Online Appendix: Alternative Monetary Policies under Keynesian Animal Spirits

Tai-kuang HO

Department of Quantitative Finance, National Tsing Hua University, Taiwan

Ya-chi LIN

School of Economics and Environmental Resources,

Hubei University of Economics, China

Kuo-chun YEH

Graduate Institute of National Development, National Taiwan University, Taiwan

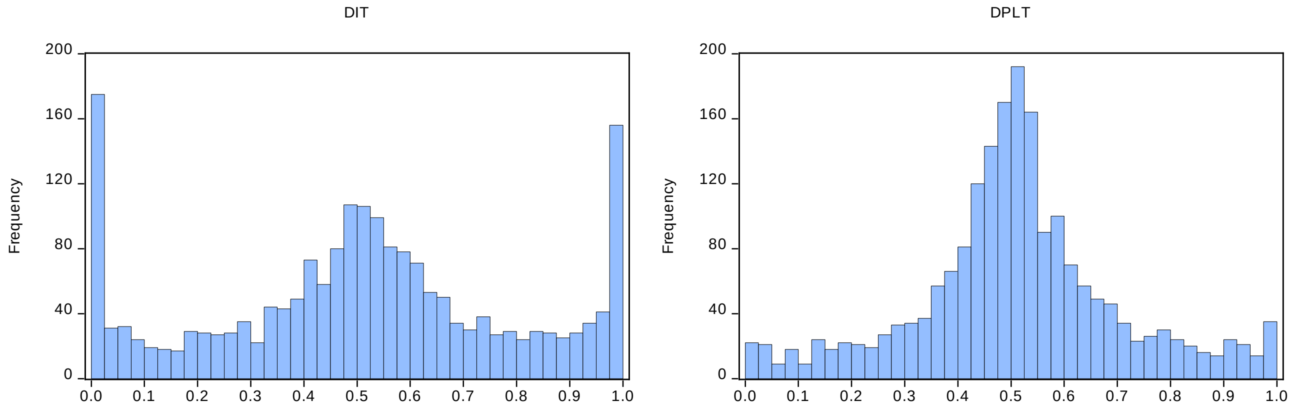
July 17, 2018

- Using Figure 1, we test the hypothesis that the samples of DIT and DPLT come from the same distribution using the Kolmogorov-Smirnov test. The p-value is 0.00, strongly rejecting the null hypothesis.

- In Figure 2 we also plot the time-varying shares of agents with extrapolative inflation forecasting. For monetary policies that do not respond to inflation or price level (e.g., PEG), the share is equal to one. Such policies put zero weight on price stability and it is optimal for the agents to ignore entirely the fundamental forecasting rule. Compared to inflation targeting (DIT and CPIT), the share of extrapolative forecasting agents under strict inflation targeting (SDIT and SCPIT) shows a highly persistent pattern. The share of extrapolative forecasting increases somehow under price-level targeting. For example, the average share of extrapolative forecasting increases from 55% under DIT to 67% under DPLT. On the other hand, the time-varying share becomes less volatile as the range of the share decreases from 0.70 to 0.57, implying that under price-level targeting, agents are less subject to waves of optimism and pessimism.

- We also construct the impulse responses to monetary shock and to demand shock, respectively. Figure 3 and Figure 4 indicate the mean responses of output and of inflation following either a monetary policy shock or a demand shock are almost identical. However, the confidence interval under price-level targeting is much narrower than that under inflation targeting. As in the case of supply shock, this indicates that price-level targeting is better able to stabilize expectations and thus reduce macroeconomic volatility.

Figure 1: Distribution of Animal Spirits under DIT and DPLT



Note: We test the hypothesis that the two samples come from the same distribution using the Kolmogorov-Smirnov test. The p-value is 0.00, strongly rejecting the null hypothesis.

Figure 2: Time-Varying Shares of Agents with Extrapolative Inflation Forecasting

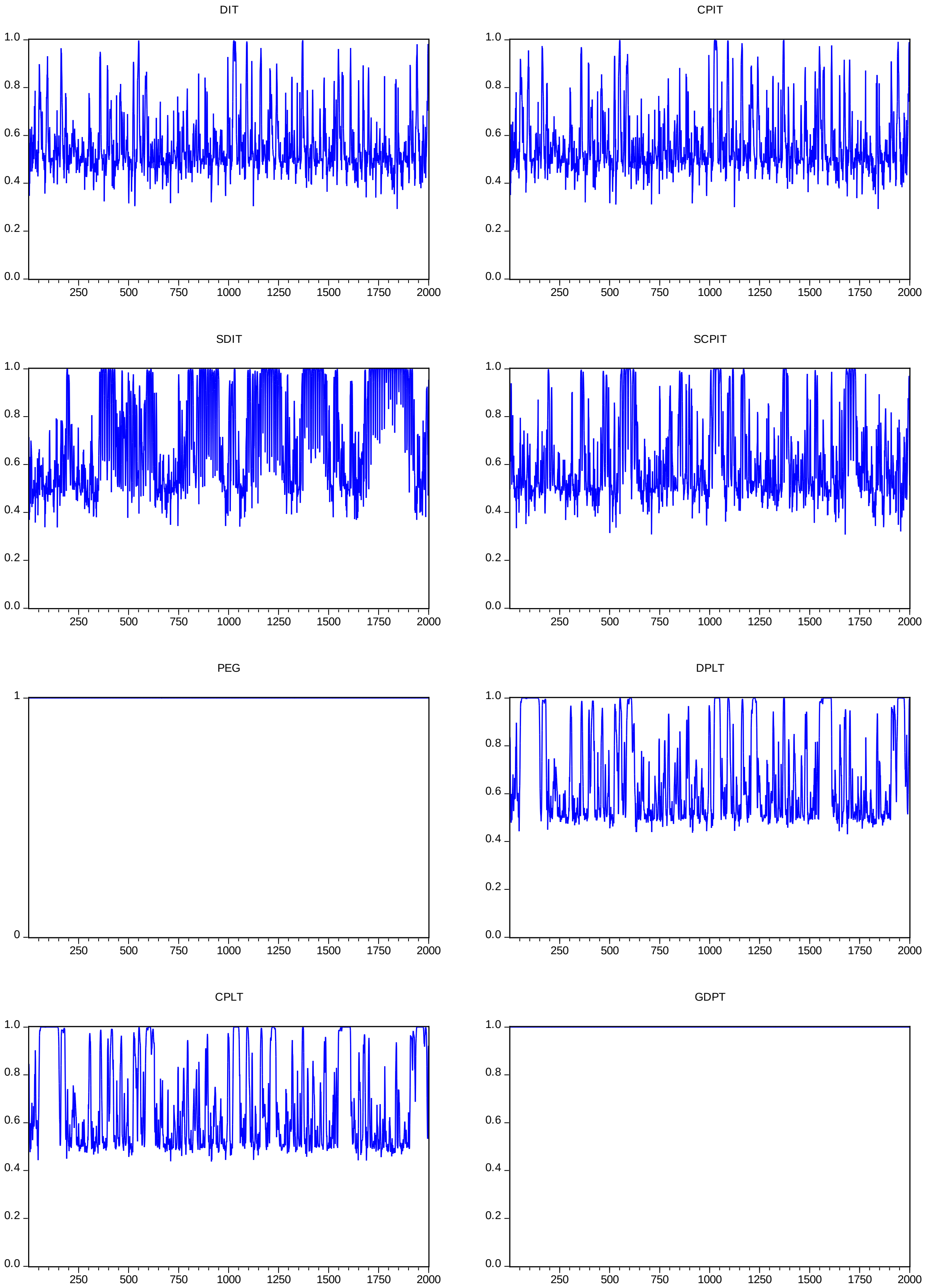


Figure 3: Impulse Response of Output and Inflation to Demand Shock

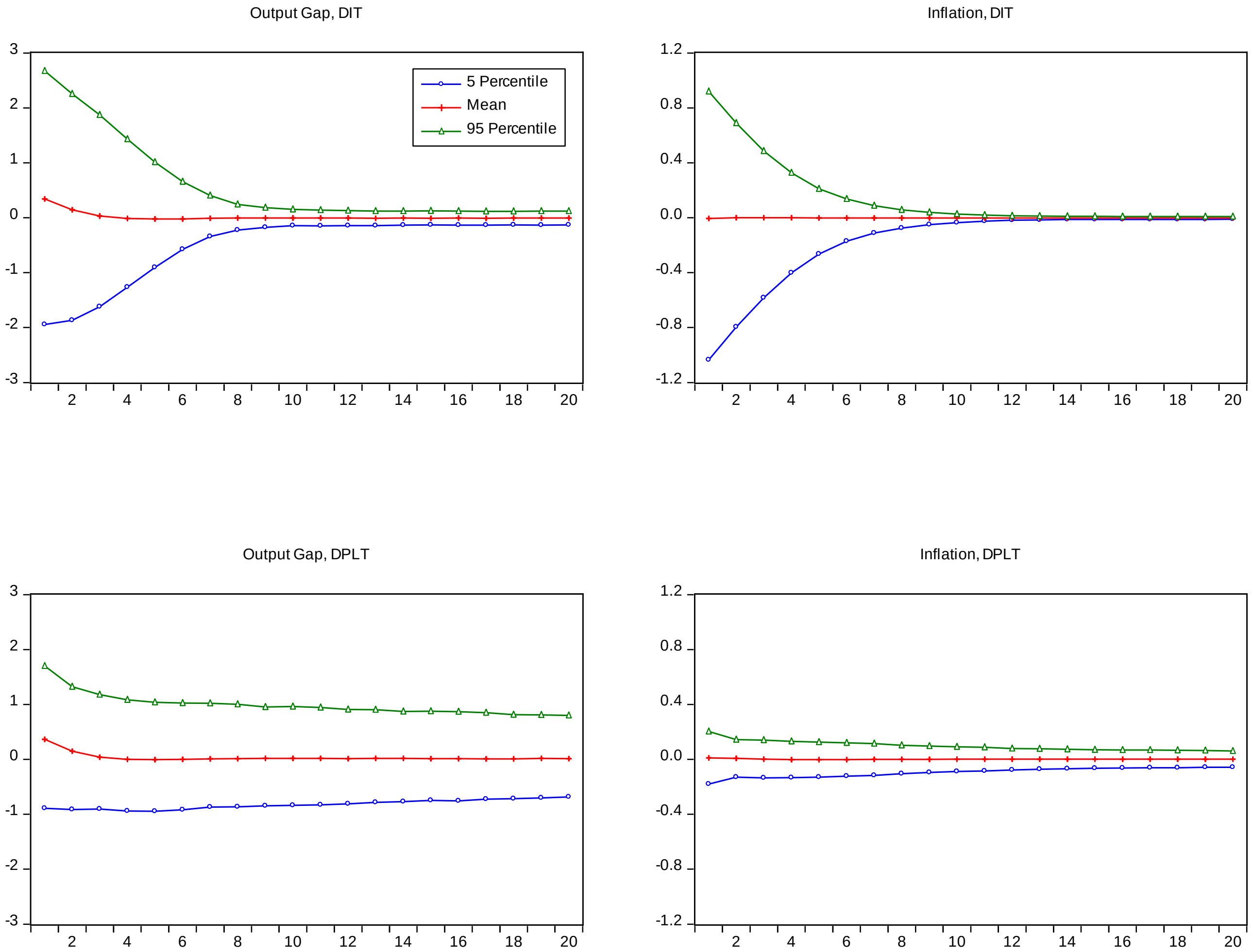
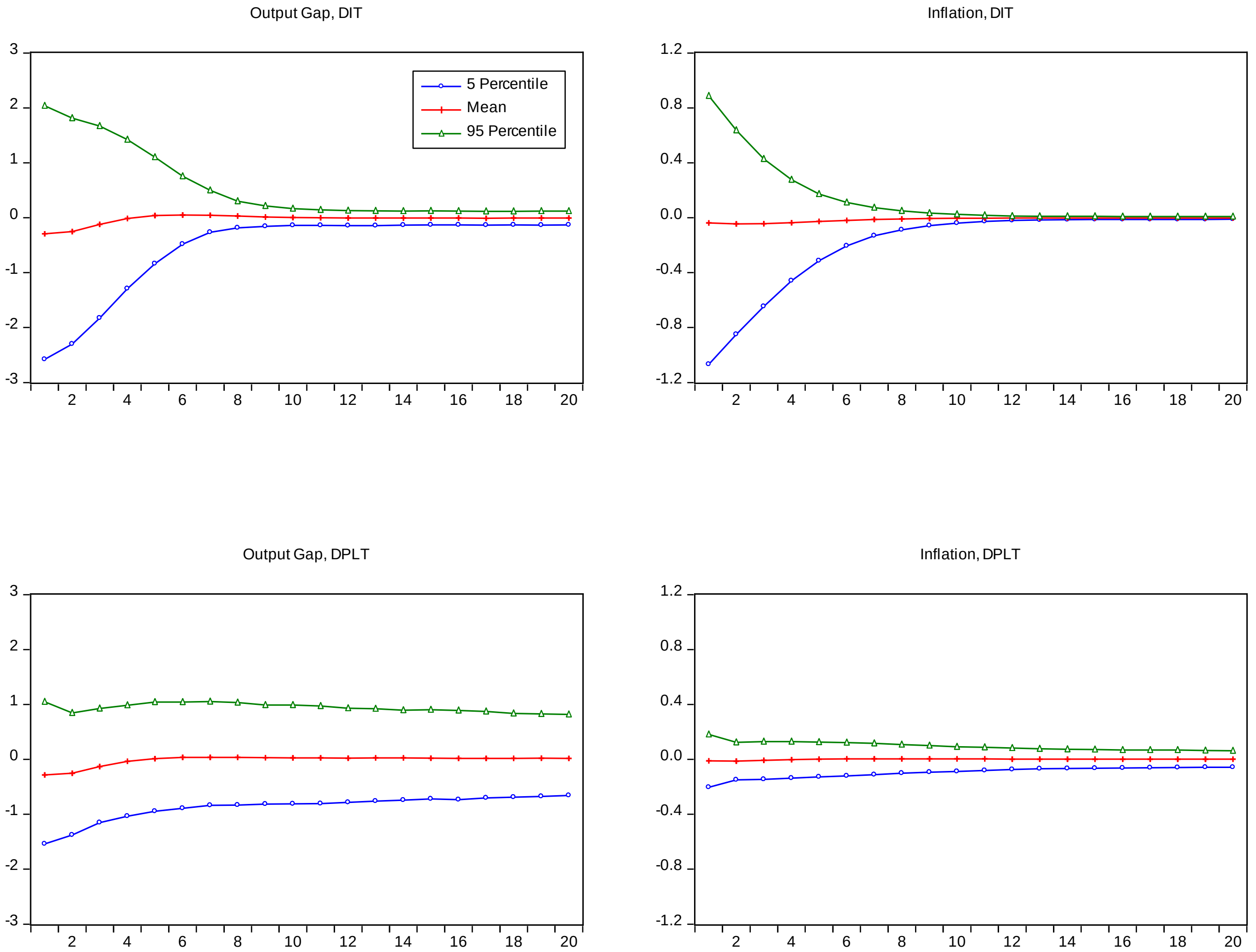


Figure 4: Impulse Response of Output and Inflation to Monetary Policy Shock



Technical Details of the Model

Tai-kuang Ho   Kuo-chun YEH   Ya-chi LIN

September 2017

Abstract

This technical note gives a detailed derivation of the model used in the simulation. The final section of the note considers 8 scenarios and the related mathematical expressions employed in coding the program.

1 Basic setup

- In this note, the notations are corrected to make them consistent throughout the text.
- IS equation:

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\,\tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n \right) + \varepsilon_t \tag{1a}$$

- $\tilde{y}_t = y_t - y_t^n$: domestic output gap
- Remark: de Grauwe (2012) uses y_t instead of $\tilde{y}_t$ to denote output gap, while here y_t denotes output (not gap).
- i_t : nominal interest rate
- π_t : domestic inflation
- $\tilde{E}_t$: expectations operator formed by the Non-Rational Expectation hypothesis
- i_t^n : natural rate of interest
- ε_t : white-noise disturbance
- Natural rate of interest in an open economy is expressed as:

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^* \tag{1b}$$

- $\rho \equiv -\log \beta$: time discount rate
- A_t : technology, which follows an AR(1) process
- Δy_{t+1}^{*} : changes in foreign output

$$a_2 = -\frac{1 + \alpha\left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{\sigma}$$

$$a_3 = \frac{a_2 + 1}{a_2\left(a_2\varphi - 1\right)}$$

$$a_4 = -\frac{\alpha\varphi\left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{a_2\varphi - 1}$$

- α : measure of openness
- σ : inter-temporal elasticity with respect to consumption
- γ : substitutability of goods produced in different foreign countries
- η : substitutability between domestic and foreign goods in the domestic consumption bundle
- φ : inverse of labor supply elasticity
- The IS curve can be rearranged to show the relationship between domestic output and terms of trade.
- s_t : terms of trade

$$y_t = y_t^{*} + \frac{1 + \alpha\left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{\sigma} s_t$$

- $s_t = p_t^{*} - p_t$: terms of trade means the price of foreign goods in terms of domestic goods.

$$y_t^{n} = \Gamma_0 + \Gamma_1 A_t + \Gamma_2 y_t^{*}$$

$$\Gamma_1 = \frac{a_2\left(1 + \varphi\right)}{a_2\varphi - 1}$$

$$\Gamma_2 = \frac{\alpha\left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{a_2\varphi - 1}$$

- $\Gamma_0 = \frac{\upsilon - \mu}{\sigma\alpha + \varphi}$. See Galí (2008), page 164. *Monetary policy, inflation, and the business cycle : an introduction to the New Keynesian framework.*

- It follows:

$$\Delta y^*_{t+1} = a_5 \left(\tilde{y}_{t+1} - \tilde{y}_t\right) + a_5 \Gamma_1 \left(A_{t+1} - A_t\right) + a_2 a_5 \Delta s_{t+1}$$

$$a_5 = \frac{1}{1 - \Gamma_2}$$

- Some algebraic derivations:

$$y_t = y^*_t + \frac{1 + \alpha \left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{\sigma} s_t = y^*_t - a_2 s_t$$

$$\because a_2 = -\frac{1 + \alpha \left[(\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)\right]}{\sigma}$$

$$y^n_t = \Gamma_0 + \Gamma_1 A_t + \Gamma_2 y^*_t$$

$$\tilde{y}_t = y_t - y^n_t = y^*_t - a_2 s_t - \Gamma_0 - \Gamma_1 A_t - \Gamma_2 y^*_t = (1 - \Gamma_2) y^*_t - a_2 s_t - \Gamma_0 - \Gamma_1 A_t$$

$$\tilde{y}_t = (1 - \Gamma_2) y^*_t - a_2 s_t - \Gamma_0 - \Gamma_1 A_t$$

$$\tilde{y}_{t+1} = (1 - \Gamma_2) y^*_{t+1} - a_2 s_{t+1} - \Gamma_0 - \Gamma_1 A_{t+1}$$

$$\Delta\tilde{y}_{t+1} = (1 - \Gamma_2) \Delta y^*_{t+1} - a_2 \Delta s_{t+1} - \Gamma_1 \Delta A_{t+1}$$

$$(1 - \Gamma_2) \Delta y^*_{t+1} = \Delta\tilde{y}_{t+1} + a_2 \Delta s_{t+1} + \Gamma_1 \Delta A_{t+1}$$

$$\Delta y^*_{t+1} = \frac{1}{1 - \Gamma_2} \Delta\tilde{y}_{t+1} + \frac{1}{1 - \Gamma_2} a_2 \Delta s_{t+1} + \frac{1}{1 - \Gamma_2} \Gamma_1 \Delta A_{t+1}$$

$$a_5 = \frac{1}{1 - \Gamma_2}$$

$$\Delta y^*_{t+1} = a_5 \Delta\tilde{y}_{t+1} + a_2 a_5 \Delta s_{t+1} + a_5 \Gamma_1 \Delta A_{t+1}$$

$$\Delta y^*_{t+1} = a_5 \left(\tilde{y}_{t+1} - \tilde{y}_t\right) + a_5 \Gamma_1 \left(A_{t+1} - A_t\right) + a_2 a_5 \Delta s_{t+1}$$

- Substituting Δy^*_{t+1} into the definition of i^n_t, and then substituting i^n_t into equation (1a), yields the following rearranged IS curve:

$$\tilde{y}_t = a_6 \tilde{E}_t \tilde{y}_{t+1} + a_7 \tilde{y}_{t-1} + a_8 \left(i_t - \tilde{E}_t \pi_{t+1} - \rho \right) + a_9 A_t + a_{10} \tilde{E}_t A_{t+1} + a_{11} \tilde{E}_t \Delta s_{t+1} + \frac{1}{(1 - a_2 a_4 a_5)} \varepsilon_t \tag{1c}$$

$$a_6 = \frac{a_1 - a_2 a_4 a_5}{1 - a_2 a_4 a_5}$$

$$a_7 = \frac{1 - a_1}{1 - a_2 a_4 a_5}$$

$$a_8 = \frac{a_2}{1 - a_2 a_4 a_5}$$

$$a_9 = \frac{a_2 a_4 a_5 \Gamma_1 - a_2 a_5}{1 - a_2 a_4 a_5}$$

$$a_{10} = -\frac{a_2 a_4 a_5 \Gamma_1}{1 - a_2 a_4 a_5}$$

$$a_{11} = -\frac{a_2^2 a_4 a_5}{1 - a_2 a_4 a_5}$$

- Some algebraic derivations:

$$i^n_t = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y^*_{t+1}$$

$$\Delta y^*_{t+1} = a_5 \left(\tilde{y}_{t+1} - \tilde{y}_t \right) + a_5 \Gamma_1 \left(A_{t+1} - A_t \right) + a_2 a_5 \Delta s_{t+1}$$

$$i^n_t = \rho + a_3 A_t + a_4 \tilde{E}_t \left[a_5 \left(\tilde{y}_{t+1} - \tilde{y}_t \right) + a_5 \Gamma_1 \left(A_{t+1} - A_t \right) + a_2 a_5 \Delta s_{t+1} \right]$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i^n_t \right) + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 i_t - a_2 \tilde{E}_t \pi_{t+1} - a_2 i^n_t + \varepsilon_t$$

$$\begin{aligned} \tilde{y}_t &= a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 i_t - a_2 \tilde{E}_t \pi_{t+1} \\ &\quad - a_2 \left\{ \rho + a_3 A_t + a_4 \tilde{E}_t \left[a_5 \left(\tilde{y}_{t+1} - \tilde{y}_t \right) + a_5 \Gamma_1 \left(A_{t+1} - A_t \right) + a_2 a_5 \Delta s_{t+1} \right] \right\} + \varepsilon_t \end{aligned}$$

$$\tilde{y}_t = a_1\tilde{E}_t\tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} + a_2 i_t - a_2\tilde{E}_t\pi_{t+1}$$
$$-a_2\rho - a_2a_3A_t - a_2a_4\tilde{E}_t\left[a_5\left(\tilde{y}_{t+1}-\tilde{y}_t\right) + a_5\Gamma_1\left(A_{t+1}-A_t\right) + a_2a_5\Delta s_{t+1}\right] + \varepsilon_t$$

$$\tilde{y}_t = a_1\tilde{E}_t\tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} + a_2 i_t - a_2\tilde{E}_t\pi_{t+1} - a_2\rho - a_2a_3A_t$$
$$-a_2a_4a_5\tilde{E}_t\tilde{y}_{t+1} + a_2a_4a_5\tilde{y}_t - a_2a_4a_5\Gamma_1\tilde{E}_tA_{t+1} + a_2a_4a_5\Gamma_1A_t - a_2^2a_4a_5\tilde{E}_t\Delta s_{t+1} + \varepsilon_t$$

$$(1-a_2a_4a_5)\,\tilde{y}_t = (a_1-a_2a_4a_5)\,\tilde{E}_t\tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} + a_2\left(i_t - \tilde{E}_t\pi_{t+1} - \rho\right)$$
$$+\left(a_2a_4a_5\Gamma_1 - a_2a_3\right)A_t - a_2a_4a_5\Gamma_1\tilde{E}_tA_{t+1} - a_2^2a_4a_5\tilde{E}_t\Delta s_{t+1} + \varepsilon_t$$

$$\tilde{y}_t = \frac{(a_1-a_2a_4a_5)}{(1-a_2a_4a_5)}\tilde{E}_t\tilde{y}_{t+1} + \frac{(1-a_1)}{(1-a_2a_4a_5)}\tilde{y}_{t-1} + \frac{a_2}{(1-a_2a_4a_5)}\left(i_t - \tilde{E}_t\pi_{t+1} - \rho\right)$$
$$+\frac{(a_2a_4a_5\Gamma_1 - a_2a_3)}{(1-a_2a_4a_5)}A_t + \frac{-a_2a_4a_5\Gamma_1}{(1-a_2a_4a_5)}\tilde{E}_tA_{t+1} + \frac{-a_2^2a_4a_5}{(1-a_2a_4a_5)}\tilde{E}_t\Delta s_{t+1} + \frac{1}{(1-a_2a_4a_5)}\varepsilon_t$$

$$\tilde{y}_t = a_6\tilde{E}_t\tilde{y}_{t+1} + a_7\tilde{y}_{t-1} + a_8\left(i_t - \tilde{E}_t\pi_{t+1} - \rho\right) + a_9A_t + a_{10}\tilde{E}_tA_{t+1} + a_{11}\tilde{E}_t\Delta s_{t+1} + \frac{1}{(1-a_2a_4a_5)}\varepsilon_t$$

- The last term should be $\frac{1}{(1-a_2a_4a_5)}\varepsilon_t$.
- The relationship between changes in the effective real exchange rate and terms of trade is expressed as:
- $\Delta q_t = (1-\alpha)\,\Delta s_t$

$$\tilde{y}_t = a_6\tilde{E}_t\tilde{y}_{t+1} + a_7\tilde{y}_{t-1} + a_8\left(i_t - \tilde{E}_t\pi_{t+1} - \rho\right) + a_9A_t + a_{10}\tilde{E}_tA_{t+1} + a_{12}\tilde{E}\Delta q_{t+1} + \frac{1}{(1-a_2a_4a_5)}\varepsilon_t \tag{1d}$$

$$a_{12} = \frac{a_{11}}{1-\alpha}$$

- Some algebraic derivations:

$$\tilde{y}_t = a_6 \tilde{E}_t \tilde{y}_{t+1} + a_7 \tilde{y}_{t-1} + a_8 \left(i_t - \tilde{E}_t \pi_{t+1} - \rho \right) + a_9 A_t + a_{10} \tilde{E}_t A_{t+1} + a_{11} \tilde{E}_t \Delta s_{t+1} + \frac{1}{(1 - a_2 a_4 a_5)} \varepsilon_t$$

$$\Delta q_t = (1 - \alpha) \Delta s_t$$

$$\tilde{y}_t = a_6 \tilde{E}_t \tilde{y}_{t+1} + a_7 \tilde{y}_{t-1} + a_8 \left(i_t - \tilde{E}_t \pi_{t+1} - \rho \right) + a_9 A_t + a_{10} \tilde{E}_t A_{t+1} + a_{11} \frac{1}{1 - \alpha} \tilde{E}_t \Delta q_{t+1} + \frac{1}{(1 - a_2 a_4 a_5)} \varepsilon_t$$

$$\tilde{y}_t = a_6 \tilde{E}_t \tilde{y}_{t+1} + a_7 \tilde{y}_{t-1} + a_8 \left(i_t - \tilde{E}_t \pi_{t+1} - \rho \right) + a_9 A_t + a_{10} \tilde{E}_t A_{t+1} + a_{12} \tilde{E}_t \Delta q_{t+1} + \frac{1}{(1 - a_2 a_4 a_5)} \varepsilon_t$$

- The last term should be $\frac{1}{(1-a_2 a_4 a_5)} \varepsilon_t$.
- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 \tilde{y}_t + \eta_t \tag{2}$$

- η_t : white-noise disturbance
- Now consider different monetary policy rules.
- DIT, Domestic Inflation Targeting

$$i_t = c_1 (\pi_t - \bar{\pi}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t \tag{3a}$$

- $\bar{\pi}$: the inflation target
- μ_t : an exogenous component
- $c_1, c_2 \geq 0$
- CPIT, CPI Inflation Targeting
- $\pi_t^{CPI} = \pi_t + \alpha \Delta s_t$

$$i_t = c_1 \left(\pi_t^{CPI} - \bar{\pi} \right) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t \tag{3b}$$

- SDIT, Strict Domestic Inflation Targeting:

$$i_t = c_1\left(\pi_t - \bar{\pi}\right) + \underbrace{c_2}_{=0}\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- SCPIT, Strict CPI Inflation Targeting:

$$i_t = c_1\left(\pi_t^{CPI} - \bar{\pi}\right) + \underbrace{c_2}_{=0}\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- PEG, Exchange Rate Peg:

$$i_t = i_t^* + \rho + \mu_t \tag{3c}$$

- i_t^* : interest rate of a foreign country
- DPLT, Domestic Price Level Targeting:

$$i_t = c_1\left(p_t - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t \tag{3d}$$

- CPLT, Consumer Price Level Targeting:

$$i_t = c_1\left(p_t^{CPI} - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t \tag{3e}$$

- $\bar{p}$: price level target
- GDPT, Output Gap Targeting:

$$i_t = \underbrace{c_1}_{=0}\left(p_t - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

$$i_t = \underbrace{c_1}_{=0}\left(p_t^{CPI} - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

2 Heuristic forecast

- Expected welfare loss:

$$V = -\frac{1-\alpha}{2}\left[\frac{\xi\theta}{\left(1-\beta\theta\right)\left(1-\theta\right)} var\left(\pi_t\right) + \left(1+\varphi\right) var\left(\tilde{y}_t\right)\right] \tag{4}$$

- $\xi > 1$: elasticity of substitution between varieties produced within a given country
- θ : price stickiness
- Forecast rule of fundamentalists:

$$\tilde{E}_t^f \tilde{y}_{t+1} = 0 \tag{5a}$$

- Extrapolative forecast rule:

$$\tilde{E}_t^e \tilde{y}_{t+1} = \tilde{y}_{t-1} \tag{5b}$$

- Expected output gap:

$$\tilde{E}_t \tilde{y}_{t+1} = \alpha_t^f \cdot \tilde{E}_t^f \tilde{y}_{t+1} + \alpha_t^e \cdot \tilde{E}_t^e \tilde{y}_{t+1} = \alpha_t^f \cdot 0 + \alpha_t^e \cdot \tilde{y}_{t-1}$$

$$\alpha_t^f + \alpha_t^e = 1$$

- Mean squared forecast errors:

$$\omega_k = (1 - \delta)\, \delta^k$$

$$U_t^f = -\sum_{k=0}^{\infty} \omega_k \left[\tilde{y}_{t-k-1} - \tilde{E}_{t-k-2}^f \tilde{y}_{t-k-1} \right]^2 = \delta U_{t-1}^f - (1 - \delta) \left[\tilde{y}_{t-1} - \tilde{E}_{t-2}^f \tilde{y}_{t-1} \right]^2 \tag{6a}$$

$$U_t^e = -\sum_{k=0}^{\infty} \omega_k \left[\tilde{y}_{t-k-1} - \tilde{E}_{t-k-2}^e \tilde{y}_{t-k-1} \right]^2 = \delta U_{t-1}^e - (1 - \delta) \left[\tilde{y}_{t-1} - \tilde{E}_{t-2}^e \tilde{y}_{t-1} \right]^2 \tag{6b}$$

- Probability of choosing one forecast rule over the other:

$$\alpha_t^f = \Pr \left[U_t^f + \varsigma_t^f > U_t^e + \varsigma_t^e \right]$$

- $\varsigma_t^f, \varsigma_t^e$: random components assumed to be logistically distributed.
- It can be shown that the probability is expressed as:

$$\alpha_t^f = \frac{\exp\left(\lambda U_t^f\right)}{\exp\left(\lambda U_t^f\right) + \exp\left(\lambda U_t^e\right)} \tag{7a}$$

$$\alpha_t^e = \frac{\exp\left(\lambda U_t^e\right)}{\exp\left(\lambda U_t^f\right) + \exp\left(\lambda U_t^e\right)} \tag{7b}$$

- The learning mechanism in inflation forecasting is defined analogously.
- Forecast rule of fundamentalists:

$$\tilde{E}_t^f \pi_{t+1} = \bar{\pi} \tag{8a}$$

- Extrapolative forecast rule:

$$\tilde{E}_t^e \pi_{t+1} = \pi_{t-1} \tag{8b}$$

- Expected inflation:

$$\tilde{E}_t \pi_{t+1} = \beta_t^f \cdot \tilde{E}_t^f \pi_{t+1} + \beta_t^e \cdot \tilde{E}_t^e \pi_{t+1} = \beta_t^f \cdot \bar{\pi} + \beta_t^e \cdot \pi_{t-1}$$

$$\beta_t^f + \beta_t^e = 1$$

- Probability of choosing one forecast rule over the other:

$$U_t^{DIT} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \tilde{E}_{t-k-2}^f \pi_{t-k-1}\right]^2 = \delta U_{t-1}^{DIT} - (1-\delta)\left[\pi_{t-1} - \tilde{E}_{t-2}^f \pi_{t-1}\right]^2$$

$$U_t^{EXT} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \tilde{E}_{t-k-2}^e \pi_{t-k-1}\right]^2 = \delta U_{t-1}^{EXT} - (1-\delta)\left[\pi_{t-1} - \tilde{E}_{t-2}^e \pi_{t-1}\right]^2$$

$$\beta_t^f = \frac{\exp\left(\lambda U_t^{DIT}\right)}{\exp\left(\lambda U_t^{DIT}\right) + \exp\left(\lambda U_t^{EXT}\right)} \tag{9a}$$

$$\beta_t^e = \frac{\exp\left(\lambda U_t^{EXT}\right)}{\exp\left(\lambda U_t^{DIT}\right) + \exp\left(\lambda U_t^{EXT}\right)} \tag{9b}$$

3 Model solution

- The complete model consists of IS curve, New Keynesian Philipps curve, and the monetary policy rule. To solve the model, we substitute the monetary policy rule into the IS curve and combine the IS curve with the New Keynesian Philipps curve to form a system of two linear equations.

$$AZ_t = B\tilde{E}_t Z_{t+1} + CZ_{t-1} + bi_{t-1} + \nu_t$$

$$Z_t = A^{-1}\left[B\tilde{E}_t Z_{t+1} + CZ_{t-1} + bi_{t-1} + \nu_t\right]$$

$$Z_t = \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix}$$

- Given Z_t, interest rate i_t is obtained by substituting into the monetary policy rule.
- More details concerning each scenario (monetary policy rule) are given in the final section.
- How to compute impulse response function for the system?
- The computation of IRF is the same as simulation. The main difference is that we shut down all shocks, and allow for one-time shock of a specific disturbance and run the simulation of the model.

4 Animal spirits

- Animal spirits variable is defined as the evolution of the probabilities by which agents extrapolate a positive output gap.

$$\begin{aligned} AS_t = \alpha_t^e \quad & if \quad \tilde{y}_{t-1} > 0 \\ AS_t = 1 - \alpha_t^e = \alpha_t^f \quad & if \quad \tilde{y}_{t-1} < 0 \end{aligned}$$

5 Other variables

- CPI inflation:

$$\pi_t^{CPI} = \pi_t + \alpha \Delta s_t$$

- Changes in nominal exchange rate:

$$\Delta e_t = \Delta s_t + \pi_t - \pi_t^*$$

- Changes in effective real exchange rate:

$$\Delta q_t = (1-\alpha)\,\Delta s_t$$

- Real interest rate (expected):

$$r_t = i_t - \tilde{E}_t \pi_{t+1}$$

- Real interest rate (in program):

$$r_t = i_t - \pi_t$$

- Net export:

$$nx_t = \alpha \left[\frac{\sigma\gamma + (1-\alpha)(\sigma\eta - 1)}{\sigma} - 1 \right] s_t$$

6 Parameters

- $\alpha = 0.4$
- $a_1 = 0.5$
- $\sigma = \gamma = \eta = 1$
- $\varphi = 3$
- $\beta = 0.99$
- $\xi = 6$
- $\theta = 0.75$
- $b_1 = 0.5$
- $b_2 = 0.05$
- $c_1 = 1.5$

- $c_2 = 0.5$
- $c_3 = 0.5$
- In practice, the effects of technology shocks and changes in the foreign output are ignored for simplicity. This means that $a_3 = a_4 \equiv 0$.

7 Program details

7.1 DIT

- DIT, Domestic Inflation Targeting

$$i_t = c_1 (\pi_t - \bar{\pi}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- IS curve

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n \right) + \varepsilon_t$$

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left[c_1 (\pi_t - \bar{\pi}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t - \tilde{E}_t \pi_{t+1} - \rho \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left[c_1 (\pi_t - \bar{\pi}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t \pi_{t+1} \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 c_1 (\pi_t - \bar{\pi}) + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t \pi_{t+1} + \varepsilon_t$$

$$(1 - a_2 c_2) \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\, \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\, \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$(1 - a_2 c_2)\, \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

- In matrix notation:

$$\begin{aligned} \begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & (1 - a_2 c_2) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} &= \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1 - b_1) & 0 \\ 0 & (1 - a_1) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} i_{t-1} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix} + \begin{bmatrix} 0 \\ -a_2 c_1 \end{bmatrix} \bar{\pi} \end{aligned}$$

- $\bar{\pi} = 0$

$$\begin{aligned} \underbrace{\begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & (1 - a_2 c_2) \end{bmatrix}}_{A} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} &= \underbrace{\begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix}}_{B} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \underbrace{\begin{bmatrix} (1 - b_1) & 0 \\ 0 & (1 - a_1) \end{bmatrix}}_{C} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} \\ &+ \underbrace{\begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix}}_{SMOOTH} i_{t-1} + \underbrace{\begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix}}_{SHOCKS} \end{aligned}$$

7.2 CPIT

- CPIT, CPI Inflation Targeting:

$$i_t = c_1 \left(\pi_t^{CPI} - \bar{\pi} \right) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

$$y_t = y_t^* - a_2 s_t$$

$$y_t^n = \Gamma_0 + \Gamma_1 A_t + \Gamma_2 y_t^*$$

- Ignore the effects of technology shocks and changes in the foreign output:

$$y_t = -a_2 s_t$$

$$y_t^n = \Gamma_0$$

$$\tilde{y}_t = y_t - y_t^n = -a_2 s_t - \Gamma_0$$

$$\tilde{y}_t + \Gamma_0 = -a_2 s_t$$

$$s_t = -\frac{1}{a_2}\tilde{y}_t - \frac{1}{a_2}\Gamma_0 = \sigma_5 \tilde{y}_t + \sigma_5 \Gamma_0$$

$$\Delta s_t = -\frac{1}{a_2}\Delta \tilde{y}_t$$

- $\pi_t^{CPI} = \pi_t + \alpha \Delta s_t$

$$\pi_t^{CPI} = \pi_t + \alpha \Delta s_t = \pi_t - \frac{\alpha}{a_2}\Delta \tilde{y}_t$$

- IS curve:

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n\right) + \varepsilon_t$$

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\tilde{y}_{t-1} + a_2 \left[c_1 \left(\pi_t^{CPI} - \bar{\pi}\right) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t - \tilde{E}_t \pi_{t+1} - \rho\right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\tilde{y}_{t-1} + a_2 \left[c_1 \left(\pi_t^{CPI} - \bar{\pi}\right) + c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t \pi_{t+1}\right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} + a_2 c_1 \left(\pi_t^{CPI} - \bar{\pi}\right) + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t \pi_{t+1} + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t^{CPI} + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \left(\pi_t - \frac{\alpha}{a_2} \Delta \tilde{y}_t\right) + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t - \alpha c_1 \tilde{y}_t + \alpha c_1 \tilde{y}_{t-1} + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

$$(1 + \alpha c_1 - a_2 c_2)\,\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t + \alpha c_1 \tilde{y}_{t-1} + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

$$(1 + \alpha c_1 - a_2 c_2)\,\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1 + \alpha c_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

- $a_2 \sigma_5 = -1$
- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1-b_1)\,\pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1-b_1)\,\pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$(1 + \alpha c_1 - a_2 c_2)\,\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1 + \alpha c_1)\,\tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 c_1 \pi_t + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t - a_2 c_1 \bar{\pi}$$

- In matrix notation:

$$
\begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & (1+\alpha c_1 - a_2 c_2) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1-b_1) & 0 \\ 0 & (1-a_1+\alpha c_1) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}
+ \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} i_{t-1} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix} + \begin{bmatrix} 0 \\ -a_2 c_1 \end{bmatrix} \bar{\pi}
$$

- $\bar{\pi} = 0$

$$
\begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & (1+\alpha c_1 - a_2 c_2) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1-b_1) & 0 \\ 0 & (1-a_1+\alpha c_1) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}
+ \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} i_{t-1} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix}
$$

7.3 SDIT

- SDIT, Strict Domestic Inflation Targeting
- Set $c_2 = 0$ in the DIT case.

7.4 SCPIT

- SCPIT, Strict CPI Inflation Targeting
- Set $c_2 = 0$ in the CPIT case.

7.5 PEG

- PEG, Exchange Rate Peg:

$$
i_t = i_t^* + \pi_t^{CPI} - \pi_t^{CPI*} + \rho + \mu_t
$$

- IS curve:

$$
\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\, \tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n \right) + \varepsilon_t
$$

$$
i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*
$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left(i_t^* + \pi_t^{CPI} - \pi_t^{CPI*} + \rho + \mu_t - \tilde{E}_t \pi_{t+1} - \rho \right) + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 i_t^* + a_2 \pi_t^{CPI} - a_2 \pi_t^{CPI*} + a_2 \mu_t - a_2 \tilde{E}_t \pi_{t+1} + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 \pi_t^{CPI} + a_2 \mu_t + \varepsilon_t + a_2 i_t^* - a_2 \pi_t^{CPI*}$$

$$\pi_t^{CPI} = \pi_t + \alpha \Delta s_t = \pi_t - \frac{\alpha}{a_2} \Delta \tilde{y}_t = \pi_t + \alpha \sigma_5 \Delta \tilde{y}_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 \pi_t + \alpha a_2 \sigma_5 \Delta \tilde{y}_t + a_2 \mu_t + \varepsilon_t + a_2 i_t^* - a_2 \pi_t^{CPI*}$$

$$(1 - \alpha a_2 \sigma_5) \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1 - \alpha a_2 \sigma_5) \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 \pi_t + a_2 \mu_t + \varepsilon_t + a_2 i_t^* - a_2 \pi_t^{CPI*}$$

- $a_2 \sigma_5 = -1$
- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$(1 - \alpha a_2 \sigma_5) \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1 - \alpha a_2 \sigma_5) \tilde{y}_{t-1} - a_2 \tilde{E}_t \pi_{t+1} + a_2 \pi_t + a_2 \mu_t + \varepsilon_t + a_2 i_t^* - a_2 \pi_t^{CPI*}$$

- In matrix notation:

$$\begin{bmatrix} 1 & -b_2 \\ -a_2 & (1-\alpha a_2 \sigma_5) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1-b_1) & 0 \\ 0 & (1-a_1-\alpha a_2 \sigma_5) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 i_t^* - a_2 \pi_t^{CPI*} \end{bmatrix}$$

- $i_t^* = \pi_t^{CPI*} = 0$

$$\begin{bmatrix} 1 & -b_2 \\ -a_2 & (1-\alpha a_2 \sigma_5) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1-b_1) & 0 \\ 0 & (1-a_1-\alpha a_2 \sigma_5) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix}$$

- Further simplification:

$$\begin{bmatrix} 1 & -b_2 \\ -a_2 & (1+\alpha) \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (1-b_1) & 0 \\ 0 & (1-a_1+\alpha) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2 \mu_t \end{bmatrix}$$

7.6 DPLT

- DPLT, Domestic Price Level Targeting:

$$i_t = c_1 (p_t - \bar{p}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- IS curve:

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1) \tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n \right) + \varepsilon_t$$

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2 \left[c_1 (p_t - \bar{p}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t - \tilde{E}_t \pi_{t+1} - \rho \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2 \left[c_1 (p_t - \bar{p}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t \pi_{t+1} \right] + \varepsilon_t$$

- $\tilde{E}_t \pi_{t+1} = \tilde{E}_t (p_{t+1} - p_t) = \tilde{E}_t p_{t+1} - p_t$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2 \left[c_1 (p_t - \bar{p}) + c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t p_{t+1} + p_t \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2 c_1 (p_t - \bar{p}) + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t p_{t+1} + a_2 p_t + \varepsilon_t$$

$$(1 - a_2 c_2)\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + (a_2 c_1 + a_2) p_t + a_2 c_3 i_{t-1} - a_2 \tilde{E}_t p_{t+1} - a_2 c_1 \bar{p} + \varepsilon_t + a_2 \mu_t$$

$$(1 - a_2 c_2)\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} - a_2 \tilde{E}_t p_{t+1} + (a_2 c_1 + a_2) p_t + a_2 c_3 i_{t-1} - a_2 c_1 \bar{p} + \varepsilon_t + a_2 \mu_t$$

- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$p_t - p_{t-1} = b_1 \tilde{E}_t p_{t+1} - b_1 p_t + (1-b_1)(p_{t-1} - p_{t-2}) + b_2 \tilde{y}_t + \eta_t$$

$$(1+b_1) p_t = b_1 \tilde{E}_t p_{t+1} + (2-b_1) p_{t-1} - (1-b_1) p_{t-2} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$(1+b_1) p_t = b_1 \tilde{E}_t p_{t+1} + (2-b_1) p_{t-1} - (1-b_1) p_{t-2} + b_2 \tilde{y}_t + \eta_t$$

$$(1 - a_2 c_2)\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} - a_2 \tilde{E}_t p_{t+1} + (a_2 c_1 + a_2) p_t + a_2 c_3 i_{t-1} - a_2 c_1 \bar{p} + \varepsilon_t + a_2 \mu_t$$

- In matrix notation:

$$\begin{bmatrix} (1+b_1) & -b_2 \\ -(a_2c_1+a_2) & (1-a_2c_2) \end{bmatrix} \begin{bmatrix} p_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t p_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (2-b_1) & 0 \\ 0 & (1-a_1) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} -(1-b_1) & 0 \\ 0 & a_2c_3 \end{bmatrix} \begin{bmatrix} p_{t-2} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2\mu_t \end{bmatrix} + \begin{bmatrix} 0 \\ -a_2c_1 \end{bmatrix} \bar{p}$$

- $\bar{p} = 0$

$$\begin{bmatrix} (1+b_1) & -b_2 \\ -(a_2c_1+a_2) & (1-a_2c_2) \end{bmatrix} \begin{bmatrix} p_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t p_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (2-b_1) & 0 \\ 0 & (1-a_1) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} -(1-b_1) & 0 \\ 0 & a_2c_3 \end{bmatrix} \begin{bmatrix} p_{t-2} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2\mu_t \end{bmatrix}$$

7.7 CPLT

- CPLT, Consumer Price Level Targeting:

$$i_t = c_1 \left(p_t^{CPI} - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- IS curve:

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2\left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n\right) + \varepsilon_t$$

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2\left[c_1\left(p_t^{CPI} - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t - \tilde{E}_t\pi_{t+1} - \rho\right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1-a_1)\tilde{y}_{t-1} + a_2\left[c_1\left(p_t^{CPI} - \bar{p}\right) + c_2\tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t\pi_{t+1}\right] + \varepsilon_t$$

- $\tilde{E}_t \pi_{t+1} = \tilde{E}_t (p_{t+1} - p_t) = \tilde{E}_t p_{t+1} - p_t$
- $p_t^{CPI} = p_t + \alpha s_t = p_t + \alpha (\sigma_5 \tilde{y}_t + \sigma_5 \Gamma_0)$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 \left\{ c_1 [p_t + \alpha (\sigma_5 \tilde{y}_t + \sigma_5 \Gamma_0) - \bar{p}] + c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t p_{t+1} + p_t \right\} + \varepsilon_t$$

$$\begin{aligned} \tilde{y}_t &= a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 c_1 (p_t + \alpha \sigma_5 \tilde{y}_t + \alpha \sigma_5 \Gamma_0 - \bar{p}) + a_2 c_2 \tilde{y}_t \\ &\quad + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t p_{t+1} + a_2 p_t + \varepsilon_t \end{aligned}$$

$$\begin{aligned} (1 - a_2 c_2) \tilde{y}_t &= a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} + a_2 c_1 p_t + \alpha a_2 c_1 \sigma_5 \tilde{y}_t + \alpha a_2 c_1 \sigma_5 \Gamma_0 - a_2 c_1 \bar{p} \\ &\quad + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t p_{t+1} + a_2 p_t + \varepsilon_t \end{aligned}$$

$$\begin{aligned} (1 - a_2 c_2 - \alpha a_2 c_1 \sigma_5) \tilde{y}_t &= a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} - a_2 \tilde{E}_t p_{t+1} + (a_2 c_1 + a_2) p_t \\ &\quad + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t + \alpha a_2 c_1 \sigma_5 \Gamma_0 - a_2 c_1 \bar{p} \end{aligned}$$

- $a_2 \sigma_5 = -1$
- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$p_t - p_{t-1} = b_1 \tilde{E}_t p_{t+1} - b_1 p_t + (1 - b_1) (p_{t-1} - p_{t-2}) + b_2 \tilde{y}_t + \eta_t$$

$$(1 + b_1) p_t = b_1 \tilde{E}_t p_{t+1} + (2 - b_1) p_{t-1} - (1 - b_1) p_{t-2} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$(1 + b_1) p_t = b_1 \tilde{E}_t p_{t+1} + (2 - b_1) p_{t-1} - (1 - b_1) p_{t-2} + b_2 \tilde{y}_t + \eta_t$$

$$\begin{aligned} (1 - a_2 c_2 - \alpha a_2 c_1 \sigma_5) \tilde{y}_t &= a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1) \tilde{y}_{t-1} - a_2 \tilde{E}_t p_{t+1} + (a_2 c_1 + a_2) p_t \\ &\quad + a_2 c_3 i_{t-1} + a_2 \mu_t + \varepsilon_t + \alpha a_2 c_1 \sigma_5 \Gamma_0 - a_2 c_1 \bar{p} \end{aligned}$$

- In matrix notation:

$$\begin{bmatrix} (1+b_1) & -b_2 \\ -(a_2c_1+a_2) & (1-a_2c_2-\alpha a_2c_1\sigma_5) \end{bmatrix} \begin{bmatrix} p_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t p_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (2-b_1) & 0 \\ 0 & (1-a_1) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} -(1-b_1) & 0 \\ 0 & a_2c_3 \end{bmatrix} \begin{bmatrix} p_{t-2} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2\mu_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ \alpha a_2c_1\sigma_5\Gamma_0 - a_2c_1\bar{p} \end{bmatrix}$$

- $\bar{p} = 0$

$$\begin{bmatrix} (1+b_1) & -b_2 \\ -(a_2c_1+a_2) & (1-a_2c_2-\alpha a_2c_1\sigma_5) \end{bmatrix} \begin{bmatrix} p_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t p_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (2-b_1) & 0 \\ 0 & (1-a_1) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} -(1-b_1) & 0 \\ 0 & a_2c_3 \end{bmatrix} \begin{bmatrix} p_{t-2} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2\mu_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ \alpha a_2c_1\sigma_5\Gamma_0 \end{bmatrix}$$

- Further simplification:

$$\begin{bmatrix} (1+b_1) & -b_2 \\ -(a_2c_1+a_2) & (1-a_2c_2+\alpha c_1) \end{bmatrix} \begin{bmatrix} p_t \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t p_{t+1} \\ \tilde{E}_t \tilde{y}_{t+1} \end{bmatrix} + \begin{bmatrix} (2-b_1) & 0 \\ 0 & (1-a_1) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} -(1-b_1) & 0 \\ 0 & a_2c_3 \end{bmatrix} \begin{bmatrix} p_{t-2} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t + a_2\mu_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ -\alpha c_1\Gamma_0 \end{bmatrix}$$

7.8 GDPT

- Output Gap Targeting:

$$i_t = c_2\tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t$$

- IS curve:

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 \left(i_t - \tilde{E}_t \pi_{t+1} - i_t^n \right) + \varepsilon_t$$

$$i_t^n = \rho + a_3 A_t + a_4 \tilde{E}_t \Delta y_{t+1}^*$$

- $a_3 = a_4 = 0$

$$i_t^n = \rho$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 \left[c_2 \tilde{y}_t + c_3 i_{t-1} + \rho + \mu_t - \tilde{E}_t \pi_{t+1} - \rho \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 \left[c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t \pi_{t+1} \right] + \varepsilon_t$$

- $\tilde{E}_t \pi_{t+1} = \tilde{E}_t p_{t+1} - p_t$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 \left[c_2 \tilde{y}_t + c_3 i_{t-1} + \mu_t - \tilde{E}_t p_{t+1} + p_t \right] + \varepsilon_t$$

$$\tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 c_2 \tilde{y}_t + a_2 c_3 i_{t-1} + a_2 \mu_t - a_2 \tilde{E}_t p_{t+1} + a_2 p_t + \varepsilon_t$$

$$(1 - a_2 c_2)\, \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} + a_2 p_t + a_2 c_3 i_{t-1} - a_2 \tilde{E}_t p_{t+1} + \varepsilon_t + a_2 \mu_t$$

$$(1 - a_2 c_2)\, \tilde{y}_t = a_1 \tilde{E}_t \tilde{y}_{t+1} + (1 - a_1)\, \tilde{y}_{t-1} - a_2 \tilde{E}_t p_{t+1} + a_2 p_t + a_2 c_3 i_{t-1} + \varepsilon_t + a_2 \mu_t$$

- New Keynesian Phillips curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\, \pi_{t-1} + b_2 \tilde{y}_t + \eta_t$$

$$p_t - p_{t-1} = b_1 \tilde{E}_t p_{t+1} - b_1 p_t + (1 - b_1)\, (p_{t-1} - p_{t-2}) + b_2 \tilde{y}_t + \eta_t$$

$$(1 + b_1)\, p_t = b_1 \tilde{E}_t p_{t+1} + (2 - b_1)\, p_{t-1} - (1 - b_1)\, p_{t-2} + b_2 \tilde{y}_t + \eta_t$$

- Combined Philipps curve and IS curve:

$$(1+b_1)\,p_t = b_1\tilde{E}_t p_{t+1} + (2-b_1)\,p_{t-1} - (1-b_1)\,p_{t-2} + b_2\tilde{y}_t + \eta_t$$

$$(1-a_2c_2)\,\tilde{y}_t = a_1\tilde{E}_t\tilde{y}_{t+1} + (1-a_1)\,\tilde{y}_{t-1} - a_2\tilde{E}_t p_{t+1} + a_2 p_t + a_2c_3 i_{t-1} + \varepsilon_t + a_2\mu_t$$

- In matrix notation:

$$\begin{bmatrix}(1+b_1) & -b_2\\ -a_2 & (1-a_2c_2)\end{bmatrix}\begin{bmatrix}p_t\\ \tilde{y}_t\end{bmatrix} = \begin{bmatrix}b_1 & 0\\ -a_2 & a_1\end{bmatrix}\begin{bmatrix}\tilde{E}_t p_{t+1}\\ \tilde{E}_t\tilde{y}_{t+1}\end{bmatrix} + \begin{bmatrix}(2-b_1) & 0\\ 0 & (1-a_1)\end{bmatrix}\begin{bmatrix}p_{t-1}\\ \tilde{y}_{t-1}\end{bmatrix}$$
$$+\begin{bmatrix}-(1-b_1) & 0\\ 0 & a_2c_3\end{bmatrix}\begin{bmatrix}p_{t-2}\\ i_{t-1}\end{bmatrix} + \begin{bmatrix}\eta_t\\ \varepsilon_t + a_2\mu_t\end{bmatrix}$$

- As s shortcut, one can use the formulation of DPLT or CPLT and simply set $c_1 = 0$.