Online Appendix: The Government Spending Multiplier in a Model with the Cost Channel

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1. Deriving the Government Spending Multiplier

1.1. Normal Times

To derive the spending multiplier analytically, I use the following system of equations:

$$[\gamma(1-\sigma)-1]\widehat{c_t} - (1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{n_t} = \widehat{R_t} - \mathbb{E}_t\left[\widehat{\pi_{t+1}} - [\gamma(1-\sigma)-1]\widehat{c_{t+1}} + (1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{n_{t+1}}\right] \quad (1)$$

$$\widehat{c_t} + \frac{\overline{n}}{1-\overline{n}}\widehat{n_t} = \widehat{w_t} \quad (2)$$

$$\widehat{\pi_t} = \beta\,\mathbb{E}_t\,\widehat{\pi_{t+1}} + \kappa\widehat{mc_t} \quad (3)$$

$$\widehat{y_t} = (1-g)\widehat{c_t} + g\widehat{g_t} \quad (4)$$

$$\widehat{R_t} = \phi_\pi\widehat{\pi_t} + \phi_y\widehat{y_t} \quad (5)$$

$$\widehat{mc_t} = \widehat{w_t} + \delta\widehat{R_{L,t}} \quad (6)$$

$$\widehat{l_t} = \widehat{w_t} + \widehat{n_t} \quad (7)$$

$$\widehat{R_{L,t}} = \Psi_R\widehat{R_t} + \Psi_l\widehat{l_t} \quad (8)$$

$$\widehat{y_t} = \widehat{n_t} \quad (9)$$

$$\widehat{g_t} = \rho\widehat{g_{t-1}} + u_t \quad (10)$$

which constitutes a system of 10 equations with 10 variables ($\widehat{c_t}, \widehat{g_t}, \widehat{l_t}, \widehat{mc_t}, \widehat{n_t}, \widehat{w_t}, \widehat{y_t}, \widehat{\pi_t}, \widehat{R_t}, \widehat{R_{L,t}}$).

Through substitutions, this system of equations can be reduced to:

$$[\gamma(1-\sigma)-1]\widehat{c_t} - (1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_t} = \widehat{R_t} - \mathbb{E}_t\left[\widehat{\pi_{t+1}} - [\gamma(1-\sigma)-1]\widehat{c_{t+1}} + (1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_{t+1}}\right] \quad (11)$$

$$\widehat{\pi_t} = \beta\,\mathbb{E}_t\,\widehat{\pi_{t+1}} + \kappa\left(\widehat{c_t} + \frac{\overline{n}}{1-\overline{n}}\widehat{y_t} + \delta\widehat{R_{L,t}}\right) \quad (12)$$

$$\widehat{y_t} = (1-g)\widehat{c_t} + g\widehat{g_t} \quad (13)$$

$$\widehat{R_t} = \phi_\pi\widehat{\pi_t} + \phi_y\widehat{y_t} \quad (14)$$

$$\widehat{R_{L,t}} = \Psi_R\widehat{R_t} + \Psi_l\left(\frac{(1-g)+(1-\overline{n})}{(1-g)(1-\overline{n})}\right)\widehat{y_t} - \Psi_l\frac{g}{1-g}\widehat{g_t} \quad (15)$$

$$\widehat{g_t} = \rho\widehat{g_{t-1}} + u_t \quad (16)$$

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Substituting (15) into (12), this system of equations can be further reduced to:

$$[\gamma(1-\sigma)-1]\widehat{c_t}-(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_t}=\widehat{R_t}-\mathbb{E}_t\left[\widehat{\pi_{t+1}}-[\gamma(1-\sigma)-1]\widehat{c_{t+1}}+(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_{t+1}}\right] \quad (17)$$

$$\widehat{\pi_t}=\beta\,\mathbb{E}_t\,\widehat{\pi_{t+1}}+\kappa\left(\widehat{c_t}+\frac{\overline{n}}{1-\overline{n}}\widehat{y_t}+\delta\Psi_R\widehat{R_t}+\delta\Psi_l\left(\frac{(1-g)+(1-\overline{n})}{(1-g)(1-\overline{n})}\right)\widehat{y_t}-\delta\Psi_l\frac{g}{1-g}\widehat{g_t}\right) \quad (18)$$

$$\widehat{y_t}=(1-g)\widehat{c_t}+g\widehat{g_t} \quad (19)$$

$$\widehat{R_t}=\phi_\pi\widehat{\pi_t}+\phi_y\widehat{y_t} \quad (20)$$

$$\widehat{g_t}=\rho\widehat{g_{t-1}}+u_t \quad (21)$$

Next, substitute (20) into (17)-(18) to obtain:

$$[\gamma(1-\sigma)-1]\widehat{c_t}-(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_t}=\phi_\pi\widehat{\pi_t}+\phi_y\widehat{y_t}-\mathbb{E}_t\left[\widehat{\pi_{t+1}}-[\gamma(1-\sigma)-1]\widehat{c_{t+1}}+(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\widehat{y_{t+1}}\right] \quad (22)$$

$$\widehat{\pi_t}=\beta\,\mathbb{E}_t\,\widehat{\pi_{t+1}}+\kappa\left(\widehat{c_t}+\frac{\overline{n}}{1-\overline{n}}\widehat{y_t}+\delta\Psi_R(\phi_\pi\widehat{\pi_t}+\phi_y\widehat{y_t})+\delta\Psi_l\left(\frac{(1-g)+(1-\overline{n})}{(1-g)(1-\overline{n})}\right)\widehat{y_t}-\delta\Psi_l\frac{g}{1-g}\widehat{g_t}\right) \quad (23)$$

$$\widehat{y_t}=(1-g)\widehat{c_t}+g\widehat{g_t} \quad (24)$$

$$\widehat{g_t}=\rho\widehat{g_{t-1}}+u_t \quad (25)$$

which is a system of 4 equations with 4 variables $(\widehat{c_t},\widehat{g_t},\widehat{y_t},\widehat{\pi_t})$.

To derive the multiplier using the Method of Undetermined Coefficients, we make the following definitions for the responses of variables to a government spending shock:

$$\widehat{y_t}=A_y\widehat{g_t} \quad (26)$$

$$\widehat{c_t}=A_c\widehat{g_t} \quad (27)$$

$$\widehat{\pi_t}=A_\pi\widehat{g_t} \quad (28)$$

The set of equations that is used to derive the spending multiplier does not include any endogenous state variable, which both simplifies the analysis and allows us to express each variable as in definitions (26)-(28). Also, for any variable x_t, we have $\mathbb{E}_t\,\widehat{x_{t+1}}=\mathbb{E}_t(A_x\widehat{g_{t+1}})$. Using the exogenous process for government spending, this condition can be written as $\mathbb{E}_t\,\widehat{x_{t+1}}=A_x\,\mathbb{E}_t(\rho\widehat{g_t}+u_{t+1})$, or:

$$\mathbb{E}_t\,\widehat{x_{t+1}}=\rho A_x\widehat{g_t} \quad (29)$$

Then, the system (22)-(24) can be written as:

$$[\gamma(\sigma-1)+1]A_c-(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}A_y=\phi_\pi A_\pi+\phi_y A_y$$
$$-\left[\rho A_\pi-[\gamma(\sigma-1)+1]\rho A_c+(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}\rho A_y\right] \quad (30)$$

$$A_\pi=\beta\rho A_c+\kappa\left(A_c+\frac{\overline{n}}{1-\overline{n}}A_y+\delta\Psi_R(\phi_\pi A_\pi+\phi_y A_y)+\delta\Psi_l\left(\frac{(1-g)+(1-\overline{n})}{(1-g)(1-\overline{n})}\right)A_y-\delta\Psi_l\frac{g}{1-g}\right) \quad (31)$$

$$A_y=(1-g)A_c+g \quad (32)$$

which is a system of 3 equations with 3 unknowns (A_y,A_c,A_π). Next, eliminate A_c using (32) and re-arrange to get:

$$\Big[(1-\rho)[\gamma(1-\sigma)+1]-(1-g)(1-\rho)(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}-(1-g)\phi_y\Big]A_y = (1-g)(\phi_\pi-\rho)A_\pi+(1-\rho)[\gamma(1-\sigma)+1]g \tag{33}$$

$$A_\pi = \frac{\kappa}{1-\beta-\delta\Psi_R\kappa\phi_\pi}\left[\left(\frac{1+\delta\Psi_l}{1-g}+\frac{\overline{n}+\delta\Psi_l}{1-\overline{n}}+\delta\Psi_R\phi_y\right)A_y-(1+\delta\Psi_l)\frac{g}{1-g}\right] \tag{34}$$

Substituting (34) in (33) and re-arranging then gives the government spending multiplier of normal times (condition (23) in the text).

1.2. Liquidity Trap

I follow similar steps to derive the spending multiplier for the model with constant nominal interest rate (the liquidity-trap multiplier). In this case, one can obtain:

$$[\gamma(\sigma-1)+1]A_c-(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}A_y = -\left[pA_\pi-[\gamma(\sigma-1)+1]pA_c+(1-\gamma)(1-\sigma)\frac{\overline{n}}{1-\overline{n}}pA_y\right] \tag{35}$$

$$A_\pi = \beta pA_c+\kappa\left(A_c+\frac{\overline{n}}{1-\overline{n}}A_y+\delta\Psi_l\left(\frac{(1-g)+(1-\overline{n})}{(1-g)(1-\overline{n})}\right)A_y-\delta\Psi_l\frac{g}{1-g}\right) \tag{36}$$

$$A_y = (1-g)A_c+g \tag{37}$$

The solution to this system of equations gives condition (24) in the text.

2. The Expected Real Interest Rate

2.1. Normal Times

This appendix outlines the effects of the cost channel on the response of the expected real interest rate and, consequently, the spending multiplier. Since $\widehat{\pi_t} = A_\pi\widehat{g_t}$, the expected inflation rate reads:

$$\mathbb{E}_t\,\widehat{\pi_{t+1}} = A_\pi\,\mathbb{E}_t\,\widehat{g_{t+1}} \tag{38}$$

or, using the AR(1) process of government spending:

$$\mathbb{E}_t\,\widehat{\pi_{t+1}} = \rho A_\pi\widehat{g_t} \tag{39}$$

With no response to output in the interest-rate rule, which is the assumption in Christiano et al. (2011) among others, we have:

$$\widehat{R_t} = \phi_\pi\widehat{\pi_t} \tag{40}$$

The expected real interest rate:

$$\mathbb{E}_t\,\widehat{r_{t+1}} = \widehat{R_t}-\mathbb{E}_t\,\widehat{\pi_{t+1}} \tag{41}$$

which, using $\widehat{\pi_t} = A_\pi\widehat{g_t}$ and (39)-(40), gives condition (25) in the text:

$$\mathbb{E}_t\,\widehat{r_{t+1}} = (\phi_\pi-\rho)A_\pi\widehat{g_t}, \tag{42}$$

The actual real interest rate is given by:

$$\widehat{r_t} = \widehat{R_t}-\widehat{\pi_t} \tag{43}$$

or, using condition (40):

$$\widehat{r_t} = (\phi_\pi-1)A_\pi\widehat{g_t} \tag{44}$$

Therefore, to the extent that $\phi_\pi > 1$ and $A_\pi > 0$, the actual real interest rate will rise following government spending shocks.

With $\widehat{R_t} = \phi_\pi \widehat{\pi_t} + \phi_y \widehat{y_t}$, the corresponding expected real interest rate reads:

$$\mathbb{E}_t \, \widehat{r_{t+1}} = \left[(\phi_\pi - \rho) A_\pi + \phi_y A_y \right] \widehat{g_t} \tag{45}$$

which is as in the text.

2.2. Liquidity Traps

When the nominal interest rate is fixed, the expected inflation rate is given by:

$$\mathbb{E}_t \, \widehat{\pi_{t+1}} = p \widehat{\pi_t} \tag{46}$$

or:

$$\mathbb{E}_t \, \widehat{\pi_{t+1}} = p A_\pi \widehat{g_t} \tag{47}$$

with:

$$A_\pi^{LT} = \frac{\kappa g \left[\frac{1}{1-g} + \frac{\overline{n}}{1-\overline{n}} \right] \left[\gamma(\sigma - 1) + 1 \right] - \frac{\kappa g}{1-g} + \kappa \delta \lambda A_s^{LT}}{1 - \beta p - \frac{\kappa p (1-g)}{1-p} \left[\frac{1}{1-g} + \frac{\overline{n}}{1-\overline{n}} \right]} \tag{48}$$

which, because $A_s^{LT} > 0$ is clearly larger in the presence of the cost channel.

Moreover, since $\widehat{R_t} = 0$, the expected real interest rate will be given by:

$$\mathbb{E}_t \, \widehat{r_{t+1}} = -p A_\pi \widehat{g_t} \tag{49}$$

which is condition (26) in the text. Similarly, the real interest rate is given by $\widehat{r_t} = -\widehat{\pi_t}$, or:

$$\widehat{r_t} = -A_\pi \widehat{g_t}. \tag{50}$$

The behavior of the actual real interest rate is thus similar to the behavior of the expected real interest rate, but with differences in the magnitudes because $p < 1$.

3. The Credit Spread in a Model with Costly State Verification

For liquidity traps, I briefly illustrate that credit frictions, as in the Costly State Verification (CSV) model of Carlstrom and Fuerst (1997) and Carlstrom and Fuerst (1998), imply a higher credit spread. Unlike the textbook CSV model, I abstract from capital and provide a simplified version of that model. To this end, assume that a firm borrows $e_t = \alpha w_t n_t$ to finance part of its labor input. Furthermore, assume that the firm has internal assets of $\overline{k}$ (which in the original model are tied to the value of capital, but assumed here to be fixed for simplicity). The firm then needs to borrow $\alpha w_t n_t - \overline{k}$. The firm produces using a modified production function, $y_t = \omega_t n_t$, with ω_t being idiosyncratic productivity.

In this model, default is possible. For this reason, the lender monitor the borrower, and the monitoring cost is a fraction of output, namely μy_t, with $0 < \mu < 1$. The borrower will default if output is less than the amount that needs to be repaid (which is the loan plus the interest on the loan). This occurs if productivity falls short of a certain threshold ($\overline{\omega_t}$). This threshold constitutes the minimum level of productivity so that default does not occur, and it satisfies:

$$\overline{\omega_t} n_t = R_{L,t} (\alpha w_t n_t - \overline{k}) \tag{51}$$

Then, the average (or expected) real income of the firm at each point in time is given by:

$$n_t \Big[\int_{\overline{\omega_t}}^{\infty} \omega \; \mathrm{d}\Phi(\overline{\omega_t}) - \overline{\omega_t} [1 - \mu \Phi(\overline{\omega_t})] \Big] \tag{52}$$

or $n_t f(\overline{\omega_t})$, with $f(\overline{\omega_t})$ being the share of the firm in the expected output outcome.

The average (or expected) real income of the lender (net of monitoring costs):

$$n_t\Big[\int_0^{\overline{\omega_t}} \omega \; \mathrm{d}\Phi(\overline{\omega_t}) - \mu\Phi(\overline{\omega_t}) + \overline{\omega_t}[1 - \mu\Phi(\overline{\omega_t})]\Big] \tag{53}$$

or $n_t u(\overline{\omega_t})$, with $u(\overline{\omega_t})$ being the share of the lender. These fractions satisfy $f(\overline{\omega_t}) + u(\overline{\omega_t}) = 1 - \mu\Phi(\overline{\omega_t})$.

The lender will participate in the financial contract if its expected income at least covers the loan. Formally, the lender's participation condition is given by:

$$n_t u(\overline{\omega_t}) \geq \alpha w_t n_t - \overline{k} \tag{54}$$

Combining (51) and (54) gives:

$$R_{L,t} = \frac{\overline{\omega_t}}{u(\overline{\omega_t})} \tag{55}$$

The risk premium is given by $R_{L,t} - 1$, or:

$$R_{L,t} - 1 = \frac{\overline{\omega_t}}{u(\overline{\omega_t})} - 1 \tag{56}$$

Therefore, a positive shock that raises the threshold level of productivity $\overline{\omega_t}$ would also raise the risk premium. This is the standard result in the CSV literature. See, for example, Gomes et al. (2003) for similar analysis.

When the policy interest rate is fixed at its steady-sate level ($\overline{R}$), the spread is given by $s_t = R_{L,t} - \overline{R}$. Then, we have:

$$s_t = \frac{\overline{\omega_t}}{u(\overline{\omega_t})} - \overline{R} \tag{57}$$

which can be shown to be increasing in $\overline{\omega_t}$. For illustration, assume that ω_t is uniformly distributed between [a,b]. Since ω_t is non-negative, $a = 0$. Therefore, the cumulative distribution is given by:

$$\Phi(\omega_t) = \frac{\omega_t}{b} \tag{58}$$

Using the expressions above, the spread in liquidity traps is given by:

$$s_t = \frac{1}{1 - \mu/b - \overline{\omega_t}/2b} - \overline{R} \tag{59}$$

Then, a positive shock that raises the threshold level of productivity $\overline{\omega_t}$ would also raise the credit spread. Therefore, the rise in the credit spread following a positive government spending shock could also result from a standard agency cost problem.

4. Separable Utility Function

The period utility function in this setup is given by:

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu} \tag{60}$$

with σ being the consumption curvature parameter and ν the inverse of the labor supply elasticity. The corresponding Euler and labor supply conditions are given by:

$$\widehat{R_t} - \mathbb{E}_t\,\widehat{\pi_{t+1}} = \sigma(\mathbb{E}_t\,\widehat{c_{t+1}} - \widehat{c_t}) \tag{61}$$

$$\nu\widehat{n_t} + \sigma\widehat{c_t} = \widehat{w_t}. \tag{62}$$

Under the assumption $\phi_\pi > 1$, the spending multiplier of normal times:

$$M^{NT} = \frac{\kappa(\phi_\pi - \rho)\sigma(1+\delta\Psi_l) + \sigma(1-\rho)(1-\beta\rho-\delta\Psi_r\kappa\phi_\pi)}{\kappa(\phi_\pi-\rho)\left[(1+\delta\Psi_l)(\sigma+\nu(1-g)) + \delta\Psi_l(1-g)\right] + \sigma(1-\rho)(1-\beta\rho-\delta\Psi_r\kappa\phi_\pi)} \tag{63}$$

And, in liquidity traps:

$$M^{LT} = \frac{\sigma[(1-\beta p)(1-p) - p\kappa(1+\delta\Psi_l)]}{\sigma(1-\beta p)(1-p) - p\kappa[(1+\delta\Psi_l)(\sigma+\nu(1-g)) + (1-g)\delta\Psi_l]} \tag{64}$$

With $\delta = 0$, the liquidity-trap spending multiplier restores that of Carlstrom et al. (2014). Once more, the spending multiplier is affected by the cost channel, κ matters markedly, and the cost channel will not affect the size of the multiplier when $\kappa = 0$. Figure D.1 provides numerical evaluation; in normal times, the introduction of the cost channel has a slight negative effect on the spending multiplier, while in liquidity traps the cost channel induces a rise in the spending multiplier. These findings confirm the findings with the non-separable utility function, albeit the effects of the cost channel are smaller than in the former case.

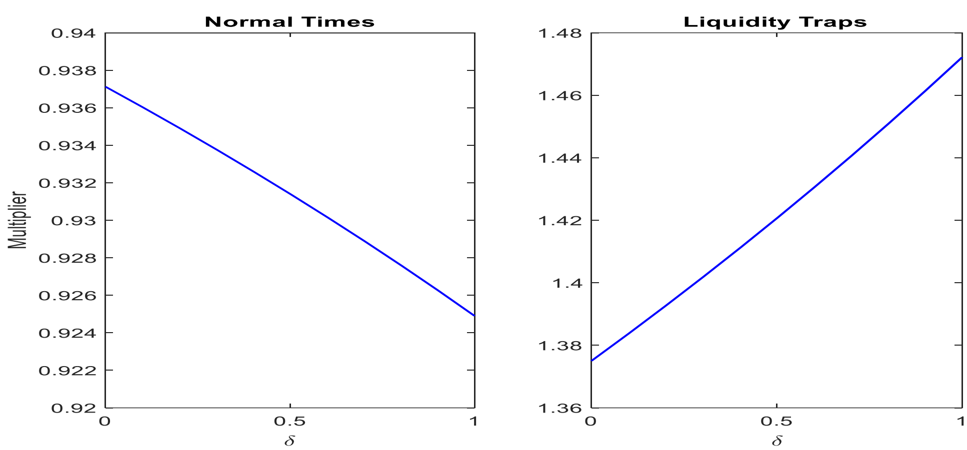

Figure D.1: The government spending multiplier for various values of δ with separable utility function.
Notes: $\nu = 0.50$. All other parameter values are as in the text.

5. A Medium-Scale DSGE Model

In this appendix, I outline a model that includes capital, habit persistence in consumption, labor taxation, capital taxation and government bonds. The model is then solved numerically and the results are presented in Section 5.7 of the paper.

5.1. Households

The problem of the representative household is to:

$$\max_{\{c_t, B_t, D_t, M_t, n_t, k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1}, n_t) \tag{65}$$

The utility function satisfies: $\frac{\partial u}{\partial c} > 0$, $\frac{\partial^2 u}{\partial c^2} < 0$ and $\frac{\partial u}{\partial n} < 0$ and $\frac{\partial^2 u}{\partial n^2} < 0$.

Maximization is subject to the sequence of households' budget constraints is given by:

$$P_t c_t + M_{t+1} + B_{t+1} + D_t + P_t i_t = M_t + R_{D,t} D_t + R_t B_t + (1 - \tau_t^n) W_t n_t + T_t + \Pi_t \tag{66}$$

and the CIA constraint:

$$P_t c_t = M_t + (1 - \tau_t^n) W_t n_t - D_t \tag{67}$$

and the law of motion of capital:

$$k_{t+1} = i_t + [r_t + 1 - \delta^k - \tau_t^k (r_t - \delta^k)] k_t - \frac{\phi^k}{2} \Big(\frac{k_{t+1}}{k_t} - 1 \Big)^2 \tag{68}$$

where B_t denotes nominal bonds, i_t is investment in physical capital, k_t is capital, R_t is the gross nominal interest rate on bonds, $R_{D,t}$ is the gross nominal interest rate on deposits, τ_t^n is the labor-income tax rate, τ_t^k is the capital-income tax rate, δ^k is the depreciation rate of capital and ϕ^k governs the adjustment cost of capital. All other variables are as defined in the text.

Optimization by households leads to the following first-order conditions:

$$u_{c,t} = \beta R_{D,t} \, \mathbb{E}_t \left(\frac{u_{c,t+1}}{\pi_{t+1}} \right) \tag{69}$$

$$-\frac{u_{n,t}}{u_{c,t}} = (1 - \tau_t^n) w_t \tag{70}$$

$$u_{c,t} = \beta \, \mathbb{E}_t \left(R_{t+1} \frac{u_{c,t+1}}{\pi_{t+1}} \frac{R_{D,t}}{R_{D,t+1}} \right) \tag{71}$$

$$u_{c,t} \Big[1 + \phi^k \Big(\frac{k_{t+1}}{k_t} - 1 \Big) \frac{1}{k_t} \Big] = \beta \, \mathbb{E}_t \Big[u_{c,t+1} \frac{R_{D,t}}{R_{D,t+1}} \big[r_{t+1} + 1 - \delta^k - \tau_{t+1}^k (r_{t+1} - \delta^k) + \phi^k \Big(\frac{k_{t+2}}{k_{t+1}} - 1 \Big) \Big(\frac{k_{t+2}}{k_{t+1}^2} \Big) \big] \Big] \tag{72}$$

where $u_{c,t}$ is the marginal utility of consumption and $u_{c,t}$ is the marginal utility of labor.

Furthermore, I use the following period utility function:

$$u(c_t, c_{t-1}, n_t) = \frac{(c_t - h c_{t-1})^{1-\sigma}}{1 - \sigma} - \chi \frac{n_t^{1+\nu}}{1 + \nu} \tag{73}$$

with h being the parameter that governs the strength of the habit persistence in consumption.

With this functional form, the marginal utility of consumption is given by:

$$u_{c,t} = (c_t - h c_{t-1})^{-\sigma} - \beta h E_t (c_{t+1} - h c_t)^{-\sigma} \tag{74}$$

Clearly, with $h = 0$, we obtain the standard marginal utility in consumption of separable preferences.

5.2. Production sector

Each intermediate-good firm hires labor and rents capital to produce output using the following technology:

$$y_{j,t} = k_{j,t}^{\varsigma} n_{j,t}^{1-\varsigma} \tag{75}$$

with ς being the elasticity of output with respect to capital.

Since I solve a non-linear version of the model (with which Calvo price rigidity is not tractable), I assume that firms face a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final good. Then, the problem of the firm it then to choose $k_{j,t}, n_{j,t}$ and $P_{j,t}$ to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{u_{c,t}}{u_{c,0}} \right) \left[\frac{P_{j,t}}{P_t} y_{j,t} - R_{L,t} \alpha w_t n_{j,t} - (1-\alpha) w_t n_{j,t} - r_t k_{j,t} - \frac{\varphi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - \overline{\pi} \right)^2 y_t \right] \tag{76}$$

with $\beta \frac{u_{c,t+1}}{u_{c,t}}$ being the stochastic discount factor, φ being the price adjustment cost parameter and $\overline{\pi}$ is the steady-state gross inflation rate. Maximization is subject to the demand curve for the firm's product and the production technology. Taking first-order conditions and imposing symmetry across firms then gives the following factor demands:

$$(1-\varsigma) k_t^{\varsigma} n_t^{-\varsigma} mc_t = (1 - \alpha + \alpha R_{L,t}) w_t \tag{77}$$

$$\varsigma k_t^{\varsigma-1} n_t^{1-\varsigma} mc_t = r_t \tag{78}$$

where mc_t is the real marginal cost. As expected, each firm hires labor and rents capital so that the marginal product of each input is a markup over its factor price.

In a symmetric equilibrium, in which all firms set the same price, Rotemberg pricing leads to the following forward-looking Phillips curve:

$$\varepsilon(1 - mc_t) = 1 - \varphi(\pi_t - \overline{\pi})\pi_t + \beta\varphi \, \mathbb{E}_t \left[\left(\frac{u_{c,t+1}}{u_{c,t}} \right) (\pi_{t+1} - \overline{\pi}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \tag{79}$$

The final-good firm sector remains unchanged.

5.3. Government Budget Constraint and Marking Clearing

Government expenditures are financed by taxation on labor and capital and the issuance of bonds:

$$P_t g_t + R_{t-1} B_t = B_{t+1} + P_t \tau_t^n w_t n_t + P_t \tau_t^k (r_t - \delta^k) \tag{80}$$

and government spending evolves according to:

$$\ln\left(\frac{g_t}{\overline{g}} \right) = \rho \ln\left(\frac{g_{t-1}}{\overline{g}} \right) + \zeta_t \tag{81}$$

The resource constraint of the economy is given by:

$$y_t + (1 - \delta^k) k_t = c_t + k_{t+1} + g_t + \frac{\varphi}{2} (\pi_t - \overline{\pi})^2 y_t + \frac{\phi^k}{2} \Big(\frac{k_{t+1}}{k_t} - 1 \Big)^2 \tag{82}$$

5.4. Intermediation sector

The deviation between the nominal policy interest rate and the nominal loan rate continue to be given by:

$$s_t = R_{L,t} - R_t. \tag{83}$$

5.5. Fiscal Policy Rules

The tax rates are assumed to respond to deviations of output and the public debt from their steady state values, as follows:

$$\ln\left(\frac{\tau_t^n}{\overline{\tau^n}}\right) = \rho_n \ln\left(\frac{\tau_{t-1}^n}{\overline{\tau^n}}\right) + (1-\rho_n)\lambda_n \ln\left(\frac{y_t}{\bar{y}}\right) + (1-\rho_n)\delta_n \ln\left(\frac{b_{t-1}}{\bar{b}}\right) \tag{84}$$

$$\ln\left(\frac{\tau_t^k}{\overline{\tau^k}}\right) = \rho_k \ln\left(\frac{\tau_{t-1}^k}{\overline{\tau^k}}\right) + (1-\rho_k)\lambda_k \ln\left(\frac{y_t}{\bar{y}}\right) + (1-\rho_k)\delta_k \ln\left(\frac{b_{t-1}}{\bar{b}}\right) \tag{85}$$

This concludes the description of the model.

5.6. Parameterization of the Model

: The parameter values that I use in the DSGE model are as follows: $\varsigma = 0.33, \beta = 0.99, \alpha = 0.50, \sigma = 2.00, \varphi = 18.47, \delta^k = 0.026, \overline{\pi} = 1.005, \overline{\tau^n} = 0.20, \overline{\tau^k} = 0.30, \phi_\pi = 1.50, \phi_y = 0.00, h = 0.80, \rho = 0.80,$ $g = 0.20$, $\phi^k = 95$ and $\varepsilon = 6.00$. The coefficients of the tax rate rule rules are set to: $\rho_k = 0.82$, $\rho_n = 0.82$, $\lambda_k = 0.66$, $\lambda_n = 0.65$, $\delta_k = 0.39$ and $\delta_n = 0.18$. Some of these parameter values are as in the text. The value of $\overline{\pi}$ implies an annual inflation rate of 2%, ε suggests a steady-state price markup of 20% in line with the literature, δ^k is set so that the annual depreciation rate of capital is roughly 11%, h is in line with most studies with habit persistence and the value of φ is consistent with a price duration of 2.5 quarters. Following Faia and Monacelli (2007), this value of φ is obtained by letting the slope of the Phillips curve under Calvo price rigidity be equal to the slope of the Phillips curve under Rotemberg price rigidity. The coefficients of the tax rate rules are obtained from Abo-Zaid et al. (2017), and they are based on empirical evidence.

5.7. Quantitative Results

The model is solved numerically to evaluate the size of the government spending multiplier. Figure E.2 displays the response of the economy to a government spending shock under the assumption that the nominal interest rate is free to adjust. For each variable, the response of that variable is normalized to the change in government spending (i.e. the vertical axes display $\frac{\Delta x_{t+h}}{\Delta g_t}$ for each variable x); therefore, for output, the figure essentially displays the impact spending multiplier. The rise in government spending induces a rise in inflation and consequently the real interest rate. In this case, investment responds strongly and consumption displays a small decline (but remains mostly unchanged due to habit persistence). This effect is magnified by the cost channel. The behavior of output mostly follows the behavior of investment and the government spending multiplier with the cost channel is smaller than in the alternative model.

For liquidity traps, the drop in the real interest rate is magnified by the cost channel (as inflation rises more), which induces a larger increase in investment and a smaller drop in consumption relative to a model without the cost channel (Figure E.3). As a result, the government spending multiplier is larger on impact. This result reaffirms the findings from the benchmark model.[1]

In the baseline model, the size of the multiplier is determined by the behavior of consumption: the multiplier is larger than one if consumption rises after a spending shock, less than one if the opposite occurs and exactly one if consumption is irresponsive to spending shocks. In the expanded model, however, the spending multiplier depends also on the behavior of investment and on the habit persistence. The latter reduces the response of consumption to spending shocks. In fact, the response of consumption in this model

[1] The goal of this subsection is to study the effectiveness of fiscal policy when the nominal interest rate is fixed. Therefore, unlike in Section 4, I do not impose that the probability that the nominal interest is fixed (p) to equal the persistence of government spending (ρ). Instead, we let $p \to 1$ in this experiment so that R_t is essentially irresponsive to spending shocks. By letting the nominal interest rate be virtually constant, we can focus on the effects of the cost channel on the effectiveness of fiscal policy without being concerned about the degree to which monetary policy is accommodative.

is very small and does not drive the dynamics of output. Investment reacts very sharply to a spending shock, which reflects the sharp decline in the real interest rate (in liquidity traps) or the sharp rise in the real interest rate (in normal times). Clearly, the behavior of output on impact resembles the behavior of investment.

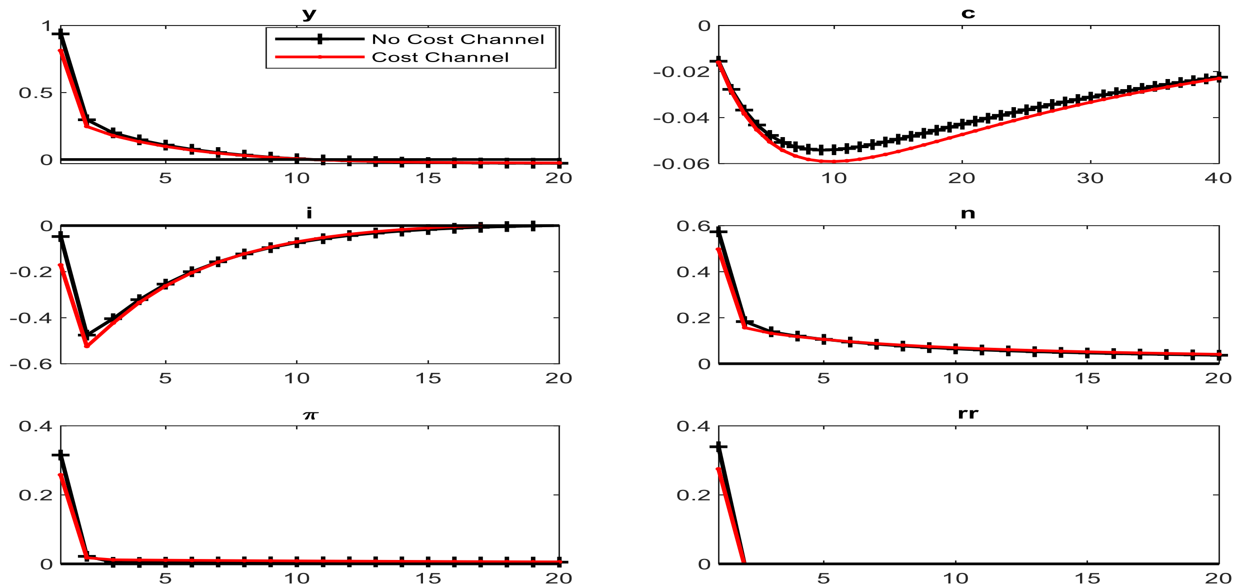

Figure E.2: Impulse responses to a positive shock to government spending. Notes: normal times, where the nominal policy interest rate follows an interest-rate rule. The response of each variable is normalized to the change in government spending. With the cost channel, the estimated value of A_s^{NT} on impact is 0.013 (and the peak is 0.027). Time unit: quarters.

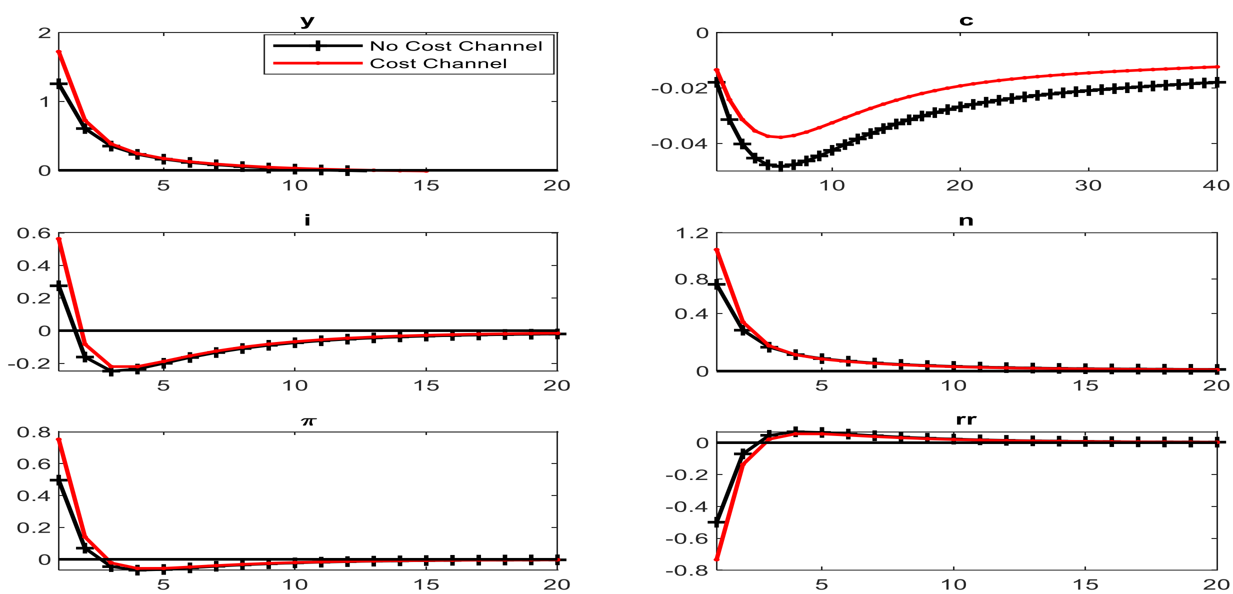

Figure E.3: Impulse responses to a positive shock to government spending. Notes: liquidity traps. The response of each variable is normalized to the change in government spending. With the cost channel, the estimated value of A_s^{LT} on impact is 0.24. Time unit: quarters.

6. Supplemental Empirical Analyses

This section provides additional analysis that are related to the response of the credit spread to government spending shocks (Subsection 6.1 in the text).

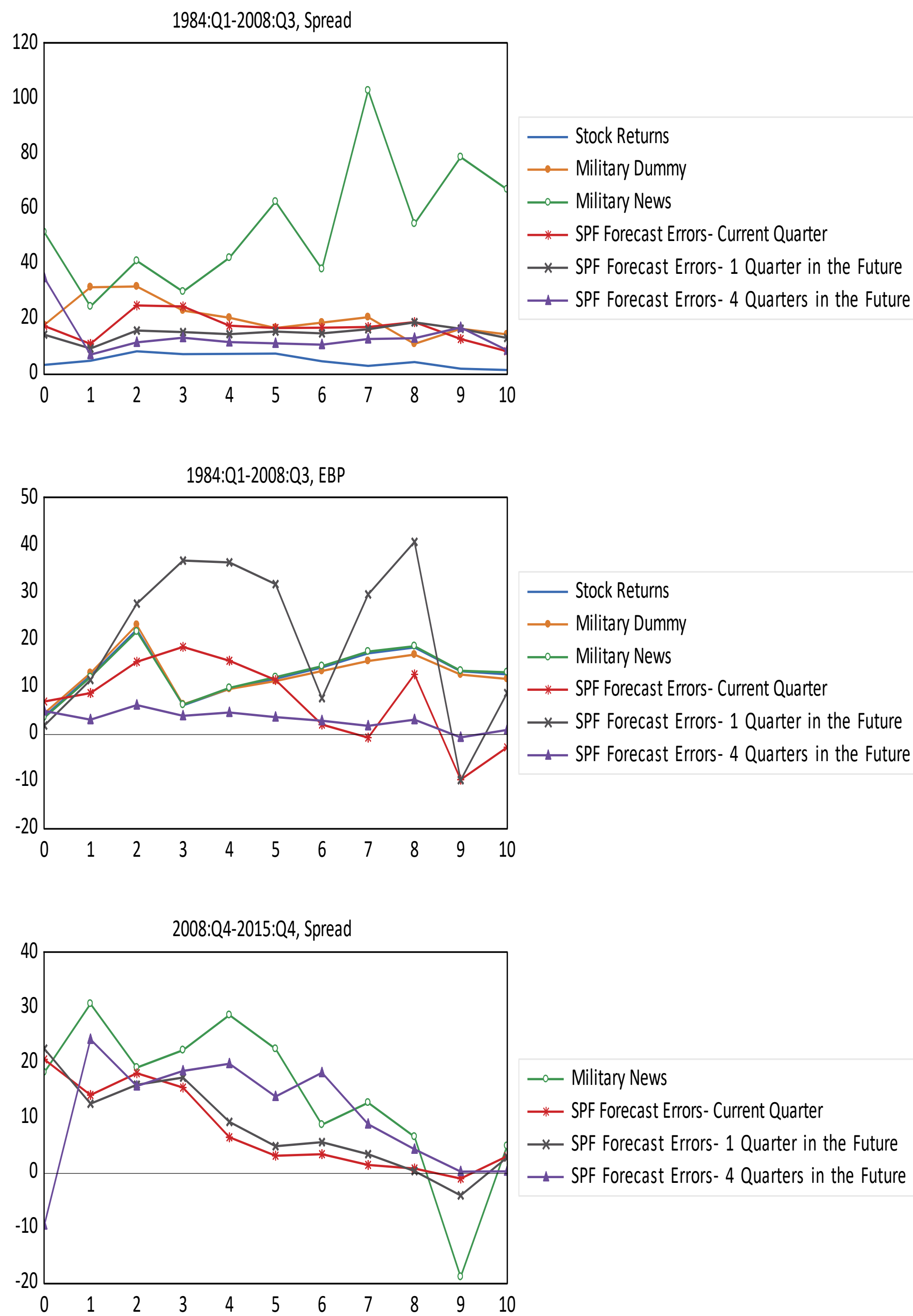

Figure F.4: The estimated values of A_s for each type of shock and at each horizon between 0 and 10, calculated as in condition (34) in the text. The medians across shocks at each horizon are reported in Figure B.7.

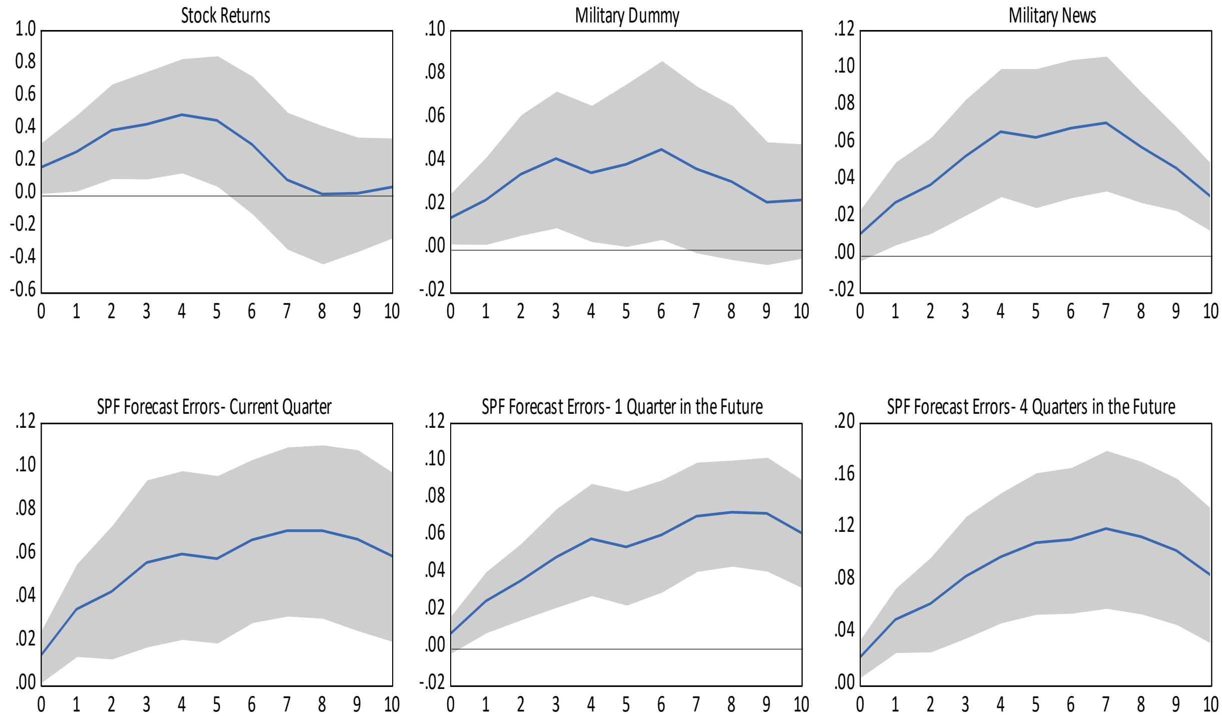

Figure F.5: Impulse responses of the corporate credit spread (relative to its steady-state value) to a positive government spending shock using local projections. Sample period: 1984:Q1-2008:Q3. Notes: shaded areas indicate the 95% confidence intervals. $\overline{s} = 0.0093, \overline{g} = 0.2216$. I include a fiscal policy shock, government spending, credit spread, output, tax receipts-GDP ratio and the 10-year minus 3-month U.S. government bond spread. Government spending and GDP are scaled by potential GDP.

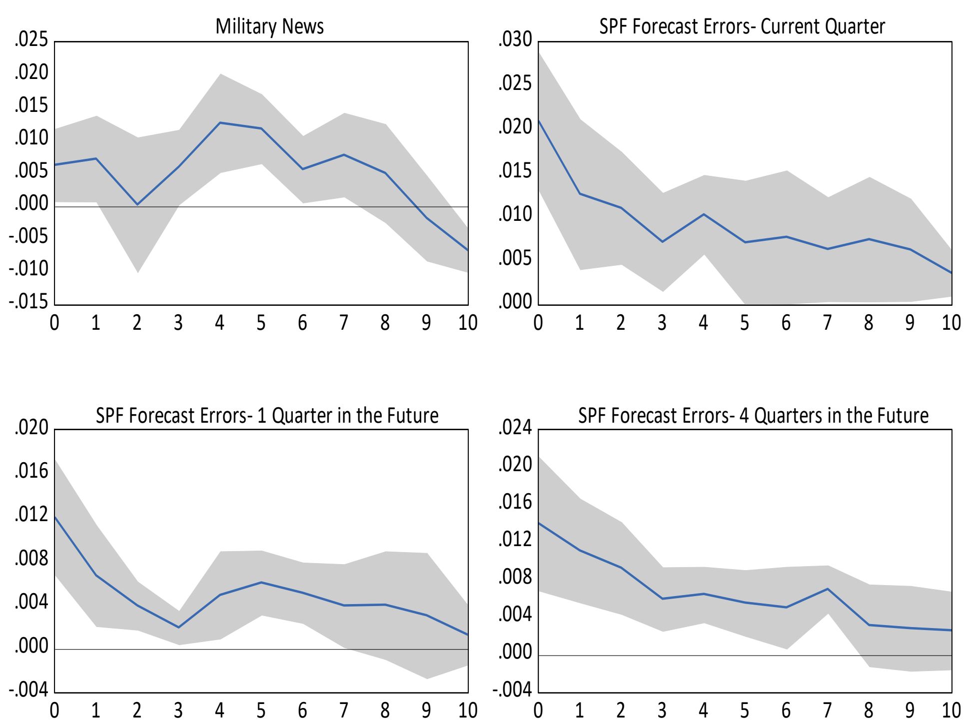

Figure F.6: Impulse responses of the corporate credit spread (relative to its steady-state value) to a positive government spending shock using local projections. Sample period: 2008:Q4-2015:Q4. Notes: $\overline{s} = 0.0136, \overline{g} = 0.1859$. See figure F.5 for more details.

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