Supplemental Appendix to
"Liquidity Regulation and Financial Stability"

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Abstract

This appendix provides additional derivations, proofs, and extensions of the analysis in the main paper. In part A, I derive the best responses of financial intermediaries and the policymaker to the strategy profile (3) under each policy regime. In part B, I provide proofs of the propositions presented in the paper. In part C, I analyze the impact of changes in the liquidation cost on the desirability of adding liquidity regulation.

A The best-response allocation

In this part, I derive the best responses of financial intermediaries and the policymaker to the strategy profile (3) under each policy regime. The expressions derived here are used in the proofs of the propositions given in the following parts.

A.1 The policy regime with early payments restriction alone

The first order conditions (8)-(10), (18) and (19), combined with the resource constraints (4), (5), and (11)-(13) define the allocation of resources that results from the best response by intermediaries and the policymaker to the strategy profile (3) under the regime with early payments restriction alone. The allocation $\mathcal{A}^I$ will lie in different cases, as defined in Table 1 of the main text, depending on the value of π. Note that the first order condition (19) implies that $\mu_1^I > 0$ always holds (i.e. intermediaries will never hold excess liquidity in equilibrium). In other words, the equilibrium allocation will never be in Case B or Case C but may lie in Case A and Case D. I define

$$\pi^I = \frac{1}{2}\left(1 + (\delta/r)^{\frac{1}{\gamma}} - \{[1 + (\delta/r)^{\frac{1}{\gamma}}]^2 - 4[(\delta/r)^{\frac{1}{\gamma}} - \delta^{\frac{1}{\gamma}}]\}^{\frac{1}{2}}\right).$$

Case A: If $\pi > \pi^I$, then there is no excess liquidity, liquidation occurs, and the solution is given by:

$$\tau^I_\alpha/\tau^I_\beta = [(1-\pi)\pi R^{1-\frac{1}{\gamma}} + (1-\pi)^2 r^{1-\frac{1}{\gamma}} + R^{1-\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}]/[(1-\pi)r + R^{1-\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}}] > 1$$

$$c^I_{1\beta}/c^I_{2\beta} = (r/R)^{\frac{1}{\gamma}} < 1$$

$$c^I_{2\alpha}/c^I_{2\beta} = [(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}]/[(1-\pi)(r/R) + \delta^{\frac{1}{\gamma}}R^{-\frac{1}{\gamma}}] \quad (24)$$

$$c^I_1/c^I_{2\alpha} = [(1-q)R + q(R/r)(c^I_{2\alpha}/c^I_{2\beta})^{\gamma}]^{-\frac{1}{\gamma}} \quad (25)$$

$$c^I_1/c^I_{2\beta} = [(1-q)R(c^I_{2\alpha}/c^I_{2\beta})^{-\gamma} + q(R/r)]^{-\frac{1}{\gamma}} = [(1-q)v'(\tau^I_\alpha)/u'(c^I_{2\beta}) + q(R/r)]^{-\frac{1}{\gamma}} \quad (26)$$

Note that if $\pi > \pi^I$, then this solution has $\mu^I_{1\beta} = 0$ (i.e. intermediaries will liquidate some units of the illiquid asset). Combing the equations (24) and (25), I can see that $c^I_{2\alpha}/c^I_{2\beta} > r^{\frac{1}{\gamma}}$ holds, which in turn implies $c^I_1 < c^I_{2\alpha}$.

Case D: If $\pi \leq \pi^I$, then there is no excess liquidity and no liquidation, and the solution is given by:

$$\tau^I_\alpha/\tau^I_\beta = 1 + (1-\pi)\pi\delta^{-\frac{1}{\gamma}} > 1 \quad (27)$$

$$u'(c^I_1)/v'(\tau^I_\alpha) = (1-q) + q[v'(\tau^I_\beta)/v'(\tau^I_\alpha)] > 1 \text{ since } \tau^I_\alpha > \tau^I_\beta \quad (28)$$

$$v'(\tau^I_\alpha)/u'(c^I_{2\alpha}) = R > 1 \quad (29)$$

$$u'(c^I_1)/u'(c^I_{1\beta}) = (1-q)[v'(\tau^I_\alpha)/v'(\tau^I_\beta)] + q < 1 \text{ since } \tau^I_\alpha > \tau^I_\beta \quad (30)$$

$$c^I_{2\alpha}/c^I_{2\beta} = 1-\pi < 1 \quad (31)$$

Note that if $\pi \leq \pi^I$, then this solution has $\mu^I_{1\beta} > 0$ (i.e. intermediaries will not liquidate any units of the illiquid asset). Combing these above equations, it is straightforward to show that $c^I_{2\beta} > c^I_{2\alpha} > c^I_1 > c^I_{1\beta}$ holds if the solution lies in Case D, which in turn implies that the financial system is always stable if $\mathcal{A}^I$ lies in Case D.

A.2 The policy regime with both regulatory tools

The first order conditions (8)-(10) and (23), combined with the resource constraints (4), (5), and (11)-(13) define the allocation of resources that results from the best response by intermediaries and the policymaker to the strategy profile (3) under the regime with both regulatory tools.

The best-response allocation $\mathcal{A}^{II}$ can lie in each of four cases described in Table 1, depending on the value of q. I define

$$q_l^{II} = \left\{1 + \tfrac{1-r}{R-1}\left[\tfrac{(1-\pi)\pi+(1-\pi)^2(r/R)^{1-\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}}{(1-\pi)(r/R)+\delta^{\frac{1}{\gamma}}}\right]^{\gamma}\right\}^{-1}, \; q_u^{II} = \left\{1 + \tfrac{1-r}{R-1}\left[\tfrac{(1-\pi)\pi+(1-\pi)^2(r/R)^{-\frac{1}{\gamma}}+\delta^{\frac{1}{\gamma}}}{1-\pi+\delta^{\frac{1}{\gamma}}}\right]^{\gamma}\right\}^{-1};$$

$$q_{ll}^{II} = \left\{1 + \tfrac{(1-r)(1-\pi)^{\gamma}[(1-\pi)\pi+\delta^{\frac{1}{\gamma}}]^{\gamma}}{r[(1-\pi)\pi+\delta^{\frac{1}{\gamma}}]^{\gamma}-(1-\pi)^{\gamma}\delta}\right\}^{-1}, q_{uu}^{II} = \left\{1 + \tfrac{[(1-\pi)\pi+\delta^{\frac{1}{\gamma}}]^{\gamma}-(1-\pi)^{\gamma}\delta}{(R-1)\delta}\right\}^{-1};$$

$$\pi^{II} = \tfrac{1}{2}\left([1+(\delta R/r)^{\frac{1}{\gamma}}] - \{[1+(\delta R/r)^{\frac{1}{\gamma}}]^2 - 4\delta^{\frac{1}{\gamma}}[(R/r)^{\frac{1}{\gamma}}-1]\}^{\frac{1}{2}}\right).$$

Case A: If $q < \begin{Bmatrix} q_l^{II} \\ q_{ll}^{II} \end{Bmatrix}$ as $\pi \begin{Bmatrix} > \\ \leq \end{Bmatrix} \pi^{II}$, then there is no excess liquidity, liquidation occurs, and the solution is given by:

$$\begin{aligned} c_{1\beta}^{II}/c_{2\beta}^{II} &= (r/R)^{\frac{1}{\gamma}} < 1 \\ \tau_\alpha^{II}/\tau_\beta^{II} &= [R/\delta(c_{2\alpha}^{II}/\tau_\beta^{II})^{-\gamma} - q(1-r)]^{-\frac{1}{\gamma}} \\ &= 1 + [(1-\pi)\pi + (1-\pi)^2(r/R)^{1-\frac{1}{\gamma}}]\delta^{-\frac{1}{\gamma}} - (1-\pi)(r/R)(c_{2\alpha}^{II}/\tau_\beta^{II}) \qquad (32) \\ v'(\tau_\alpha^{II}) &= \mu_1 + u'(c_{2\alpha}^{II}) \qquad (33) \\ u'(c_1^{II})/v'(\tau_\alpha^{II}) &= (1-q) + qv'(\tau_\beta^{II})/v'(\tau_\alpha^{II}) \qquad (34) \\ c_1^{II}/c_{2\beta}^{II} &= [(1-q)v'(\tau_\alpha^{II})/u'(c_{2\beta}^{II}) + q(R/r)]^{-\frac{1}{\gamma}} \qquad (35) \end{aligned}$$

Note that if $q < \begin{Bmatrix} q_l^{II} \\ q_{ll}^{II} \end{Bmatrix}$ as $\pi \begin{Bmatrix} > \\ \leq \end{Bmatrix} \pi^{II}$, then this solution has $\mu_1^{II} > 0$ (i.e. $v'(\tau_\alpha^{II}) > u'(c_{2\alpha}^{II})$) and $\mu_{1\beta}^{II} = 0$ (i.e. intermediaries will not hold excess liquidity but liquidate some units of the illiquid asset). Using (32), it is then straightforward to show that $\tau_\alpha^{II}/\tau_\beta^{II}$ is strictly increasing in q, which implies that $\tau_\alpha^{II} > \tau_\beta^{II}$ holds. Combined with (34) and the condition $v'(\tau_\alpha^{II}) > u'(c_{2\alpha}^{II})$, I have $u'(c_1^{II}) > v'(\tau_\alpha^{II}) > u'(c_{2\alpha}^{II})$, which in turn implies $c_1^{II} < c_{2\alpha}^{II}$.

Case B: If $q_l^{II} \leq q \leq q_u^{II}$ and $\pi > \pi^{II}$, then there is excess liquidity and liquidation, and the solution is given by:

$$\begin{aligned} \tau_\alpha^{II}/\tau_\beta^{II} &= [(1-q)(R-1)]^{\frac{1}{\gamma}}/[q(1-r)]^{\frac{1}{\gamma}} \\ c_{1\beta}^{II}/c_{2\beta}^{II} &= (r/R)^{\frac{1}{\gamma}} < 1 \\ c_1^{II}/c_{2\alpha}^{II} &= (1-r)^{\frac{1}{\gamma}}/[(1-q)(R-r)]^{\frac{1}{\gamma}} \\ c_1^{II}/c_{2\beta}^{II} &= [r(R-1)]^{\frac{1}{\gamma}}/[qR(R-r)]^{\frac{1}{\gamma}} \end{aligned}$$

Note that if $q_l^{II} \leq q \leq q_u^{II}$ and $\pi > \pi^{II}$, then this solution has $\mu_1^{II} = 0$ and $\mu_{1\beta}^{II} = 0$ (i.e. financial intermediaries will hold excess liquidity and liquidate some units of the illiquid asset). It is straightforward to show that $q_u^{II} < (R-1)/(R-r)$, which implies that $c_1^{II} < c_{2\alpha}^{II}$ and $\tau_\alpha^{II} > \tau_\beta^{II}$.

Case C: If $\begin{Bmatrix} q_u^{II} \\ q_{uu}^{II} \end{Bmatrix} < q < 1$ as $\pi \begin{Bmatrix} > \\ \leq \end{Bmatrix} \pi^{II}$, then there is excess liquidity but no liquidation, and the solution is given by:

$$\begin{aligned}
c_{1\beta}^{II}/c_{2\beta}^{II} &= [R + (1-q)/q(R-1)(c_{2\beta}^{II}/c_{2\alpha}^{II})^{\gamma}]^{-\frac{1}{\gamma}} < 1 \\
\tau_{\alpha}^{II}/\tau_{\beta}^{II} &= [(1-\pi)\pi + \delta^{\frac{1}{\gamma}} + (1-\pi)^2 \cdot (c_{2\beta}^{II}/c_{1\beta}^{II})]/(1-\pi+\delta^{\frac{1}{\gamma}}) > 1 \text{ since } c_{2\beta}^{II} > c_{1\beta}^{II} \\
c_{2\alpha}^{II}/c_{2\beta}^{II} &= \{[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}] \cdot (c_{1\beta}^{II}/c_{2\beta}^{II}) + (1-\pi)^2\}/(1-\pi+\delta^{\frac{1}{\gamma}}) < 1 \text{ since } c_{1\beta}^{II} < c_{2\beta}^{II} \\
v'(\tau_{\alpha}^{II}) &= u'(c_{2\alpha}^{II}) \\
u'(c_1^{II})/v'(\tau_{\alpha}^{II}) &= (1-q) + qv'(\tau_{\beta}^{II})/v'(\tau_{\alpha}^{II}) > 1 \text{ since } \tau_{\alpha}^{II} > \tau_{\beta}^{II}
\end{aligned}$$

Note that If $\begin{Bmatrix} q_u^{II} \\ q_{uu}^{II} \end{Bmatrix} < q < 1$ as $\pi \begin{Bmatrix} > \\ \leq \end{Bmatrix} \pi^{II}$, then this solution has $\mu_1^{II} = 0$ and $\mu_{1\beta}^{II} > 0$ (i.e., intermediaries will hold excess liquidity but not liquidate assets). Combining these above equations, I can see that $c_{2\beta}^{II} > c_{2\alpha}^{II} > c_1^{II} > c_{1\beta}^{II}$. Thus, the financial system is always stable if $\mathcal{A}^{II}$ lies in Case C.

Case D: If $q_{ll}^{II} \leq q \leq q_{uu}^{II}$ and $\pi \leq \pi^{II}$, then there is no excess liquidity and no liquidation, and the solution is given by equations (27) - (31). Note that if $q_{ll}^{II} \leq q \leq q_{uu}^{II}$ and $\pi \leq \pi^{II}$, then this solution has $\mu_1^{II} > 0$ and $\mu_{1\beta}^{II} > 0$. Combined with these above equations, I have $c_{2\beta}^{II} > c_{2\alpha}^{II} > c_1^{II} > c_{1\beta}^{II}$. Thus, the financial system is always stable if $\mathcal{A}^{II}$ lies in Case D.

B Proofs of Propositions

Proposition 1. If the financial system is *fragile* under the regime with a cap on early payments, then $\mathcal{A}^I$ must lie in Case A of Table 1.

Proof. First, note that the first order condition (19) implies that $\mu_1^I > 0$ always holds (i.e. intermediaries will never hold excess liquidity in equilibrium). In other words, the equilibrium allocation under the regime with early payments restriction alone will never be in Case B or Case C of Table 1 but may lie in Case A or Case D. According to Supplemental Appendix A.1, I have $c_1^I < c_{2\beta}^I$ whenever the economy lies in Case D. The run equilibrium can only exist, therefore, if the allocation $\mathcal{A}^I$ lies in Case A. ■

Proposition 2. If the economy lies in Φ^I, then $\begin{pmatrix} c_{1\beta}^I/c_{2\beta}^I \\ c_1^I/c_{1\beta}^I \end{pmatrix}$ is strictly $\begin{pmatrix} \text{increasing} \\ \text{decreasing} \end{pmatrix}$ in r.

Proof. By Proposition 1, if the financial system is fragile under the regime with an early payments restriction alone, then the equilibrium allocation must lie in Case A. Using the best-response allocation from Supplemental Appendix A.1 for the allocation in Case A, I have

$$c^I_{1\beta}/c^I_{2\beta} = (r/R)^{\frac{1}{\gamma}}$$
$$c^I_1/c^I_{1\beta} = [(1-q)g(r)^{-\gamma} + q]^{-\frac{1}{\gamma}}$$

where $g(r) = [(1-\pi)\pi + \delta^{\frac{1}{\gamma}} + (1-\pi)^2(r/R)^{1-\frac{1}{\gamma}}]/[(1-\pi)R^{\frac{1}{\gamma}-1}r + \delta^{\frac{1}{\gamma}}]$. Differentiating these expressions with respect to r, I have $c^I_{1\beta}/c^I_{2\beta}$ is strictly increasing in r and the derivative of $c^I_1/c^I_{1\beta}$ has the same sign of the derivative of $g(r)$, which is given by

$$g'(r) = (1-\pi)R^{\frac{1}{\gamma}-1}[A(r) + B(r)]/[(1-\pi)R^{\frac{1}{\gamma}-1}r + \delta^{\frac{1}{\gamma}}]^2$$

where $A(r) = (1-\pi)^2R^{\frac{1}{\gamma}-1}r(1-1/\gamma-r^{-\frac{1}{\gamma}})$ and $B(r) = (1-1/\gamma)(1-\pi)\delta^{\frac{1}{\gamma}} - [(1-\pi)\pi+\delta^{\frac{1}{\gamma}}]$. It is straightforward to show that $A(r) < 0$. In addition, recall that if $\pi > \pi^I$ then the best-response allocation $\mathcal{A}^I$ lies in Case A. Rewriting this inequality, I have $[(1-\pi)\pi + \delta^{\frac{1}{\gamma}}] > r^{-\frac{1}{\gamma}}(1-\pi)\delta^{\frac{1}{\gamma}}$, which in turn implies that $B(r) < 0$ holds. Taken together, I have this proposition as desired. ■

Proposition 3. If the financial system is *fragile* under the regime with both regulatory tools, then $\mathcal{A}^{II}$ must lie in either Case A or Case B of Table 1.

Proof. According to Supplemental Appendix A.2, I have $c^{II}_1 < c^{II}_{2\beta}$ whenever the economy lies in Cases C and D. The run equilibrium can only exist, therefore, if the allocation $\mathcal{A}^{II}$ lies in Cases A and B. ■

Proposition 4. Suppose the economy is in both Φ^I and Φ^{II} and $q > 0$. Then $\mathcal{W}^{II} > \mathcal{W}^I$.

Proof. I first show that the equilibrium allocation of resources under the regime with both regulatory tools attains the highest possible level of welfare conditional on investors following strategy profile (3). It then follows that the welfare associated with $\mathcal{A}^I$ is strictly lower.

Suppose a benevolent planner could control all endowments and operate both financial intermediation and the public sector. However, this planner cannot control investors' withdrawal decisions and faces the same form of limited commitment as do financial intermediaries and the policymaker. At date 0, the planner makes a portfolio choice $(1-x^*, x^*)$. Before the realization of the state, the planner chooses to give the same amount of consumption, c^*_1, to each investor who withdraws early. Once she has observed the state s, the planner will choose to give common amounts $c^*_{1\beta}$ to each of the remaining impatient investors in state β and c^*_{2s} to each of the remaining patient investors. In addition, she will provide an amount of g^*_s of the public good. Thus, the best response of the planner to the strategy profile (3) can be summarized by a vector

$$\mathcal{A}^* \equiv \left(x^*, c^*_1, c^*_{1\beta}, \{c^*_{2s}, g^*_s\}_{s=\alpha,\beta}\right).$$

The elements of this vector will be chosen to maximize

$$\pi u(c_1) + (1-q)[(1-\pi)u(c_{2\alpha}) + v(g_\alpha)] + q\left\{(1-\pi)\left[\pi u(c_{1\beta}) + (1-\pi)u(c_{2\beta})\right] + v(g_\beta)\right\}$$

subject to the resource constraints

$$\begin{aligned}
\pi c_1 &\leq 1 - x - g_\alpha, \\
(1-\pi)c_{2\alpha} &= 1 - x - g_\alpha - \pi c_1 + Rx, \\
1 - x - g_\beta - \pi c_1 &\leq (1-\pi)\pi c_{1\beta}, \\
(1-\pi)^2 c_{2\beta} &= R\left\{x - \tfrac{1}{r}\left[(1-\pi)\pi c_{1\beta} - (1 - x - g_\beta - \pi c_1)\right]\right\}.
\end{aligned}$$

Letting $\mu_1, \mu_{2\alpha}, \mu_{1\beta}$ and $\mu_{2\beta}$ denote the multipliers on the constraints, the solution to the problem is characterized by the first order conditions

$$\begin{aligned}
u'(c_1) &= \mu_1 + \mu_{2\alpha} + \tfrac{R}{r}\mu_{2\beta} - \mu_{1\beta} = R(\mu_{2\alpha} + \mu_{2\beta}) \\
(1-q)u'(c_{2\alpha}) &= \mu_{2\alpha} \\
qu'(c_{1\beta}) &= \tfrac{R}{r}\mu_{2\beta} - \mu_{1\beta} \\
qu'(c_{2\beta}) &= \mu_{2\beta} \\
(1-q)v'(g_\alpha) &= \mu_{1\alpha} + \mu_{2\alpha} \\
qv'(g_\beta) &= \tfrac{R}{r}\mu_{2\beta} - \mu_{1\beta} \\
(1 - x - g_\alpha - \pi c_1)\mu_1 &= 0 \\
[1 - x - g_\beta - \pi c_1 - (1-\pi)\pi c_{1\beta}]\mu_{1\beta} &= 0
\end{aligned}$$

$$\begin{aligned}
(1-\pi)c_{2\alpha} &= 1 - x - g_\alpha - \pi c_1 + Rx \\
(1-\pi)^2 c_{2\beta} &= R\left\{x - \tfrac{1}{r}\left[(1-\pi)\pi c_{1\beta} - (1 - x - g_\beta - \pi c_1)\right]\right\}
\end{aligned}$$

The solution to the problem will lie in one of four cases as described in Table 1, depending on the value of q. It is straightforward to show that the equilibrium allocation vector $\mathcal{A}^{II}$ solves the problem of the benevolent planner and, since this solution is unique, must create strictly higher welfare than that of the equilibrium allocation $\mathcal{A}^{I}$. ■

Proposition 6. For all $q > 0$, if $0 < f(r) \leq (R^2 - 2rR + r)/(R - Rr)[(1-\pi)(r/R) + \delta^{\frac{1}{\gamma}}]^\gamma$, then the set Φ^I is strictly contained in Φ^{II}.

Proof. Recall that if the economy lies in Φ^I, then the equilibrium allocation lies in Case A. Suppose that the economy under the regime with two regulatory tool also lies in Case A. In this situation, by the conditions (26) and (35), the ratio $c_1/c_{2\beta}$ for both policy regimes can be written as $c_1/c_{2\beta} = [(1-q)v'(\tau_\alpha)/u'(c_{2\beta}) + q(R/r)]^{-\frac{1}{\gamma}}$.

Using the first order conditions (19) and (22), it is straightforward to show that the ratio $\tau_\alpha/c_{2\beta}$ is identical under both regimes as $q \to 0$. Using the resource constraints, if the economy lies in Case A under the regime with both regulatory tools, I then have

$$\tau_\alpha^{II}/c_{2\beta}^{II} = [(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}] - (1-\pi)(r/R)\cdot(c_{2\alpha}^{II}/c_{2\beta}^{II}).$$

Recall that $\tau_\alpha^{II}/c_{2\beta}^{II}$ is strictly increasing in q. Taken together, I have $\tau_\alpha^{II}/c_{2\beta}^{II} > \tau_\alpha^{II}/c_{2\beta}^{II}|_{q\to 0} = \tau_\alpha^{I}/c_{2\beta}^{I}$, and, hence, $c_1^{II}/c_{2\beta}^{II} > c_1^{I}/c_{2\beta}^{I}$.

The proposition will be established, therefore, if I can show that the fragile set Φ^I is strictly contained in the region of Case A under the regime with both regulatory tools.

Now, I restrict attention to the equilibrium allocation in Case A under the regime with early payments restriction alone. It is straightforward to show that the ratio $c_1^I/c_{2\beta}^I$ is strictly decreasing in q in this case. Thus, I can characterize the fragile set Φ^I by looking at the condition $c_1^I \geq c_{2\beta}^I$, which yields

$$q \leq f(r)/\{R/r[(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}]^\gamma - [(1-\pi)rR^{\frac{1}{\gamma}-1} + \delta^{\frac{1}{\gamma}}]^\gamma\}. \quad (36)$$

If $0 < f(r) \leq (R^2 - 2rR + r)/(R - Rr)[(1-\pi)(r/R) + \delta^{\frac{1}{\gamma}}]^\gamma$, then the condition (36) implies $q < q_l^{II}$. In other words, if the economy lies in Φ^I then it must lie in Case A under the regime with both regulatory tools once the above condition is satisfied. Hence, the economy is also lies in Φ^{II} because $c_1^{II}/c_{2\beta}^{II} > c_1^I/c_{2\beta}^I$ always holds in this scenario.

To verify that the inclusion is strict, it is easy to find examples of economies that belong to Φ^{II} but not to Φ^I; see the gray region of Figure 4. Together, these above steps establish the result. ■

C The impact of liquidation cost

This part of the appendix contains a more detailed analysis of the relationship between the liquidation cost and the desirability of adding liquidity regulation. This analysis was omitted from the main paper to save space.

For notational convenience, I define the following critical values and expressions, which depend on the specific features of the economy.

$$h(r) = (1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}} - [(1-\pi)r/R + \delta^{\frac{1}{\gamma}}][\tfrac{R^2-2rR+r}{R(1-r)}]^{\frac{1}{\gamma}},$$

$$k(r) = (1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}} - [(1-\pi)rR^{\frac{1}{\gamma}-1} + \delta^{\frac{1}{\gamma}}][\tfrac{R^2-2rR+r}{R(1-r)}]^{\frac{1}{\gamma}},$$

$$e(q) = (1-\pi)rR^{\frac{1}{\gamma}-1}(\tfrac{1-q}{1-qR})^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}(\tfrac{1-q}{1-qR/r})^{\frac{1}{\gamma}} - [(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2 r/R + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}],$$

$$\bar{q}_1 = f(r)/\{R/r[(1-\pi)\pi(r/R)^{\frac{1}{\gamma}} + (1-\pi)^2(r/R) + \delta^{\frac{1}{\gamma}}(r/R)^{\frac{1}{\gamma}}]^\gamma - [(1-\pi)rR^{\frac{1}{\gamma}-1} + \delta^{\frac{1}{\gamma}}]^\gamma\},$$

$$\bar{q}_2 = \begin{cases} \{q \mid e(q) = 0; q \in (0, q_l^{II})\}, & \text{if } h(r) \leq 0 \\ \frac{r(R-1)}{R(R-r)}, & \text{if } h(r) > 0 \end{cases}.$$

I then have the following result.

Proposition 1. Assume $q > 0$ and $f(r) > 0$.

- For any economy with $k(r) \geq 0$, adding liquidity regulation is always desirable;
- For any economy with $k(r) < 0$, adding liquidity regulation is $\left\{\begin{matrix} \text{always} \\ \textbf{never} \end{matrix}\right\}$ desirable, if $\left\{\begin{matrix} q \leq \bar{q}_1 \\ \bar{q}_1 < q \leq \bar{q}_2 \end{matrix}\right\}$.

Proof. First, note that in equilibrium the ratio of $c_1/c_{2\beta}$ under both policy regimes is strictly decreasing in q. This fact implies that there exists a threshold value of q below which a run can occur in equilibrium. The proof is divided into three steps, which are addressed in separate lemmas below. First, Lemmas 1 and 2 derive the unique maximum probability with which financial fragility arises under both policy regimes. Lemma 3 then compares policy options in terms of financial fragility. The result in the proposition follows immediately from these lemmas.

Definition 1. Given $(R, r, \gamma, \pi, \delta)$, let $\bar{q}^I$ be the maximum value of q such that $c_1^I \geq c_{2\beta}^I$ holds. If $c_1^I \geq c_{2\beta}^I$ does not hold for any value of q, then define $\bar{q}^I = 0$.

In this way, the measure of financial fragility can be formalized in a general formula. I then have the following result.

Lemma 1. Given $R, r, \gamma, \pi, \delta$,

- if $f(r) \leq 0$, then the financial system is stable for all q under the regime with early payments restriction alone and, therefore, $\bar{q}^I = 0$;
- if $f(r) > 0$, then $\bar{q}^I = \bar{q}_1$.

proof of the Lemma. According to Proposition 1, the best-response allocation $\mathcal{A}^I$ is always in Case A in equilibrium. There are two scenarios needed to be considered.

Scenario (i): If $f(r) \leq 0$, using Supplemental Appendix A.1, then I see that $c_1^I < c_{2\beta}^I$ holds in Case A under this condition. Hence, I have $\bar{q}^I = 0$.

Scenario (ii): If $f(r) > 0$, using Supplemental Appendix A.1, it is straightforward to show that the ratio $c_1^I/c_{2\beta}^I$ is strictly decreasing in q. In this case, $c_1^I \geq c_{2\beta}^I$ if and only if $q \leq \bar{q}_1$, which establishes the Lemma. □

Definition 2. Given $(R, r, \gamma, \pi, \delta)$, let $\bar{q}^{II}$ be the maximum value of q such that $c_1^{II} \geq c_{2\beta}^{II}$ holds. If $c_1^{II} \geq c_{2\beta}^{II}$ does not hold for any value of q, then define $\bar{q}^{II} = 0$.

Lemma 2. Given $R, r, \gamma, \pi, \delta$,

- if $f(r) \leq 0$, then the financial system is stable for all q under the regime with both regulatory tools and, therefore, $\bar{q}^{II} = 0$;
- if $f(r) > 0$ and $h(r) \leq 0$, then $\bar{q}^{II} = \{q \mid e(q) = 0; q \in (0, q_l^{II})\}$;
- if $h(r) > 0$ (i.e. $f(r) > 0$), then $\bar{q}^{II} = \frac{r(R-1)}{R(R-r)}$.

Proof of the Lemma. According to Proposition 2, the best-response allocation $\mathcal{A}^{II}$ must lie in either Case A or Case B in equilibrium. In other words, the measure of financial fragility $\bar{q}^{II}$ will lie in Cases A and B depending on parameter values. There are three scenarios needed to be considered.

Scenario (i): If $f(r) \leq 0$, using Supplemental Appendix A.2, then I see that $c_1^{II} < c_{2\beta}^{II}$ holds in Cases A, and B under this condition. Hence, I have $\bar{q}^{II} = 0$.

Scenario (ii): If $f(r) > 0$ and $h(r) \leq 0$, using Supplemental Appendix A.2, then I see that $c_1^{II} < c_{2\beta}^{II}$ holds in Case B under this condition. It is straightforward to show that the ratio $c_1^{II}/c_{2\beta}^{II}$ is strictly decreasing in q when the solution lies in Case A. In this case, there exists a unique value of q_m such that $(c_1^{II}/c_{2\beta}^{II})(q_m) = 1$, where $e(q_m) = 0$. Then, I have $c_1^{II} \geq c_{2\beta}^{II}$ if and only if $q \leq q_m$. Hence, I have $\bar{q}^{II} = \{q \mid e(q) = 0; q \in (0, q_l^{II})\}$.

Scenario (iii): If $h(r) > 0$ (i.e. $f(r) > 0$), using Supplemental Appendix A.2, then I see that $c_1^{II} > c_{2\beta}^{II}$ always holds in Case A under this condition. It is straightforward to show that the ratio $c_1^{II}/c_{2\beta}^{II}$ is strictly decreasing in q when the solution lies in Case B. Then, I see that $c_1^{II} \geq c_{2\beta}^{II}$ if and only if $q \leq \frac{r(R-1)}{R(R-r)}$. Hence, I have $\bar{q}^{II} = \frac{r(R-1)}{R(R-r)}$, the Lemma has been established. □

If adopting an alternative policy regime decreases the critical value of q, I say that it makes the financial system less fragile.

Lemma 3. Assume $f(r) > 0$. $\bar{q}_1 \begin{Bmatrix} \geq \\ < \end{Bmatrix} \bar{q}_2$ if $k(r) \begin{Bmatrix} \geq \\ < \end{Bmatrix} 0$.

Proof of the Lemma. Combing Lemmas 1 and 2, there are three scenarios needed to be considered.

Scenario (i): If $f(r) > 0$ and $h(r) \leq 0$ (i.e. $k(r) < 0$), straightforward algebra shows that $\bar{q}_1 < \bar{q}^{II} = \{q \mid e(q) = 0; q \in (0, q_l^{II})\}$.

Scenario (ii): If $f(r) > 0$, $h(r) > 0$, and $k(r) < 0$, straightforward algebra shows that $\bar{q}_1 < \bar{q}^{II} = \frac{r(R-1)}{R(R-r)}$.

Scenario (iii): If $f(r) > 0$, and $k(r) \geq 0$ (i.e. $h(r) > 0$), straightforward algebra shows that $\bar{q}_1 \geq \bar{q}^{II} = \frac{r(R-1)}{R(R-r)}$.

Together, the results above establish the Lemma. □

If the economy is fragile under one policy regime but not the other, the optimal policy is to select the non-fragile regime. If the economy is fragile under both regimes, the policymaker chooses the higher-welfare regime by comparing the welfare level $\mathcal{W}$ conditional on the financial system being fragile.

Recall that the equilibrium allocation vector $\mathcal{A}^{II}$ attains the highest possible level of welfare conditional on investors following strategy profile (3). Consider all economies with $k(r) \geq 0$, in this case, $\bar{q}_2 \leq \bar{q}_1$ always holds. In other words, adopting the regime with both regulatory tools can both promote financial stability and improve welfare. Therefore, adding liquidity regulation is always desirable. Consider all economies with $k(r) < 0$ (i.e. $\bar{q}_1 < \bar{q}_2$), if $q \leq \bar{q}_1$ then adding liquidity regulation is desirable because it raises welfare. However, it is never desirable if $\bar{q}_1 < q \leq \bar{q}_2$ because it introduces the bad equilibrium. ■