Online Appendix to "Intellectual Property and Product Market Competition Regulations in a Model with Two R&D Performing Sectors"

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A Online Technical Appendix

Definition of Equilibrium

The decentralized equilibrium in this model is the paths of the quantities

$$\left\{C, A, \left\{X_k, \left\{x_{k,j}, L_{x_{k,j}}, L_{r_{k,j}}, \lambda_{k,j}, \bar{\lambda}_{k,j}\right\}_{j=1}^{N_k}, \left\{u_{k,i,j}, u_{k,j,i}\right\}_{j,i=1(j\neq i)}^{N_k}\right\}_{k=1,2}\right\}$$

and prices

$$\left\{r, w, \left\{\left\{p_{x_{k,j}}, p_{\lambda_{k,j}}, p_{\lambda_{k,i}}\right\}_{j,i=1}^{N_k}\right\}_{k=1,2}\right\}$$

such that:

- The household chooses C, $\left\{X_k, \left\{x_{k,j}, L_{x_{k,j}}, L_{r_{k,j}}\right\}_{j=1}^{N_k}\right\}_{k=1,2}$, and the evolution of A to maximize its utility, given $r, w, \left\{\left\{p_{x_{k,j}}\right\}_{j=1}^{N_k}\right\}_{k=1,2}$ and the current value of A.

- The firm $j = 1, ..., N_k$ in sector $k = 1, 2$ maximizes its value, given the current value of $\lambda_{k,j}$ and $\left\{p_{\lambda_{k,j}}, p_{\lambda_{k,i}}\right\}_{j,i=1(j\neq i)}^{N_k}$.

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– It chooses $\left\{L_{x_{k,j}}, L_{r_{k,j}}\right\}_{j=1}^{N_k}$ and $\{u_{k,i,j}, u_{k,j,i}\}_{j,i=1(j\neq i)}^{N_k}$ subject to the inverse demand for its product under Cournot competition.

– It chooses $\left\{p_{x_{k,j}}, L_{r_{k,j}}\right\}_{j=1}^{N_k}$ and $\{u_{k,i,j}, u_{k,j,i}\}_{j,i=1(j\neq i)}^{N_k}$ subject to the demand for its product under Bertrand competition.

- Labor market clears:

$$L = \sum_{k=1}^{2} \left(N_k L_{x_k} + N_k L_{r_k}\right). \tag{1}$$

- Knowledge market in each sector $k = 1, 2$ clears:

$$\sum_{j=1}^{N_k} \sum_{i=1, i\neq j}^{N_k} u_{k,i,j}\lambda_{k,j} = \sum_{j=1}^{N_k} \sum_{i=1, i\neq j}^{N_k} u_{k,j,i}\lambda_{k,i}. \tag{2}$$

- Intermediate goods and asset markets clear $\left(\dot{A} = 0\right)$.

- Spillovers are firm independent and are given by $\bar{\lambda}_k = \frac{1}{N_k} \sum_{j=1}^{N_k} \lambda_{k,j}$.

Proof of Proposition 1

I use equations (7) and (13) to obtain a relation between labor force allocations in sectors 1 and 2 in a symmetric equilibrium in these sectors:

$$N_2 L_{x_2} = D N_1 L_{x_1}, \tag{3}$$

where D is given by equation (23). This relation, together with the labor market clearing condition (24) implies that labor force allocations to production in sectors

1 and 2 are given by

$$N_1 L_{x_1} = (1+D)^{-1}\left(L - \sum_{k=1}^{2} N_k L_{r_k}\right), \tag{4}$$

$$N_2 L_{x_2} = D\,(1+D)^{-1}\left(L - \sum_{k=1}^{2} N_k L_{r_k}\right). \tag{5}$$

All variables grow at constant rates on a balanced growth path. From equations (9), (4), and (5), it follows that labor allocations are constant on that path.

I use equations (9), (13), (14), (18), (17), (20), and (21) to rewrite relation (19) in the following way:

$$\frac{\dot{q}_{\lambda_k}}{q_{\lambda_k}} = r - \frac{\dot{\lambda}_k}{\lambda_k}\left(\gamma_k \frac{N_k L_{x_k}}{N_k L_{r_k}} + 1 - \alpha_{k,1}\right). \tag{6}$$

From the Euler equation (3) and equations (13), (14), and (7), it follows that another relation for the returns on knowledge accumulation is

$$\frac{\dot{q}_{\lambda_k}}{q_{\lambda_k}} = r - \rho - \frac{\dot{\lambda}_k}{\lambda_k}. \tag{7}$$

I combine these two relations to obtain

$$0 = \rho - \frac{\dot{\lambda}_k}{\lambda_k}\left(\gamma_k \frac{N_k L_{x_k}}{N_k L_{r_k}} - \alpha_{k,1}\right). \tag{8}$$

This expression, together with equations (9), (4) and (5), determines labor force allocations in the balanced growth path equilibrium. The labor force allocations are given by

$$N_1 L_{x_1} = \frac{\xi_1 \xi_2 \frac{\alpha_{1,1}}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2} L + \left(\xi_2 \frac{\alpha_{2,1}}{\gamma_2}\frac{1}{\gamma_1} + \xi_1 \frac{\alpha_{1,1}}{\gamma_1}\frac{1}{\gamma_2}\right)\rho}{\xi_1 \xi_2 \left\{\frac{\alpha_{1,1}}{\gamma_1}\left[\frac{\alpha_{2,1}}{\gamma_2}(1+D) + D\right] + \frac{\alpha_{2,1}}{\gamma_2}\right\}}, \tag{9}$$

$$N_2 L_{x_2} = D N_1 L_{x_1}, \tag{10}$$

and

$$N_1 L_{r_1} = \frac{\xi_1 \xi_2 \frac{\alpha_{2,1}}{\gamma_2} L - \xi_2 \left[\frac{\alpha_{2,1}}{\gamma_2} (1 + D) + D \right] \frac{\rho}{\gamma_1} + \xi_1 \frac{\rho}{\gamma_2}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1 + D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}}, \tag{11}$$

$$N_2 L_{r_2} = \frac{D \xi_1 \xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1} (1 + D) + 1 \right] \frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1 + D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}}. \tag{12}$$

I assume that parameter values are such that $N_1 L_{r_1}$ and $N_2 L_{r_2}$ are positive so that both sectors innovate in equilibrium.

In order to obtain relation (33), I use equation (8) and the fact that labor productivity growth in sector k is given by

$$g_k = \gamma_k g_{\lambda_k}, \tag{13}$$

where g denotes growth rate.

Proof of Corollary 1

The growth rate of λ_k can be derived from equation (9):

$$g_{\lambda_k} = \xi_k N_k L_{r_k}, \tag{14}$$

where $N_1 L_{r_1}$ and $N_2 L_{r_2}$ are given by equations (11) and (12).

Using equations (9)-(12), it can be shown that

$$\frac{\partial}{\partial D} N_1 L_{x_1} = - \frac{\xi_1 \xi_2 \frac{\alpha_{1,1}}{\gamma_1} \frac{\alpha_{2,1}}{\gamma_2} L + \xi_1 \frac{\alpha_{1,1}}{\gamma_1} \frac{\rho}{\gamma_2} + \xi_2 \frac{\alpha_{2,1}}{\gamma_2} \frac{\rho}{\gamma_1}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1 + D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} \frac{\alpha_{1,1}}{\gamma_1} \left(\frac{\alpha_{2,1}}{\gamma_2} + 1 \right) < 0, \tag{15}$$

$$\frac{\partial}{\partial D} N_1 L_{r_1} = - \frac{\xi_1 \xi_2 \frac{\alpha_{1,1}}{\gamma_1} \frac{\alpha_{2,1}}{\gamma_2} L + \xi_1 \frac{\alpha_{1,1}}{\gamma_1} \frac{\rho}{\gamma_2} + \xi_2 \frac{\rho}{\gamma_1} \frac{\alpha_{2,1}}{\gamma_2}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1 + D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} \left(\frac{\alpha_{2,1}}{\gamma_2} + 1 \right) < 0, \tag{16}$$

and

$$\frac{\partial}{\partial D}N_2L_{x_2}=\frac{\xi_1\xi_2\frac{\alpha_{1,1}}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2}L+\xi_1\frac{\alpha_{1,1}}{\gamma_1}\frac{\rho}{\gamma_2}+\xi_2\frac{\alpha_{2,1}}{\gamma_2}\frac{\rho}{\gamma_1}}{\xi_1\xi_2\left\{\frac{\alpha_{1,1}}{\gamma_1}\left[\frac{\alpha_{2,1}}{\gamma_2}(1+D)+D\right]+\frac{\alpha_{2,1}}{\gamma_2}\right\}^2}\frac{\alpha_{2,1}}{\gamma_2}\left(\frac{\alpha_{1,1}}{\gamma_1}+1\right)>0, \tag{17}$$

$$\frac{\partial}{\partial D}N_2L_{r_2}=\frac{\xi_1\xi_2\frac{\alpha_{1,1}}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2}L+\xi_1\frac{\alpha_{1,1}}{\gamma_1}\frac{\rho}{\gamma_2}+\xi_2\frac{\rho}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2}}{\xi_1\xi_2\left\{\frac{\alpha_{1,1}}{\gamma_1}\left[\frac{\alpha_{2,1}}{\gamma_2}(1+D)+D\right]+\frac{\alpha_{2,1}}{\gamma_2}\right\}^2}\left(\frac{\alpha_{1,1}}{\gamma_1}+1\right)>0. \tag{18}$$

According to equation (23), D declines with e_1 and increases with e_2. Therefore, output, R&D, and growth in sector k increase with the level of competition in sector k and decline with the level of competition in the other sector. A uniform increase in competition in both sectors can either increase or reduce D depending on the values of e_1 and e_2. Let $\mathbf{e}=(e_1,e_2)$,

$$\frac{\partial}{\partial \mathbf{e}}D=\frac{1-\sigma}{\sigma}\frac{1}{e_2(e_1-1)}\frac{e_1(e_1-1)-e_2(e_2-1)}{e_2(e_1-1)}. \tag{19}$$

The partial derivative of the growth rate of consumption goods (final output) with respect to D can be derived from equations (4), (5), (8), (16), and (18). It is given by

$$\begin{aligned}\frac{\partial}{\partial D}g_C=&-\frac{\xi_1\xi_2\frac{\alpha_{1,1}}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2}L+\xi_1\frac{\alpha_{1,1}}{\gamma_1}\frac{\rho}{\gamma_2}+\xi_2\frac{\rho}{\gamma_1}\frac{\alpha_{2,1}}{\gamma_2}}{\xi_1\xi_2\left\{\frac{\alpha_{1,1}}{\gamma_1}\left[\frac{\alpha_{2,1}}{\gamma_2}(1+D)+D\right]+\frac{\alpha_{2,1}}{\gamma_2}\right\}^2}\\&\times\left[\sigma_1\gamma_1\xi_1\left(\frac{\alpha_{2,1}}{\gamma_2}+1\right)-(1-\sigma_1)\gamma_2\xi_2\left(\frac{\alpha_{1,1}}{\gamma_1}+1\right)\right].\end{aligned} \tag{20}$$

The sign of this expression depends on the values of the model parameters. This means that the effect of changing the level of competition in sector k and/or uniformly changing the level of competition in both sectors on long-run growth depends on the model parameters. For example, $\partial g_C/\partial D$ is negative (positive) when $\sigma_1>1/2$ ($\sigma_1<1/2$) and the effect of changing the level of competition on growth in sector 2 is higher (lower) than this effect in sector 1. It is necessarily negative (positive) if $\sigma_1=1/2$, $\gamma_1=\gamma_2$, $\alpha_{2,1}=\alpha_{1,1}$, and $\xi_1>\xi_2$ ($\xi_1<\xi_2$).[1]

In a special case when $e_1 = e_2$, D does not depend on the levels of competition in sectors 1 and 2. This implies that the level of competition does not matter for resource allocations in the economy and imperfect/oligopolistic competition does not distort them. Such a result holds because all price levels are equally affected by imperfect competition when $e_1 = e_2$ and the relative prices are not. Decentralized equilibrium allocations are socially optimal in a similar one-sector model when relative prices are not distorted and $\alpha_{\cdot,1} = 0$ (e.g., when $\sigma = 1$ in the model of Jerbashian, 2016).[2] In contrast, in this two sector model allocations in decentralized equilibrium are not socially optimal. Both sectors innovate in decentralized equilibrium because of private incentives. However, the social planner would choose $L_r = 0$ and no innovation in the sector which has the lowest contribution to growth. It would do so because R&D process (9) is linear in labor input.

The partial derivatives of labor force allocations with respect to $\alpha_{k,1}$ can be readily derived from equations (9)-(12). The partial derivatives with respect to $\alpha_{1,1}$ are given by

$$\frac{\partial}{\partial \alpha_{1,1}} N_1 L_{x_1} = \frac{\alpha_{2,1}}{\gamma_2} \frac{1}{\gamma_1} \frac{\xi_1 \xi_2 L \frac{\alpha_{2,1}}{\gamma_2} - \left\{ \xi_2 \frac{1}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] - \xi_1 \frac{1}{\gamma_2} \right\} \rho}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0, \tag{21}$$

$$\begin{aligned} \frac{\partial}{\partial \alpha_{1,1}} N_1 L_{r_1} = &- \frac{\xi_1 \xi_2 \frac{\alpha_{2,1}}{\gamma_2} L - \xi_2 \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] \frac{\rho}{\gamma_1} + \xi_1 \frac{\rho}{\gamma_2}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} \frac{1}{\gamma_1} \\ &\times \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] < 0, \end{aligned} \tag{22}$$

$$\frac{\partial}{\partial \alpha_{1,1}} N_2 L_{x_2} = D \frac{\alpha_{2,1}}{\gamma_2} \frac{1}{\gamma_1} \frac{\xi_1 \xi_2 L \frac{\alpha_{2,1}}{\gamma_2} - \left\{ \xi_2 \frac{1}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] - \xi_1 \frac{1}{\gamma_2} \right\} \rho}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0, \tag{23}$$

$$\frac{\partial}{\partial \alpha_{1,1}} N_2 L_{r_2} = \frac{1}{\gamma_1} D \frac{\xi_1 \xi_2 \frac{\alpha_{2,1}}{\gamma_2} L - \xi_2 \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] \frac{\rho}{\gamma_1} + \xi_1 \frac{\rho}{\gamma_2}}{\xi_1 \xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0. \tag{24}$$

In turn, the partial derivatives with respect to $\alpha_{2,1}$ are given by

$$\frac{\partial}{\partial \alpha_{2,1}} N_1 L_{x_1} = \frac{1}{\gamma_2} \frac{\alpha_{1,1}}{\gamma_1} \frac{D\xi_1\xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1} (1+D) + 1 \right] \frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1\xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0, \tag{25}$$

$$\frac{\partial}{\partial \alpha_{2,1}} N_1 L_{r_1} = \frac{1}{\gamma_2} \frac{D\xi_1\xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1} (1+D) + 1 \right] \frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1\xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0, \tag{26}$$

$$\frac{\partial}{\partial \alpha_{2,1}} N_2 L_{x_2} = D \frac{1}{\gamma_2} \frac{\alpha_{1,1}}{\gamma_1} \frac{D\xi_1\xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1} (1+D) + 1 \right] \frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1\xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} > 0, \tag{27}$$

$$\begin{aligned} \frac{\partial}{\partial \alpha_{2,1}} N_2 L_{r_2} = & - \frac{D\xi_1\xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1} (1+D) + 1 \right] \frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1\xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} \\ & \times \frac{1}{\gamma_2} \left[\frac{\alpha_{1,1}}{\gamma_1} (1+D) + 1 \right] < 0. \end{aligned} \tag{28}$$

These results imply that reducing $\alpha_{k,1}$ increases R&D and growth in sector k and reduces R&D and growth in the other sector.

The effect of a uniform change in $\alpha_{1,1}$ and $\alpha_{2,1}$ on the growth rate in sector k is given by the sum of the partial derivatives of $N_k L_{r_k}$ with respect to $\alpha_{1,1}$ and $\alpha_{2,1}$. The sign and the magnitude of this effect depend on the model parameters.

The partial derivatives of the growth rate of consumption goods (final output) with respect to $\alpha_{1,1}$ and $\alpha_{2,1}$ can be derived from equations (4), (5), (8), and (21)-(28). They are given by

$$\begin{aligned} \frac{\partial}{\partial \alpha_{1,1}} g_C = & - \frac{1}{\gamma_1} \frac{\xi_1\xi_2 \frac{\alpha_{2,1}}{\gamma_2} L - \xi_2 \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] \frac{\rho}{\gamma_1} + \xi_1 \frac{\rho}{\gamma_2}}{\xi_1\xi_2 \left\{ \frac{\alpha_{1,1}}{\gamma_1} \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] + \frac{\alpha_{2,1}}{\gamma_2} \right\}^2} \\ & \times \left\{ \sigma_1\gamma_1\xi_1 \left[\frac{\alpha_{2,1}}{\gamma_2} (1+D) + D \right] - (1-\sigma_1)\gamma_2\xi_2 D \right\}, \end{aligned} \tag{29}$$

and

$$\frac{\partial}{\partial \alpha_{2,1}} g_C = \frac{1}{\gamma_2} \frac{D\xi_1\xi_2 \frac{\alpha_{1,1}}{\gamma_1} L - \xi_1 \left[\frac{\alpha_{1,1}}{\gamma_1}(1+D)+1\right]\frac{\rho}{\gamma_2} + \xi_2 D \frac{\rho}{\gamma_1}}{\xi_1\xi_2 \left\{\frac{\alpha_{1,1}}{\gamma_1}\left[\frac{\alpha_{2,1}}{\gamma_2}(1+D)+D\right]+\frac{\alpha_{2,1}}{\gamma_2}\right\}^2} \times \left\{\sigma_1\gamma_1\xi_1 - \left[\frac{\alpha_{1,1}}{\gamma_1}(1+D)+1\right](1-\sigma_1)\gamma_2\xi_2\right\}. \tag{30}$$

The signs of these expressions depend on the values of the model parameters. This means that the effects of changing $\alpha_{1,1}$ and $\alpha_{2,1}$ on long-run growth depend on the model parameters. For example, $\partial g_C/\partial \alpha_{1,1}$ is negative (positive) when $\sigma_1 > 1/2$ ($\sigma_1 < 1/2$) and the effect of changing $\alpha_{1,1}$ on growth in sector 1 is higher (lower) than this effect on growth in sector 2. Both these expressions are negative when $\sigma_1 = 1/2$, $\xi_1 = \xi_2$, and $\gamma_1 = \gamma_2$.

The effect of a uniform change in $\alpha_{1,1}$ and $\alpha_{2,1}$ on g_C is given by the sum of the partial derivatives of g_C with respect to $\alpha_{1,1}$ and $\alpha_{2,1}$. The sign and the magnitude of this effect depend on the model parameters.

B Online Appendix - Further Results

Table A: The Growth Effects of Weakening Intellectual Property Rights and Reducing Product Market Competition

	A.1: 10%Δ in $\alpha_{1,1}$			A.2: 10%Δ in $\alpha_{2,1}$			A.3: 10%Δ in $\alpha_{1,1}$ and $\alpha_{2,1}$		
%Δ in	g_1	g_2	g_C	g_1	g_2	g_C	g_1	g_2	g_C
Germany	-8.724	0.583	-3.375	1.628	-6.952	-3.303	-7.229	-6.401	-6.753
UK	-8.824	0.408	-3.440	1.507	-7.187	-3.563	-7.444	-6.802	-7.070
US	-8.761	0.563	-3.110	1.518	-6.953	-3.616	-7.368	-6.421	-6.794
	B.1: 10%Δ in $1/e_1$			B.2: 10%Δ in $1/e_2$			B.3: 10%Δ in $1/e_1$ and $1/e_2$		
%Δ in	g_1	g_2	g_C	g_1	g_2	g_C	g_1	g_2	g_C
Germany	-1.223	0.024	-0.506	2.437	-1.431	0.214	1.638	-0.962	0.144
UK	-1.494	0.130	-0.547	1.435	-0.546	0.280	0.244	-0.093	0.047
US	-1.856	0.241	-0.585	2.711	-1.171	0.358	1.217	-0.526	0.161
	C: 10%Δ in $\alpha_{1,1}$, $\alpha_{2,1}$, $1/e_1$, and $1/e_2$								
%Δ in	g_1	g_2	g_C						
Germany	-5.721	-7.303	-6.630						
UK	-7.220	-6.888	-7.027						
US	-6.247	-6.912	-6.650						

Note: This table offers the effects of weakening intellectual property rights (10% increase in $\alpha_{k,1}$) and reducing product market competition (10% increase in $1/e_k$) on labor productivity growth rates in the goods and services sectors (g_1 and g_2) and on the growth rate of the economy [$g_C = \sigma_1 g_1 + (1 - \sigma_1) g_2$]. The effects are computed as percentage changes in the values of the growth rates presented in Table 1. The goods sector is sector 1 and the services sector is sector 2.

B.1 An Extension of Aghion, Bloom, Blundell, Griffith, and Howitt (2005)

In this section, I use the data and an extension of the empirical methodology developed by Aghion et al. (2005) and present evidence that innovation in an industry can be affected by competition in closely related industries.

Aghion et al. (2005) aim to identify the effect of competition in industries on innovation and growth. They use data from the UK for seventeen 2-digit SIC manufacturing industries for the period 1973–1994. They use the number of citation-weighted patents in each industry as an indicator of innovation/R&D. In turn, they compute the intensity of competition in an industry as follows:

$$c_{jt} = 1 - \frac{1}{N_{jt}} \sum_{i \in j} li_{it}, \tag{31}$$

where N_{jt} is the number of firms in industry j at time t, i indexes firms, and li is the price-cost margin/Learner index. Aghion et al. (2005) compute it as

$$li_{it} = \frac{\text{operating profits - financial costs}}{\text{sales}}. \tag{32}$$

They run a regression of the following form:

$$\mathbb{E}\left[p_{jt} | c_{jt}, x_{jt}\right] = \exp\left(\beta_1 c_{jt} + \beta_2 c_{jt}^2 + x'_{jt}\Gamma\right), \tag{33}$$

where p_{jt} is the citation-weighted number of patents, β_1, β_2 and Γ are parameters and x'_{jt} are control variables. To alleviate reverse causality concerns, Aghion et al. (2005) use the control function approach. They find that $\beta_1 > 0$ and $\beta_2 < 0$ and that the relationship between competition and innovation has an inverted-U shape. In column 1 of Table B, I present their preferred results from column 4 of Table 1 of their paper.[3]

In the main text, I show that competition for factor inputs across two industries can create a link between competition in one industry and innovation in the other. I utilize the 2-digit SIC symmetric input-output table and develop a measure of proximity between industries in terms of factor inputs to formally test this in a setting with multiple industries. From the input-output table, I obtain the share of compensation for each input out of the total input compensation in the 2-digit

SIC industries in the UK in 1984.[4] For each industry, I compute the Euclidean distances between the vector of its input compensation shares and the vectors of input compensation shares of the remainder of industries. The distances of these vectors are a measure of dissimilarity between industries, and I take their inverse to obtain a measure of proximity between industries. Let θ_{jm} be the values of this proximity measure between industries j and m. I replace $\theta_{jj} = 0$ and compute for industry j the interaction between its proximity to other industries and competition in those industries,

$$\hat{c}_{jt} = \sum_{m} \theta_{jm} c_{mt}. \tag{34}$$

The data used by Aghion et al. (2005) are unbalanced and many (non-overlapping) years are missing for SIC industries 23, 35, 37 and 49. I drop these industries from the sample because keeping them severely restricts the number of observations when computing $\hat{c}_{jt}$. Column 2 of Table B offers the results from the estimation of specification (33) for the restricted sample.

I augment specification (33) with additional terms and estimate the following regression:

$$\mathbb{E}\left[p_{jt} | c_{jt}, x_{jt}\right] = \exp\left(\beta_1 c_{jt} + \beta_2 c_{jt}^2 + \delta_1 \hat{c}_{jt} + \delta_2 \hat{c}_{jt}^2 + x_{jt}' \Gamma\right). \tag{35}$$

According to the theoretical model developed in the main text, the estimate of β_1 is expected to be positive, while that of δ_1 is expected to be negative. It can also be expected that the estimate of β_2 is negative so that the relationship between competition and innovation in an industry has a shape resembling an inverted-U. This is because, in this model, the relationship between competition and innovation in an industry is increasing and concave, as long as there is a positive amount of innovation. Moreover, increasing competition in an industry reduces profits in equation (12) and there is a level of competition at which profits are equal to zero. Innovation increases with competition till this level and ceases when the level of competition increases above this level. In the same vein, the estimate of δ_2 can be expected to be positive since resources that can be devoted to R&D decline with competition in rival industries at a declining rate. This is because of the concave relationship between competition and innovation in an industry. Moreover, they increase in an industry if some of the rival industries stop innovating.

Table B: The Effects of Competition on Innovation

Dependent variable: citation-weighted count of patents in industry j at time t	(1)	(2)	(3)	(4)
c_{jt}	386.592***	246.337***		220.652**
	(67.611)	(93.873)		(95.365)
c_{jt}^2	-205.320***	-127.915***		-114.630**
	(36.105)	(50.346)		(51.124)
$\hat{c}_{jt}$			-104.314***	-72.159*
			(40.486)	(41.811)
$\hat{c}_{jt}^2$			38.222***	26.223*
			(15.185)	(15.736)
Observations	354	286	286	286

Note: This table presents the results from the estimation of specification (35). Column 1 reports the results from column 4 of Table 1 of Aghion et al. (2005). These results can be obtained estimating specification (35) for the full sample of industries and parameter restriction $\delta_1 = \delta_2 = 0$. Column 2 reports the results when I drop SIC industries 23, 35, 37 and 49 from the sample and keep $\delta_1 = \delta_2 = 0$. In columns 3 and 4, SIC industries 23, 35, 37 and 49 are dropped from the sample. Columns 3 and 4 report the results from the estimation of specification (35) with and without parameter restriction $\beta_1 = \beta_2 = 0$, correspondingly. All regressions include industry and year dummies and use the Poisson regression framework. Moreover, all regressions are carried out using the control function method. To implement it, c_{jt} and $\hat{c}_{jt}$ are linearly projected on a set of exogenous instruments (see, for the list of instruments Aghion et al., 2005). The residuals from these projections are added in specification (35) as independent variables. The exogenous instruments are jointly significant in these projections and R-squares are higher than 0.8. Standard errors are reported in parentheses. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

Column 3 of Table B reports the results from the estimation of specification (35) under the restriction $\beta_1 = \beta_2 = 0$. Column 4 of Table B reports the results without this restriction. As expected, the estimate of δ_1 is negative. which suggests that innovation in an industry can decline with higher competition in other and closely related industries. The estimates of β_1, β_2, and δ_2 also have the expected signs.

According to Column 4 of Table B, it is important to control for $\hat{c}$ and $\hat{c}^2$ in (35) for the identification of the magnitude of estimates of β_1 and β_2. These estimates change by about 10 percent when $\hat{c}$ and $\hat{c}^2$ are controlled for.[5]

The results reported in columns 3 and 4 of Table B constitute a first attempt to show that competition in an industry can affect innovation and growth in other industries. They outline an area of potentially fruitful future research.

B.2 Calibration of the Chu (2011) and Goh and Olivier (2002) Models

The model of this paper and the model developed by Chu (2011) feature two horizontally related sectors, whereas the model developed by Goh and Olivier (2002) features two vertically related sectors. Therefore, it is straightforward to calibrate the model of Chu (2011) for the goods and services sectors as it is done in this paper. I discuss the calibration and results for the model of Chu (2011) first because of this. I change the notations of the models of Chu (2011) and Goh and Olivier (2002) to align them more closely with the notation used in this paper.

Chu (2011) considers a two-sector version of the canonical Schumpeterian growth model. In this model, equilibrium labor force allocations to production and R&D in sector k ($k = 1, 2$) are given by

$$L_{x_k} = \sigma_k \left(L + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2} \right) \frac{1}{\mu_k}, \tag{36}$$

$$L_{r_k} = \sigma_k \left(L + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2} \right) \left(\frac{\mu_k - 1}{\mu_k} \right) - \frac{\rho}{\varphi_k}, \tag{37}$$

where σ_k is the share of expenditures on sector k out of expenditures on final consumption goods, ρ is the discount rate, and φ_k is a technological opportunity parameter. A higher value of φ_k increases the arrival rate of innovations. Conceptually, φ_k is similar to ξ_k and γ_k since increasing φ_k increases the sectoral growth rate for a given amount of labor allocated to R&D. In turn, μ_k is the patent breadth parameter. It characterizes the strength of property rights by establishing the minimal difference between an idea and current patents such that the idea can be patented. At the same time, it measures the market power of producers (patent holders) because it defines their power to exclude certain ideas from being patented and produced. It is given by

$$\mu_k = \frac{1}{1 - 1/e_k}. \tag{38}$$

The growth rates of sectors and final output are given by

$$g_k = \varphi_k L_{r_k} \ln z, \tag{39}$$

$$g_C = \sigma_1 g_1 + (1 - \sigma_1) g_2, \tag{40}$$

where z is the exogenous step size of productivity improvement from an innovation.

I calibrate this model for the goods and services sectors. The values of L, ρ, σ_k, and $1/e_k$ are from Table 1. I set z to be equal to the base of the natural logarithm so that $\ln z = 1$ and derive the values of μ_k from equation (38). Finally, I calibrate φ_k using the values of sectoral growth rates. Panel A of Table C offers the values of μ_k and φ_k for $k = 1, 2$, where the goods sector is sector 1 and the services sector is sector 2.

Table D reports the effects of a 10 percent increase in $1/e_k$ in the goods and services sectors on sectoral growth rates, as well as on the growth rate of final output. As a policy, these comparative statics correspond to increasing patent breadth and in that sense they correspond to strengthening property rights in the goods and services sectors in the model of Chu (2011). Sectoral and aggregate growth increase with patent breadth in this model.

These results differ from the results of the model of this paper in a few notable ways. Changes in $1/e_k$ have no cross-sectoral effects in the model of Chu (2011). This is because $1/e_k$ does not affect labor force allocation to sector k in the model of Chu (2011). It affects labor force allocation across production and R&D activities within sector k. This can be clearly seen by summing up L_{x_k} and L_{r_k} from equations

Table C: The Values of Parameters of the Chu (2011) and Goh and Olivier (2002) Models

A. Chu (2011)

z	2.718

	μ_1	μ_2	φ_1	φ_2
Germany	1.098	1.298	1.587	0.215
UK	1.136	1.163	1.729	0.306
US	1.159	1.287	1.231	0.162

B. Goh and Olivier (2002)

	1/e
Germany	0.162
UK	0.132
US	0.188

	χ	0.200	0.300	0.400	0.500	0.600	0.700	0.800
Germany	b_i and b_j	1.737	0.772	0.450	0.290	0.193	0.129	0.083
	A	1.112	0.908	0.851	0.874	0.974	1.189	1.665
	ϵ	5.000	3.333	2.500	2.000	1.667	1.429	1.250
UK	b_i and b_j	1.372	0.610	0.356	0.229	0.152	0.102	0.065
	A	1.267	1.039	0.976	1.005	1.122	1.371	1.920
	ϵ	5.000	3.333	2.500	2.000	1.667	1.429	1.250
US	b_i and b_j	2.088	0.928	0.541	0.348	0.232	0.155	0.099
	A	0.737	0.614	0.587	0.614	0.696	0.863	1.227
	ϵ	5.000	3.333	2.500	2.000	1.667	1.429	1.250

Note: Panel A in this table offers the calibrated values of parameters of the model of Chu (2011). The goods sector is sector 1 and the services sector is sector 2. Panel B offers the calibrated values of parameters of the model of Goh and Olivier (2002). See Table 1 for sample periods, the values of growth rates and the reminder of parameters.

Table D: The Growth Effects of Increasing Patent Breadth in the Chu (2011) Model

	A: 10%Δ in $1/e_1$			B: 10%Δ in $1/e_2$			C: 10%Δ in $1/e_1$ and $1/e_2$		
%Δ in	g_1	g_2	g_C	g_1	g_2	g_C	g_1	g_2	g_C
Germany	16.920	0.000	7.197	0.000	21.338	12.262	16.920	21.338	19.459
UK	15.919	0.000	6.635	0.000	23.272	13.572	15.919	23.272	20.207
US	18.102	0.000	7.132	0.000	27.379	16.591	18.102	27.379	23.724

Note: This table offers the effects of increasing patent breadth (10% increase in $1/e_k$) on labor productivity growth rates in the goods and services sectors (g_1 and g_2) and on the growth rate of the economy $[g_C = \sigma_1 g_1 + (1-\sigma_1)\, g_2]$ in Chu (2011) model. The effects are computed as percentage changes from the values of growth rates offered in Table 1. The goods sector is sector 1 and the services sector is sector 2.

(36) and (37). Moreover, innovation in sector k increases with $1/e_k$ in the model of Chu (2011). Such an inference holds because innovation is carried out by entrants in this model, and increasing $1/e_k$ increases entrants' post innovation profits and value. In contrast, innovation declines with $1/e_k$ in the model of the current paper. This is because increasing $1/e_k$ reduces competition, sales, and the marginal product of innovation.[6]

Goh and Olivier (2002) consider a two-sector version of the Romer (1990) model, where the sectors of the economy are vertically related. In this model, the growth rate of final output is given by

$$g_C = \chi g_i, \tag{41}$$

where g_i is the rate of innovation in the intermediate goods sector,

$$g_i = \frac{A_i \chi b_i \left(L + \frac{\rho}{A_j} + \frac{\rho}{A_i}\right)}{\left(1 + \frac{b_j}{\epsilon - 1}\right)\left(1 + \frac{\chi b_i}{1-\chi}\right)} - \rho, \tag{42}$$

χ is the share of labor compensation in the final goods sector, ϵ is the elasticity of substitution between final goods, ρ is discount rate, and A_i and A_j are R&D productivity parameters in the R&D labs of the intermediate and final goods sectors.[7] These parameters are conceptually similar to ξ_k and γ_k. In turn, b_i and b_j are patent breadth parameters for the intermediate and final goods sectors. Similarly to Chu (2011), they are related to competition and mark-ups in the corresponding sectors:

$$\frac{\chi b_i}{1-\chi} = \frac{1}{e_i - 1}, \tag{43}$$

$$\frac{b_j}{\epsilon - 1} = \frac{1}{e_j - 1}. \tag{44}$$

To calibrate the values of the model parameters, I assume that mark-ups in the intermediate and final goods sectors are equal and that initially there are no differences between b_i and b_j: $\chi b_i/(1-\chi) = b_j/(\epsilon - 1)$ and $b_i = b_j$. I also assume that $A_i = A_j$ and denote $e \equiv e_i = e_j$.

I compute the price-cost margin at the economy-level, take its average over time and use this average $(1/e)$ to compute the values of $\chi b_i/(1-\chi)$ and $b_j/(\epsilon - 1)$. The share of labor compensation in the final goods sector χ is allowed to vary freely in $(0.2, 0.8)$ interval. For a given value of χ, the value of A is computed using the value of g_C from Table 1. Panel B of Table C offers the calibrated values of these parameters.

Table E reports the effects of a 10 percent increase in b_i and b_j on the growth rate of final output. As a policy, this corresponds to increasing the strength of property rights in the intermediate and final goods sectors in the model of Goh and Olivier (2002).

The growth rate of final output increases with the strength of patent breadth in the intermediate goods sector and declines with the strength of patent breadth in the final goods sector. The latter result holds because, similarly to the model of the current paper, competition and property rights regulations have cross-sectoral effects in the model of Goh and Olivier (2002). Within a sector, innovation increases with b and mark-ups in the model of Goh and Olivier (2002).[8] This is similar to the model of Chu (2011) and in contrast to the model of the current paper. Again,

Table E: The Growth Effects of Increasing Patent Breadth in the Goh and Olivier (2002) Model

%Δ in g_C	A: 10%Δ in b_i	B: 10%Δ in b_j	C: 10%Δ in b_i and b_j
Germany	12.962	-2.502	10.254
UK	11.435	-2.089	11.435
US	10.748	-3.327	10.748

Note: This table offers the effects of increasing patent breadth (10% increase in b_i and b_j) on the growth rate of the economy in the Goh and Olivier (2002) model. The effects are computed as percentage changes from the value of growth rate offered in Table 1 and are invariant to the value of χ.

such an inference holds because increasing b and mark-ups increases the entrants' post innovation profits and value in the model of Goh and Olivier (2002).

Notes

[1]This derivative is also negative (positive) if $\sigma_1 = 1/2$, $\gamma_1 = \gamma_2$, $\xi_1 = \xi_2$, and $\alpha_{2,1} > \alpha_{1,1}$ ($\alpha_{2,1} < \alpha_{1,1}$). Therefore, the strength of property rights can play an important role for the effect of product market competition in an industry on economic growth.

[2]In the one-sector model developed by Chantrel, Grimaud, and Tournemaine (2012), resource allocation and growth are socially optimal when licensors can verify the use of their patents and have the bargaining power to exclude the use (i.e., $\alpha_{\cdot,2} = 1$). In the current model, this happens only when $e_1 = e_2$.

[3]The authors have recently corrected that table and the corrections are available online.

[4]I use the 1984 input-output tables because the industry classification in this table directly matches with the industry classification used by Aghion et al. (2005). Using a fixed year for shares should not be a major issue because usually the shares of compensations change very little over time.

[5]It has to be noted that these changes are not statistically significant, even though they are economically sizeable.

[6]The previous section of this appendix provides empirical evidence that competition can have cross-sector effects. In turn, Blundell, Griffith, and van Reenen (1999) provide evidence that innovation increases with competition. The model of this paper can also generate an inverted-U shape like relationship between competition and innovation (Aghion et al., 2005).

[7]I consider the case when both the final and intermediate input sectors innovate.

[8]The rate of innovation in the final goods sector is given by $g_j = [A_j \frac{b_j}{\epsilon-1}(L+\frac{\rho}{A_j}+\frac{\rho}{A_i})/(1+\frac{b_j}{\epsilon-1})]-\rho$.

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