

# SUPPLEMENTARY APPENDIX

## A.1 STRUCTURAL PARAMETERS

We employ the Sbordone (2007) and Cogley and Sbordone (2008) models. Kobayashi and Muto (2013) also employ the Cogley and Sbordone's (2008) model as the full NKPC model. Expression of output gap and natural rate in NKPC follows Section A in Kobayashi and Muto's (2013) Supplementary appendix. That is,

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (\text{A.1})$$

$$\begin{aligned} \pi_t = & [\kappa(\Pi) - (1 - \phi)^{-1} \chi(\Pi)] x_t + \beta \xi(\Pi) \mathbb{E}_t \pi_{t+1} + \beta \chi(\Pi) \mathbb{E}_t x_{t+1} \\ & + \beta \chi(\Pi) \mathbb{E}_t \sum_{j=2}^{\infty} \phi^{j-1} x_{t+j} + \beta \zeta(\Pi) \mathbb{E}_t \sum_{j=2}^{\infty} \phi^{j-1} \pi_{t+j} + \gamma(\Pi) r_t^n + u_t \end{aligned} \quad (\text{A.2})$$

$x_t$ ,  $\pi_t$ , and  $r_t^n$  denote output gap, inflation rate, and natural rate of interest. Equation (A.1) denotes the IS curve, which is derived from households' intertemporal consumption decisions. Equation (A.2) denotes the new Keynesian Phillips curve (NKPC), which is derived from the optimal price setting of firms. The NKPC is different from the standard one in that the trend inflation is included. Structural parameters are as follows:

$$\begin{aligned} \bar{\alpha}(\Pi) &= \alpha \Pi^{\epsilon-1}, \quad \vartheta(\Pi) = \Pi^{1+\omega\epsilon}, \quad \phi = \bar{\alpha}(\Pi) \beta \\ \xi(\Pi) &= \bar{\alpha}(\Pi) \vartheta(\Pi) + (1 - \bar{\alpha}(\Pi)) \left( \frac{\epsilon \vartheta(\Pi)(1 + \omega)}{1 + \omega\epsilon} - \frac{\epsilon - 1}{1 + \omega\epsilon} \right) \\ \zeta(\Pi) &= \frac{(\vartheta(\Pi) - 1)(1 - \bar{\alpha}(\Pi))(\epsilon - 1)}{1 + \omega\epsilon}, \quad \chi(\Pi) = \zeta(\Pi) \frac{(1 - \sigma^{-1})(1 - \phi)}{\epsilon - 1} \\ \kappa(\Pi) &= \frac{(1 - \bar{\alpha}(\Pi))(1 - \phi \vartheta(\Pi)) \sigma^{-1} + \omega}{\bar{\alpha}(\Pi) (1 + \omega\epsilon)}, \quad \gamma(\Pi) = \frac{\zeta(\Pi)(\sigma - 1)}{(\epsilon - 1)(1 - \phi \rho_r)} \end{aligned}$$

$\alpha$ ,  $\beta$ ,  $\omega$ ,  $\sigma$ ,  $\epsilon$ , and  $\Pi$  denote rate of keeping price, discount factor, elasticity of real marginal cost to its own output, intertemporal substitution of elasticity, elasticity of substitution among differentiated goods, and trend inflation rate in gross term. These parameters satisfy  $0 < \alpha < 1$ ,  $0 < \beta < 1$ ,  $\omega > 0$ ,  $\sigma > 0$ ,  $\epsilon > 1$ , and  $\Pi > 0$ .

To compute the model, we change the form of the NKPC simply by introducing the auxiliary variable  $h_t$ . The expression with the misspecification term is

$$x_t = \mathbb{E}_t x_{t+1} - \sigma [i_t - \mathbb{E}_t \pi_{t+1} - (r_t^n + v_t^x)], \quad (\text{A.3})$$

$$\begin{aligned} \pi_t = & [\kappa(\Pi) - (1 - \phi)^{-1} \chi(\Pi)] x_t + \beta [\xi(\Pi) - \zeta(\Pi)] \mathbb{E}_t \pi_{t+1} + h_t \\ & + \gamma(\Pi)(r_t^n + v_t^x) + (u_t + v_t^\pi), \end{aligned} \quad (\text{A.4})$$

$$h_t = \beta \chi(\Pi) \mathbb{E}_t x_{t+1} + \beta \zeta(\Pi) \mathbb{E}_t \pi_{t+1} + \phi \mathbb{E}_t h_{t+1} + v_t^h. \quad (\text{A.5})$$

## A.2 DERIVATION OF POLICY FUNCTION

We solve the model analytically for the case of discretion with the undetermined coefficient method. Combining first order conditions (5)-(9), we first erase the Lagrange multipliers.

$$x_t = - \frac{\kappa(\Pi) - \chi(\Pi)(1 - \phi)^{-1}}{\lambda(\Pi)} \pi_t, \quad (\text{A.6})$$

$$v_t^x = \frac{\gamma}{\theta} \pi_t, \quad (\text{A.7})$$

$$v_t^\pi = \frac{1}{\theta} \pi_t, \quad (\text{A.8})$$

$$v_t^h = \frac{1}{\theta} \pi_t. \quad (\text{A.9})$$

Second, inserting these conditions into Eqs.(A.4) and (A.5), we express the model with only terms  $\pi_t$  and  $h_t$ , except for the exogenous variables:

$$\begin{aligned} -S(\Pi, \theta) \pi_t + \beta [\xi(\Pi) - \zeta(\Pi)] \mathbb{E}_t \pi_{t+1} + h_t + \gamma(\Pi) r_t^n + u_t &= 0, \\ -T(\Pi) \mathbb{E}_t \pi_{t+1} + \phi \mathbb{E}_t h_{t+1} + \frac{1}{\theta} \pi_t - h_t &= 0, \end{aligned}$$

where we define  $S(\Pi, \theta)$  and  $T(\Pi)$  as follows:

$$\begin{aligned} S(\Pi, \theta) &\equiv \frac{[\kappa(\Pi) - (1 - \phi)^{-1} \chi(\Pi)]^2}{\lambda(\Pi)} - \frac{1}{\theta} (\gamma^2 + 1) + 1, \\ T(\Pi) &\equiv \frac{\beta \chi(\Pi) [\kappa(\Pi) - (1 - \phi)^{-1} \chi(\Pi)]}{\lambda(\Pi)} - \beta \zeta(\Pi). \end{aligned}$$

We guess policy functions for  $\pi_t$  and  $h_t$  as follows:

$$\pi_t = a_r r_t^n + a_u u_t, \quad (\text{A.10})$$

$$h_t = d_r r_t^n + d_u u_t. \quad (\text{A.11})$$

Inserting these policy function guesses, we obtain the following equations:

$$\begin{aligned} -S(\Pi, \theta)(a_r r_t^n + a_u u_t) + d_r r_t^n + d_u u_t + \gamma(\Pi) r_t^n + u_t &= 0, \\ \frac{1}{\theta}(a_r r_t^n + a_u u_t) - (d_r r_t^n + d_u u_t) &= 0. \end{aligned}$$

Collecting the coefficients for  $r_t^n$  and  $u_t$ :

$$\begin{aligned} -S(\Pi, \theta)a_r + d_r + \gamma(\Pi) &= 0, \\ -S(\Pi, \theta)a_u + d_u + 1 &= 0, \\ \frac{1}{\theta}a_r - d_r &= 0, \\ \frac{1}{\theta}a_u - d_u &= 0. \end{aligned}$$

We then obtain solutions:

$$a_r = \frac{1}{A(\Pi, \theta)}\gamma(\Pi), \quad a_u = \frac{1}{A(\Pi, \theta)}, \quad (\text{A.12})$$

$$d_r = -\frac{1/\theta}{A(\Pi, \theta)}\gamma(\Pi), \quad d_u = -\frac{1/\theta}{A(\Pi, \theta)}, \quad (\text{A.13})$$

where

$$A(\Pi, \theta) = -1/\theta + S(\Pi, \theta).$$

Inserting Eq.(A.12) into Eq.(A.6), we obtain the policy function of the output gap.

$$\begin{aligned} x_t &= -\frac{\kappa(\Pi) - \chi(\Pi)(1 - \phi)^{-1}}{\lambda(\Pi)}(a_r r_t^n + a_u u_t), \\ &= b_r r_t^n + b_u u_t. \end{aligned} \quad (\text{A.14})$$

Finally, inserting the policy functions of inflation and output gap into the dynamic IS equation, we obtain the policy function of the nominal interest rate.

$$\begin{aligned}
i_t &= \frac{1}{\sigma}(\mathbb{E}_t x_{t+1} - x_t) + \mathbb{E}_t \pi_{t+1} + r_t^n + v_t^x, \\
&= -\frac{1}{\sigma}[b_r r_t^n + b_u u_t] + r_t^n + \frac{1}{\theta} \pi_t, \\
&= -\frac{b_r}{\sigma} r_t^n - \frac{b_u}{\sigma} u_t + r_t^n + \frac{1}{\theta}(a_r r_t^n + a_u u_t), \\
&= -\left(\frac{b_r}{\sigma} - \frac{a_r}{\theta} - 1\right) r_t^n - \left(\frac{b_u}{\sigma} - \frac{a_u}{\theta}\right) u_t, \\
&= c_r r_t^n + c_u u_t.
\end{aligned} \tag{A.15}$$

### A.3 HOW POLICY FUNCTION DEPENDS ON TREND INFLATION

We derive cross derivatives of coefficients on policy function respect to  $\theta$  and  $\Pi$ . Derivatives depend on  $\lambda(\Pi)$ ,  $\mathcal{K}(\Pi)$ ,  $\gamma(\Pi)$ , and  $A(\Pi, \theta)$ . We calculate derivatives of these structural parameters with respect to  $\Pi$ . A derivative of  $\mathcal{K}(\Pi)$  with respect to  $\Pi$  is

$$\begin{aligned}
\frac{\partial \mathcal{K}(\Pi)}{\partial \Pi} &= -\kappa(\Pi) \times \left[ \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} + \frac{\epsilon - 1}{\Pi} \right] \\
&\quad - \kappa(\Pi) \times \frac{1 - \sigma^{-1}}{\sigma^{-1} + \omega} \times \left[ \frac{\alpha(1 + \omega\epsilon)\Pi^{\epsilon+\omega-1}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha^2(\epsilon - 1)\Pi^{2\omega-3}(\Pi^{1+\omega\epsilon} - 1)}{(1 - \alpha\Pi^{\epsilon-1})(1 - \alpha\beta\Pi^{\epsilon(1+\omega)})} \right].
\end{aligned}$$

The second bracket term corresponds to  $\partial\zeta(\Pi)/\partial\Pi$ . Under  $\sigma > 1$ , the derivative is non-positive if

$$\frac{\alpha(\epsilon - 1)\Pi^{2\omega - \epsilon(\omega+1) - 2}(\Pi^{1+\omega\epsilon} - 1)}{(1 - \alpha\beta\Pi^{\epsilon(1+\omega)})} \leq 1 + \omega\epsilon. \tag{A.16}$$

Under the calibrated parameter, Eq.(A.16) is satisfied. If  $\sigma = 1$ , the second bracket term is diminished as follows.

$$\frac{\partial \mathcal{K}(\Pi)}{\partial \Pi} = -\kappa(\Pi) \times \left[ \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} + \frac{\epsilon - 1}{\Pi} \right] \leq 0.$$

The derivative of  $\lambda(\Pi)$  with respect to  $\Pi$  is

$$\frac{\partial \lambda(\Pi)}{\partial \Pi} = -\lambda(\Pi) \times \left[ \frac{2\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \frac{\epsilon - 1}{\Pi} + \frac{1 - \sigma^{-1}}{\sigma^{-1} + \omega} \times \left( \frac{\alpha(1 + \omega\epsilon)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha^2(\epsilon - 1)(\Pi^{1+\omega\epsilon} - 1)\Pi^{2\omega-3}}{(1 - \alpha\Pi^{\epsilon-1})(1 - \alpha\beta\Pi^{\epsilon(1+\omega)})} \right) \right].$$

The derivative is non-positive if Condition (A.16) is satisfied. Similar to  $\mathcal{K}(\Pi)$ , the second bracket term diminished if  $\sigma = 1$ .

$$\frac{\partial \lambda(\Pi)}{\partial \Pi} = -\lambda(\Pi) \times \left( \frac{2\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \frac{\epsilon - 1}{\Pi} \right) \leq 0.$$

The derivative of  $\gamma(\Pi)$  respect to  $\Pi$  is

$$\frac{\partial \gamma(\Pi)}{\partial \Pi} = \gamma(\Pi) \times \left[ \frac{(1 + \omega\epsilon)\Pi^{\epsilon\omega}}{\Pi^{1+\omega\epsilon} - 1} - \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\epsilon - 1}{\Pi} + \frac{(\epsilon - 1)\alpha\beta\rho_r}{1 - \phi\rho_r}\Pi^{\epsilon-2} \right].$$

The sign of the derivative can be positive and negative under  $\sigma > 1$ . Under  $\sigma = 1$ ,  $\gamma(\Pi) = 0$ .

The derivative of  $A(\Pi, \theta)$  with respect to  $\Pi$  is as follows.

$$\begin{aligned} \frac{\partial A(\Pi, \theta)}{\partial \Pi} &= \left( \frac{\mathcal{K}(\Pi)}{\lambda(\Pi)} \right)^2 \left[ \left( 2 - \frac{2\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \left( 1 - \frac{2\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \frac{\epsilon - 1}{\Pi} \right. \\ &\quad - \frac{2\kappa(\Pi)}{\mathcal{K}(\Pi)} \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} + \frac{1 - \sigma^{-1}}{\sigma^{-1} + \omega} \times \left( 1 - \frac{2\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \\ &\quad \times \left. \left( \frac{\alpha(1 + \omega\epsilon)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha^2(\epsilon - 1)(\Pi^{1+\omega\epsilon} - 1)\Pi^{2\omega-3}}{(1 - \alpha\Pi^{\epsilon-1})(1 - \alpha\beta\Pi^{\epsilon(1+\omega)})} \right) \right] \\ &\quad - \frac{2\gamma(\Pi)^2}{\theta} \times \left[ \frac{(1 + \omega\epsilon)\Pi^\omega}{\Pi^{1+\omega\epsilon} - 1} - \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\epsilon - 1}{\Pi} + \frac{(\epsilon - 1)\alpha\beta\rho_r}{1 - \phi\rho_r}\Pi^{\epsilon-2} \right], \end{aligned}$$

since  $\kappa(\Pi) \geq \mathcal{K}(\Pi)$ ,  $(2 - 2\kappa(\Pi)/\mathcal{K}(\Pi))$  and  $(1 - 2\kappa(\Pi)/\mathcal{K}(\Pi))$  are negative. If Eq.(A.16) is satisfied, the first bracket term is negative. However, the sign in the derivative of  $\gamma(\Pi)$  is ambiguous, and the sign in the derivative of  $A(\Pi, \theta)$  is also ambiguous. Under  $\sigma = 1$ ,  $\kappa(\Pi) = \mathcal{K}(\Pi)$  and  $\gamma(\Pi) = 0$ . The derivative of  $A(\Pi, \theta)$  is reduced to the following.

$$\frac{\partial A(\Pi, \theta)}{\partial \Pi} = -\frac{\kappa(\Pi)^2}{\lambda(\Pi)} \times \left[ \frac{2\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} + \frac{\epsilon - 1}{\Pi} \right] \leq 0$$

We calculate the cross derivative of the coefficients of inflation,  $a_j(\Pi, \theta)$ ,  $j = r, u$ . The effects of the central bank's robustness to  $a_u(\Pi, \theta)$  is

$$-\frac{\partial |a_u(\Pi, \theta)|}{\partial \theta} = \frac{[\gamma(\Pi)]^2 + 2}{[\theta A(\Pi, \theta)]^2}$$

The cross derivative is

$$-\frac{\partial^2 |a_u(\Pi, \theta)|}{\partial \theta \partial \Pi} = \frac{2\gamma(\Pi)}{[\theta A(\Pi, \theta)]^2} \frac{\partial \gamma(\Pi)}{\partial \Pi} - 2 \frac{[\gamma(\Pi)]^2 + 2}{\theta^2 [A(\Pi, \theta)]^3} \frac{\partial A(\Pi, \theta)}{\partial \Pi}$$

The sign of the cross derivative is ambiguous. If the derivative of  $\partial \gamma(\Pi)/\partial \Pi \geq 0$  and condition (A.16) is satisfied, the sign of the cross derivative of  $a_u(\Pi, \theta)$  is non-negative.

Under  $\sigma = 1$ , the cross derivative is reduced to the following form.

$$-\frac{\partial^2 |a_u(\Pi, \theta)|}{\partial \theta \partial \Pi} = -\frac{4}{\theta^2 [A(\Pi, \theta)]^3} \frac{\partial A(\Pi, \theta)}{\partial \Pi} \geq 0$$

The cross derivative of  $b_u(\Pi, \theta)$  is as follows.

$$\begin{aligned} -\frac{\partial^2 |b_u(\Pi, \theta)|}{\partial \theta \partial \Pi} &= \frac{\mathcal{K}(\Pi)}{\lambda(\Pi)} \frac{[\gamma(\Pi)]^2 + 2}{[\theta A(\Pi, \theta)]^2} \times \\ &\left[ \left( 2 - \frac{\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} + \left( 1 - \frac{\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \frac{\epsilon - 1}{\Pi} - \frac{\kappa(\Pi)}{\mathcal{K}(\Pi)} \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} \right. \\ &\left. + \frac{1 - \sigma^{-1}}{\sigma^{-1} + \omega} \left( 1 - \frac{\kappa(\Pi)}{\mathcal{K}(\Pi)} \right) \left( \frac{\alpha(1 + \omega\epsilon)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha^2(\epsilon - 1)(\Pi^{1+\omega\epsilon-1})\Pi^{2\omega-3}}{(1 - \alpha\Pi^{\epsilon-1})(1 - \alpha\beta\Pi^{\epsilon(1+\omega)})} \right) \right] \\ &- \frac{\mathcal{K}(\Pi)}{\lambda(\Pi)} \frac{\partial^2 |a_u(\Pi, \theta)|}{\partial \theta \partial \Pi} \end{aligned}$$

Similar to  $a_u(\Pi, \theta)$ , the sign of the cross derivative of  $b_u(\Pi, \theta)$  is non-negative if the derivative of  $\partial \gamma(\Pi)/\partial \Pi \geq 0$  and condition (A.16) is satisfied.

Under  $\sigma = 1$ , the derivative is reduced to the following. The cross derivative of  $b_u(\Pi, \theta)$  is ambiguous.

$$\begin{aligned} &-\frac{\partial^2 |b_u(\Pi, \theta)|}{\partial \theta \partial \Pi} \\ &= \frac{\kappa(\Pi)}{\lambda(\Pi)} \frac{[\gamma(\Pi)]^2 + 2}{[\theta A(\Pi, \theta)]^2} \times \left[ \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} \right] - \frac{\kappa(\Pi)}{\lambda(\Pi)} \frac{4}{\theta^2 [A(\Pi, \theta)]^3} \frac{\partial A(\Pi, \theta)}{\partial \Pi} \end{aligned}$$

This term can be positive and negative. The term  $\frac{2}{A(\Pi, \theta)} \frac{\partial A(\Pi, \theta)}{\partial \Pi}$  also can be positive and negative, because  $A(\Pi, \theta)$  can be positive and negative depending on the value of  $\theta$ .

The cross derivative of  $c_u(\Pi, \theta)$  is as follows.

$$-\frac{\partial^2 |c_u(\Pi, \theta)|}{\partial \theta \partial \Pi} = -\frac{1}{[\theta A(\Pi, \theta)]^2} \frac{\partial A(\Pi, \theta)}{\partial \Pi} - \frac{1}{\sigma} \frac{\partial^2 |b_u(\Pi, \theta)|}{\partial \theta \partial \Pi}$$

Similarly, the cross derivative of  $c_u(\Pi, \theta)$  is also ambiguous.

$$-\frac{\partial^2 |c_u(\Pi, \theta)|}{\partial \theta \partial \Pi} = -\frac{1}{[\theta A(\Pi, \theta)]^2} \frac{\partial A(\Pi, \theta)}{\partial \Pi} + \frac{\kappa(\Pi)}{\sigma \lambda(\Pi)} \frac{2}{[\theta A(\Pi, \theta)]^2} \left[ \frac{\alpha(\epsilon - 1)\Pi^{\epsilon-2}}{1 - \alpha\Pi^{\epsilon-1}} - \frac{\alpha\beta\epsilon(1 + \omega)\Pi^{\epsilon(1+\omega)-1}}{1 - \alpha\beta\Pi^{\epsilon(1+\omega)}} - \frac{2}{A(\Pi, \theta)} \frac{\partial A(\Pi, \theta)}{\partial \Pi} \right]$$

The sign of first term is positive, because  $\frac{1}{[\theta A(\Pi, \theta)]^2} \frac{\partial A(\Pi, \theta)}{\partial \Pi}$  is negative. However, the rest of the terms are ambiguous, analogous to the case of  $b_u(\Pi, \theta)$ .

## A.4 ADDITIONAL ANALYSIS

### A.4.1 Log Utility for Consumption

This subsection derives the effects of robustness for the case where we set  $\sigma = 1$ . Structural parameters  $\gamma(\Pi)$  and  $\chi(\Pi)$  become zero and  $\mathcal{K}(\Pi)$  is reduced to  $\kappa(\Pi)$  under  $\sigma = 1$ . Tables A1-A3 show policy functions for  $i_t$ ,  $\pi_t$ , and  $x_t$  when  $\sigma = 1$ . Robust control does not affect coefficients on natural rate, because the monetary policy does not face the trade-off. The tendency for the coefficients on cost-push shock does not differ from Tables 3-5. Deviations from the rational expectation increase in tandem with the degree of trend inflation.

### A.4.2 Robustness Analysis

For baseline calibration, we apply Cogley and Sbordone's (2008) parameter values. However, various parameter values have been proposed by empirical research. Following Coibion and Gorodnichenko's (2011) analysis for robustness, we set price stickiness  $\alpha$  as 0.4 following Bilts and Klenow (2004) and 0.7 following Nakamura and Steinsson (2008). Bilts and Klenow (2004) find a firm set price every 4 to 5 months on average. On the other hand, Nakamura and Steinsson (2008) show a firm set price every 9 to 11 months on average.

Tables A4-A6 and A7-A9 show policy functions for  $i_t$ ,  $\pi_t$ , and  $x_t$  when  $\alpha = 0.4$  and  $\alpha = 0.7$ , respectively. The tendency for coefficient on cost-push shock does not differ from Tables 3-5. Deviations from the rational expectation increase with the degree of trend inflation.

$\bar{\pi}$	$\beta\xi(\Pi)$	$\beta\chi(\Pi)$	$\beta\zeta(\Pi)$	$\lambda(\Pi)$
0%	0.99	0	0	0.0062903
1%	1.0117	0.00013398	0.0087346	0.0057769
2%	1.033	0.0002519	0.016975	0.0052884
3%	1.054	0.00035389	0.024695	0.0048242

Table 1: Coefficients on Forward Terms for Degree of Trend Inflation

Parameters	Values	Explanation
$\beta$	0.99	Discount Factor
$\sigma$	1.5	Intertemporal Substitution of Consumption
$\omega$	0.429	Sensitivity of Real Marginal Cost to Output
$\alpha$	0.588	Price Stickiness
$\epsilon$	9.8	Elasticity of Substitution for Individual Goods
$\sigma_r$	1.0160	Standard Deviation of Natural Rate Shock
$\sigma_u$	0.154	Standard Deviation of Cost-Push Shock

Table 2: Parameter Values



(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	4.0729	4.1696	4.2485	4.3033
$i_t$ (Worst-Case)	4.1481	4.2845	4.4089	4.5051
Change in Percent	1.8471	2.7559	3.7771	4.6915

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1.0028	1.0062	1.0106
$i_t$ (Worst-Case)	1	1.0031	1.0105	1.0346
Change in Percent	0	0.030722	0.42327	2.3739

Table 3: Policy Function for  $i_t$  under Discretion

(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.6234	0.65499	0.68771	0.72137
$\pi_t$ (Worst-Case)	0.72214	0.76588	0.81325	0.85645
Change in Percent	15.8379	16.9303	18.2538	18.7261

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0.0013414	0.0030664	0.0053243
$\pi_t$ (Worst-Case)	0	0.0040958	0.013599	0.041227
Change in Percent	0	205.3302	343.4835	674.3166

Table 4: Policy Function for  $\pi_t$  under Discretion

(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-6.1093	-6.2545	-6.3728	-6.455
$x_t$ (Worst-Case)	-7.0769	-7.3134	-7.5361	-7.6637
Change in Percent	15.8379	16.9303	18.2538	18.7261

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	-0.012809	-0.028415	-0.047643
$x_t$ (Worst-Case)	0	-0.03911	-0.12601	-0.36891
Change in Percent	0	205.3302	343.4835	674.3166

Table 5: Policy Function for  $x_t$  under Discretion

(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	4.0729	4.2912	4.5218	4.7652
$i_t$ (Worst-Case)	4.1481	4.3491	4.5602	4.7814
Change in Percent	1.8471	1.3495	0.84766	0.33947

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1	1	1
$i_t$ (Worst-Case)	1	1	1	1
Change in Percent	0	0	0	0

Table 6: Policy Function for  $i_t$  under Discretion ( $\Pi_{\text{str}} = 1$ )

(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.6234	0.60322	0.58188	0.55938
$\pi_t$ (Worst-Case)	0.72214	0.69697	0.67077	0.64235
Change in Percent	15.8379	15.5419	15.2747	14.8319

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0	0	0
$\pi_t$ (Worst-Case)	0	0	0	0
Change in Percent	0	0	0	0

Table 7: Policy Function for  $\pi_t$  under Discretion ( $\Pi_{\text{str}} = 1$ )

(a) Coefficient on cost-push shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-6.1093	-6.4368	-6.7828	-7.1478
$x_t$ (Worst-Case)	-7.0769	-7.4372	-7.8189	-8.208
Change in Percent	15.8379	15.5419	15.2747	14.8319

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	0	0	0
$x_t$ (Worst-Case)	0	0	0	0
Change in Percent	0	0	0	0

Table 8: Policy Function for  $x_t$  under Discretion ( $\Pi_{\text{str}} = 1$ )

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	4.0689	3.9362	3.7546	3.5253
$i_t$ (Worst Case)	4.1446	4.066	3.9376	3.7588
Change in Percent	1.8589	3.2988	4.8757	6.6224

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1.002	1.0037	1.005
$i_t$ (Worst Case)	1	1.002	1.0038	1.0053
Change in Percent	0	0.0064964	0.017769	0.032925

Table 9: Policy Function for  $i_t$  under Discretion ( $\Pi_{\text{loss}} = 1$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.62376	0.67431	0.72402	0.77174
$\pi_t$ (Worst Case)	0.72278	0.7909	0.85984	0.92898
Change in Percent	15.8735	17.291	18.7584	20.3741

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0.00033803	0.00070537	0.0010938
$\pi_t$ (Worst Case)	0	0.00039648	0.00083768	0.0013167
Change in Percent	0	17.291	18.7584	20.3741

Table 10: Policy Function for  $\pi_t$  under Discretion ( $\Pi_{\text{loss}} = 1$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-6.1035	-5.9044	-5.6319	-5.2881
$x_t$ (Worst Case)	-7.0723	-6.9253	-6.6884	-6.3655
Change in Percent	15.8735	17.291	18.7584	20.3741

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	-0.0029599	-0.0054868	-0.0074949
$x_t$ (Worst Case)	0	-0.0034716	-0.006516	-0.0090219
Change in Percent	0	17.291	18.7584	20.3741

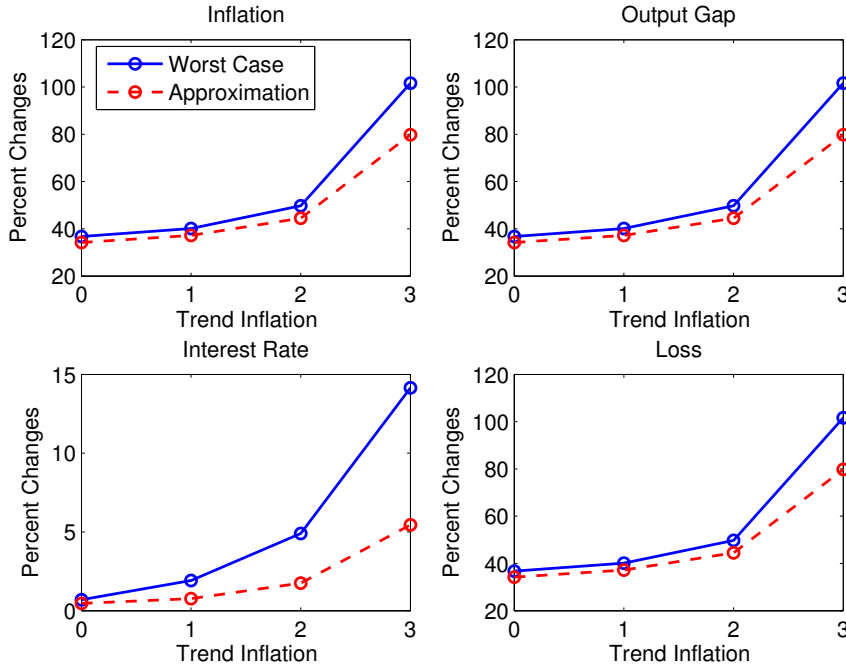
Table 11: Policy Function for  $x_t$  under Discretion ( $\Pi_{\text{loss}} = 1$ ).

Figure 1: Percent Changes in Variances and Losses

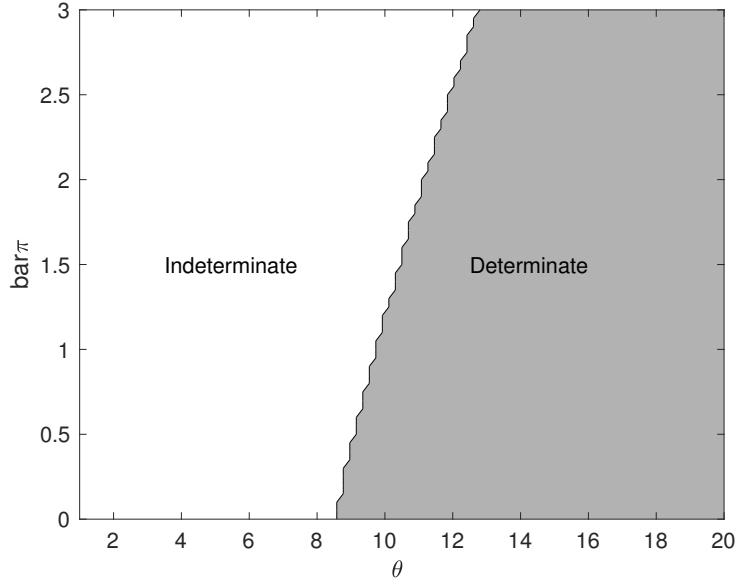


Figure 2: Determinate Region for  $\bar{\pi}$  and  $\theta$ . Grid interval for  $\bar{\pi}$  and  $\theta$  are 0.01 and 0.1, respectively.

(a) Coefficient on cost shock ( $u_t$ )				
Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	5.4813	5.6665	5.8389	5.9918
$i_t$ (Worst Case)	5.486	5.7172	5.9421	6.1546
Change in Percent	0.086709	0.89392	1.7668	2.7167

(b) Coefficient on natural rate shock ( $r_t^n$ )				
Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1	1	1
$i_t$ (Worst Case)	1	1	1	1
Change in Percent	0	0	0	0

Table A1: Policy Function for  $i_t$  under Discretion ( $\sigma = 1$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.55932	0.58985	0.62187	0.65528
$\pi_t$ (Worst Case)	0.64224	0.68228	0.72565	0.77201
Change in Percent	14.8256	15.6702	16.6881	17.8152

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0	0	0
$\pi_t$ (Worst Case)	0	0	0	0
Change in Percent	0	0	0	0

Table A2: Policy Function for  $\pi_t$  under Discretion ( $\sigma = 1$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-5.4813	-5.6666	-5.839	-5.9919
$x_t$ (Worst Case)	-6.294	-6.5546	-6.8134	-7.0594
Change in Percent	14.8256	15.6702	16.6881	17.8152

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	0	0	0
$x_t$ (Worst Case)	0	0	0	0
Change in Percent	0	0	0	0

Table A3: Policy Function for  $x_t$  under Discretion ( $\sigma = 1$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	2.277	2.3615	2.448	2.5363
$i_t$ (Worst Case)	2.1861	2.2774	2.3729	2.4728
Change in Percent	-3.9913	-3.559	-3.0692	-2.502

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1.0018	1.0036	1.0055
$i_t$ (Worst Case)	1	1.0017	1.0035	1.0054
Change in Percent	0	-0.0062318	-0.011009	-0.013771

Table A4: Policy Function for  $i_t$  under Discretion ( $\alpha = 0.4$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.34852	0.36576	0.38415	0.40375
$\pi_t$ (Worst Case)	0.38035	0.40125	0.42364	0.4473
Change in Percent	9.1345	9.7027	10.2806	10.7876

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0.00027168	0.00056489	0.00088106
$\pi_t$ (Worst Case)	0	0.00029804	0.00062296	0.0009761
Change in Percent	0	9.7027	10.2806	10.7876

Table A5: Policy Function for  $\pi_t$  under Discretion ( $\alpha = 0.4$ ).



(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-3.4155	-3.5422	-3.6721	-3.8044
$x_t$ (Worst Case)	-3.7275	-3.8859	-4.0496	-4.2148
Change in Percent	9.1345	9.7027	10.2806	10.7876

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	-0.0026311	-0.0053998	-0.008302
$x_t$ (Worst Case)	0	-0.0028864	-0.005955	-0.0091976
Change in Percent	0	9.7027	10.2806	10.7876

Table A6: Policy Function for  $x_t$  under Discretion ( $\alpha = 0.4$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	5.1385	5.1291	5.0551	4.8994
$i_t$ (Worst Case)	5.4285	5.4775	5.4581	5.3542
Change in Percent	5.645	6.792	7.9709	9.2829

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$i_t (\theta = \infty)$	1	1.0018	1.0034	1.0047
$i_t$ (Worst Case)	1	1.002	1.0037	1.0051
Change in Percent	0	0.012429	0.027249	0.043531

Table A7: Policy Function for  $i_t$  under Discretion ( $\alpha = 0.7$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0.78651	0.82006	0.85287	0.88429
$\pi_t$ (Worst Case)	0.94594	0.99695	1.047	1.0978
Change in Percent	20.2697	21.5713	22.7625	24.1392

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$\pi_t (\theta = \infty)$	0	0.00029312	0.00057874	0.00085037
$\pi_t$ (Worst Case)	0	0.00035635	0.00071047	0.0010556
Change in Percent	0	21.5713	22.7625	24.1392

Table A8: Policy Function for  $\pi_t$  under Discretion ( $\alpha = 0.7$ ).

(a) Coefficient on cost shock ( $u_t$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	-7.7078	-7.6938	-7.5829	-7.3492
$x_t$ (Worst Case)	-9.2702	-9.3534	-9.3089	-9.1233
Change in Percent	20.2697	21.5713	22.7625	24.1392

(b) Coefficient on natural rate shock ( $r_t^n$ )

Trend Inflation	0%	1%	2%	3%
$x_t (\theta = \infty)$	0	-0.00275	-0.0051455	-0.0070673
$x_t$ (Worst Case)	0	-0.0033432	-0.0063168	-0.0087733
Change in Percent	0	21.5713	22.7625	24.1392

Table A9: Policy Function for  $x_t$  under Discretion ( $\alpha = 0.7$ ).