t} - \frac{\alpha}{D_t}\right] \\
+\lambda_{4,t} & \left[\frac{C_t}{D_t} - \frac{1-\alpha}{\alpha} Q_t\right] \\
+\lambda_{5,t} & \left[w_t^C + \frac{U_{N,t}}{U_{C,t}}\right] \\
+\lambda_{6,t} & \left[U_{N,t} + \nu \left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^C\right)^{\frac{1}{\lambda}} N_t^{\varphi - \frac{1}{\lambda}}\right] \\
+\lambda_{7,t} & \left[\frac{w_t^C}{w_t^D} - \left(\frac{\chi^C}{1-\chi^C}\right)^{-\frac{1}{\lambda}} \left(\frac{N_t^C}{N_t^D}\right)^{\frac{1}{\lambda}}\right] \\
+\lambda_{8,t} & \left[\Lambda_{t,t+1} - \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{e_{t+1}^B}{e_t^B}\right] \\
+\lambda_{9,t} & \left[1 - \Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}^C}\right] \\
+\lambda_{10,t} & \left[\Pi_t^D - \Pi_t^C \frac{Q_t}{Q_{t-1}}\right] \\
+\lambda_{11,t} & \left[R_t^{\text{real}} - \frac{R_t}{\Pi_{t+1}^C}\right] \\
+\lambda_{12,t} & \left[Y_t^C - e_t^A e_t^{A,C} N_t^C\right] \\
+\lambda_{13,t} & \left[Y_t^D - e_t^A e_t^{A,D} N_t^D\right] \\
+\lambda_{14,t} & \Big[\epsilon_c MC_t^C - (\epsilon_c - 1) - \vartheta_c \left(\Pi_t^C - \Pi^C\right) \Pi_t^C + \vartheta_c E_t \Big[\Lambda_{t,t+1} \frac{Y_{t+1}^C}{Y_t^C} \left(\Pi_{t+1}^C - \Pi^C\right) \Pi_{t+1}^C\Big] \\
+\lambda_{15,t} & \Big[\epsilon_d MC_t^D - (\epsilon_d - 1) - \vartheta_d \left(\Pi_t^D - \Pi^D\right) \Pi_t^D + \vartheta_d E_t \Big[\Lambda_{t,t+1} \frac{Q_{t+1}}{Q_t} \frac{Y_{t+1}^D}{Y_t^D} \left(\Pi_{t+1}^D - \Pi^D\right) \Pi_{t+1}^D\Big]\Big] \\
+\lambda_{16,t} & \left[MC_t^C - \frac{w_t^C}{e_t^A e_t^{A,C}}\right] \\
+\lambda_{17,t} & \left[MC_t^D - \frac{w_t^D}{e_t^A Q_t}\right] \\
+\lambda_{18,t} & \left[Y_t^C - C_t - \frac{\vartheta_c}{2} \left(\Pi_t^C - \Pi^C\right)^2 Y_t^C\right] \\
+\lambda_{19,t} & \left[Y_t^D - D_t - \frac{\vartheta_d}{2} \left(\Pi_t^D - \Pi^D\right)^2 Y_t^D\right] \\
+\lambda_{20,t} & \left[Y_t - Y_t^C - Q_t Y_t^D\right] \\
+\lambda_{21,t} & \left[N_t - \left[\left(\chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^C\right)^{\frac{1+\lambda}{\lambda}} + \left(1 - \chi^C\right)^{-\frac{1}{\lambda}} \left(N_t^D\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}}\right]
\end{array}
\right\}
$$

B.2 Ramsey planner's first-order conditions

Differentiating the Lagrangian function reported in Section B.1 with respect to all the endogenous variables $X_t, C_t, D_t, U_{C,t}, U_{D,t}, Q_t, U_{N,t}, W_t^C, W_t^D, N_t, \Lambda_{t,t+1}, R_t, R_t^{\text{real}}, \Pi_t^C, \Pi_t^D, Y_t^C, N_t^C, Y_t^D, N_t^D$,

[1] To derive the Ramsey first-order conditions we use the toolbox provided by Lopez-Salido and Levin (2004) and Levin et al. (2006).

MC_t^C,MC_t^D, Y_t and setting the first derivatives equal to zero yields the following first-order conditions:

$$0 = \lambda_{3,t}\frac{\alpha}{D_t^2} - \lambda_{4,t}\frac{C_t}{D_t^2} - \lambda_{19,t} - \lambda_{1,t}\alpha D_t^{\alpha-1}C_t^{1-\alpha} \tag{B.1}$$

$$0 = \lambda_{8,t} - \lambda_{9,t}R_t^{\text{real}} + \frac{\lambda_{14,t}\vartheta_c\left(\Pi_{t+1}^C - \Pi^C\right)\Pi_{t+1}^C Y_t^C}{Y_t^C} + \frac{\lambda_{15,t}\vartheta_d\left(\Pi_{t+1}^D - \Pi^C\right)\Pi_{t+1}^D Q_{t+1}Y_t^D}{Q_t Y_t^C} \tag{B.2}$$

$$0 = \lambda_{16,t} + \lambda_{14,t}\epsilon_c \tag{B.3}$$

$$0 = \lambda_{17,t} + \lambda_{15,t}\epsilon_d \tag{B.4}$$

$$0 = \lambda_{21,t} - \nu N_t^{\varphi} + \frac{\lambda_{6,t}\nu\left(\varphi - \frac{1}{\lambda}\right)\left(N_t^C\right)^{\frac{1}{\lambda}} N_t^{\varphi-\frac{1}{\lambda}-1}}{\left(\chi^C\right)^{\frac{1}{\lambda}}} \tag{B.5}$$

$$\begin{aligned}0 = &\frac{\lambda_{6,t}\nu\left(N_t^C\right)^{\frac{1}{\lambda}-1} N_t^{\varphi-\frac{1}{\lambda}}}{\lambda\left(\chi^C\right)^{\frac{1}{\lambda}}} - \frac{\lambda_{21,t}\left(N_t^C\right)^{\frac{1}{\lambda}}\left(\frac{\left(N_t^C\right)^{\frac{1+\lambda}{\lambda}}}{\left(\chi^C\right)^{\frac{1}{\lambda}}} + \frac{\left(N_t^D\right)^{\frac{1+\lambda}{\lambda}}}{\left(1-\chi^C\right)^{\frac{1}{\lambda}}}\right)^{-\frac{1}{1+\lambda}}}{\left(\chi^C\right)^{\frac{1}{\lambda}}} - \\ &- \lambda_{12,t}e_t^A e_t^{A,C} + \frac{\lambda_{7,t}N_t^D\left(\frac{N_t^D}{N_t^C}\right)^{\frac{1}{\lambda}-1}}{\lambda\left(N_t^C\right)^2\left(-\frac{\chi^C-1}{\chi^C}\right)^{\frac{1}{\lambda}}}\end{aligned} \tag{B.6}$$

$$0 = -\lambda_{13,t}e_t^A e_t^{A,D} - \frac{\lambda_{21,t}\left(N_t^D\right)^{\frac{1}{\lambda}}\left(\frac{\left(N_t^C\right)^{\frac{1+\lambda}{\lambda}}}{\left(\chi^C\right)^{\frac{1}{\lambda}}} + \frac{\left(N_t^D\right)^{\frac{1+\lambda}{\lambda}}}{\left(1-\chi^C\right)^{\frac{1}{\lambda}}}\right)^{-\frac{1}{1+\lambda}}}{\left(1-\chi^C\right)^{\frac{1}{\lambda}}} - \lambda_{7,t}\frac{\lambda_{7,t}\left(\frac{N_t^D}{N_t^C}\right)^{\frac{1}{\lambda}-1}}{\lambda N_t^C\left(-\frac{\chi^C-1}{\chi^C}\right)^{\frac{1}{\lambda}}} \tag{B.7}$$

$$\begin{aligned}0 = &\frac{\lambda_{14,t-1}\left[\frac{\vartheta_c\Lambda_{t-1,t}\Pi_t^C Y_t^C}{Y_{t-1}^C} + \frac{\vartheta_c\Lambda_{t-1,t}\left(\Pi_t^C-\Pi^C\right)Y_t^C}{Y_{t-1}^C}\right]}{\beta} - \lambda_{14,t}\left(\vartheta_c\Pi_t^C + \vartheta_c\left(\Pi_t^C - \Pi^C\right)\right) - \\ &- \lambda_{10,t}\frac{Q_t}{Q_{t-1}} - \lambda_{18,t}\vartheta_c\left(\Pi_t^C - \Pi^C\right)Y_t^C + \frac{\lambda_{11,t-1}R_{t-1}}{\beta\left(\Pi_t^C\right)^2}\end{aligned} \tag{B.8}$$

$$\begin{aligned}0 = &\lambda_{10,t} - \lambda_{15,t}\left(\vartheta_d\Pi_t^D + \vartheta_d\left(\Pi_t^D - \Pi^C\right)\right) - \frac{\lambda_{15,t-1}\left[\frac{\vartheta_d\Lambda_{t-1,t}\Pi_t^D Q_t Y_t^D}{Q_{t-1}Y_{t-1}^D} + \frac{\vartheta_d\Lambda_{t-1,t}\left(\Pi_t^D-\Pi^C\right)Y_t^D}{Q_{t-1}Y_{t-1}^D}\right]}{\beta} - \\ &- \lambda_{19,t}\left(\vartheta_d Y_t^D\left(\Pi_t^D - \Pi^C\right)\right)\end{aligned} \tag{B.9}$$

$$0 = \lambda_{4,t}\frac{\alpha-1}{\alpha} - \lambda_{10,t}\frac{\Pi_t^C}{Q_{t-1}} + \lambda_{17,t}\frac{w_t^D}{e_t^A e_t^{A,D}Q_t^2} + \lambda_{10,t}\frac{\beta\Pi_{t+1}^C Q_{t+1}}{Q_t^2} \tag{B.10}$$

$$0 = -\frac{\lambda_{11,t}}{\Pi^C_{t+1}} - 2w_r\left(R_t - R\right) \tag{B.11}$$

$$0 = \lambda_{11,t} - \lambda_{9,t}\Lambda_{t,t+1} \tag{B.12}$$

$$0 = \lambda_{3,t} \tag{B.13}$$

$$0 = \lambda_{6,t} + \frac{\lambda_{5,t}}{U_{C,t}} \tag{B.14}$$

$$0 = \lambda_{2,t} - \lambda_{5,t}\frac{U_{N,t}}{U^2_{C,t}} + \lambda_{8,t}\beta\frac{e^B_{t+1}}{e^B_t}\frac{U_{C,t+1}}{U^2_{C,t}} - \lambda_{8,t-1}\frac{e^B_t}{e^B_{t-1}U_{C,t-1}} \tag{B.15}$$

$$0 = \lambda_{5,t} - \frac{\lambda_{16,t}}{e^A_t e^{A,C}_t} - \lambda_{7,t}\frac{w^D_t}{\left(w^C_t\right)^2} \tag{B.16}$$

$$0 = \frac{\lambda_{7,t}}{w^C_t} - \frac{\lambda_{17,t}}{e^A_t e^{A,D}_t Q_t} \tag{B.17}$$

$$0 = \lambda_{1,t} + \frac{1}{X_t} \tag{B.18}$$

$$0 = \lambda_{20,t} \tag{B.19}$$

$$\begin{aligned} 0 = &\lambda_{12,t} - \lambda_{20,t} - \lambda_{18,t}\left[\frac{\vartheta_c\left(\Pi^C_t - \Pi^C\right)^2}{2} - 1\right] - \lambda_{14,t-1}\left[\frac{\vartheta_c\Lambda_{t,t+1}\left(\Pi^C_{t+1} - \Pi^C\right)\Pi^C_{t+1}Y^C_{t+1}}{\left(Y^C_t\right)^2}\right] + \\ &+ \frac{\lambda_{14,t-1}\left[\vartheta_c\Lambda_{t-1,t}\left(\Pi^C_t - \Pi^C\right)\Pi^C_t\right]}{\beta Y^C_{t-1}} \end{aligned} \tag{B.20}$$

$$\begin{aligned} 0 = &\lambda_{13,t} - \lambda_{20,t}Q - \lambda_{19,t}\left[\frac{\vartheta_d\left(\Pi^D_t - \Pi^C\right)^2}{2} - 1\right] - \lambda_{15,t-1}\left[\frac{\vartheta_d\Lambda_{t,t+1}\left(\Pi^D_{t+1} - \Pi^C\right)\Pi^D_{t+1}Q_{t+1}Y^D_{t+1}}{Q_t\left(Y^D_t\right)^2}\right] + \\ &+ \frac{\lambda_{15,t-1}\left[\vartheta_d\Lambda_{t-1,t}\left(\Pi^D_t - \Pi^C\right)\Pi^D_t Q_t\right]}{\beta Q_{t-1}Y^D_{t-1}} \end{aligned} \tag{B.21}$$

$$0 = \frac{\lambda_{4,t}}{D_t} - \lambda_{18,t} - \lambda_{2,t}\frac{\alpha - 1}{C^2_t} + \lambda_{1,t}\frac{(\alpha - 1)\,D^\alpha_t}{C^\alpha_t} \tag{B.22}$$

The Ramsey's first-order conditions together with the 21 equations characterizing the symmetric equilibrium reported in Section A.1 (excluding the Taylor rule, the inflation aggregator and the processes for the exogenous shocks) make a system of 43 dynamic equations in 43 unknowns (22 endogenous variables and 21 Lagrange multipliers). We approximate the solution to this system by using the Dynare solver that takes a second-order Taylor expansion around the Ramsey-optimal steady state, which we compute numerically as described in Section B.3.

B.3 Steady-state

The steady-state values of all endogenous variables and Lagrange multipliers in the Ramsey equilibrium are found simultaneously using a numerical procedure. In particular, the procedure is designed to choose the values of C and Π^C that simultaneously solve equations (A.22) and (B.11) evaluated at the steady state. The value of the remaining endogenous variables is found recursively by evaluating equations in Section A.1 at the steady state, while the steady state values of the 21 Lagrange multipliers of the Ramsey problem are found by solving the system of 21 equations (linear in the Lagrange multipliers) in 21 unknowns, obtained by evaluating equations in Section (B.2) at the steady state. Note that value of Π^C defines the optimal steady state inflation under the Ramsey policy.

C Robustness analysis in the stylized model

In Table C.1 we provide an analysis of robustness to different assumptions on the level and sectoral distribution of price stickiness, and to different degrees of labor mobility in the stylized model (building on cases *(ii)* and *(iii)* reported in Table 1). The inverse relationship between the optimal τ and λ continues to hold.

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	τ	$100 \times \omega$
(i) Heterogeneous price stickiness $\vartheta_c = 90, \vartheta_d = 30$						
∞	1.0000	0.0081	0.0216	0.0000	0.3143	0.0003
3	1.0000	0.0080	0.0227	0.0000	0.3295	0.0003
1	1.0000	0.0084	0.0210	0.0000	0.3847	0.0005
0.5	1.0000	0.0090	0.0211	0.0000	0.4402	0.0005
0.10	1.0000	0.0102	0.0202	0.0000	0.5779	0.0009
(ii) Heterogeneous price stickiness $\vartheta_c = 60, \vartheta_d = 0$						
∞	1.0000	0.0040	0.0215	0.0000	0.0000	0.0002
3	1.0000	0.0041	0.0217	0.0000	0.0225	0.0002
1	1.0000	0.0042	0.0221	0.0000	0.0373	0.0002
0.5	1.0000	0.0043	0.0225	0.0000	0.0514	0.0003
0.10	1.0000	0.0044	0.0231	0.0000	0.0709	0.0003
(iii) Heterogeneous price stickiness $\vartheta_c = 120, \vartheta_d = 0$						
∞	1.0000	0.0076	0.0197	0.0000	0.0000	0.0002
3	1.0000	0.0077	0.0198	0.0000	0.0000	0.0003
1	1.0000	0.0079	0.0202	0.0000	0.0184	0.0003
0.5	1.0000	0.0082	0.0206	0.0000	0.0375	0.0003
0.10	1.0000	0.0085	0.0213	0.0000	0.0710	0.0003

Table C.1: Optimized monetary policy rules: robustness in the stylized model

D Data

We define the durables sector as the a composite of durable goods and residential investments whereas the nondurables sector comprises nondurables goods and services.

Series	Definition	Source	Mnemonic
DUR^N	Nominal Durable Goods	BEA	Table 2.3.5 Line 3
RI^N	Nominal Residential Investment	BEA	Table 1.1.5 Line 13
ND^N	Nominal Nondurable Goods	BEA	Table 2.3.5 Line 8
S^N	Nominal Services	BEA	Table 2.3.5 Line 13
P_{DUR}	Price Deflator, Durable Goods	BEA	Table 1.1.9 Line 4
P_{RI}	Price Deflator, Residential Investment	BEA	Table 1.1.9 Line 13
P_{ND}	Price Deflator, Nondurable Goods	BEA	Table 1.1.9 Line 5
P_S	Price Deflator, Services	BEA	Table 1.1.9 Line 6
Y^N	Nominal GDP	BEA	Table 1.1.5 Line 1
P_Y	Price Deflator, GDP	BEA	Table 1.1.9 Line 1
FFR	Effective Federal Funds Rate	FRED	FEDFUNDS
N^C	Average Weekly Hours: Nondurable Goods and Services	FRED	CES3200000007-CES0800000007
N^D	Average Weekly Hours: Durable Goods and Construction	FRED	CES3100000007-CES2000000007
W^C	Average Hourly Earnings: Nondurable Goods and Services	FRED	CES3200000008-CES0800000008
W^D	Average Hourly Earnings: Durable Goods and Construction	FRED	CES3100000008-CES2000000008
POP	Civilian Non-institutional Population, over 16	FRED	CNP16OV
CE	Civilian Employment, 16 over	FRED	CE16OV

Table D.1: Data Sources

D.1 Durables and Residential Investments

1. Sum nominal series: $DUR^N + RI^N = DR^N$
2. Calculate sectoral weights of deflators: $\omega^D = \frac{DUR^N}{DR^N}$; $\omega^{RI} = \frac{RI^N}{DR^N}$
3. Calculate Deflator: $P_D = \omega^D P_{DUR} + \omega^{RI} P_{RI}$
4. Calculate Real Durable Consumption: $D = \frac{DUR^N + RI^N}{P_D}$

D.2 Nondurables and Services

1. Sum nominal series: $ND^N + S^N = NS^N$
2. Calculate sectoral weights of deflators: $\omega^{ND} = \frac{ND^N}{NS^N}$; $\omega^S = \frac{S^N}{NS^N}$
3. Calculate Deflator: $P_C = \omega^{ND} P_{ND} + \omega^S P_S$
4. Calculate Real Nondurable Consumption: $C = \frac{ND^N + S^N}{P_C}$

D.3 Data transformation for Bayesian estimation

Variable	Description	Construction
POP_{index}	Population index	$\frac{POP}{POP_{2009:1}}$
CE_{index}	Employment index	$\frac{CE}{CE_{2009:1}}$
Y^o	Real per capita GDP	$\ln\left(\frac{\frac{Y^N}{P_Y}}{POP_{index}}\right)100$
I_D^o	Real per capita consumption: durables	$\ln\left(\frac{D}{POP_{index}}\right)100$
C^o	Real per capita consumption: nondurables	$\ln\left(\frac{C}{POP_{index}}\right)100$
$W^{o,j}$	Real wage sector $j = C, D$	$\ln\left(\frac{W^j}{P_Y}\right)100$
$N^{o,j}$	Hours worked per capita sector $j = C, D$	$\ln\left(\frac{H^j \times CE_{index}}{POP_{index}}\right)100$
Π_C^o	Inflation: nondurables sector	$\Delta(\ln P_C)100$
Π_D^o	Inflation: durables sector	$\Delta(\ln P_D)100$
R^o	Quarterly Federal Funds Rate	$\frac{FFR}{4}$

Table D.2: Data transformation - Observables

E Bayesian impulse responses in the estimated model

The estimated model exhibits well-behaved macroeconomic dynamics. For instance, Figure E.1 shows that Bayesian impulse responses of selected macroeconomic variables to an aggregate positive technology shock are in line with the dynamics of standard models (see e.g. Kim and Katayama, 2013, for an example of a two-sector model). Labor productivity increases in both sectors, thus implying an expansion of sectoral production and aggregate output, which leads to a decline in sectoral and aggregate inflation to which the central bank responds by cutting the interest rate. Responses to the other shocks are likewise standard and are available upon request.

F The role of durability

Erceg and Levin (2006) show why durable goods are particularly important for optimal monetary policy. In general, in a two sector model, following a sector-specific shock, demand in the two sectors moves in opposite direction. The central bank should therefore increase the interest rate to stabilize the output gap in one sector while decreasing it to stabilize the output gap in the other sector. This trade-off is particularly severe when one sector produces

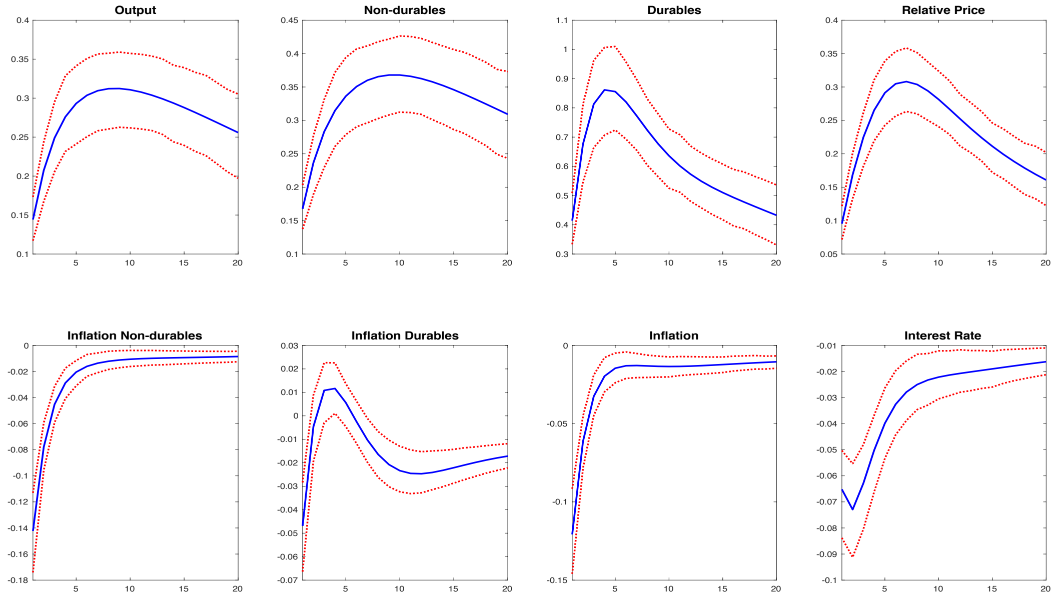

Figure E.1: Bayesian impulse responses to aggregate technology shock. Blue solid lines represent mean responses. Red dotted lines represent 90% confidence bands.

durable goods for two reasons. First, the demand for durables is for a stock, so also small changes in the demand for the stock generate large changes in the flow of newly produced durables. Then, the presence of sectoral price stickiness prevents prices from adjusting and insulate the durables sector from the shocks. Together, these two intrinsic features imply that durables are much more sensitive to the interest rate than nondurables. Therefore the same magnitude of the interest rate change generates a larger response of output in the durables sector, hence the more severe trade-off. To see this, we follow the reasoning made by Erceg and Levin (2006). The asset price equation of durables (21) requires that the marginal rate of substitution between durables and nondurables $\frac{U_{D,t}}{U_{C,t}} = \frac{\alpha}{1-\alpha}\frac{C_t}{D_t}$ equals the *user cost* of durable goods Θ_t:

$$\frac{U_{D,t}}{U_{C,t}} = \Theta_t \equiv Q_t - \beta\left(1-\delta\right)E_t\left[\frac{U_{C,t+1}}{U_{C,t}}Q_{t+1}\right], \tag{F.1}$$

which implies that

$$D_t = \frac{\alpha}{1-\alpha}\frac{C_t}{\Theta_t}, \tag{F.2}$$

or in log-linear form:

$$\hat{D}_t = \hat{C}_t - \hat{\Theta}_t. \tag{F.3}$$

Log-linearizing also the user cost of durables (F.1) and the Euler equation (4) around the steady state yields, respectively

$$\hat{\Theta}_t = \frac{\hat{Q}_t - (1-\delta)\,\beta E_t\left[\hat{U}_{C,t} - \hat{U}_{C,t+1} - \hat{Q}_{t+1}\right]}{1-(1-\delta)\,\beta}, \tag{F.4}$$

$$\hat{U}_{C,t} - \hat{U}_{C,t+1} = \hat{R}_t - E_t\hat{\Pi}^C_{t+1}, \tag{F.5}$$

combining which yields

$$\hat{\Theta}_t = \frac{\hat{Q}_t - (1-\delta)\,\beta E_t\left[\hat{R}_{r,t} - \hat{Q}_{t+1}\right]}{1-(1-\delta)\,\beta}, \tag{F.6}$$

where $\hat{R}_{r,t} = \hat{R}_t - E_t\hat{\Pi}^C_{t+1}$ is the real interest rate. Equation (F.6) shows that the user cost of durables depends on the relative price and the real interest rate. When prices are sticky, the relative price will adjust slowly to shocks so that the user cost and hence, for a sufficiently low depreciation rate δ, the demand of durables is very sensitive to the real interest rate. Note also that when there is no durability ($\delta = 1$), the output gap in the two sectors is entirely determined by the relative price. Finally, rearranging (F.6) yields equation (28).

G Robustness analysis in the fully-fledged model

In this section we perform several robustness checks. We first look at the role of sectoral shocks, nominal and real frictions, and the depreciation rate of durables. Then, we replace the monetary policy rule (13) with alternative rules and compare the results with the baseline model (top panel of Table 4). Our main findings continue to hold under all the robustness checks.

G.1 Sectoral shocks

Our model includes both aggregate (or symmetric) and sector-specific shocks. In particular, technology and preference shocks fall in the former category, while durables investment, nondurables and durables price markup and wage markup and government spending shocks fall in the latter category. In multi-sector models, aggregate shocks typically generate a comovement across sector thus inducing little labor reallocation.[2] Conversely, sectoral dis-

[2]The sectoral comovement in response to aggregate shocks is evident in our model from Figure E.1 (Appendix E), where we consider an economy-wide technology shock and, more generally, in the vast literature on the sectoral responses to a monetary policy innovation (see Cantelmo and Melina, 2018, for a detailed review).

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	τ	$100 \times \omega$
Excluding price markup shocks in nondurables						
∞	0.7586	0.7578	0.0000	0.0000	0.0000	0.0091
1.2250	0.2087	2.2056	0.0000	0.0000	0.0771	0.0633
0.1	0.8920	1.2426	0.0011	0.0000	0.6090	0.2345
Excluding price markup shocks in durables						
∞	0.0237	2.3965	0.0000	0.0000	0.0935	0.0744
1.2250	0.4514	1.2501	0.0000	0.1539	0.2387	0.1165
0.1	1.0000	1.2106	0.0026	0.0000	0.7451	0.0551
Excluding wage markup shocks in nondurables						
∞	1.0000	0.0672	0.0003	0.0702	0.2000	0.0656
1.2250	1.0000	0.2849	0.0000	0.3744	0.2212	0.0506
0.1	0.9666	0.7299	0.0013	0.3498	0.9904	0.1454
Excluding wage markup shocks in durables						
∞	0.0409	2.4523	0.0000	0.5041	0.0021	0.1044
1.2250	0.7124	0.5995	0.0000	0.2049	0.1193	0.1454
0.1	0.9141	0.8887	0.0014	0.0000	0.7726	0.2755
Excluding government spending shocks						
∞	0.0004	2.3028	0.0000	0.0352	0.0189	0.0886
1.2250	0.4759	1.0841	0.0000	0.2577	0.1481	0.1353
0.1	0.9164	0.8832	0.0014	0.0000	0.7725	0.2761
Excluding durables investment specific shocks						
∞	0.0005	2.3139	0.0000	0.0411	0.0179	0.0882
1.2250	0.3614	1.2647	0.0000	0.2239	0.1652	0.1264
0.1	0.3086	0.9622	0.0000	0.0000	0.6721	0.1477

Table G.1: Optimized monetary policy rules: robustness to the absence of sectoral shocks

turbances have the potential to generate larger labor reallocation since demand or supply in different sectors move in opposite direction. It is therefore natural to assess whether the inverse relationship between the optimal weight on durables inflation and labor mobility is driven by any specific sectoral disturbance. We thus eliminate each sectoral shock one at a time and verify that our results still hold. Table G.1 shows that our findings do not hinge on a specific sectoral disturbance. In each case, the weight placed on durables inflation is inversely related to the degree of labor mobility, while welfare losses are comparable to the baseline results. As already noted in Section 4.3, the price markup shock in the durables

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	τ	$100 \times \omega$
Flexible wages in durables sector						
∞	0.0254	0.3397	0.0000	0.0000	0.0000	0.0722
1.2250	0.2229	1.6563	0.0000	0.2437	0.0617	0.1151
0.1	0.9832	0.1289	0.0001	0.0000	0.5143	0.1220
Flexible prices and wages in durables sector						
∞	0.0370	2.3748	0.0000	0.0000	0.0000	0.0742
1.2250	0.2204	1.6011	0.0000	0.1471	0.0000	0.1150
0.1	0.7326	0.5162	0.0000	0.1901	0.0589	0.1491

Table G.2: Optimized monetary policy rules: robustness to nominal rigidities

sector matters only for the magnitude of the welfare loss, but not for the inverse relationship between labor mobility and the optimal durables inflation weight.

G.2 Nominal rigidities

We next verify whether our results still hold in counterfactual economies without nominal rigidities in prices and nominal wages in the durables sector, although the estimation suggests that both are substantially sticky. The top panel of Table G.2 shows the case of flexible wages ($\vartheta_d^w = 0$) whereas in the lower panel both durables prices and wages are flexible ($\vartheta_d = \vartheta_d^w = 0$). Relative to the baseline model, at the estimated limited degree of labor mobility, the optimal weight on durables inflation drops as wages become flexible in the durables sector (τ falls from 0.15 to 0.0617) and it becomes zero as both nominal frictions are removed. Nevertheless, a sufficiently low degree of labor mobility (e.g. $\lambda = 0.10$) still entails a positive weight on durables inflation both with flexible wages and sticky prices ($\tau = 0.5143$) and with both flexible wages and prices ($\tau = 0.0589$), meaning that imperfect sectoral labor mobility creates scope for a positive weight on durables inflation even if nominal rigidities are absent in that sector. Overall, the main conclusions drawn in the previous section are carried over with these two counterfactual economies: i) τ and λ are negatively related, hence a higher weight is assigned to durables inflation as labor becomes less mobile across sectors; ii) the interaction between labor mobility and wage stickiness is key in that sticky wages and limited labor mobility entails a higher weight on durables inflation, but flexible wages alone do not necessarily imply a zero weight on durables inflation if labor is sufficiently non-mobile.

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	τ	$100 \times \omega$
Excluding habit formation in nondurables consumption						
∞	0.1588	2.5457	0.0000	0.0000	0.0000	0.0165
1.2250	0.1988	1.6282	0.0000	0.0000	0.1636	0.0668
0.10	0.8821	1.8159	0.0014	0.9923	0.4430	0.1257
Excluding investment adjustment costs in durables						
∞	1.0000	2.0050	0.0033	1.2715	0.1859	0.1775
1.2250	0.8790	5.0000	0.0069	2.9205	0.3918	0.2463
0.10	0.5083	5.0000	0.0004	0.0000	1.0000	0.8665

Table G.3: Optimized monetary policy rule: robustness to the absence of real frictions

G.3 Real frictions

The model employed in this paper features two sources of real frictions, important to bring it to the data. In particular, households display habit formation in consumption of nondurable goods, while changing investment plans in durables goods entails a quadratic cost. In this section we verify that the inverse relationship between λ and τ continues to hold in restricted models in which we remove one real friction at a time. Table G.3 demonstrates that all the results are robust both to a calibration of the model which excludes habits in nondurables consumption ($\zeta = \rho_c = 0$), and to a model without investment adjustment costs in durables ($\phi = 0$), cases in which the inverse relationship between the optimal weight on durables and sectoral labor mobility still exists.

G.4 Depreciation rate of durable goods

Our baseline calibration, inspired by previous studies, assumes a 1% quarterly depreciation rate of durable goods. Barsky et al. (2016) study optimal monetary policy in a two-sector economy with durable goods, with price stickiness as the only source of nominal rigidity, no real frictions and a smaller set of shocks, and show how the optimal weight on durables inflation is affected by the depreciation rate of durables. We therefore check the robustness of our findings to alternative rates of depreciation of durable goods. Table G.4 reports the optimized parameters and welfare losses under higher (quarterly) depreciation rates than that assumed in the baseline calibration. We find that for higher depreciation rates, and even if durables would fully depreciate each quarter ($\delta \to 1$), the inverse relationship between the optimal weight on durables inflation and sectoral labor mobility survives.

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	τ	$100 \times \omega$
			$\delta = 0.025$			
∞	0.7098	0.6090	0.0000	0.1303	0.0125	0.1908
1.2250	0.7084	0.5799	0.0000	0.1977	0.1358	0.1627
0.1	0.9070	1.0141	0.0016	0.1066	0.7638	0.4252
			$\delta = 0.10$			
∞	0.9962	0.0133	0.0000	0.0000	0.0137	0.0994
1.2250	0.7538	0.5580	0.0000	0.2736	0.0877	0.1701
0.1	0.7558	2.1838	0.0016	0.9159	0.5698	0.4924
			$\delta \to 1$			
∞	0.7425	0.5619	0.0000	0.2548	0.0289	0.1770
1.2250	0.7469	0.5516	0.0000	0.2665	0.0523	0.1765
0.1	0.7045	0.5617	0.0000	0.2765	0.3143	0.3170

Table G.4: Optimized monetary policy rule: robustness to alternative depreciation rates of durables

G.5 Alternative interest rate rules

Implementable rules. We replace rule (13) with an interest rate rule that responds only to deviations of inflation and output from their respective steady states. Following Schmitt-Grohe and Uribe (2007) this type of interest rate rule is typically labeled as *implementable rule* and reads as follows:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \alpha_\pi \log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right) + \alpha_y \log\left(\frac{Y_t}{\bar{Y}}\right). \tag{G.1}$$

The top panel of Table G.5 demonstrates that despite these modifications, the inverse relationship between labor mobility and the optimal weight on durables inflation still hold true. In addition, the implied welfare losses are similar to the baseline model.

Responding to wages. Erceg and Levin (2006) find that rules targeting the output gap or a weighted average of price and wage inflation represent good approximations of the optimal rule. We therefore check whether the inverse relationship between labor mobility and the optimal weight on durables inflation continues to hold under interest rate rules that respond to wages.

λ	ρ_r	α_π	α_y	$\alpha_{\Delta y}$	α_w	τ	$100 \times \omega$
			Implementable rule				
∞	0.0004	2.3170	0.0000	/	/	0.0177	0.0868
1.2250	0.2358	1.5019	0.0000	/	/	0.1579	0.1264
0.1	0.9152	0.8904	0.0016	/	/	0.7729	0.2753
			Wage inflation				
∞	0.0667	2.0806	0.0000	0.0000	0.3821	0.0000	0.0893
1.2250	0.8426	0.7527	0.0002	0.0740	0.4096	0.1407	0.1121
0.1	1.0000	0.6682	0.0015	0.0000	0.4038	0.6441	0.0183
			Real wage growth				
∞	0.5931	0.9068	0.0000	0.0000	0.2061	0.0000	0.1401
1.2250	1.0000	2.9147	0.0020	0.9829	0.8763	0.0559	0.1071
0.1	1.0000	1.0715	0.0015	0.0000	0.4037	0.4017	0.0183
			Real sectoral wage growth differential				
∞	/	/	/	/	/	/	/
1.2250	0.6612	0.8592	0.0000	0.3209	0.0231	0.1243	0.1434
0.1	1.0000	0.7169	0.0016	0.0000	0.1095	0.6200	0.1167

Table G.5: Robustness to alternative optimized monetary policy rule

We start by closely following Erceg and Levin (2006) by adding a term to the interest rate rule (G.2) that responds to nominal wage inflation and optimize $\alpha_w \in [0, 5]$ along with the other policy parameters and the weight on durables inflation.[3] In accordance with the findings in Erceg and Levin (2006), the second panel of Table G.5 shows that responding to wage inflation is welfare enhancing.

$$\begin{aligned} \log\left(\frac{R_t}{\bar{R}}\right) &= \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \alpha_\pi \log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right) + \alpha_w \log\left(\frac{\Pi_t^w}{\Pi^w}\right) + \\ &+ \alpha_y \log\left(\frac{Y_t}{Y_t^f}\right) + \alpha_{\Delta y}\left[\log\left(\frac{Y_t}{Y_t^f}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-1}^f}\right)\right]. \end{aligned} \tag{G.2}$$

[3]Following Iacoviello and Neri (2010), we define an aggregate wage index $W_t = \left(\left(W_t^C\right)^{\frac{1+\lambda}{\lambda}} + \left(W_t^D\right)^{\frac{1+\lambda}{\lambda}}\right)^{\frac{\lambda}{1+\lambda}}$ and wage inflation as $\Pi_t^w = \frac{W_t}{W_{t-1}}\Pi_t^C$.

We then take a step further, following Faia (2008), by assessing whether responding to real, rather than nominal, wage growth is welfare enhancing in our model. Specifically we add a term to the interest rate rule (13) that responds to real wage growth:

$$\begin{aligned}\log\left(\frac{R_t}{\bar{R}}\right) &= \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \alpha_\pi \log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right) + \alpha_w \log\left(\frac{w_t}{w_{t-1}}\right) \\ &+ \alpha_y \log\left(\frac{Y_t}{Y_t^f}\right) + \alpha_{\Delta y}\left[\log\left(\frac{Y_t}{Y_t^f}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-1}^f}\right)\right]. \end{aligned} \tag{G.3}$$

The third panel of Table G.5 shows that responding to real wage growth slightly improves welfare relative to responding to wage inflation for limited degrees of labor mobility.

Finally, the last check we perform is optimizing a monetary rule that embeds a response to the change in the relative wage across sectors, so that equation (G.3) becomes:

$$\begin{aligned}\log\left(\frac{R_t}{\bar{R}}\right) &= \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \alpha_\pi \log\left(\frac{\tilde{\Pi}_t}{\tilde{\Pi}}\right) + \alpha_w \left[\log\left(\frac{w_t^d}{w_t^c}\right) - \log\left(\frac{w_{t-1}^d}{w_{t-1}^c}\right)\right] \\ &+ \alpha_y \log\left(\frac{Y_t}{Y_t^f}\right) + \alpha_{\Delta y}\left[\log\left(\frac{Y_t}{Y_t^f}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-1}^f}\right)\right]. \end{aligned} \tag{G.4}$$

We only consider cases of limited labor mobility as, with perfect labor mobility, wages in the two sectors are always the same by construction and the interest rate rule (G.4) collapses to the rule (13) studied in the main analysis. It turns out that it is optimal for the central bank to respond to some extent to the change in the wage differential.

Crucially, the main result on the negative relationship between sectoral labor mobility and the optimal weight on durables inflation survives in all cases considered.