A Mathematical Appendix

Here we provide all the equilibrium conditions and steady state values of the model. We also provide a sensitivity analysis for the case of international goods as complements rather than substitutes.

A.1 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union.

The equilibrium conditions of the model are grouped into the following blocks:

Aggregate Demand Block

The aggregate demand block is composed of aggregate demand in both countries H:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha+\alpha^* \frac{1-h}{h}\left(\frac{\xi_t^*}{\xi_t}\right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\eta-\frac{1}{\sigma}} \left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(\eta-1)}}\right] C_t + G_t \tag{A.1.1}$$

and F:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha^*+\alpha \frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}\right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\frac{1}{\sigma}-\eta} \left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}}\right] C_t^* + G_t^* \tag{A.1.2}$$

while the evolution of private consumption is given by the households' Euler Equation in country F:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^*}\right\} \tag{A.1.3}$$

and by the international risk-sharing condition in country H:

$$C_t = \frac{h}{1-h}\left[\frac{\xi_t}{\xi_t^*}\mathcal{S}_t\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1}{1-\eta}}\right]^{\frac{1}{\sigma}} C_t^* \tag{A.1.4}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{A.1.5}$$

in country H and:

$$\Pi_t^* = \Pi_{H,t}^* \left[\frac{1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha^* + \alpha^*(\mathcal{S}_{t-1})^{\eta-1}} \right]^{\frac{1}{1-\eta}} \tag{A.1.6}$$

in country F, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{A.1.7}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{A.1.8}$$

$$\xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t} \tag{A.1.9}$$

Aggregate Supply Block

The aggregate supply block is composed of the aggregate supply equation for country H:

$$\left(\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1} \frac{K_t}{F_t} \tag{A.1.10}$$

where:

$$K_t \equiv \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{A.1.11}$$

$$F_t \equiv \xi_t (C_t)^{-\sigma} Y_t (1-\tau_t^s) + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{A.1.12}$$

and marginal cost in country H is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1-\tau_t^w)(A_t)^{1+\varphi}(h)^{\varphi+\sigma}} \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{A.1.13}$$

and the aggregate supply equation for country F:

$$\left(\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1} \frac{K_t^*}{F_t^*} \tag{A.1.14}$$

where:

$$K_t^* \equiv \xi_t^*(C_t^*)^{-\sigma} Y_t^* MC_t^* + \beta\theta^* E_t \left\{ \frac{(\Pi_{H,t+1}^*)^{\varepsilon+1}}{\Pi_{t+1}^*} K_{t+1}^* \right\} \tag{A.1.15}$$

$$F_t^* \equiv \xi_t^*(C_t^*)^{-\sigma} Y_t^* (1-\tau_t^{*s}) + \beta\theta^* E_t \left\{ \frac{(\Pi_{H,t+1}^*)^{\varepsilon}}{\Pi_{t+1}^*} F_{t+1}^* \right\} \tag{A.1.16}$$

and marginal cost in country F is given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi}(d_t^*)^{\varphi}(C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{A.1.17}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.18}$$

for country H, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.19}$$

for country F, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{A.1.20}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{A.1.21}$$

Net Exports, Net Foreign Assets and the Balance of Payments

Real Net Exports for country H are given by:

$$\widetilde{NX}_t \equiv Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{A.1.22}$$

Real Net Foreign Assets for country H are given by:

$$\widetilde{NFA}_t \equiv \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \tag{A.1.23}$$

so that real Net Foreign Assets for country H evolve according to:

$$\widetilde{NFA}_t = (1+i_{t-1})\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t \tag{A.1.24}$$

and are in zero international net supply, as also state contingent claims:

$$\int_0^h D_t^i \, di + \int_h^1 D_t^{*i} \, di = hD_t^i + (1-h)D_t^{*i} = D_t + D_t^* = 0 \tag{A.1.25}$$

Monetary Policy

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1+i_{t-1})]^{\rho_i} \tag{A.1.26}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{A.1.27}$$

Fiscal Policy in a Pure Currency Union

Fiscal policy, in a Pure Currency Union scenario, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.28}$$

for country H, and:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.29}$$

for country F, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.30}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country F) and $1-\gamma$ ($1-\gamma^*$

for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.1.31}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.1.32}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.33}$$

$$G_t^* + \tilde{T}_t^* + i_{t-1}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.34}$$

Fiscal Policy in a Coordinated Currency Union

Fiscal policy, in a Coordinated Currency Union scenario, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.35}$$

for country H, and:

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.36}$$

for country F, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.37}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country F) and $1-\gamma$ ($1-\gamma^*$ for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.1.38}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.1.39}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} = \tau^s_t Y_t + \tau^w_t MC_t d_t Y_t + \tilde{B}^G_t - \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \tag{A.1.40}$$

$$G^*_t + \tilde{T}^*_t + i_{t-1}\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} = \tau^{*s}_t Y^*_t + \tau^{*w}_t MC^*_t d^*_t Y^*_t + \tilde{B}^{*G}_t - \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} \tag{A.1.41}$$

Fiscal Policy in a Full Fiscal Union

Fiscal policy, in a Full Fiscal Union scenario, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.42}$$

$$\frac{G^*_t}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{A.1.43}$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{T}^*_t = \tilde{T}^* \qquad \text{and} \qquad \tilde{\mathcal{B}}^G_t = \frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} \implies \tilde{B}^G_t + S_t\tilde{B}^{*G}_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + S_t\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} \tag{A.1.44}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$, while distributing equally among the two countries the cost of fiscal policy, through the following tax rules:

$$\gamma(\tau^s_t - \tau^s) = (1-\gamma)(\tau^w_t - \tau^w) \tag{A.1.45}$$

$$(\tau^s_t - \tau^s) = (\tau^{*s}_t - \tau^{*s}) \tag{A.1.46}$$

$$(\tau^w_t - \tau^w) = (\tau^{*w}_t - \tau^{*w}) \tag{A.1.47}$$

with the following consolidated budget constraint:

$$G_t + \tilde{T}_t + S_t(G^*_t + \tilde{T}^*_t) + i_{t-1}\frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} = (\tau^s_t + \tau^w_t MC_t d_t)Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t)S_t Y^*_t + \tilde{\mathcal{B}}^G_t - \frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} \tag{A.1.48}$$

We can now define an equilibrium for the Currency Union.

Definition 2 (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes*

$\mathcal{X}_t \equiv \{Y_t, Y_t^*, C_t, C_t^*, \Pi_{H,t}, \Pi_{H,t}^*, \Pi_t, \Pi_t^*, \Pi_t^U, S_t, K_t, K_t^*, F_t, F_t^*, MC_t, MC_t^*, d_t, d_t^*, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$

and exogenous disturbances

$\mathcal{Z}_t \equiv \{\xi_t, \xi_t^*, A_t, A_t^*\}$

satisfying equations A.1.1 through A.1.24 and the definition of union-wide inflation A.1.27, given initial conditions

$\mathcal{I}_{-1} \equiv \{C_{-1}, C_{-1}^*, \Pi_{H,-1}, \Pi_{H,-1}^*, S_{-1}, d_{-1}, d_{-1}^*, \widetilde{NFA}_{-1}\}$

plus monetary and fiscal policies

$\mathcal{P}_t \equiv \{i_t, G_t, G_t^*, \tilde{T}_t, \tilde{T}_t^*, \tau_t^s, \tau_t^{*s}, \tau_t^w, \tau_t^{*w}, \tilde{B}_t^G, \tilde{B}_t^{*G}\}$

specified in equation A.1.26 for monetary policy and in equations A.1.28 through A.1.48 for the various specifications of fiscal policy, for $t \geq 0$.

A.2 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in Calvo (1983).

The symmetric non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations, is defined by the following assumptions and equations. All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{A.2.1}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{A.2.2}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{A.2.3}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{A.2.4}$$

Aggregate demand in each country is given by:

$$Y = \left(1 - \alpha + \alpha^* \frac{1-h}{h}\right) C + G \tag{A.2.5}$$

$$Y^* = \left(1 - \alpha^* + \alpha \frac{h}{1-h}\right) C^* + G^* \tag{A.2.6}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \left[Y^* - G^*\right] + G \tag{A.2.7}$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{A.2.8}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^s) \tag{A.2.9}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^{*s}) \tag{A.2.10}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^{\varphi}(C)^{\sigma}}{(1-\tau^w)(h)^{\varphi+\sigma}} \tag{A.2.11}$$

$$MC^* = \frac{(Y^*)^\varphi (C^*)^\sigma}{(1-\tau^{*w})(1-h)^{\varphi+\sigma}} \tag{A.2.12}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1-\tau^s)(1-\tau^w)(h)^{\varphi+\sigma}}{(Y)^\varphi} \right]^{\frac{1}{\sigma}} \tag{A.2.13}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1-\tau^{*s})(1-\tau^{*w})(1-h)^{\varphi+\sigma}}{(Y^*)^\varphi} \right]^{\frac{1}{\sigma}} \tag{A.2.14}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)} \right]^{\frac{1}{\varphi}} Y \tag{A.2.15}$$

In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \tag{A.2.16}$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{D} + \tilde{B} - \tilde{B}^G \tag{A.2.17}$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1 \right) \widetilde{NFA} \tag{A.2.18}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \tag{A.2.19}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1 \right) \widetilde{NFA} \tag{A.2.20}$$

These relations yield an equation linking output, government consumption and household consumption in the two countries in steady state, which mainly comes from the fact that net exports are in zero international net supply in steady state:

$$Y + Y^* = C + C^* + G + G^* \tag{A.2.21}$$

The household budget constraints in steady state for countries H and F are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{D} + \tilde{B}) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \tag{A.2.22}$$

$$C^* = \left(\frac{1}{\beta} - 1\right)(\tilde{D}^* + \tilde{B}^*) + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \tag{A.2.23}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{A.2.24}$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{A.2.25}$$

A.3 Welfare with International Goods as Complements

In Subsection 4.3 we show that net exports and output are less volatile when international goods are complements after a negative technology shock in country H. The domination of the income effect on the substitution effect, when international goods are complements, could also affect the welfare analysis, both in terms of Consumption Equivalent Variations and in terms of an ad hoc loss function. When the traded goods are complements, net exports are mildly driven by relative prices, because consumers substitute less between domestic and foreign goods, and targeting net exports implies a welfare cost.

As Table 4 reports, the highest welfare gains in terms of Consumption Equivalent Variations are attained in the Pure Currency Union scenario and, since the income effect dominates the substitution effect, the largest welfare gains from output stabilization are for country H. Even if there is a welfare cost on average from stabilizing the net exports gap with a low elasticity of substitution, country H has a welfare gain from stabilizing the net exports gap with respect to the case of exogenous government consumption. When international goods are complements, the substitution effect is almost absent and stabilizing net exports makes the terms of trade more volatile. Since country F has stickier prices and greater home bias, facing higher fluctuations in relative prices without the possibility of adjusting the consumption baskets is very costly for households in this country. Finally, Table 4 shows that the consolidation of budget constraints leads to welfare losses, also when the elasticity of substitution is low. This is mainly given by the fact that the dynamics of taxes in country F are not affected by the consolidation of budget constraints, while smoothing international spillovers amplifies the dynamics of the terms of trade, generating a welfare cost for country F and a small welfare gain for country H.

Table 4: Optimal Fiscal Policy Parameters and Welfare Costs based on CEV - Complements

Policy Scenarios	Optimal Parameters*		Conditional Welfare Costs		
	ψ	ψ^*	Country H	Country F	Average
PCU (exogenous)	0	0	0%	0%	0%
PCU	0.038	0.103	-2.051%	-0.245%	-0.968%
CCU	0.083	0.084	-0.039%	0.279%	0.15%
FFU	0.083	0.084	-0.019%	0.304%	0.175%
FFU (exogenous)	0	0	-0.019%	0.304%	0.175%

*The optimal parameters have been selected by maximizing the unconditional expectation of lifetime utility.

Since there is no trade-off between price and output stabilization if the international goods are complements, fiscal policy is able to successfully stabilize the output gap. For this reason, Table 5 reports that only in the Pure Currency Union scenario with countercyclical government consumption there is a gain in terms of an ad hoc loss function, compared to the case of exogenous government consumption. The intuition behind this finding is that targeting net exports, when the international trade elasticity is low, implies a higher volatility in government consumption, which increases the volatility in prices. Since net exports move in the opposite direction with respect to the terms of trade, government consumption has the opposite effect on the terms of trade with respect to the case of international substitutes. Thus, the fluctuations in relative prices are larger, but the overall volatility of output is barely affected. Finally, consolidating budget constraints partially offsets the amplification effect due to the stabilization of net exports, which is why the Full Fiscal Union scenario, despite being less stabilizing than the Pure Currency Union scenario, has a lower cost in terms of the ad hoc loss function than the Coordinated Currency Union scenario, as we can see in Table 5.

Table 5: Welfare Gains based on an ad hoc Loss Function - Complements

Policy Scenarios	Losses			Welfare Gains*		
	Country H	Country F	Average	Country H	Country F	Average
PCU (exogenous)	0.0727	0.0634	0.0671	0%	0%	0%
PCU	0.0727	0.0630	0.0669	0%	0.63%	0.36%
CCU	0.1249	0.2479	0.1987	-71.80%	-291.01%	-196.04%
FFU	0.0812	0.1034	0.0945	-11.69%	-63.09%	-40.82%
FFU (exogenous)	0.0858	0.0723	0.0777	-18.02%	-14.04%	-15.76%

* Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss in the PCU with $\psi = \psi^* = 0$.