## **ONLINE APPENDIX**

APPENDIX A: MODEL SOLUTION To simplify the exhibition, we substitute the policy rules (3.3)-(3.4) for  $(\hat{R}_t, \hat{s}_t)$  in the model and rewrite the remaining trivariate LRE system in the canonical form (2.1)

where

$$\begin{pmatrix}
d_{M,t} \\
d_{F,t}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{1-\rho_M L} & 0 \\
0 & \frac{1}{1-\rho_F L}
\end{pmatrix} \begin{pmatrix}
\epsilon_{M,t} \\
\epsilon_{F,t}
\end{pmatrix}$$
(A.2)

and the solution  $x_t = C(L)\epsilon_t$  to (A.1) is taken to be covariance stationary. Below we closely follow the solution procedure laid out in Section 2.1 and the notations established therein to derive the content of  $C(\cdot)$ .

DETERMINACY First, transform the time-domain system (A.1) into its equivalent frequencydomain representation. Appealing to the Wiener-Kolmogorov optimal prediction formula, we can evaluate the vector of expectational errors as  $\eta_{t+1} = C_0 L^{-1} \epsilon_t$ . Define  $\Gamma(L) \equiv \Gamma_{-1} L^{-1} + \Gamma_0 + \Gamma_1 L$ and substitute  $x_t$ ,  $\eta_{t+1}$ , and (A.2) into (A.1)

$$\Gamma(L)C(L)\varepsilon_t = (\Psi_0 A(L) + \Gamma_{-1}C_0 L^{-1})\epsilon_t$$

which must hold for all realizations of  $\epsilon_t$ . Therefore, the coefficient matrices are related by the z-transform identities

$$z\Gamma(z)C(z) = z\Psi_0A(z) + \Gamma_{-1}C_0$$

where C(z) needs to have only non-negative powers of z and be analytic inside the unit circle so that its coefficients are square-summable by covariance stationarity.

Second, apply the Smith canonical factorization to the polynomial matrix  $z\Gamma(z)$ 

$$z\Gamma(z) = U(z)^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & z(z - \lambda_1)(z - \lambda_2)(z - \lambda_3) \end{pmatrix} V(z)^{-1}$$

with

$$\lambda_1 = \frac{\gamma_1 + \sqrt{\gamma_1^2 - 4\gamma_0}}{2\gamma_0}, \quad \lambda_2 = \frac{\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0}}{2\gamma_0}, \quad \lambda_3 = \frac{\beta}{1 - \gamma(1 - \beta)}$$

where  $\gamma_0 = (1 + \alpha \sigma \kappa)/\beta$  and  $\gamma_1 = (1 + \beta + \sigma \kappa)/\beta$ . The zero root arises whenever the model is forward-looking, i.e.,  $\Gamma_{-1} \neq 0.^{25}$  The root  $\lambda_3$  emerges as the reciprocal of the eigenvalue from the government budget constraint (3.5) viewed as a difference equation in  $\hat{b}$ . To see where the pair of roots  $(\lambda_1, \lambda_2)$  comes from, combine the dynamic IS equation (3.1) and the new Keynesian Phillips curve (3.2) and substitute out  $\hat{y}$  to obtain a second order expectational difference equation for inflation

$$\mathbb{E}_t \hat{\pi}_{t+2} - \frac{1 + \beta + \sigma\kappa}{\beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{1 + \alpha\sigma\kappa}{\beta} \hat{\pi}_t = -\frac{\sigma\kappa}{\beta} \epsilon_{M,t}$$

The eigenvalues governing the dynamics of this equation are exactly  $(1/\lambda_1, 1/\lambda_2)$ .

Lastly, examine the existence and uniqueness of solution. Under regime-M with  $\alpha > 1$  and  $\gamma > 1$ , it follows that  $0 < \lambda_2 < \lambda_1 < 1 < \lambda_3$ . Collect the roots inside the unit circle in S(z) and multiply both sides of the z-transform identities by  $S(z)^{-1}$ 

$$T(z)C(z) = \begin{pmatrix} U_{1.}(z) \\ \\ U_{2.}(z) \\ \\ \frac{1}{z(z-\lambda_1)(z-\lambda_2)}U_{3.}(z) \end{pmatrix} (z\Psi_0A(z) + \Gamma_{-1}C_0)$$

These identities are valid for all z on the open unit disk except for  $z = 0, \lambda_1, \lambda_2$ . But since

C(z) must be well-defined for all |z| < 1, this condition places the following restrictions on the unknown coefficient matrix  $C_0$ 

$$U_{3}(z)(z\Psi_0A(z) + \Gamma_{-1}C_0)|_{z=0,\lambda_1,\lambda_2} = 0$$

Stacking the above restrictions yields

$$-\underbrace{\begin{pmatrix}\frac{\lambda_1^2\lambda_3\kappa(\alpha\beta-1)}{(1+\alpha\sigma\kappa)\beta} & \frac{\lambda_1^2\lambda_3(\alpha\beta-1)(\sigma\kappa+\beta)}{(1+\alpha\sigma\kappa)\beta} - \frac{\lambda_1\lambda_3(\alpha\beta-1)}{1+\alpha\sigma\kappa} & 0\\ \frac{\lambda_2^2\lambda_3\kappa(\alpha\beta-1)}{(1+\alpha\sigma\kappa)\beta} & \frac{\lambda_2^2\lambda_3(\alpha\beta-1)(\sigma\kappa+\beta)}{(1+\alpha\sigma\kappa)\beta} - \frac{\lambda_2\lambda_3(\alpha\beta-1)}{1+\alpha\sigma\kappa} & 0\end{pmatrix}}_{R}C_0 = \underbrace{\begin{pmatrix}\frac{\lambda_1^3\lambda_3\sigma\kappa(\alpha\beta-1)}{(1+\alpha\sigma\kappa)\beta(1-\rho_M\lambda_1)} & 0\\ \frac{\lambda_2^3\lambda_3\sigma\kappa(\alpha\beta-1)}{(1+\alpha\sigma\kappa)\beta(1-\rho_M\lambda_2)} & 0\end{pmatrix}}_{A}$$

Apparently, the solution exists because  $\operatorname{span}(A) \subseteq \operatorname{span}(R)$  is satisfied here. In order for the solution to be unique, we must be able to pin down the terms

$$QC_0 = U_{3} \cdot (\lambda_3) \Gamma_{-1} C_0 = \left( \frac{\lambda_3^3 \kappa (\alpha \beta - 1)}{(1 + \alpha \sigma \kappa) \beta} - \frac{\lambda_3^3 (\alpha \beta - 1) (\sigma \kappa + \beta)}{(1 + \alpha \sigma \kappa) \beta} - \frac{\lambda_3^2 (\alpha \beta - 1)}{1 + \alpha \sigma \kappa} - 0 \right) C_0$$

from the knowledge of  $RC_0$ . This is tantamount to verifying  $\operatorname{span}(Q') \subseteq \operatorname{span}(R')$ , which is also satisfied here.

Now the unique solution can be computed as

$$\begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{pmatrix} = (L\Gamma(L))^{-1} (L\Psi_0 A(L) + \Gamma_{-1}C_0) \begin{pmatrix} \epsilon_{M,t} \\ \epsilon_{F,t} \end{pmatrix}$$

$$= \begin{pmatrix} C_0(1,1)\frac{1}{1-\rho_M L} & 0 \\ C_0(2,1)\frac{1}{1-\rho_M L} & 0 \\ C_0(3,1)\frac{1}{(1-\frac{1}{\lambda_3}L)(1-\rho_M L)} & C_0(3,2)\frac{1}{(1-\frac{1}{\lambda_3}L)(1-\rho_F L)} \end{pmatrix} \begin{pmatrix} \epsilon_{M,t} \\ \epsilon_{F,t} \end{pmatrix}$$

where the contemporaneous responses are given by

$$C_{0} = \begin{pmatrix} -\frac{\sigma(1-\beta\rho_{M})\lambda_{1}\lambda_{2}}{(1-\rho_{M}\lambda_{1})(1-\rho_{M}\lambda_{2})\beta} & 0 \\ -\frac{\sigma\kappa\lambda_{1}\lambda_{2}}{(1-\rho_{M}\lambda_{1})(1-\rho_{M}\lambda_{2})\beta} & 0 \\ 1 - \frac{(\alpha\beta-1)[\sigma\kappa-\rho_{M}\kappa C_{0}(1,1)-\rho_{M}(\sigma\kappa+\beta)C_{0}(2,1)]}{\beta(1+\alpha\sigma\kappa)} & \frac{\beta-1}{\beta} \end{pmatrix}$$

Setting  $\rho_M = \rho_F = 0$  gives the expression for  $C_0$  in Section 3.2.1.

Under regime-F with  $0 \leq \alpha < 1$  and  $\gamma = 0$ , it follows that  $0 < \lambda_2 < \lambda_3 = \beta < 1 < \lambda_1$ . Collect the roots inside the unit circle in S(z) and multiply both sides of the z-transform identities by  $S(z)^{-1}$ 

$$T(z)C(z) = \begin{pmatrix} U_{1.}(z) \\ \\ U_{2.}(z) \\ \\ \frac{1}{z(z-\lambda_2)(z-\lambda_3)}U_{3.}(z) \end{pmatrix} (z\Psi_0A(z) + \Gamma_{-1}C_0)$$

These identities are valid for all z on the open unit disk except for  $z = 0, \lambda_2, \lambda_3$ . But since C(z) must be well-defined for all |z| < 1, this condition places the following restrictions on the unknown coefficient matrix  $C_0$ 

$$U_{3}(z)(z\Psi_{0}A(z) + \Gamma_{-1}C_{0})|_{z=0,\lambda_{2},\lambda_{3}} = 0$$

Stacking the above restrictions yields

$$-\underbrace{\begin{pmatrix}\frac{\lambda_{2}^{2}\kappa(\alpha\beta-1)}{1+\alpha\sigma\kappa} & \frac{\lambda_{2}^{2}(\alpha\beta-1)(\sigma\kappa+\beta)}{1+\alpha\sigma\kappa} - \frac{\lambda_{2}\beta(\alpha\beta-1)}{1+\alpha\sigma\kappa} & 0\\ \\ \frac{\beta^{2}\kappa(\alpha\beta-1)}{1+\alpha\sigma\kappa} & \frac{\beta^{2}(\alpha\beta-1)(\beta-1+\sigma\kappa)}{1+\alpha\sigma\kappa} & 0\end{pmatrix}}{R}C_{0} = \underbrace{\begin{pmatrix}\frac{\sigma\kappa\lambda_{2}^{3}(\alpha\beta-1)}{(1+\alpha\sigma\kappa)(1-\rho_{M}\lambda_{2})} & 0\\ 0 & \frac{\beta^{2}\sigma\kappa(1-\beta)(\alpha\beta-1)}{(1+\alpha\sigma\kappa)(1-\rho_{F}\beta)}\end{pmatrix}}{A}$$

Apparently, the solution exists because  $\operatorname{span}(A) \subseteq \operatorname{span}(R)$  is satisfied here. In order for the solution to be unique, we must be able to pin down the terms

$$QC_0 = U_{3} \cdot (\lambda_1) \Gamma_{-1} C_0 = \left( \frac{\lambda_1^2 \kappa (\alpha \beta - 1)}{1 + \alpha \sigma \kappa} \quad \frac{\lambda_1^2 (\alpha \beta - 1) (\sigma \kappa + \beta)}{1 + \alpha \sigma \kappa} - \frac{\lambda_1 \beta (\alpha \beta - 1)}{1 + \alpha \sigma \kappa} \quad 0 \right) C_0$$

from the knowledge of  $RC_0$ . This is tantamount to verifying  $\operatorname{span}(Q') \subseteq \operatorname{span}(R')$ , which is also

satisfied here.

Now the unique solution can be computed as

$$\begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{pmatrix} = (L\Gamma(L))^{-1} (L\Psi_0 A(L) + \Gamma_{-1} C_0) \begin{pmatrix} \epsilon_{M,t} \\ \epsilon_{F,t} \end{pmatrix}$$

$$= \begin{pmatrix} C_0(1,1) \frac{1 - \frac{\beta - (1+\rho_M \beta)\lambda_2 + \rho_M \beta(1+\alpha\sigma\kappa)\lambda_2^2}{\beta\lambda_2(\beta - 1+\sigma\kappa)} L}{(1 - \frac{1}{\lambda_1}L)(1 - \rho_M L)} & C_0(1,2) \frac{1}{1 - \frac{1}{\lambda_1}L} \\ C_0(2,1) \frac{1 - \frac{(1+\rho_M \beta)\lambda_2 - \beta}{\beta\lambda_2}L}{(1 - \frac{1}{\lambda_1}L)(1 - \rho_M L)} & C_0(2,2) \frac{1}{1 - \frac{1}{\lambda_1}L} \\ C_0(3,1) \frac{1}{(1 - \frac{1}{\lambda_1}L)(1 - \rho_M L)} & C_0(3,2) \frac{1}{(1 - \frac{1}{\lambda_1}L)(1 - \rho_F L)} \end{pmatrix} \begin{pmatrix} \epsilon_{M,t} \\ \epsilon_{F,t} \end{pmatrix}$$

where the contemporaneous responses are given by

$$C_{0} = \begin{pmatrix} \frac{\sigma\lambda_{2}^{2}(\beta-1+\sigma\kappa)}{(\lambda_{2}-\beta)(1-\rho_{M}\lambda_{2})} & -\frac{(1-\beta)\sigma[(\sigma\kappa+\beta)\lambda_{2}-\beta]}{(\lambda_{2}-\beta)(1-\rho_{F}\beta)} \\ -\frac{\sigma\kappa\lambda_{2}^{2}}{(\lambda_{2}-\beta)(1-\rho_{M}\lambda_{2})} & \frac{\sigma\kappa\lambda_{2}(1-\beta)}{(\lambda_{2}-\beta)(1-\rho_{F}\beta)} \\ \frac{\beta}{\lambda_{1}} + \frac{(1-\alpha\beta)[\sigma\kappa-\rho_{M}\kappa C_{0}(1,1)-\rho_{M}(\sigma\kappa+\beta)C_{0}(2,1)]}{\lambda_{1}(1+\alpha\sigma\kappa)} & \frac{\beta-1}{\lambda_{1}} + \frac{\rho_{F}(\alpha\beta-1)[\kappa C_{0}(1,2)+(\sigma\kappa+\beta)C_{0}(2,2)]}{\lambda_{1}(1+\alpha\sigma\kappa)} \end{pmatrix}$$

Setting  $\rho_M = \rho_F = 0$  gives the expression for  $C_0$  in Section 3.2.2.

INDETERMINACY Dropping the fiscal policy (3.4) and government budget constraint (3.5) and introducing the inflation forecast error  $\eta_{\pi,t} = \hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t$  into the equilirium system, (A.1)–(A.2) can be modified as

$$\mathbb{E}_{t}\left[\underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ \hline & & & \\ & &$$

where

$$\begin{pmatrix}
d_{M,t} \\
d_{\pi,t}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{1-\rho_M L} & 0 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
\epsilon_{M,t} \\
\eta_{\pi,t}
\end{pmatrix}$$
(A.4)

and we treat  $\eta_{\pi,t}$  as a new fundamental shock.

Next, apply the Smith canonical factorization to the polynomial matrix  $z\Gamma(z)$ 

$$z\Gamma(z) = U(z)^{-1} \begin{pmatrix} 1 & 0 & & 0 \\ 0 & z & & 0 \\ 0 & 0 & z(z - \lambda_1)(z - \lambda_2) \end{pmatrix} V(z)^{-1}$$

where the roots  $(\lambda_1, \lambda_2)$  are identical to those under determinacy.

Finally, examine the existence and uniqueness of solution. Under indeterminacy with  $0 \le \alpha < 1$ , it follows that  $0 < \lambda_2 < 1 < \lambda_1$ . Collect the roots inside the unit circle in S(z) and multiply

both sides of the z-transform identities by  $S(z)^{-1}$ 

$$T(z)C(z) = \begin{pmatrix} U_{1.}(z) \\ \\ \frac{1}{z}U_{2.}(z) \\ \\ \frac{1}{z(z-\lambda_2)}U_{3.}(z) \end{pmatrix} (z\Psi_0A(z) + \Gamma_{-1}C_0)$$

These identities are valid for all z on the open unit disk except for  $z = 0, \lambda_2$ . But since C(z) must be well-defined for all |z| < 1, this condition places the following restrictions on the unknown coefficient matrix  $C_0$ 

$$U_{2}(z)(z\Psi_{0}A(z) + \Gamma_{-1}C_{0})|_{z=0} = 0$$
$$U_{3}(z)(z\Psi_{0}A(z) + \Gamma_{-1}C_{0})|_{z=0,\lambda_{2}} = 0$$

Stacking the effective restrictions yields

$$-\underbrace{\left(-\frac{\lambda_{2\kappa}}{1+\alpha\sigma\kappa} \quad 0 \quad 0\right)}_{R} C_{0} = \underbrace{\left(-\frac{\lambda_{2}^{2}\sigma\kappa}{(1+\alpha\sigma\kappa)(1-\rho_{M}\lambda_{2})} \quad -\frac{\lambda_{2}[\lambda_{2}(1+\alpha\sigma\kappa)-1]}{1+\alpha\sigma\kappa}\right)}_{A}$$

Apparently, the solution exists because  $\operatorname{span}(A) \subseteq \operatorname{span}(R)$  is satisfied here. In order for the solution to be unique, we must be able to pin down the terms

$$QC_0 = U_{3\cdot}(\lambda_1)\Gamma_{-1}C_0 = \begin{pmatrix} \lambda_{1\kappa} & 0 & 0 \end{pmatrix} C_0$$

from the knowledge of  $RC_0$ . This is tantamount to verifying  $\operatorname{span}(Q') \subseteq \operatorname{span}(R')$ , which is also

satisfied here.

Now the unique solution can be computed as

$$\begin{pmatrix} \hat{y}_{t} \\ \hat{\pi}_{t} \\ \mathbb{E}_{t} \hat{\pi}_{t+1} \end{pmatrix} = (L\Gamma(L))^{-1} (L\Psi_{0}A(L) + \Gamma_{-1}C_{0}) \begin{pmatrix} \epsilon_{M,t} \\ \eta_{\pi,t} \end{pmatrix}$$
$$= \begin{pmatrix} C_{0}(1,1) \frac{1 - \frac{1}{\beta}L}{(1 - \rho_{M}L)\left(1 - \frac{1}{\lambda_{1}}L\right)} & C_{0}(1,2) \frac{1}{1 - \frac{1}{\lambda_{1}}L} \\ C_{0}(3,1) \frac{L}{(1 - \rho_{M}L)\left(1 - \frac{1}{\lambda_{1}}L\right)} & C_{0}(2,2) \frac{1}{1 - \frac{1}{\lambda_{1}}L} \\ C_{0}(3,1) \frac{1}{(1 - \rho_{M}L)\left(1 - \frac{1}{\lambda_{1}}L\right)} & C_{0}(3,2) \frac{1}{1 - \frac{1}{\lambda_{1}}L} \end{pmatrix} \begin{pmatrix} \epsilon_{M,t} \\ \eta_{\pi,t} \end{pmatrix}$$

where the contemporaneous responses are given by

$$C_{0} = \begin{pmatrix} -\frac{\sigma\lambda_{2}}{1-\rho_{M}\lambda_{2}} & -\frac{(1+\alpha\sigma\kappa)\lambda_{2}-1}{\kappa} \end{pmatrix}$$
$$0 & 1$$
$$\frac{\sigma\kappa}{(1-\rho_{M}\lambda_{2})(1+\alpha\sigma\kappa)\lambda_{1}} & \frac{1}{\lambda_{1}} \end{pmatrix}$$

Setting  $\rho_M = 0$  gives the expression for  $C_0$  in Section 3.2.3.

APPENDIX B: DATA SET Unless otherwise stated, the following data are drawn from the National Income and Product Accounts (NIPA) released by the Bureau of Economic Analysis. All data in levels from NIPA are nominal values and divided by 4. The quarterly observable sequences in the text are constructed as follows.

1. Per capita real output growth rate, YGR. Per capita real output is obtained by dividing

the gross domestic product (Table 1.1.5, line 1) by the civilian noninstitutional population (series "CNP16OV", Federal Reserve Economic Data, St. Louis Fed) and deflating using the implicit price deflator for gross domestic product (Table 1.1.9, line 1). Growth rates are computed using quarter-to-quarter log difference and converted into percentage by multiplying by 100.

- Annualized inflation rate, INF, is defined as the quarter-to-quarter log difference of the implicit price deflator for gross domestic product and converted into percentage by multiplying by 400.
- 3. Annualized nominal interest rate, INT, corresponds to the effective federal funds rate (Board of Governors of the Federal Reserve System) and is in percentage.
- 4. Per capita real debt growth rate, BGR. Per capita real debt is obtained by dividing the market value of privately held gross federal debt (Federal Reserve Bank of Dallas) by the civilian noninstitutional population and deflating using the implicit price deflator for gross domestic product. Growth rates are computed using quarter-to-quarter log difference and converted into percentage by multiplying by 100.

APPENDIX C: SUPPLEMENTARY TABLES AND FIGURES Table 6 reports the posterior estimates of model parameters based on the full band. Figures 1–4 compare the cross-correlograms of the data (black solid line with cross) with those of regime-M (blue dashed line) and regime-F (red solid line) evaluated with the posterior mean over partial bands.

	Pre-Volcker Era					Post-Volcker Era			
	Regime-M		Regime-F		-	Regime-M		Regime-F	
Para	Mean	90% HPD	Mean	90% HPD	-	Mean	90% HPD	Mean	90% HPD
$1/\sigma$	4.92	[4.42, 5.41]	5.26	[4.77, 5.77]		4.93	[4.44, 5.42]	5.14	[4.62, 5.66]
$\kappa$	0.51	[0.42, 0.59]	0.45	[0.38, 0.53]		0.51	[0.42, 0.59]	0.39	[0.33, 0.46]
$\bar{r}$	0.50	[0.33, 0.66]	0.50	[0.33, 0.65]		0.50	[0.34, 0.66]	0.51	[0.36, 0.67]
$\alpha$	1.80	[1.57, 2.01]	0.56	[0.44, 0.69]		2.24	[1.98, 2.48]	0.47	[0.35, 0.59]
$\gamma$	1.51	[1.18, 1.82]	—	_		1.50	[1.16, 1.82]	_	_
$ ho_M$	0.93	[0.91, 0.95]	0.97	[0.95, 0.98]		0.95	[0.94, 0.97]	0.95	[0.93, 0.97]
$ ho_F$	0.49	[0.34, 0.67]	0.50	[0.34, 0.67]		0.51	[0.35, 0.67]	0.50	[0.34, 0.67]
$100\sigma_M$	0.34	[0.27, 0.42]	0.25	[0.21, 0.29]		0.27	[0.22, 0.32]	0.21	[0.18, 0.24]
$100\sigma_F$	0.43	[0.28, 0.57]	0.42	[0.28, 0.57]		0.43	[0.28, 0.57]	0.43	[0.29, 0.57]
Ave Ineff		2.3		2.3			2.2		10.1

Table 6: Full Band Posterior Estimates

NOTES: See Table 2.



Figure 1: Pre-Volcker cross-correlogram estimated on high-pass band.



Figure 2: Pre-Volcker cross-correlogram estimated on low-pass band.



Figure 3: Post-Volcker cross-correlogram estimated on high-pass band.