R&D policy in the economy with structural change and heterogeneous spillovers

Anton Bondarev*

June 4, 2019

*International Business School Suzhou, Xi'an Jiaotong-Liverpool University, 8, Chongwen Road, 215123 Suzhou, P. R. China, e-mail: anton.bondarev@xjtlu.edu.cn

This research is part of the activities of SCCER CREST (Swiss Competence Center for Energy Research), which is financially supported by the Swiss Commission for Technology and Innovation (CTI) under contract KTI.2014.0114. Preliminary version of this paper has been presented at the conference "Finance and growth in the aftermath of crisis" in Milan. Author thanks the participants of the conference and H. Dawid in particular for their helpful comments.

Running head:

Structural change and R&D spillovers

Corresponding author:

Anton Bondarev,

International Business School Suzhou, Xi'an Jiaotong-Liverpool University, 8, Chongwen Road, 215123 Suzhou, P. R. China,

e-mail: anton.bondarev@xjtlu.edu.cn

Abstract

This paper develops an endogenous growth model with doubly-differentiated R&D being the growth engine. The model incorporates dynamic structural change and heterogeneous knowledge spillovers. As a result, decentralized economy may exhibit non-monotonic growth paths and declining R&D productivity. Conditions on the knowledge spillover operator granting the existence of balanced growth for first-best and market economies are obtained. Different regulation tools helpful in achieving the sustainable path and their limits are studied.

JEL classification: C61; O32; O38; O41; H3

Keywords: endogenous structural change; knowledge spillovers; dynamic stability; government regulation; sustainable growth

1 Introduction

The process of large-scale technical change involves not only the emergence of new technologies and refinement of existing ones, but also the structural change of the economy whereas some older technologies and sectors are replaced by newer ones. This is the case with renewable energy transition being actively pursued by many European economies and the modernization of developing economies where traditional sectors are gradually replaced by the more sophisticated ones.

Once such a structural change is of an issue it is important not only to understand the drivers of this change, but also the impact of the changing structure of knowledge spillovers being experienced by the R&D sector. Existing growth models fall short of capturing both these issues. First, majority of the growth literature assumes a rather simplistic structure of knowledge spillover. In particular, the intensity of spillovers is assumed to be uniform across technologies, even if dependent on the existing number of technologies, as in Peretto and Connolly (2007), Acemoglu et al. (2012) among others. The recent exceptions are Acemoglu and Cao (2015) and Chu et al. (2017) where firms' heterogeneity is allowed for but this is not attributed to the structure of R&D spillovers as a whole.

Second, the exit of outdated technologies is rarely accounted for and the range of technologies is either stabilizing in the long-run, resulting in the vertical innovations being the primary growth driver as in Peretto and Connolly (2007), or the range of sectors grow in an unlimited way, as in Chu et al. (2012). In recent Hamano and Zanetti (2017) endogenous process of firms' entry and exit is modeled, but continuous structural change is not the growth driver there. At last, in Akcigit and Kerr (2018) heterogeneous doubly-differentiated R&D is modelled, but lacking dynamic structural change.

At the same time there is increasing evidence that growth rates of the modern economy may be non-monotonic or declining (see e. g. Storper (2011), Fernald and Jones (2014), Gordon (2016)). Many papers recently tried to explain these growth fluctuations through different channels: labor and financial frictions (Mumtaz and Zanetti (2016)), financial regulations (e. g. Van der Hoog and Dawid (2019)), job destruction shocks (e. g. Zanetti (2019)). Still no fundamental theory for non-monotonic growth is present.

Moreover the R&D productivity as the primary driver of growth may be experiencing a decline. Conventional growth theories are not capable of explaining these phenomena. This paper proposes one potential source of such non-monotonic growth: heterogeneous and varying in time cross-technologies spillovers. Recent empirical findings suggest that it is mainly the *structure* and not the *intensity* of cross-technologies spillovers that governs growth, see Acemoglu et al. (2016). It is found that the structure of cross-sectoral spillovers is neither global (i. e. uniform as in standard endogenous growth models) nor local (i. e. intra-sectoral) and thus the network effects are important. It is then natural to capture those effects through a technology diffusion operator and its spectral properties.

In this paper, the model of cross-technology interactions that is more general than existing models regarding possible interdependencies of technological developments is developed. In particular I use the term of *dynamic structural change* to describe the economy with continuous reallocation of labor from outdated sectors towards emerging ones and continuous process of horizontal innovations combined with outdating of older technologies, as in Bondarev and Greiner (2019).

The contribution of this paper is threefold. First, the general properties of such a heterogeneous spillovers are studied. It is shown under which conditions the underlying economy would possess balanced growth paths in decentralized and centralized cases. It turns out that for market economy those conditions are very restrictive, whereas for a social planner they are more relaxed.

Second I find out that the first-best solution for the economy is not always feasible even without liquidity constraints for the government. Conditions for the feasibility of the sustainable structural change are established based on the spectral properties of cross-technologies interactions.

At last the characterization of the size and duration of regulation necessary to grant dynamic consistency ot such an economy in technological transition are obtained.

In the next section, the model is described. Section 3 contains main results of the paper. Section 4 studies limits of efficient regulation for this economy. Section 5 concludes. Majority of auxiliary derivations and definitions as well as proofs are to be found in the Online Appendices[1].

2 Economy

The economy consists of households supplying labor and assets, manufacturing sector supplying consumption goods and R&D sector inventing and developing new technologies. There is a continuum of sectors as well as technologies and these become outdated because of competitive pressure from newer technologies and limited resources (labour) reallocated to more profitable technologies. The economy constantly experiences a structural change in the form of emergence of new sectors and obsolence of older ones and all existing technologies are subject to cross-technologies spillovers, which make them heterogeneous.

In particular, the economy has the following ingredients:

2.1 Households

The representative household maximizes utility from consumption of a dynamic range of final products,

$$J^H = \int_0^\infty \mathbf{e}^{-\rho t} U(C) dt, \tag{1}$$

with $U(C) = \ln C$ being the utility function from composite consumption C consisting of the continuum of products,

$$C = \left[\int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{2}$$

with $1 < \varepsilon < \infty$ being the elasticity of substitution between goods.

It supplies labor to a variety of final producers

$$L = \int_{N_{min}(t)}^{N_{max}(t)} L(i,t) di,$$
$$N_{min}(t) < N_{max}(t) < N(t), \tag{3}$$

where:

- L is the total labour in the economy (equal to population),
- $L(i)$ is the employment in sector i,
- $N(t)$ is the number of products or technologies (range) invented up to time t,
- $N_{max}(t)$ is the range of manufacturing sectors with positive operating profit (any new technology does not immediately yield positive productivity),

- $N_{min}(t)$ is the range of sectors, which have disappeared from the economy up to time t.

It also supplies accumulated assets to the R&D sector, given by the flow budget constraint

$$\dot{a} = ra + L - \int_{N_{min}}^{N_{max}} P_i C_i di - Ta, \tag{4}$$

with L the numeraire so that the wage rate is equal to one and where:

- a is the value of assets being hold by the households
- r is the interest rate
- T is the income tax rate (time-varying or not)

see Online Appendix A for derivations of expenditures $E = \int_{N_{min}}^{N_{max}} P_i C_i di$. dynamics.

2.2 Final producers

Final product $C = Y = \int_{N_{min}(t)}^{N_{max}(t)} Y_i di$ is a composite of products $i \in [N_{min}(t); N_{max}(t)]$ each of which is produced in a monopolistically competitive fashion by separate firm with production technology linear in labour with productivity defined by the state of the associated technology i (its quality).

$$\forall i \in [N_{min}(t); N_{max}(t)] : \; Y_i = A_i^{\alpha} L_i \;, \tag{5}$$

where $0 < \alpha < 1$ determines the marginal product of the technological input. The productivity A_i is the result of vertical innovations that raise the quality of a given technology and that are generated by the R&D sector.

The range of operational sectors is dynamic reflecting the continuous process of arrival of new technologies and outdating of older ones. This last happens because of positive operational costs Ψ experienced by every manufacturing firm. The profit of firm i is

$$\Pi_i = P_i Y_i - L_i - \Psi \ , \tag{6}$$

yielding demand for labor and price for product i as functions of productivity A_i within the operational time $t \in [\tau_{max}(i); \tau_{min}(i)]$ with $\tau_{max}(i)$ being the time technology i becomes operational and $\tau_{min}(i)$ the time it becomes non-competitive. See derivations of labour demand L_i^D and price for final product P_i in Online Appendix B.

2.3 R&D sector

Every technology is first invented through horizontal innovations at a frontier $N(t)$ with zero productivity. It is then gradually improved until it reaches the operational frontier, $N_{max}(t)$ and keeps improving until technology is no longer profitable and reaches the obsolence frontier $N_{min}(t)$.

Both horizontal and vertical R&D are financed through assets being held by households and incentivized by patent payment $p_A(i)$ (defined by (B.6) in Online Appendix B) from manufacturing firms which captures all potential profit from subsequent production.

Thus, the total sum of both kinds of R&D investments at any time forms the demand for assets in the economy:

$$u(t) + \int\limits_{N_{min}(t)}^{N(t)} g(i,t)di = a^D(t) \ , \tag{7}$$

where

- $u(t)$ are horizontal innovations investments at time t;

- $g(i,t)$ are vertical innovations investments at time t for technology i within the range of invented and not out-dated technologies, $[N_{min}(t), N(t)]$;

- $a^D(t)$ is the total demand for assets.

At the horizontal level there is a free entry and every successful innovator is granted an exclusive patent for the technology and hence is the only firm allowed to further develop it.

The creation of new technologies (horizontal innovations) in general follows the setup of Peretto and Connolly (2007) and closely of Bondarev and Greiner (2019). Assume that new technologies appear due to knowledge creation mechanisms that are governed by private initiatives of competitive R&D firms. New technologies are created through R&D investments, $u(t)$, chosen optimally by the firms[2]:

$$\dot{N} = u(t) \ , \tag{8}$$

These are financed from the assets of the households $a(t)$ and represent a part of the total assets demand a^D in (7).

The value of horizontal R&D consists solely in expected future profits from vertical innovations:

$$V_N = \max_{u(\bullet)} \int_0^{\infty} \mathrm{e}^{-rt} \left(\pi^R(i)|_{i=N} u(t) - \frac{1}{2} u^2(t) \right) dt. \tag{9}$$

Here, the profit of developing the next technology $i = N$, $\pi^R(i)|_{i=N}$, equals the value of vertical innovations into technology i, which is given by:

$$\pi^R(i) = p_A(i) - \frac{1}{2}\int\limits_{\tau_0}^{\tau_{min}} \mathrm{e}^{-r(t-\tau_0)} g^2(i,t)dt, \tag{10}$$

with investments $g(i,t)$ going into the increase of productivity $A(i,t)$ as long as the technology is operational.

Each R&D firm derives profit from capturing all monopolistic rents from the manufacturing sector and optimally designs inter temporally investments into the development of the subsequent technology.

Apart from optimally controlled R&D efforts every technology is subject to uncontrolled *spillovers* coming from all other existing technologies (including intra-technology spillover $\theta(i,i)$), so that the *total* evolution of $A(i,t)$ is governed by two components[3]:

$$\dot{A}(i,t) = \dot{A}^P(i,t) + \dot{A}^{SP}(i,t) = g^P(i,t) + \int_{N_{min}(\tau_0(i))}^{N(\tau_{min}(i))} \theta(i,j)A(j,t)dj - A(i,t) \tag{11}$$

So the total level $A(i,t)$ which is utilized by manufacturing firms and which affects through spillovers other sectors, contains both individual investments and the spillover part. See further derivations for R&D in Online Appendix C.

Firms do not account for their impact on other technologies but only for the spillover received by the firm itself, whereas the social planner does[4]. The total knowledge spillover experienced by the economy is given by the operator Θ with entries $\theta(i,j)$ measuring individual impacts.

This operator is the central part of the analysis. It is the continuous counterpart of the technology diffusion matrix. As such it shares some properties with usual matrices but is a much more general object. In particular, it may possess much richer spectral properties. The main contribution of a paper is to find a correspondence between properties of the economy's growth rates (size, sign, sustainability, balanced growth path properties, etc.) and spectral properties of this operator Θ. At this point I set no restrictions on the structure of technological spillovers; it turns out that the more specific is the operator, the closer is the economy to a standard one.

At last decentralized equilibrium is given by all markets clearing: expenditures are constant due to price dynamics, labour is redistributed towards newer sectors from older ones, assets are growing with rate r once there is sufficient initial endowment (otherwise no growth is possible) and thus resource constraint for the R&D is not binding, i. e. there is always sufficient capital to finance R&D investments. See Online Appendix E for market clearing conditions.

3 Results

In this section I study the model described above. First the long-run behavior of both decentralized and socially optimal economies are described and then I discuss the necessity, scale and duration of interventions.

3.1 The BGP existence conditions

I first briefly state results similar for benchmark and heterogeneous cases and then add some new insights appearing due to the spillover operator Θ.

Proposition 1. *The productivity of the oldest operational sector, $A_{N_{min}}$, is equal to the productivity of the newest operational sector, $A_{N_{max}}$, at the time when the first is leaving* *the economy and the latter is entering its operational phase:*

$$A_{N_{min}} = \left((\Psi/L)(\epsilon - 1) \int_{N_{min}}^{N_{max}} A_j^{\alpha(\epsilon-1)} dj \right)^{1/\alpha(\epsilon-1)} = A_{N_{max}}. \tag{12}$$

At the same time, the productivity of each sector grows within its operational phase,

$$A_i(\tau_{min}(i)) > A_i(\tau_{max}(i)). \tag{13}$$

Proof. This follows from the zero profit condition defining the operational phase for any i (see Lemma C.2 in Online Appendix C). □

Since profit is zero on both ends of the operational phase and it is nonnegative in between it follows that

Lemma 1. *For any technology i exists at least one point $\bar{\tau}(i)$ such that the profit of the manufacturing sector is at maximum and it holds*

$$\dot{\Pi}(i) = 0 \Leftrightarrow \left(\frac{\dot{A}(i,t)}{A(i,t)} - \int_{N_{min}}^{N_{max}} \frac{\dot{A}(j,t)}{A(j,t)} dj \right) = \frac{\Psi}{\alpha L}(\dot{N}_{max} - \dot{N}_{min}). \tag{14}$$

Proof. By mean value theorem, see any analysis textbook, as Stromberg (1981). □

By Lemma 1 there are exactly two options for (14) to hold: either both sides simultaneously equal zero (which is the case for the baseline symmetric model) or generically,

$$\forall i \in [0, \infty] : \; \exists \bar{\tau}(i) : \; \frac{g_{A(i)}(\bar{\tau}(i)) - \bar{g}_A(\bar{\tau}(i))}{\dot{\mathcal{O}}(\bar{\tau}(i))} = \frac{\Psi}{\alpha L} \tag{15}$$

where we denote $g_{A(i)} \overset{def}{=} \frac{\dot{A}(i,t)}{A(i,t)}$ the growth rate of productivity for technology i, $\bar{g}_A \overset{def}{=} \int\limits_{N_{min}}^{N_{max}} \frac{\dot{A}(j,t)}{A(j,t)} dj$ the average growth rate of productivities and $\dot{\mathcal{O}} \overset{def}{=} \dot{N}_{max} - \dot{N}_{min}$ the shift describing the change in the size of the economy.

We call the technology, which reaches at t maximum profit the *leading* technology. We infer the following observation from (15):

Corollary 1. *As soon as the growth rate of the leading technology is below average, the economy shrinks in size and vice versa.*

Proof. Indeed, since $\frac{\Psi}{\alpha L}$ is always positive, it follows that once the numerator of the righthandside in (15) is negative, the denominator is negative too, implying $\dot{N}_{max}(\bar{\tau}(i)) < \dot{N}_{min}(\bar{\tau}(i))$. This means the economy shrinks in size. With above average growth rate the opposite holds. □

It then follows that the output growth rate in the heterogeneous economy includes additional element defined by the Corollary 1 and can be non-monotonic and varying in sign:

Corollary 2. *The output growth may be positive or negative in the economy with heterogeneous spillovers and is given by:*

$$g_Y \overset{def}{=} \frac{\dot{Y}}{Y} = \alpha \bar{g}_A (N_{max} - N_{min}) + \frac{\Psi}{\alpha L}(\dot{N}_{max} - \dot{N}_{min}) \gtreqless 0 \tag{16}$$

Proof. Follows the same lines as in the baseline model for positive growth except that condition $\dot{N}_{max} = \dot{N}_{min}$ does not always hold. It then follows that once economy shrinks, growth may be negative. □

In particular, if the growth rate is positive together with the core expansion $\dot{\mathcal{O}} > 0$, such type of growth is not sustainable: as long as the expansion happens long enough as for $t^E : N_{max}(t^E) - N_{min}(t^E) \to \infty$ to realize, the growth becomes undefined, since the economy would consist of the infinitely many infinitely small sectors with infinitely low productivity.

On the other hand, if the second term is negative, implying core of the economy shrinks, this type of growth, even with positive growth rates, is also unsustainable in the long-run: as soon as for $t^S : N_{max}(t^S) - N_{min}(t^S) = 0$ new sectors become outdated at the very same moment they become operational. Then the long-term growth rate is zero, since $N_{max} \geq N_{min}$ cannot be violated by definition of these quantities.

Next define the balanced growth path (BGP) based on previous discussion:

Definition 1. *The BGP of the economy described in Section 2 is the path along which two conditions hold:*

1. *The output growth rate is positive and constant*

$$\forall t \in [t^0, \infty) : \; g_Y \geq 0, \dot{g}_Y = 0 \tag{17}$$

2. *The economy's size stays positive and finite:*

$$\forall t \in [t^0, \infty) : \; 0 < \mathcal{O}(t) < \infty \tag{18}$$

The first condition (17) is the standard one, implying the economy grows in the long run. It does not require all the variables to grow at the same rate, but only the output, since assets and productivity would then automatically follow some balanced growth pattern (see e. g. Barbier (1999) for close definition of BGP).

The second condition is novel and reflects the importance of structural change: as soon as the number of technologies becoming operational exceeds the number of becoming outdated on a regular basis, the size of $N_{max} - N_{min} = \mathcal{O}$ grows without bounds and the growth becomes unsustainable. One may speak of the *over-diversification* of the economy: there are too many different technologies/sectors so the limited labour force is not enough to keep them running. At the same time, if the opposite happens, $N_{max} = N_{min}$, there is only one technology present in manufacturing at any time and it varies continuously, making it impossible to invest into the rise of productivity. In this case although vertical R&D takes place rising the productivity of new technologies, these technologies cannot stay operational long enough to provide positive profits for manufacturers and thus a stimulus for further inventions. This is the situation of *overburning*: the structural change intensifies so much, that older technologies are scrapped faster, then the economy may compensate them with newer ones.

We thus see than the only usual BGP in this economy is exactly the one described by the benchmark model with constant size of the core, $\dot{\mathcal{O}} = 0$. Still, this may happen only in a very special case as the following Proposition 2 shows.

In what follows I use:

Assumption 1. *The spillover operator* Θ *is a spectral one.*

For definitions of spectral properties of Θ used throughout the paper see Appendix F. Roughly speaking the requirement to Θ to be spectral just restricts analysis to spillover operators which are the closest to finite-dimensional ones but not necessarily compact.

We are now ready to characterize the BGP existence of the decentralized economy.

Proposition 2 (On BGP existence for decentralised economy with spillovers)**.** *Assume* $\Theta \neq 0$ *and all spillovers are non-negative,* $\forall\{i,j\} : \theta_{i,j} \geq 0$. *Then:*

1. *As long as* Θ *is a scalar operator (see Definition F.2), the decentralized economy always possesses a BGP with constant growth rates. It is defined by the spillover*

size as

$$\bar{g}_A^{\theta} = \begin{cases} \theta - 1, & \textit{if}\, \theta > 1; \\ 1, & \textit{if}\, \theta = 1; \\ r, & \textit{if}\, \theta < 1. \end{cases} \tag{19}$$

2. *As long as Θ is a scalar-type operator (see Definition F.3), the decentralised economy possesses a BGP with constant growth rate independent of the spillover size $\bar{g}_A^0 = r$ if $\rho(\Theta) \leq 1$ and no BGP in the sense of Definition 1 otherwise;*

3. *If Θ is not a scalar-type operator, there is no BGP type Definition 1 in the decentralized economy with heterogeneous spillovers.*

Proof. see Online Appendix G. □

This proposition tells us that the market economy may converge to a BGP only if the spillovers are either intra-sectoral (spillover matrix is diagonal) or if there are cross-sectoral spillovers, they should be limited in their intensity (spectral radius does not exceed 1, i.e all entries of Θ are bounded).

The Figure 1 illustrates the conceptual reconstruction of the economy with balanced growth path for the case of balanced structural change.

There $Q(t)$ denotes the overall productivity of the economy and ellipses represent the total product of existing range of sectors. Since the variety expansion and obsolence rates are balanced, the range of existing sectors remains unchanged, but productivity of newer sectors grows due to higher abundance of assets, thus growth is balanced and sustainable.

Observe that for the social planner problem the BGP can be achieved in a wider variety of situations than for the decentralized economy:

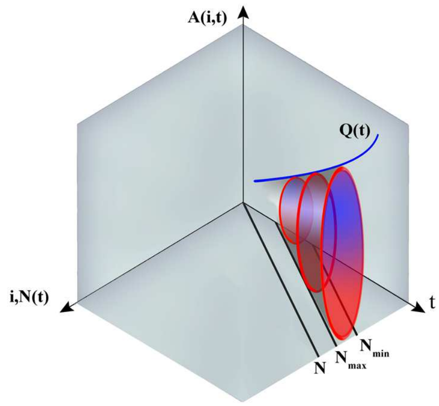

Figure 1: The BGP for the economy with $\dot{\mathcal{O}} = 0$

Proposition 3. *The socially optimal R&D system* (D.3) *admits the balanced growth path in a sense of Definition 1 as soon as either:*

1. *Operator Θ is of the scalar-type*

2. *Operator Θ is compact (see Definition F.4).*

Otherwise no socially optimal BGP exists.

Proof. see Online Appendix H □

So the economy with a social planner would converge to a BGP either if spillovers if heterogeneous, may be internalized (matrix is diagonizable) to appropriate sectors/firms or if this is not the case, at least the spillover experienced by any technology is bounded and can be traced back to the source technology.

If this is not the case, and there are distributed or non-point source spillovers in the economy, long-run stability of the BGP cannot be granted even under optimally planned development.

Define (a weaker notion of) sustained growth path:

Definition 2. *The economy follows a sustained growth path if* (18) *holds, but not necessarily* (17)*.*

It is immediate to observe that once the economy is on the sustainable growth path (SGP), it might experience prolonged periods of negative and positive growth, but the requirement (18) implies that long run growth stays positive and finite *on average*, so this economy may sustain growth for infinite time.

Figure 2 illustrates cases of unsustainable and sustainable, but non-monotonic growth paths.

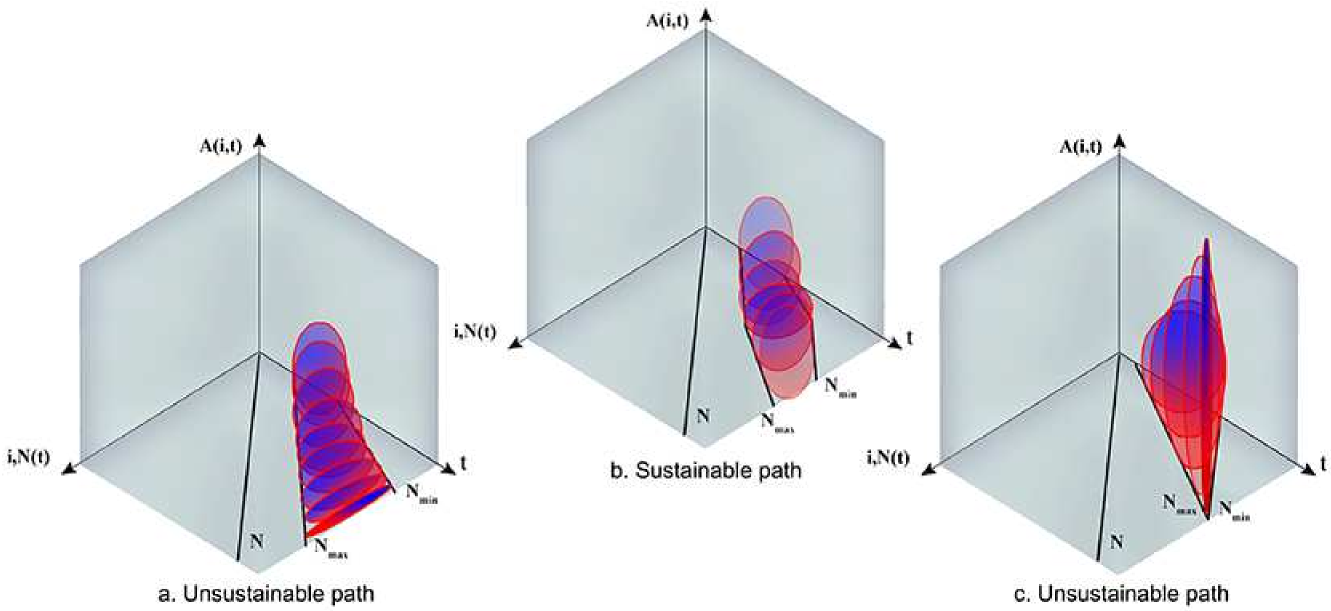

Figure 2: Non-balanced growth paths for $\dot{\mathcal{O}} \lesseqgtr 0$

In case a. $\dot{\mathcal{O}} > 0$ constantly and the economy reaches the state with infinitely many sectors of zero productivity. In case c. $\dot{\mathcal{O}} < 0$ continuously and economy ends up with a single infinitely productive technology. In case b. the growth is non-monotonic but sustainable: the size of economy neither reaches infinity nor zero. As it will be shown in the Section 3 such trajectory may be reached only with the help of the government and the market economy alone cannot return to the balanced (or sustained) path by itself once some technological shock has driven the economy away from the equilibrium.

3.2 Dynamic inefficiency

In this part of the paper the potential inefficiencies of both decentralized and centralized economy are studied.

3.2.1 Market inefficiency

Proposition 2 states that the decentralized economy admits the constant sustained growth path only as long as spillovers operator Θ is scalar or of the scalar-type and has limited size (spectral radius is small enough). Now observe that typically dynamic nature of technologies' spectrum implies that the spillovers' structure changes continuously. We thus need some additional machinery to tackle with this issue.

Consider $\Theta|_{t^0}$: restriction of the spillover operator to the (fixed) time instant t^0. This restriction is a standard operator over the space of technologies, independent of time and thus it makes sense to define the compactness[5] of this restriction:

Definition 3. *Operator Θ is said to be t^0-compact if its restriction to t^0 is compact in all existing at t^0 technologies $\mathcal{J}|_{t^0} := \{i|N_{min}(t^0) \leq i \leq N(t^0)\}$.*

Operator Θ is said to be t^0-scalar(-type) if its restriction to t^0 is scalar(-type) in all existing at t^0 technologies $\mathcal{J}|_{t^0}$.

Following Proposition 2 denote by $\mathcal{F} \subseteq \mathbb{R}_+$ those time instances t when operator Θ is t-scalar-type with small spectral radius or just scalar:

$$\mathcal{F} \stackrel{def}{=} \{t: \ \rho(\Theta(i,j)|_t) \leq 1\} \tag{20}$$

and by $\mathcal{G} \subseteq \mathbb{R}_+$ those time instances when the operator does not admit t-scalar form. Then it follows that at any time t the spillover may take one of two forms: either it admits consistent decentralised solution or not.

Denote further by $\mathbf{S}$ the form of the spillover operator at t which admits dynamically consistent market solution (e. g. scalar or scalar-type with spectral radius smaller then 1):

$$\mathbf{S} \stackrel{def}{=} \Theta|_{t\in\mathcal{F}} \tag{21}$$

Proposition 4 (Market failure)**.**
For any $t \in [0, \infty)$:

1. *As long as $\forall t \in \mathcal{F} \subseteq [0, \infty) : \ \Theta|_{t\in\mathcal{F}} = \mathbf{S}$ market solution grants sustained positive long-run growth rates to the economy and no government intervention is needed.*

2. *As soon as $\exists t_S \in \mathcal{G} \subseteq [0, \infty) : \ \Theta|_{t\geq t_S} \neq \mathbf{S}$, market solution leads to the collapse of the economy in finite time and government intervention is necessary for some positive duration starting from t_S onwards.*

Proof. See Online Appendix I □

Comment: The last property is widely known as the knife-edge property of endogenous growth models: once finely tuned conditions are violated, the economy cannot return to the balanced growth path. For discussion of these see e. g. Peretto and Valente (2015). The main novelty of the Proposition 4 lies in its dynamic nature: market economy can be efficient and sustainable for some time, but only until essentially cross-sectoral spillovers would appear in the economy. Once this is the case, such cross-sectoral interactions have to be dealt with. Moreover even when the technology causing initial spillover would be scrapped, it is not the case that the distortion caused by it will immediately stop, since there could be cascading persistent effects (for details see Section 4).

From the other hand, Proposition 4 establishes the conditions for market interventions. These need not to be constant or even continuous. Rather the government should additionally intervene in a timely manner only at those time instances $t \in \mathcal{G}$ when homogeneity condition for technologies is violated. Such a result is possible only for a dynamic range of technologies. Indeed, once we set $\dot{N} = 0$ the interactions operator Θ is fixed and its current structure fully defines whether there is a need in the regulation or not. This regulation is then permanently in place (although the scale of intervention may decrease over time) and its efficiency fully resembles that of Bondarev and Krysiak (2017) where the cross-technologies interactions are time-invariant.

3.2.2 Government inefficiency

We next tackle the question under which conditions the government regulation may help the economy to achieve the sustained growth path. Observe that the Definition 2 does not imply the uniqueness of the SGP. On the contrary there might exist a lot of evolution paths of the economy which make the $\mathcal{O}$ operator positive and finite. I refer to the SGP as the *optimal* if it yields maximal social welfare among possible SGPs. With this in mind let us consider the dynamic efficiency of the government in achieving the optimal SGP.

Proposition 5 (Government efficiency)**.**
For any $t \in [0, \infty)$:

1. *As long as* $t \in \mathcal{G}$*, but* $\Theta_{t \in \mathcal{G}}$ *is compact, government subsidies may implement the first-best solution and sustained BGP is achieved;*

2. *As soon as* $\Theta_{t\in\mathcal{G}}$ *is not compact, but its residual spectrum is null, the government policy may help the economy to approach the optimal SGP with approximation error increasing in the size of continuous spectrum of* $\Theta_{t\in\mathcal{G}}$*;*

3. *As soon as* $\Theta_{t\in\mathcal{G}}$ *is not compact and its residual spectrum is non-empty, only the economy-wide average subsidy is welfare improving, but the economy will not converge to the optimal SGP.*

Proof. see Online Appendix J □

The Proposition 5 illustrates the fact that government has limited influence on the economy: at some times it can improve upon the market failure and return the economy on the BGP, but at other times it could be the case that *any* government intervention cannot help to stabilize the economy and economy-wide crisis follows. Apparently this would be the case when some fundamentally new technology appears (like those studied in the literature on general purpose technologies (GPT), see Bresnahan (2010) for an overview) which has impact on a broad range of dispersed sectors. This would be the case 3. of the Proposition 5. If this new technology's impact is limited and affects some isolated group of industries, the case 2. realizes and the government may at least smooth away part of this influence. In normal situation case 1. realizes, when new technologies influence existing structure of the economy in a limited way[6].

In particular, both Propositions 4 and 5 pave the way to obtain non-monotonic growth rates in the otherwise fully analytic endogenous growth model. It thus may be applied to observed stylized facts concerning growth: it can be non-monotonic, growth rates may diverge across countries, government interventions are necessary but not sufficient to smooth away all fluctuations along the growth path.

4 Policy implications: Size and duration of regulation

I next ask the question on the duration of government intervention for a particular externality caused by the new technology. Let us again limit the attention to spectral operators only (see Definition F.6), which is the fairly general class of operators for which the spectral theory is well established.

Any compact operator is spectral but not vice versa. The scalar-type operator is the immediate infinite-dimensional extension of what is called semi-simple operator, that is, the one without defective eigenvalues. The nilpotent part thus would contain all of potential complexities of the operator.

First the compact case is studied, where I understand compactness in the sense of Definition 3.

4.1 t-compact case

Assume economy is evolving in such a way that Θ is compact w. r. t. $\mathcal{J}|_t$ for all time $t \in [0, t_0]$. Compact operators possess point-wise non-zero spectrum and zero as a continuous spectrum, see Kolmogorov and Fomin (1999) for details. Thus this situation falls into case 1 of Proposition 5 and we may apply the property rights reform as defined below.

Definition 4. *The property rights reform in economy is given by canonical form of the operator Θ. In particular, it assigns to each technology i all of its externalities according to the spectrum of Θ*

In particular this is the restatement of the result that operator Θ has resolution of identity and can be 'diagonalized' in a sense that it is unitary similar to the multiplication operator. The multiplication operator is an infinite-dimensional equivalent of a diagonal matrix. Thus this reform just reassigns shares of different technologies in such a way that newly defined entities do not have cross-technologies interactions, i. e. the spillover operator is 'diagonal'.

This property rights reform obviously makes sense only for the operator with non-zero pointwise spectrum, since otherwise it is not clear where to attribute some of externalities.

If we consider the case of compact operator we immediately see (with the help of Lemma F.1, see Online Appendix F) that government intervention always have only two components:

1. Rearranging property rights via the spectral decomposition of Θ
2. Subsidies/taxes for technologies with $Re(\lambda_i) > 1$
3. Subsidies/taxes for the technologies which have spillovers entering the nilpotent part of Θ

The first part is always possible once operator is compact and Lemma F.1 holds. Indeed, for any semi-simple operator in finite dimensions the Jordan canonical form (JCF, see Weintraub (2008) for example) is diagonal. Then for compact spectral infinite-dimensional case we get equivalent as the resolution of identity with pointwise eigenvalues as a diagonal infinite-dimensional matrix. Moreover, the scalar-type part then exactly corresponds to the part of cross-technologies interactions which does not require any additional intervention: the reformed operator is already a scalar one. So the only instability source left is the possibly high impact of knowledge spillover, case 2 and the complexity impacts, reflected by the nilpotent part.

Now the duration of these two types of intervention is different: the knowledge spillover $\theta(i,i)$ has to be taken care of only within the operational phase of technology i. Indeed, this is the spillover affecting only this given technology (after redefinition of property rights) and thus once it becomes non-profitable there is no need in further regulation.

The nilpotent part, however, has to be regulated for a longer time: even once technology becomes outdated, its impact on other technologies persists through cascading impacts. This cascades are long-term persistent effects which slowly deteriorate over time. The duration of these post-effects is proportional to the number of technologies being affected (size of affected cluster) and fully vanishes only once all the technologies in this affected cluster are outdated. Denote, following Bondarev and Krysiak (2017), the size of the nilpotent part by the number of eigenvalues entering it:

Definition 5. *The t-complexity of the (t-compact) operator Θ is the number of eigenvalues with different algebraic and geometric multiplicities:*

$$\chi(\Theta|_t) \stackrel{def}{=} \sum_{i}^{K} (\mu^a(\lambda_i - \mu^g(\lambda_i))) \tag{22}$$

where $\mu^g(\lambda_i), \mu^a(\lambda_i)$ are geometric and algebraic multiplicities of a given eigenvalue λ_i.

Observe that this definition makes sense only for the operator with point-wise spectrum (except zero), since for continuous and residual spectra notions of algebraic and geometric multiplicities are not well-defined. Still as long as we are in the compact case the notion of complexity is useful, as the following demonstrates

Proposition 6. *Assume $\Theta|_t$ is compact. Then:*

1. *As long as $\Theta|_t$ is of scalar-type and $\rho(\Theta|_t) \leq 1$ the government intervention for each t consists solely of rearranging property rights via canonical decomposition of Θ;*

2. *As soon as* $\Theta|_t$ *is of scalar-type but* $\rho(\Theta|_t) > 1$ *government regulation includes additionally subsidies* $\forall i_1 : \lambda(i_1) > 1$ *size the impact for the duration* $\tau_{min}(i_1) - \tau_0(i_1)$*;*

3. *As soon as* $\Theta|_t$ *contains non-zero nilpotent part and* $\infty > \chi(\Theta|_t) > 0$*, exactly* $\chi(\Theta|_t)$ *additional cascading subsidies are necessary with the duration for each cluster being* $\forall k \in \chi(\Theta|_t) : \max_k \tau_{min}(i_k) - \max_k \tau_0(i_k)$

4. *As soon as* $\chi(\Theta|_t) \to \infty$ *the regulation is permanent and continuous as long as complexity stays infinite.*

Proof. see Online Appendix K. □

So even in the best possible world of compact cross-technologies interactions there are cases when the first-best outcome may be achieved only with the help of continuous regulation. Still there are multiple instances where the time-limited and technology-specific intervention is sufficient. Observe that not only the size (intensity) of technological spillovers, but the structure (through the complexity measure) has crucial importance for the size and duration of these interventions.

However the interactions operator needs not to be compact. For example, the emergence of the drastic innovation (or GPT) would violate compactness assumption. I study what can be done in the case of a non-compact operator Θ in the concluding part of this section.

4.2 Noncompact interactions

Specify within the set $\mathcal{G}$ those time instances when Θ is t-compact and when it is not. Denote

$$
\begin{aligned}
&\mathcal{G}^1 := \{t \in \mathcal{G}\} : \sigma(\Theta|_t) = \sigma_p(\Theta|_t),\\
&\mathcal{G}^2 := \{t \in \mathcal{G}\} : \sigma(\Theta|_t) = \sigma_p(\Theta|_t) \cup \sigma_c(\Theta|_t),\\
&\mathcal{G}^3 := \{t \in \mathcal{G}\} : \sigma_r(\Theta|_t) \neq \emptyset,\\
&\mathcal{G} = \mathcal{G}^1 \cup \mathcal{G}^2 \cup \mathcal{G}^3,\ \mathcal{T} := [0, \infty) = \mathcal{G} \cup \mathcal{F}
\end{aligned}
\tag{23}
$$

where $\sigma_p(\Theta|_t), \sigma_c(\Theta|_t), \sigma_r(\Theta|_t)$ denote point-wise, continuous and residual components of the spectrum $\sigma(\Theta|_t)$ respectively.

Once $\Theta|_t$ is compact we apply Proposition 6 from above. Once it is not compact but self-adjoint, it may possess continuous, but not the residual spectrum. In other words the operator Θ at $t \in \mathcal{G}^2$ remains the spectral operator, but is no longer compact. Thus it admits the canonical decomposition into scalar-type and nilpotent parts, but not the JCD-type decomposition (formulated by Lemma F.1 in Online Appendix F), since some part of the spectrum is continuous.

In this case the new technology appearing at some $t^b \in \mathcal{G}^2$ has substantial impact on an open set of pre-existing sectors, so that the boundaries of this impact cannot be determined precisely. The government regulation would include additional corrective subsidies for a group of affected technologies up to a point when the technology generating this spillover will become outdated (and thus t-compactness is restored). This additional subsidy however cannot be finely tuned as to grant the first-best allocation, since it is not clear to what extent the technologies in the affected group experience the externality.

Thus this additional subsidy is group-specific but uniform within the affected group. This inefficiency comes into being because for the case of continuous spectrum the complexity $\chi(\Theta|_t)$ is not well-defined: one could count the number of affected clusters, but not the number of affected technologies within each cluster. The canonical decomposition yields the sum of multiplication-similar operator (which is then subject to property rights reform) and the nilpotent operator which contains all essentially complex interactions, but they are not isolated as in the compact case.

At last, once at some $t^n \in \mathcal{G}^3$ the new technology appears which is GPT-like and affects the whole structure of the economy, the interactions operator exhibits residual spectral component (i. e. is not self-adjoint and not compact). In this case there is no government policy in the class of subsidies which would allow the economy to approach the first-best BGP (since this does not exist at all).

5 Conclusion

In this paper the novel endogenous growth framework with dynamic structural change is used to study the role of cross-technologies interactions. These interactions are represented as a general infinite-dimensional matrix of pairwise technology interaction intensities. The overall knowledge spillover experienced by a given technology is then a result of the summing up individual impacts of all existing technologies weighted by intensity of the influence and the level of development of those technologies themselves. Such a shape of the spillover is fairly flexible and can capture the standard type of knowledge spillovers studied previously in the literature (like Peretto and Smulders (2002)) but also new types of spillovers. For example it allows for one-way (asymmetric) spillovers and heterogeneity of interactions.

It turns out that the decentralized economy would possess a balanced growth path exhibiting constant growth rates only for the very special type of knowledge spillover, which is by no coincidence the only one previously studied. These are scalar interactions describing equal spillover intensities for all the technologies dependent only of the technology itself. In all other cases the market cannot provide the dynamically consistent way of technological transition without government interventions. This result is independent of the specific model at hand, since unequal spillovers lead to competitive advantage of some of the technologies and the capital mobility ensures these are the only surviving technologies in the long run. On the other hand the first-best solution is dynamically consistent in a wider variety of cases. Namely it suffices for cross-technologies interactions to form a compact operator, i. e. all interactions are well defined on closed sets and are bounded in size. If this is the case the socially optimal solution may be achieved through a set of taxes/subsidies on the R&D eliminating those disbalancing competitive advantages.

In particular, two different regulation tools are suggested: the redefinition of property rights such that every technology becomes separated and independent of all the others and additional taxation of those sectors which grow faster then the average growth rate of the economy.

The second contribution of the paper is the general characterization of those cases, when not only the market but the social planner's solution cannot grant the dynamical efficiency to the economy. These are cases when the spillover operator is more complex and includes spillovers affecting a significant range of technologies and those for which the source cannot be identified precisely. In the latter case the balanced growth may be at least approximated whereas in the former case there is no way to achieve the first-best growth rates.

At last the characterization of the size and duration of interventions has been carried out. The time-varying economic policy impulses which depend on the spectral structure of the spillover operator in a general way are introduced. The periods of qualitatively different regulation are defined solely by the spectral structure of the spillover operator.

Despite their abstract nature, the results of this paper have immediate policy implications. First, it is crucial to take into account not only the intensity, but also the scope and structure of knowledge spillovers and technologies' interactions when designing the R&D policy.

Second, if the economy is undergoing structural change such that older technologies and sectors disappear and newer ones emerge (as is the case with large-scale renewable energy transitions) the decentralized economy is unlikely to achieve the balanced growth without government interventions. However some sustained growth may be achieved at least temporarily, but the balanced growth cannot be restored automatically due to distortions being brought into the system by the advance of newer technologies.

Third, the efficiency of government regulation of a large scale structural change depends crucially on the scale of such changes and the scope of affected sectors. If only isolated sectors/technologies are affected by new technologies, the conventional subsidizing policy would be efficient in restoring the equilibrium path of the economy. If significant clusters of sectors are under impact of emerging technologies, the government may be able to achieve the sustainable, but not the balanced growth and at last if the entire economy is affected by some general purpose technology, the optimal path cannot be sustained and (temporary) crisis is unavoidable. It thus seems that the large scale technical change comes at the cost of temporary slowdown in economic performance.

There are still many directions in the suggested framework open for future research with two of them being particularly intriguing. First, the impact of free-riding technologies may turn the dynamics to be even more complicated and become unpredictable, justifying the robust policy tools choice on the macroeconomic level. Second, the distribution of some technologies spillovers may be unbounded (consider fire or electricity as examples) making some spillovers non-local. Both these directions require far more complicated analytical tools to study and are left for future research.

Notes

[1]Any additional equations, definitions, lemmas and assumptions used in Appendices are referenced with letters and numbers to separate them from main text objects. The Online Appendix is available at University of Liverpool repository at https://elements.liverpool.ac.uk/repository.html?pub=384615

[2]of course the horizontal dimension may include knowledge spillovers as in standard endogenous growth models. This is assumed away to streamline the exposition.

[3]Observe that the actual decay rate of technology i equals $(1-\theta(i,i))A(i,t)$ and can be positive or negative (intra-technology spillover)

[4]See Online Appendix D for the social planner's problem formulation.

[5]The linear operator is compact if it maps bounded subsets of the domain into relatively compact subsets of its range, see e. g. Kolmogorov and Fomin (1999).

[6]Compactness of the operator means it maps bounded sequences to bounded sequences, thus spillovers are smoothly distributed and bounded.

References

Acemoglu, D., Akcigit, U. and Kerr, W. R. (2016) Innovation Network. *Proceedings of the National Academy of Sciences*, 113(41): 11483–11488.

Acemoglu, D. and Cao, D. (2015) Innovation by entrants and incumbents. *Journal of Economic Theory*, 157(C): 255–294.

Acemoglu, D., G. Gancia, and F. Zilibotti (2012) Competing engines of growth: Innovation and standardization *Journal of Economic Theory*, 147 (2):570–601.e3. Issue in honor of David Cass.

Akcigit, U. and W. R. Kerr (2018) Growth through heterogeneous innovations. *Journal of Political Economy*, 126(4): 1374–1443.

Barbier, E. (1999) Endogenous growth and natural resource scarcity. *Environmental and Resource Economics*, 14(1): 51–74.

Bondarev, A. and A. Greiner (2019) Endogenous growth and structural change through vertical and horizontal innovations. *Macroeconomic Dynamics*, 23(1): 52–79.

Bondarev, A. and F. Krysiak (2017) Dynamic heterogeneous r&d with cross-technologies interactions. WWZ Working paper 2017/13, University of Basel.

Bresnahan, T. (2010) General purpose technologies. In B. H. Hall and N. Rosenberg (eds.), *Handbook of the Economics of Innovation, Volume 2*, pp. 761–791. North-Holland.

Chu, A. C., G. Cozzi, Y. Furukawa, and C.-H. Liao (2017) Inflation and economic growth in a schumpeterian model with endogenous entry of heterogeneous firms. *European Economic Review*, 98: 392–409.

Chu, A. C., G. Cozzi, and S. Galli (2012) Does intellectual monopoly stimulate or stifle innovation? *European Economic Review*, 56(4): 727–746.

Fernald, J. G. and C. I. Jones (2014) The future of US economic growth. *American Economic Review*, 104(5): 44–49.

Gordon, R. (2016) *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War.* Princeton: Princeton University Press.

Hamano, M. and F. Zanetti (2017) Endogenous product turnover and macroeconomic dynamics. *Review of Economic Dynamics*, 26: 263–279.

Kolmogorov, A. and S. Fomin (1999) *Elements of the Theory of Functions and Functional Analysis.* Dover: Dover Publications; Dover Books on Mathematics edition.

Mumtaz, H., and Zanetti, F. (2016) The effect of labor and financial frictions on aggregate fluctuations. *Macroeconomic Dynamics*, 20(1):313–341.

Peretto, P. and M. Connolly (2007) The manhattan metaphor. *Journal of Economic Growth*, 12 (4): 329–350.

Peretto, P. and S. Smulders (2002) Technological distance, growth and scale effects. *The Economic Journal*, 112(481): 603–624.

Peretto, P. and S. Valente (2015) Growth on a finite planet: resources, technology and population in the long run. *Journal of Economic Growth*, 20(3): 305–331.

Storper, M. (2011) Justice, efficiency and economic geography: should places help one another to develop? *European Urban and Regional Studies*, 18(1): 3–21.

Stromberg, K. (1981) *Introduction to classical real analysis.* Wadsworth: Wadsworth International Group.

Van der Hoog, S. and Dawid, H. (2019) Bubbles, crashes, and the financial cycle: the impact of banking regulation on deep recessions. *Macroeconomic Dynamics*, 23(3): 1205–1246.

Weintraub, S. (2008) *Jordan Canonical Form: Application to Differential Equations.* San Rafael: Morgan & Claypool Publishers.

Zanetti, F. (2019) Financial shocks, job destruction shocks and labor market fluctuations. *Macroeconomic Dynamics*, 23(3):1137–1165. doi:10.1017/S1365100517000190