Online Appendix to "Investigating the Role of Money in the Identification of Monetary Policy Behavior: A Bayesian DSGE Perspective"

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June 10, 2019

This note gathers supplementary material to Li, Liu, and Pei (2019).

Contents

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1 Equilibrium Conditions

The equilibrium system of the baseline model consists of the following conditions:

$$u_t^d \mathbb{U}_{1t} - \beta h E_t u_{t+1}^d \mathbb{U}_{1t+1} = \lambda_t \tag{1}$$

$$u_t^d \mathbb{U}_{2t} \frac{1}{\nu_t} - \lambda_t + \beta E_t \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0 \tag{2}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \tag{3}$$

$$r_t = \mu_t^{-1} a'[v_t] \tag{4}$$

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\delta) q_{t+1} + r_{t+1} v_{t+1} - \mu_{t+1}^{-1} a[v_{t+1}] \right\} \right\} \tag{5}$$

$$1 = q_t \mu_t \left\{ 1 - S\left[\frac{i_t}{i_{t-1}}\right] - S'\left[\frac{i_t}{i_{t-1}}\right] \frac{i_t}{i_{t-1}} \right\} + \beta E_t q_{t+1} \mu_{t+1} \frac{\lambda_{t+1}}{\lambda_t} S'\left[\frac{i_{t+1}}{i_t}\right] \left(\frac{i_{t+1}}{i_t}\right)^2 \tag{6}$$

$$\begin{aligned} 0 = {} & E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left\{ (1-\eta_{t+\tau}) \lambda_{t+\tau} \left[\prod_{s=1}^{\tau} \left(\frac{\pi_{t+s-1}}{\bar{\pi}}\right)^{\chi_w} \left(\frac{\bar{\pi}}{\pi_{t+s}}\right) \right]^{1-\eta_{t+\tau}} \left(\frac{w_t^*}{w_{t+\tau}}\right)^{-\eta_{t+\tau}} l_{t+\tau}^d \right\} + \\ & E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left\{ \frac{\eta_{t+\tau} \psi u_{t+\tau}^d}{w_t^*} \left[\prod_{s=1}^{\tau} \left(\frac{\pi_{t+s-1}}{\bar{\pi}}\right)^{\chi_w} \left(\frac{\bar{\pi}}{\pi_{t+s}}\right) \right]^{-\eta_{t+\tau}(1+\gamma_l)} \left(\frac{w_t^*}{w_{t+\tau}}\right)^{-\eta_{t+\tau}(1+\gamma_l)} \left(l_{t+\tau}^d\right)^{1+\gamma_l} \right\} \end{aligned} \tag{7}$$

$$\frac{m_t}{m_{t-1}} \pi_t = \omega_t \tag{8}$$

$$\begin{aligned} & E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} (\pi_t^*)^{-\varepsilon_{t+\tau}} \left[\prod_{s=1}^{\tau} \left(\frac{\pi_{t+s-1}}{\bar{\pi}}\right)^{\chi_p} \left(\frac{\bar{\pi}}{\pi_{t+s}}\right) \right]^{-\varepsilon_{t+\tau}} y_{t+\tau}^d \\ & * \left[(1-\varepsilon_{t+\tau}) \pi_t^* \prod_{s=1}^{\tau} \left(\frac{\pi_{t+s-1}}{\bar{\pi}}\right)^{\chi_p} \left(\frac{\bar{\pi}}{\pi_{t+s}}\right) + \varepsilon_{t+\tau} \Psi_{t+\tau} \right] = 0 \end{aligned} \tag{9}$$

$$\frac{v_t k_{t-1}}{l_t^d} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \tag{10}$$

$$\Psi_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} A_t^{\alpha-1} (w_t)^{1-\alpha} (r_t)^\alpha \tag{11}$$

$$1 = \theta_w \left[\left(\frac{w_{t-1}}{w_t}\right) \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{\chi_w} \left(\frac{\bar{\pi}}{\pi_t}\right) \right]^{1-\eta_t} + (1-\theta_w)(\pi_t^{w*})^{1-\eta_t} \tag{12}$$

$$\pi_t^{w*} = \frac{w_t^*}{w_t} \tag{13}$$

$$1 = \theta_p \left[\left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{\chi_p} \left(\frac{\bar{\pi}}{\pi_t}\right) \right]^{1-\varepsilon_t} + (1-\theta_p)(\pi_t^*)^{1-\varepsilon_t} \tag{14}$$

$$y_t^d = \frac{A_t^{1-\alpha} (v_t k_{t-1})^\alpha (l_t^d)^{1-\alpha}}{\Delta_t^p} \tag{15}$$

$$y_t^d = c_t + i_t + \mu_t^{-1} a[v_t] k_{t-1} + G_t \tag{16}$$

$$l_t = \Delta_t^w l_t^d \tag{17}$$

$$\Delta_t^p = \theta_p \left[\left(\frac{\pi_{t-1}}{\bar{\pi}} \right)^{\chi_p} \left(\frac{\bar{\pi}}{\pi_t} \right) \right]^{-\varepsilon_t} \Delta_{t-1}^p + (1-\theta_p)(\pi_t^*)^{-\varepsilon_t} \tag{18}$$

$$\Delta_t^w = \theta_w \left[\left(\frac{w_{t-1}}{w_t} \right) \left(\frac{\pi_{t-1}}{\bar{\pi}} \right)^{\chi_w} \left(\frac{\bar{\pi}}{\pi_t} \right) \right]^{-\eta_t} \Delta_{t-1}^w + (1-\theta_w)(\pi_t^{w*})^{-\eta_t} \tag{19}$$

$$k_t - (1-\delta)k_{t-1} - \mu_t \left\{ 1 - S\left[\frac{i_t}{i_{t-1}} \right] \right\} i_t = 0 \tag{20}$$

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\gamma_\pi^R} \left(\frac{y_t^d}{\bar{y}^d} \right)^{\gamma_y^R} \left(\frac{\omega_t}{\bar{\omega}} \right)^{\gamma_\omega^R} \right]^{1-\rho_R} \exp(\varepsilon_t^R) \tag{21}$$

$$\ln(\varepsilon_t) = (1-\rho_\varepsilon)\ln(\bar{\varepsilon}) + \rho_\varepsilon \ln(\varepsilon_{t-1}) + \varepsilon_t^\varepsilon \tag{22}$$

$$\ln(A_t) = (1-\rho_a)\ln(\bar{A}) + \rho_a \ln(A_{t-1}) + \varepsilon_t^a \tag{23}$$

$$\ln(\eta_t) = (1-\rho_\eta)\ln(\bar{\eta}) + \rho_\eta \ln(\eta_{t-1}) + \varepsilon_t^\eta \tag{24}$$

$$\ln(u_t^d) = \rho_d \ln(u_{t-1}^d) + \varepsilon_t^d \tag{25}$$

$$\ln(\nu_t) = (1-\rho_\nu)\ln(\bar{\nu}) + \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu \tag{26}$$

$$\ln(\mu_t) = (1-\rho_\mu)\ln(\bar{\mu}) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu \tag{27}$$

$$\ln(g_t) = (1-\rho_g)\ln(\bar{g}) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \tag{28}$$

where $w_t \equiv W_t/P_t$, $m_t \equiv M_t/P_t$ denote real wage and real money balance, respectively. In (1) and (2), $\mathbb{U}_{1t}$ and $\mathbb{U}_{2t}$ denote the partial derivatives of the utility with respect to the first and the second argument, respectively. (1) to (7) are from the first-order conditions of the households, (8) is an identity, (9) to (11) are from the intermediate goods producers' optimization problem, (12) to (14) are from the laws of motion of aggregate wage and price indexes, (15) to (19) are from aggregation and market clearing, (20) is from the law of motion for capital and (21) is from the monetary policy rule. (22) to (28) are stochastic processes of the exogenous shocks. Note, Ψ_t is the real marginal cost of the intermediate goods producers, which does not depend on the index i.

2 Steady State

Evaluating the equilibrium conditions of the baseline model in the steady state where $\bar{v} = 1$ and $a[1] = 0$, we obtain the following equations:

$$\mathbb{U}_c - \beta h \mathbb{U}_c = \bar{\lambda} \tag{29}$$

$$\frac{\mathbb{U}_m}{\bar{\nu}} - \bar{\lambda} + \frac{\beta\bar{\lambda}}{\bar{\pi}} = 0 \tag{30}$$

$$\bar{R} = \frac{\bar{\pi}}{\beta} \tag{31}$$

$$\bar{r}\bar{\mu} = \gamma_1 \tag{32}$$

$$\bar{r}\bar{\mu} = \frac{1}{\beta} - (1-\delta) \tag{33}$$

$$\bar{q}\bar{\mu} = 1 \tag{34}$$

$$(1-\bar{\eta})\bar{\lambda}\bar{w}^{*}+\bar{\eta}\psi\left(\bar{l}^{d}\right)^{\gamma_l}=0 \tag{35}$$

$$\bar{\pi}=\bar{\omega} \tag{36}$$

$$\bar{\Psi}=\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}} \tag{37}$$

$$\frac{\bar{k}}{\bar{l}^{d}}=\frac{\alpha}{1-\alpha}\frac{\bar{w}}{\bar{r}} \tag{38}$$

$$\bar{\Psi}=\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\bar{A}^{\alpha-1}\bar{w}^{1-\alpha}\bar{r}^{\alpha} \tag{39}$$

$$\bar{\pi}^{w*}=1 \tag{40}$$

$$\bar{\pi}^{*}=1 \tag{41}$$

$$\bar{\Delta}^{p}\bar{y}^{d}=\bar{A}^{1-\alpha}\bar{k}^{\alpha}(\bar{l}^{d})^{1-\alpha} \tag{42}$$

$$\bar{c}+\bar{i}=\frac{1}{\bar{g}}\bar{y}^{d} \tag{43}$$

$$\bar{l}=\bar{l}^{d} \tag{44}$$

$$\bar{\Delta}^{p}=1 \tag{45}$$

$$\bar{\Delta}^{w}=1 \tag{46}$$

$$\bar{i}=\delta\bar{k} \tag{47}$$

where variables with bar, e.g., $\bar{x}$ for any generic variable x_t, denote steady state values. In (29) and (30), $\mathbb{U}_c$ and $\mathbb{U}_m$ denote the steady state values of the partial derivatives of $\mathbb{U}$ with respect to c_t and m_t/ν_t, respectively.

Given the fixed parameters and the parameters to be estimated as shown in the paper, we calcuate the steady state in the following manner. By assuming $\bar{\mu}=1$, we first calculate $[\bar{r},\bar{R}]$ from (33) and (31). Second, we calculate $[\bar{\Psi},\bar{w},\bar{k}/\bar{l}]$ from (37) to (39). Third, we use $\bar{k}/\bar{l}$ to calculate $[\bar{y}^{d}/\bar{l},\bar{i}/\bar{l},\bar{c}/\bar{l}]$ according to (42), (47) and (43).

3 Log-Linearized System

Let $\hat{x}_t$ denote the log deviation of a generic variable x_t from its steady state value $\bar{x}$, i.e., $\hat{x}_t\equiv\ln(x_t)-\ln(\bar{x})$. The stationary equilibrium of the baseline model in log-linearized form consists of the following equations:

$$(1-\beta h)\sigma\hat{\lambda}_t=\beta hE_t\hat{c}_{t+1}-(1+\beta h^2)\hat{c}_t+h\hat{c}_{t-1}+\tau(\hat{m}_t-\hat{\nu}_t)-\beta h\tau(E_t\hat{m}_{t+1}-E_t\hat{\nu}_{t+1})$$
$$+\sigma\hat{u}^{d}_t-\beta h\sigma E_t\hat{u}^{d}_{t+1} \tag{48}$$

$$\hat{m}_t=\eta_c(\hat{c}_t-h\hat{c}_{t-1})-\sigma_m\hat{\lambda}_t-\frac{\beta}{\bar{\pi}-\beta}\sigma_m\hat{R}_t+(1-\sigma_m)\hat{\nu}_t+\sigma_m\hat{u}^{d}_t \tag{49}$$

$$E_t\hat{\lambda}_{t+1}-E_t\hat{\pi}_{t+1}=\hat{\lambda}_t-\hat{R}_t \tag{50}$$

$$\hat{r}_t+\hat{\mu}_t=\frac{\gamma_2}{\gamma_1}\hat{v}_t \tag{51}$$

$$E_t\hat{\lambda}_{t+1}+(1-\delta)\beta E_t\hat{q}_{t+1}+\left[1-(1-\delta)\beta\right]E_t\hat{r}_{t+1}=\hat{\lambda}_t+\hat{q}_t \tag{52}$$

$$E_t\hat{i}_{t+1}=\left(\frac{1}{\beta}+1\right)\hat{i}_t-\frac{1}{\beta}\hat{i}_{t-1}-\frac{1}{\beta\kappa}\hat{q}_t-\frac{1}{\beta\kappa}\hat{\mu}_t \tag{53}$$

$$E_t\hat{w}^*_{t+1} + E_t\hat{\pi}_{t+1} = -\kappa_w\gamma_l\hat{l}^d_t + \kappa_w\hat{\lambda}_t - \kappa_w\eta\gamma_l\hat{w}_t + \frac{1}{\beta\theta_w}\hat{w}^*_t + \chi_w\hat{\pi}_t - \kappa_w\hat{u}^d_t + \frac{\kappa_w}{\eta-1}\hat{\eta}_t \tag{54}$$

$$\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t = \hat{\omega}_t \tag{55}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\chi_p}E_t\hat{\pi}_{t+1} + \frac{\chi_p}{1+\beta\chi_p}\hat{\pi}_{t-1} + \kappa_p\hat{\Psi}_t + \frac{\kappa_p}{1-\varepsilon}\hat{\varepsilon}_t \tag{56}$$

$$\hat{v}_t + \hat{k}_{t-1} - \hat{l}^d_t = \hat{w}_t - \hat{r}_t \tag{57}$$

$$\hat{\Psi}_t = (\alpha-1)\hat{A}_t + (1-\alpha)\hat{w}_t + \alpha\hat{r}_t \tag{58}$$

$$\hat{w}_t + \hat{\pi}_t - \left(\frac{1-\theta_w}{\theta_w}\right)\hat{\pi}^{w*}_t = \hat{w}_{t-1} + \chi_w\hat{\pi}_{t-1} \tag{59}$$

$$\hat{\pi}^{w*}_t = \hat{w}^*_t - \hat{w}_t \tag{60}$$

$$\hat{\pi}^*_t = \frac{\theta_p}{1-\theta_p}\left(\hat{\pi}_t - \chi_p\hat{\pi}_{t-1}\right) \tag{61}$$

$$\hat{y}^d_t = (1-\alpha)\hat{A}_t + \alpha\hat{v}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{l}^d_t \tag{62}$$

$$\frac{\bar{c}}{\bar{l}}\hat{c}_t + \frac{\bar{i}}{\bar{l}}\hat{i}_t + \gamma_1\frac{\bar{k}}{\bar{l}}\hat{v}_t = \frac{1}{\bar{g}}\frac{\bar{y}^d}{\bar{l}}\hat{y}^d_t - \frac{1}{\bar{g}}\frac{\bar{y}^d}{\bar{l}}\hat{g}_t \tag{63}$$

$$\hat{l}_t = \hat{l}^d_t \tag{64}$$

$$\hat{\Delta}^p_t = 0 \tag{65}$$

$$\hat{\Delta}^w_t = 0 \tag{66}$$

$$\hat{k}_t - (1-\delta)\hat{k}_{t-1} - \delta\hat{\mu}_t - \delta\hat{i}_t = 0 \tag{67}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\gamma^R_\pi\hat{\pi}_t + \gamma^R_y\hat{y}^d_t + \gamma^R_\omega\hat{\omega}_t\right] + \varepsilon^R_t \tag{68}$$

$$\hat{\varepsilon}_t = \rho_\varepsilon\hat{\varepsilon}_{t-1} + \varepsilon^\varepsilon_t \tag{69}$$

$$\hat{A}_t = \rho_a\hat{A}_{t-1} + \varepsilon^a_t \tag{70}$$

$$\hat{\eta}_t = \rho_\eta\hat{\eta}_{t-1} + \varepsilon^\eta_t \tag{71}$$

$$\hat{u}^d_t = \rho_d\hat{u}^d_{t-1} + \varepsilon^d_t \tag{72}$$

$$\hat{\nu}_t = \rho_\nu\hat{\nu}_{t-1} + \varepsilon^\nu_t \tag{73}$$

$$\hat{\mu}_t = \rho_\mu\hat{\mu}_{t-1} + \varepsilon^\mu_t \tag{74}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon^g_t \tag{75}$$

where

$$\kappa_w = \frac{1-\beta\theta_w}{\beta\theta_w(1+\eta\gamma_l)}$$

$$\kappa_p = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p(1+\beta\chi_p)}$$

In (48), $\sigma \equiv -\mathbb{U}_c/\mathbb{U}_{cc}\bar{c}$ denotes the curvature of utility function with respect to c_t, which is the intertemporal elasticity of substitution. $\tau \equiv \sigma\chi$ measures the real balance effect upon aggregate demand, where $\chi \equiv \left(\mathbb{U}_{cm}\frac{\bar{m}}{\bar{\nu}}\right)/\mathbb{U}_c$ is the elasticity of $\mathbb{U}_c$ relative to real money balance. $\mathbb{U}_{cc}$ and $\mathbb{U}_{cm}$ denote the steady state values of the partial derivatives of $\mathbb{U}_c$ with respect to c_t and m_t/ν_t, respectively. In (49), $\eta_c \equiv -\mathbb{U}_{mc}\bar{c}/\left(\mathbb{U}_{mm}\frac{\bar{m}}{\bar{\nu}}\right)$ denotes the consumption elasticity of money demand and $\sigma_m \equiv -\mathbb{U}_m / \left(\mathbb{U}_{mm} \frac{\bar{m}}{\bar{\nu}}\right)$ denotes the curvature of utility function with respect to m_t/ν_t. $\mathbb{U}_{mc}$ and $\mathbb{U}_{mm}$ denote the steady state values of the partial derivatives of $\mathbb{U}_m$ with respect to c_t and m_t/ν_t, respectively.

4 Model Solution

To solve the model under each specification, we first collect the equilibrium conditions as in Section 1 under the corresponding parameter restrictions and solve for the deterministic steady state as in Section 2, around which the model is log-linearized. We then cast the linearized system as in Section 3 into the canonical form $\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Pi \xi_t + \Psi \varepsilon_t$, where X_t is a vector of model variables and ε_t is a vector of i.i.d. random innovations to the structural shocks. Variables in X_t are log deviations of the variables from their respective steady states, i.e., $\hat{x}_t \equiv \ln(x_t) - \ln(\bar{x})$, where $\hat{x}_t$ is a generic variable in X_t and $\bar{x}$ is the corresponding steady state value. Note that ξ_t is a newly defined vector that is composed of one-period-ahead endogenous forecasting errors. For example, the forecasting error of inflation is $\xi_t^\pi \equiv \hat{\pi}_t - E_{t-1}\hat{\pi}_t$. $[\Gamma_0, \Gamma_1, \Pi, \Psi]$ are matrices consisting of nonlinear functions of the model parameters. Following Sims (2001), we obtain the model solution $X_t = G_1 X_{t-1} + G_2 \varepsilon_t$, where G_1 and G_2 are matrices consisting of nonlinear functions of the model parameters.

5 Impulse Responses: Baseline Estimation

Figure 1 plots the impulse responses of all the four model specifications in the baseline estimation, computed at the posterior mean parameters. Figure 2-5 are the impulse responses of each model specification in the baseline estimation, computed at the posterior mean parameters (solid line) together with 90% HPD intervals (dashed line).

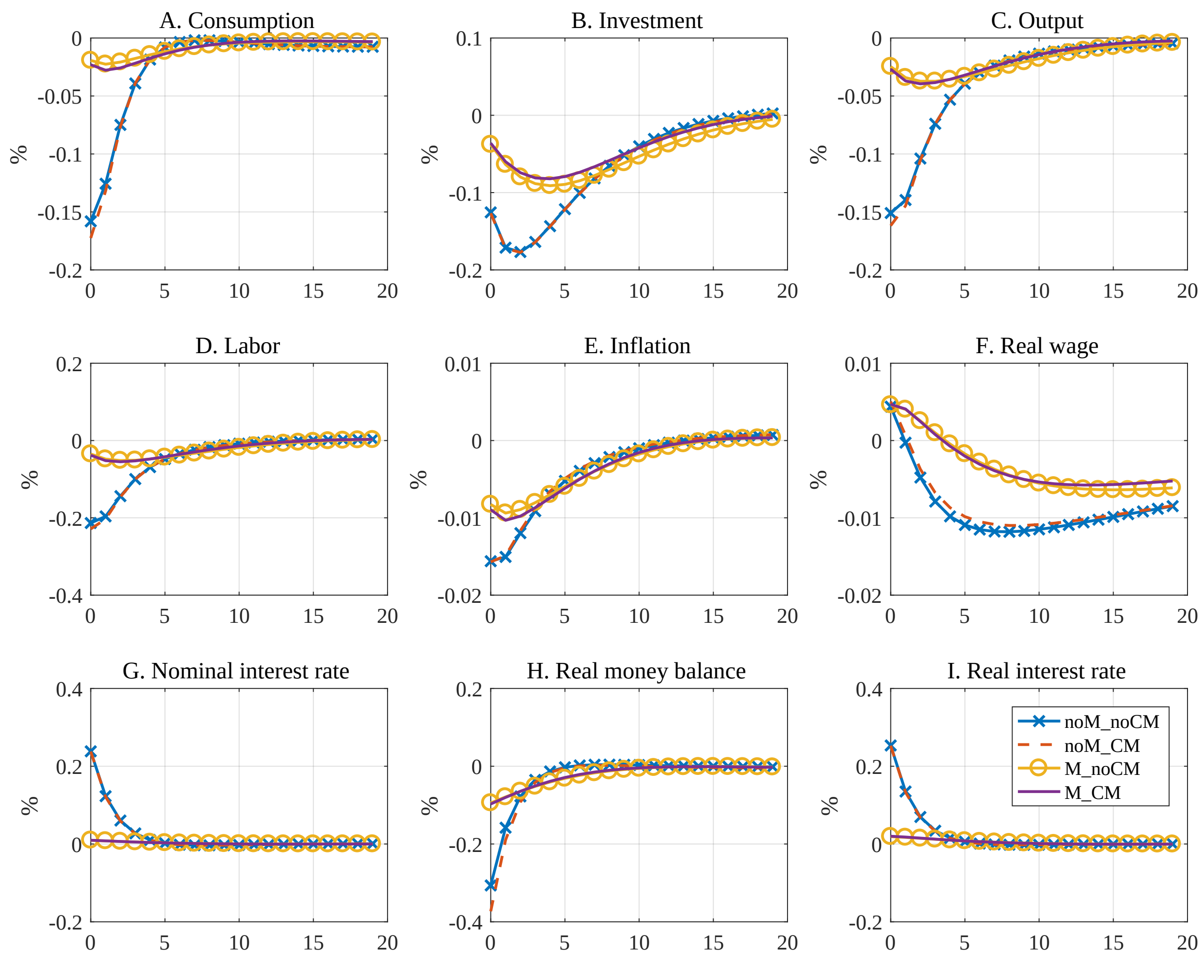

Figure 1: Impulse responses to a 25-basis-point positive monetary policy shock. Baseline model. Sample: 1967:Q1-2008:Q2. The impulse responses are computed at the posterior mean parameters. Solid line with cross: restricted model without consumption-money non-separability (no$\mathcal{M}$_no$\mathcal{CM}$); Dashed line: restricted model with consumption-money non-separability (no$\mathcal{M}$_$\mathcal{CM}$); Solid line with circle: unrestricted model without consumption-money non-separability ($\mathcal{M}$_no$\mathcal{CM}$); Solid line: unrestricted model with consumption-money non-separability ($\mathcal{M}$_$\mathcal{CM}$). The vertical axis is percentage deviation from the steady state.

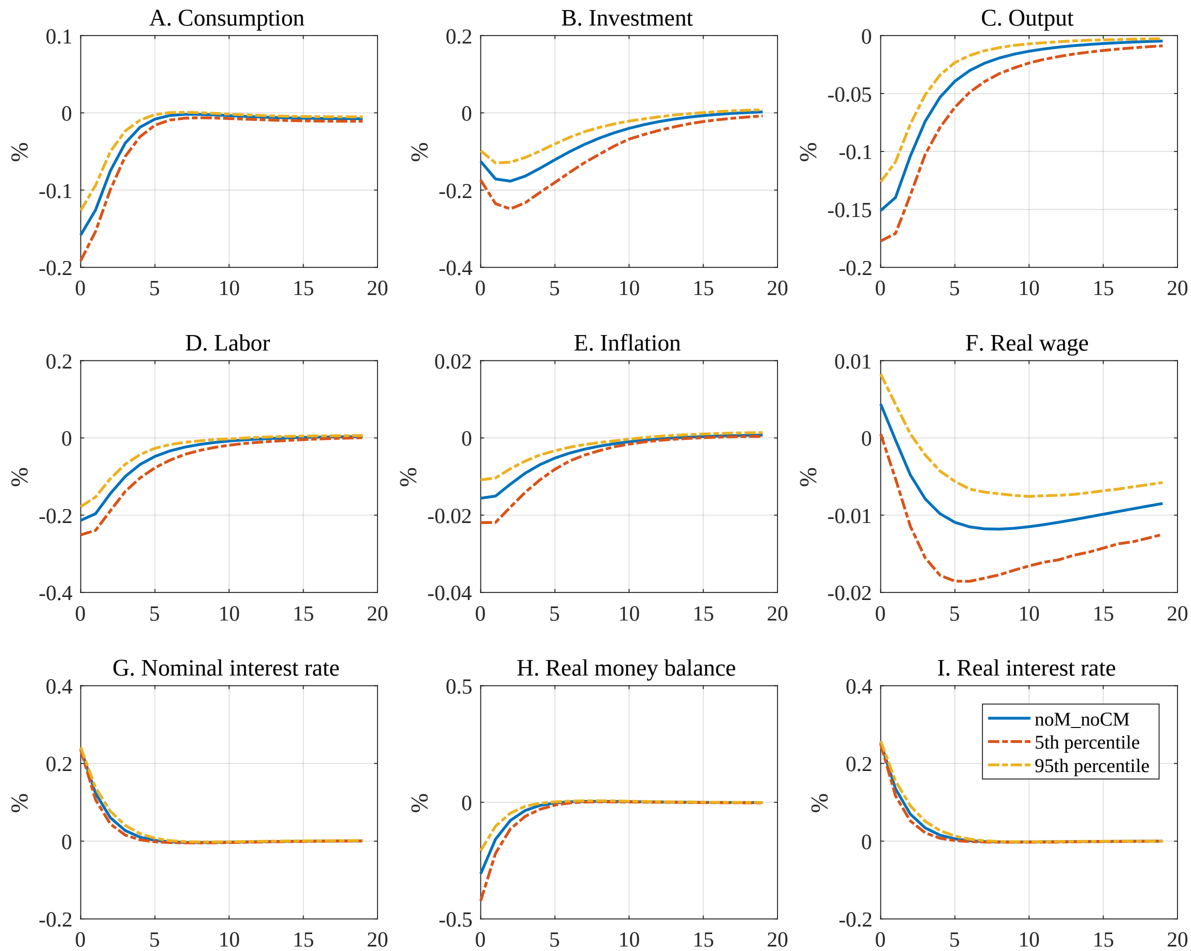

Figure 2: Impulse responses to a 25-basis-point positive monetary policy shock, baseline model, restricted model without consumption-money non-separability (no$\mathcal{M}$_no$\mathcal{CM}$). Sample: 1967:Q1-2008:Q2. Solid line: impulse response at the posterior mean parameters; Dashed line: 5th and 95th percentiles. The vertical axis is percentage deviation from the steady state.

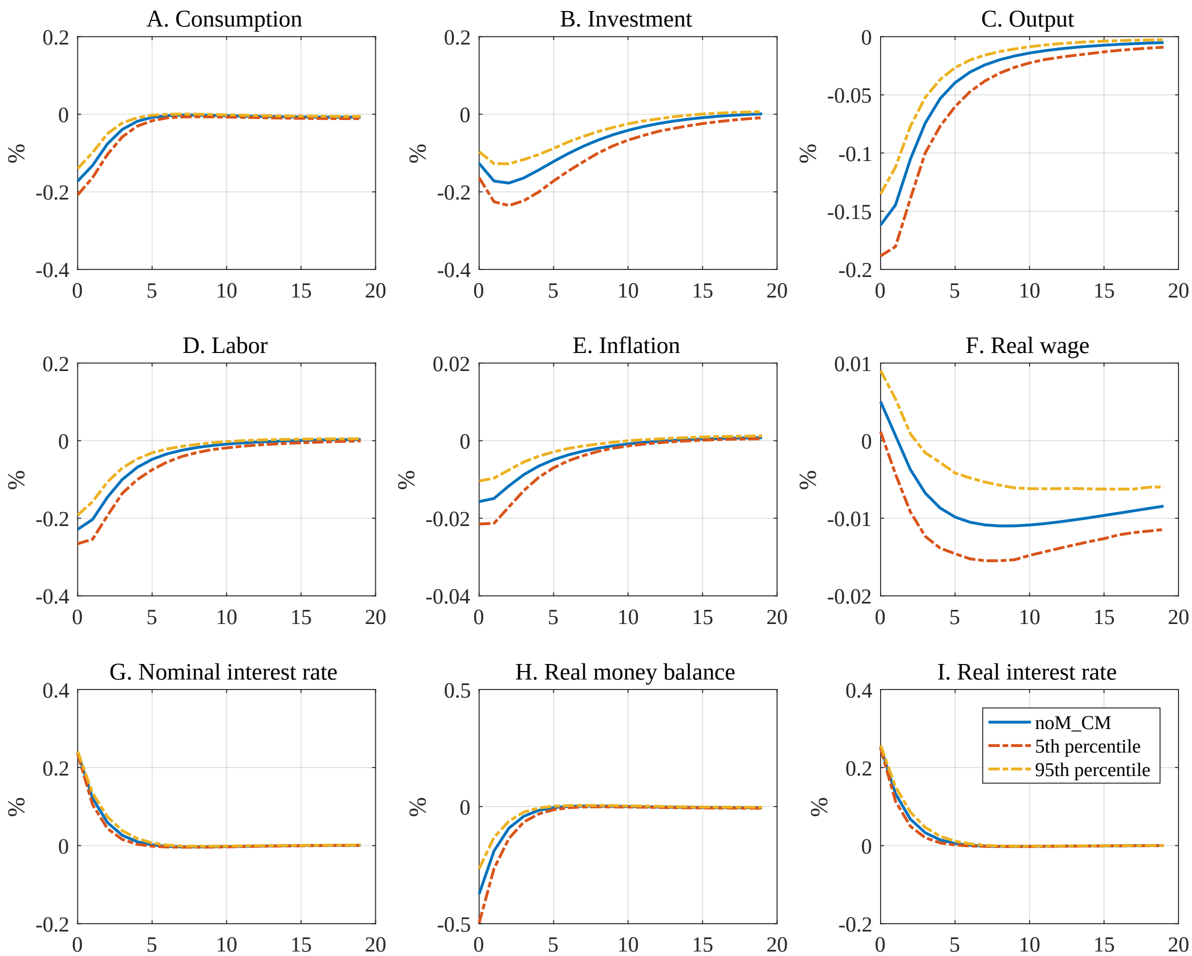

Figure 3: Impulse responses to a 25-basis-point positive monetary policy shock, baseline model, restricted model with consumption-money non-separability (no$\mathcal{M}_\mathcal{CM}$). Sample: 1967:Q1-2008:Q2. Solid line: impulse response at the posterior mean parameters; Dashed line: 5th and 95th percentiles. The vertical axis is percentage deviation from the steady state.

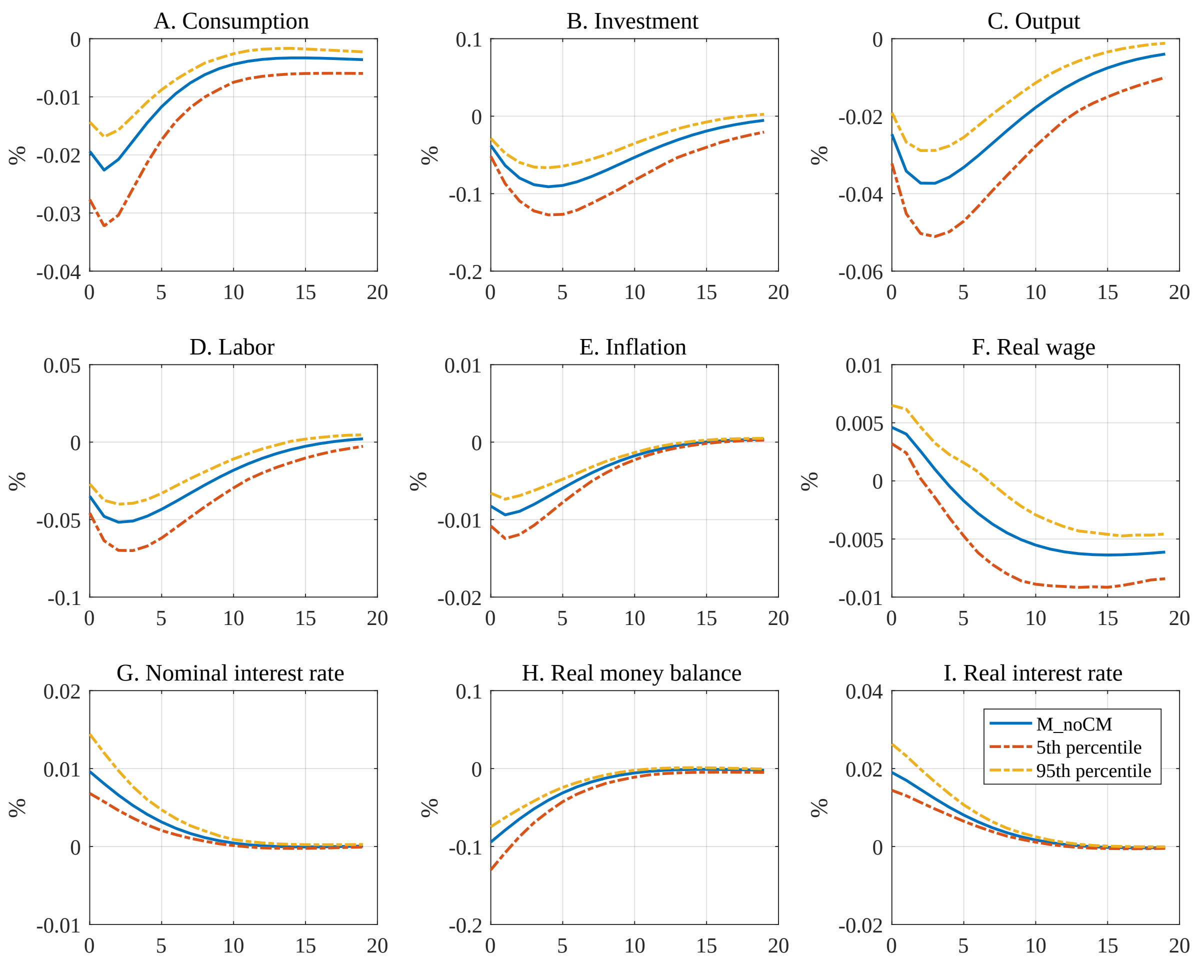

Figure 4: Impulse responses to a 25-basis-point positive monetary policy shock, baseline model, unrestricted model without consumption-money non-separability ($\mathcal{M}$_no$\mathcal{CM}$). Sample: 1967:Q1-2008:Q2. Solid line: impulse response at the posterior mean parameters; Dashed line: 5th and 95th percentiles. The vertical axis is percentage deviation from the steady state.

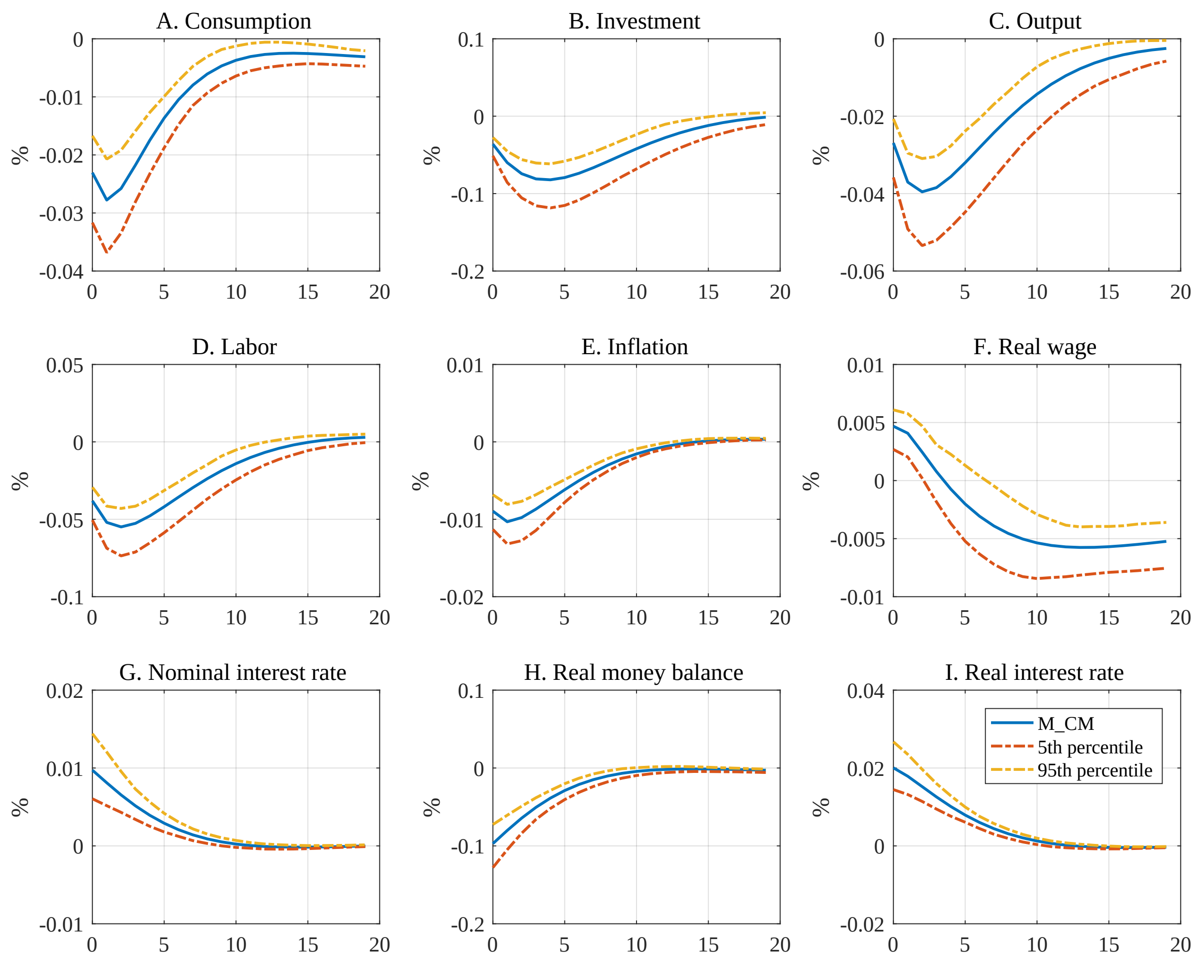

Figure 5: Impulse responses to a 25-basis-point positive monetary policy shock, baseline model, unrestricted model with consumption-money non-separability ($\mathcal{M}\_\mathcal{CM}$). Sample: 1967:Q1-2008:Q2. Solid line: impulse response at the posterior mean parameters; Dashed line: 5th and 95th percentiles. The vertical axis is percentage deviation from the steady state.

6 Forecast Error Variance Decomposition of no$\mathcal{M}_\mathcal{CM}$ and $\mathcal{M}\_\mathcal{CM}$: Baseline Estimation

Table 1-6 are the forecast error variance decompositions for output, inflation, and the nominal interest rate at six different time horizons under the restricted baseline model with consumption-money non-separability (no$\mathcal{M}_\mathcal{CM}$) and the unrestricted baseline model with consumption-money non-separability ($\mathcal{M}_\mathcal{CM}$).

Table 1: Forecast error variance decomposition, baseline model, output, no$\mathcal{M}_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	25.37	19.46	6.74	0.00	4.33	6.27	18.74	19.10
4	41.00	24.01	3.04	0.00	7.61	8.81	7.08	8.45
8	51.99	21.38	1.69	0.00	8.57	6.93	4.82	4.60
12	58.78	18.16	2.18	0.00	8.21	5.25	4.09	3.33
20	65.50	13.75	4.65	0.00	6.75	3.60	3.46	2.29
40	70.16	9.03	9.86	0.00	4.49	2.26	2.77	1.44

Table 2: Forecast error variance decomposition, baseline model, output, $\mathcal{M}\_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	8.32	18.58	7.79	6.75	2.61	5.33	46.63	4.00
4	19.56	32.06	3.36	5.69	6.63	11.81	16.73	4.15
8	28.74	33.12	1.49	3.36	9.08	12.82	8.42	2.97
12	35.40	31.34	1.32	2.40	9.84	11.37	6.02	2.32
20	43.32	28.86	1.34	1.81	9.40	8.96	4.53	1.78
40	50.96	26.44	1.21	1.43	7.86	7.07	3.61	1.42

Table 3: Forecast error variance decomposition, baseline model, inflation, no$\mathcal{M}_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	21.78	3.84	3.31	0.00	13.56	54.58	1.13	1.80
4	24.72	5.47	5.35	0.00	15.84	45.34	1.31	1.97
8	26.44	5.65	7.56	0.00	16.35	40.52	1.54	1.94
12	26.81	5.44	9.37	0.00	16.08	38.76	1.68	1.87
20	26.72	5.80	12.87	0.00	15.10	35.88	1.90	1.73
40	26.99	6.22	22.39	0.00	12.25	28.39	2.38	1.39

Table 4: Forecast error variance decomposition, baseline model, inflation, $\mathcal{M}\_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	26.56	6.27	1.57	1.69	12.36	43.98	3.76	3.81
4	27.77	10.18	2.03	2.28	12.62	33.70	4.23	7.19
8	26.95	10.40	2.03	2.26	11.97	33.05	4.17	9.18
12	25.39	12.06	1.91	2.85	11.63	33.27	3.96	8.94
20	23.41	17.12	1.77	4.01	10.83	30.94	3.70	8.23
40	23.45	18.09	1.74	4.27	10.58	30.22	3.62	8.03

Table 5: Forecast error variance decomposition, baseline model, nominal interest rate, no$\mathcal{M}_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	1.90	0.46	0.37	0.00	1.25	5.13	0.16	90.74
4	7.81	2.69	2.21	0.00	5.45	15.73	0.66	65.46
8	10.26	3.95	4.11	0.00	7.14	16.26	1.04	57.25
12	10.67	3.98	5.42	0.00	7.33	15.80	1.25	55.55
20	10.74	4.12	7.63	0.00	7.19	15.26	1.56	53.51
40	11.60	4.79	14.36	0.00	6.52	13.44	2.24	47.07

Table 6: Forecast error variance decomposition, baseline model, nominal interest rate, $\mathcal{M}\_\mathcal{CM}$, 1967Q1–2008Q2

Quarter	a	μ	d	ν	η	ε	g	R
1	0.72	0.07	22.98	66.15	0.00	0.37	0.61	9.09
4	1.09	2.85	21.98	59.29	1.30	5.44	0.28	7.78
8	3.22	9.79	19.55	49.87	2.92	8.04	0.57	6.05
12	4.18	14.67	18.15	45.72	3.33	7.71	0.89	5.35
20	4.70	16.40	17.52	44.32	3.37	7.36	1.25	5.09
40	5.91	17.67	16.80	42.57	3.37	7.08	1.73	4.87

7 Pitfall of Small-scale DSGE Models: A Simulation Study

The small-scale DSGE model in our simulation study basically follows Canova and Menz (2011, CM2011 hereafter). We keep the notation of variables and parameters as in CM2011. For the purpose of clarity, the log-linearized model equations are presented as below:

$$\begin{aligned}
\hat{y}_t &= \frac{1}{1+h}E_t\hat{y}_{t+1} + \frac{h}{1+h}\hat{y}_{t-1} - \frac{\omega_1}{1+h}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} - (\hat{a}_t - E_t\hat{a}_{t+1})\right] \\
&\quad + \frac{\omega_2}{1+h}\left[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})\right] && (76) \\
\hat{m}_t &= \gamma_1(\hat{y}_t - h\hat{y}_{t-1}) - \gamma_2\hat{R}_t + [1 - (R^s - 1)\gamma_2]\hat{e}_t && (77) \\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \psi\left[\frac{1}{\omega_1}(\hat{y}_t - h\hat{y}_{t-1}) - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t\right] && (78) \\
\hat{R}_t &= \rho_r\hat{R}_{t-1} + (1-\rho_r)[\rho_y\hat{y}_t + \rho_\pi\hat{\pi}_t + \rho_m\hat{\mu}_t] + \hat{\epsilon}_t && (79) \\
\hat{\mu}_t &= \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t && (80) \\
\hat{a}_t &= \rho_a\hat{a}_{t-1} + u_t^a && (81) \\
\hat{z}_t &= \rho_z\hat{z}_{t-1} + u_t^z && (82) \\
\hat{e}_t &= \rho_e\hat{e}_{t-1} + u_t^e && (83)
\end{aligned}$$

where $[\hat{y}_t, \hat{\pi}_t, \hat{R}_t, \hat{m}_t]$ are output, inflation, nominal interest rate, and real money balance. Variables with s as superscript denote steady-state values. (76) is the consumption Euler equation, (77) is the money demand equation, (78) is the New-Keynesian Phillips curve, (79) is the monetary policy rule, (80) is the relation defining nominal money growth rate $\hat{\mu}_t$, and (81)-(83) are the AR(1) processes of preference shock $\hat{a}_t$, productivity shock $\hat{z}_t$, and money demand shock $\hat{e}_t$. The monetary policy shock $\hat{\epsilon}_t$ is assumed to be *i.i.d.* $[\sigma_a, \sigma_z, \sigma_e, \sigma_\epsilon]$ are the standard deviations of innovations to the four structural shocks. Rather than estimating the number of lags of response in the monetary policy rule, i.e., the parameter p in CM2011, we follow most of the literature and assume that all the responses are contemporaneous. Also note that $R^s = \pi^s/\beta$ and $\gamma_1 = (R^s - 1 + R^s\omega_2\frac{y^s}{m^s})(\frac{\gamma_2}{\omega_1})$.

In the simulation study, we first simulate a typical data set by treating the case no$\mathcal{M}$_no$\mathcal{CM}$ of our medium-scale DSGE model as the data-generating process (DGP). In this specification, money does not enter the monetary policy rule. Consumption and money are additively separable in utility. So the two channels through which money plays a role are turned off in the "true" economy. The length of simulated data is the same as our baseline sample. As we use the training sample method to estimate the marginal likelihood, we simulate a sample of 198 quarters and take off the first 32 quarters as the training sample. This leaves a sample of 166 quarters, which is comparable to 1967Q1–2008Q2. The parameter values in the simulation are the posterior modes of no$\mathcal{M}$_no$\mathcal{CM}$ we obtain from the baseline estimation.

We then fit the small-scale DSGE model presented above to the simulated data. To investigate the role of money, we consider four specifications of the small-scale model, i.e., no$\mathcal{M}$_no$\mathcal{CM}$, no$\mathcal{M}$_$\mathcal{CM}$, $\mathcal{M}$_no$\mathcal{CM}$, and $\mathcal{M}$_$\mathcal{CM}$. The way we classify these cases is the same as in our medium-scale models. In the small-scale model, whether money enters the monetary policy rule depends on ρ_m. When money does not enter the policy rule, we set

$\rho_m = 0$. Otherwise, it is freely estimated. On the other hand, the non-separability of consumption and money in utility is controlled by ω_2. If they are additively separable, we set $\omega_2 = 0$. Otherwise, it is freely estimated. Following the literature with small-scale DSGE models, the estimation is based on a smaller set of observable variables, which are extracted from the simulated data set and include output, inflation, nominal interest rate, and real money balance. The measurement equations are

$$\begin{bmatrix} \mathrm{GDP}_t \\ l\mathrm{INF}_t \\ l\mathrm{INT}_t \\ \mathrm{MON}_t \end{bmatrix} = \begin{bmatrix} 0 \\ 100\log(\pi^s) \\ 100\log(R^s) \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{m}_t \end{bmatrix} \tag{84}$$

where $[\mathrm{GDP}_t, \mathrm{INF}_t, \mathrm{INT}_t, \mathrm{MON}_t]$ are variables simulated from no$\mathcal{M}$_no$\mathcal{CM}$ of our medium-scale DSGE model and the variables on the right-hand side are from the small-scale model. l stands for 100 times the log of each variable.

The four specifications of the small-scale model are estimated by Bayesian methods. As for the prior, we first fix $[\omega_1, \beta, \pi^s]$ at the values in the data-generating process. Here, ω_1 is comparable to σ in our medium-scale model. As in CM2011, we fix $\frac{y^s}{m^s}$ to 0.67. For the other parameters, we set quite loose uniform priors as in Table 7 to avoid artificial effects on the results. In addition, we require that the stationary equilibrium of the small-scale model is always determinate.

Table 8 reports posterior modes of the parameters for the four specifications of the small-scale model, together with their log marginal likelihoods. It turns out that the best fitting model is $\mathcal{M}$_no$\mathcal{CM}$. Its marginal likelihood is -692, while that of no$\mathcal{M}$_no$\mathcal{CM}$ is -698. So the Bayes factor between these two cases is exp(6), which is a very strong signal that a nontrivial role of money in the policy rule is favored by the small-scale model to match the data that is simulated from the medium-scale model no$\mathcal{M}$_no$\mathcal{CM}$. We also see that the non-separability of consumption and money in utility is not important in fitting the data, a finding similar to the results of our baseline estimation but contrary to CM2011. We have also tried to simulate data based on parameters that are randomly chosen from the posterior distribution of the medium-scale model no$\mathcal{M}$_no$\mathcal{CM}$ and conduct the same simulation as above. The qualitative results of the simulation turn out to be very robust.

Table 7: Prior distributions, small-scale model

Name	Density	Lower Bound	Upper Bound
h	Uniform	0	0.99
ρ_r	Uniform	0	0.99
ρ_y	Uniform	0	1
ρ_π	Uniform	1.01	5
ψ	Uniform	0.01	2
γ_2	Uniform	-1	5
ρ_a	Uniform	0	0.99
ρ_z	Uniform	0	0.99
ρ_e	Uniform	0	0.99
$100\sigma_a$	Uniform	0	20
$100\sigma_z$	Uniform	0	20
$100\sigma_e$	Uniform	0	20
$100\sigma_\epsilon$	Uniform	0	20
ρ_m	Uniform	0	3
ω_2	Uniform	0	5

Table 8: Posterior modes, small-scale model

	no$\mathcal{M}$_no$\mathcal{CM}$	no$\mathcal{M}$_$\mathcal{CM}$	$\mathcal{M}$_no$\mathcal{CM}$	$\mathcal{M}$_$\mathcal{CM}$
Log marginal likelihood	-698	-698	-692	-701
h	0.35	0.34	0.35	0.35
ρ_r	0.50	0.49	0.44	0.45
ρ_y	0.02	0.02	0.01	0.02
ρ_π	1.45	1.44	1.29	1.35
ψ	0.02	0.03	0.02	0.03
γ_2	1.31	1.30	1.45	1.42
ρ_a	0.94	0.94	0.94	0.94
ρ_z	0.97	0.98	0.97	0.97
ρ_e	0.96	0.97	0.97	0.97
$100\sigma_a$	3.32	3.14	3.18	3.41
$100\sigma_z$	2.90	2.85	2.94	2.90
$100\sigma_e$	1.06	1.06	1.07	1.06
$100\sigma_\epsilon$	0.64	0.64	0.70	0.69
ρ_m	0	0	0.14	0.12
ω_2	0	0.00	0	0.00

[1] no$\mathcal{M}$_no$\mathcal{CM}$: restricted model without consumption-money non-separability.
[2] no$\mathcal{M}$_$\mathcal{CM}$: restricted model with consumption-money non-separability.
[3] $\mathcal{M}$_no$\mathcal{CM}$: unrestricted model without consumption-money non-separability.
[4] $\mathcal{M}$_$\mathcal{CM}$: unrestricted model with consumption-money non-separability.

References

[1] Canova, F., and T. Menz (2011): "Does Money Matter in Shaping Domestic Business Cycles? An International Investigation." *Journal of Money, Credit and Banking*, 43(4), 577-607.

[2] Li, B., Q. Liu, and P. Pei (2019): "Investigating the Role of Money in the Identification of Monetary Policy Behavior: A Bayesian DSGE Perspective." Working paper.

[3] Sims, C.A. (2001): "Solving Linear Rational Expectations Models." *Journal of Computational Economics*, 20(1-2), 1-20.