Appendix to “Inflation Targeting, Pattern of Trade and Economic Dynamics”

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In Appendix A, we present data sources and descriptions, as well as perform some simple data analyses. In Appendix B, we provide mathematical proofs and derivations.

Appendix A: Data Sources, Descriptions and Simple Analyses

There are 28 IT countries, including Armenia, Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Finland, Hungary, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, the Philippines, Poland, Romania, Serbia, South Africa, South Korea, Spain, Sweden, Switzerland, Thailand, Turkey, and the United Kingdom. As a control group, we consider 39 non-IT countries, including Argentina, Austria, Belarus, Belgium, Bulgaria, China, Croatia, Denmark, the Dominican Republic, Ecuador, Estonia, France, Georgia, Germany, Guatemala, Hong Kong, Ireland, Italy, Jamaica, Japan, Jordan, Kazakhstan, Latvia, Lebanon, Lithuania, Morocco, the Netherlands, Panama, Paraguay, Russia, Singapore, Slovenia, Syria, Trinidad & Tobago, Tunisia, Ukraine, the United States, Uruguay, and Venezuela.

We collect GDP per capita from *World Development Indicators* to calculate the relative income of IT countries to the U.S. (relative income). The data for capital stock are directly collected from the Penn World Table version 8.0. The sources of net capital good exports, net consumption good exports, and trade openness are OECD statistics and the International Trade Centre. Based on the UN Broad Economic Category (BEC) classifications and by using the detailed classification of trade in goods of the Harmonized System (HS), OECD, bilateral flows of exports and imports are classified into three main categories: Household Consumption Goods (primary, processed unfinished, and processed finished products), Intermediate Goods (primary and processed unfinished products), and Capital Goods (processed finished products).

Among the 28 IT countries, we classify 11 countries with relative income exceeding 50% at the time of adopting IT as advanced, and 13 countries with relative income below 28% as less developed (Poland's relative income in 1998 when adopting IT was 28.46%). Among 11 advanced countries, Canada, Israel, New Zealand, and the United Kingdom are early adopters (with IT institutionalized during 1990-92) with insufficient bilateral trade data prior to the adoption of IT. The IT policy in Finland or Spain has been on an on-and-off basis, so some of the previous studies have viewed them as non-IT. Thus, after attrition, we end up with 5 advanced countries: Norway, Switzerland, Sweden, Australia, and Iceland. Among 13 less developed countries, an early adopter, Chile (with IT institutionalized in 1991), as well as late adopters, Armenia, Indonesia, Romania and Serbia (with IT institutionalized in 2005 or after), are excluded due to insufficient data prior to the adoption of IT or after the adoption of IT. We thus end up with 8 developing countries: Mexico, Brazil, South Africa, Thailand, Colombia, Peru, the Philippines, and Turkey.

We then identify 4 pairs of mutually major trading partners: Turkey-Switzerland, Turkey-Sweden, Brazil-Switzerland, and Thailand-Australia that meet three qualifications, as noted in the

Introduction. While Sweden and Switzerland have maintained their inflation targets of around 2% since 1993 and 2000, respectively, Turkey sharply lowered its targets from 20% in 2002 to around 8% by 2004, and Brazil's inflation targets fell from 8% in 1999 to below 4% by 2001. While Australia's inflation targets since adoption in 1993 have been kept at around 2% with only small adjustments, Thailand set its core inflation target at between $0.0 - 3.5\%$, and its inflation rate then fell from 5% (before the IT adoption) to 1.1% by 2004. Moreover, Turkey is the 3rd-ranked destination to which Switzerland exports and the 5th to which Sweden exports, whereas Switzerland is the 6th-ranked destination to which Turkey exports and the 5th to which Brazil exports. While Sweden is the 5th-ranked exporting destination of Turkey, Brazil is the 6th-ranked exporting destination of Switzerland. Thailand and Australia are the 1st- and the 4th-ranked exporting destinations of each other. These pairs are major trading partners with one advanced country with steady inflation targets, and another less developed country with noticeable unilateral changes in inflation targets. Note that in order to be consistent with our dynamic Heckscher-Ohlin world equilibrium, trade patterns are commonly expressed in terms of aggregate measures.

In addition, we depict the time series of three indicators (developing countries' net consumption good exports (XC), developed countries' net capital good exports (XK), and capital accumulation (K)) for comparable plots of the 13 countries as follows. We plot each indicator over the period with 5 years prior to and after the adoption of IT. There are two exceptions, Australia and Sweden, where we only have three years of data on trade patterns prior to their adoption of IT. These two countries are nonetheless included; otherwise, our advanced countries in the treatment group would have been reduced to only three.

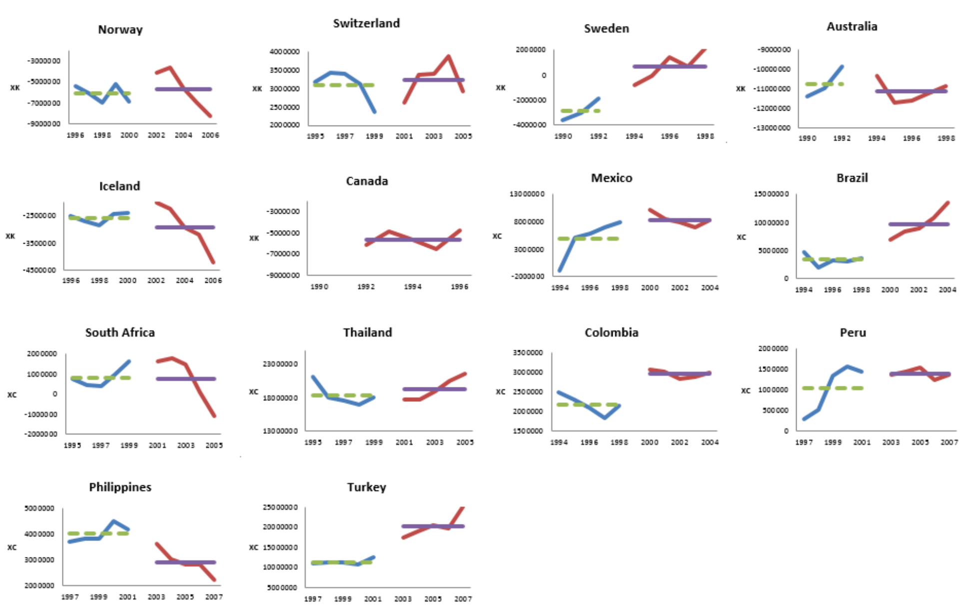

Figure A-1: Time series of capital and consumption good exports

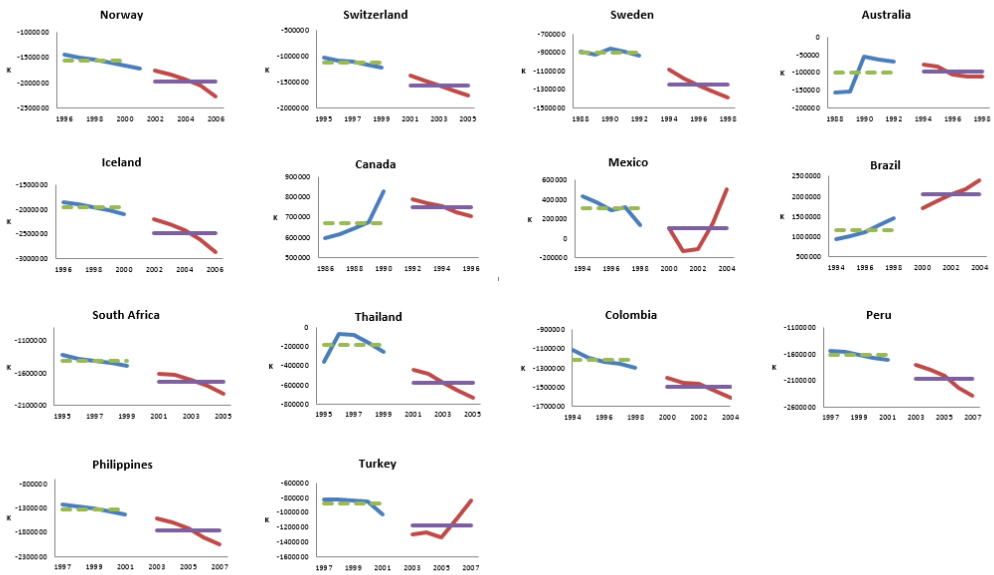

Figure A-2: Time series of capital stock

These time series observations are summarized in Table 1, which reports the respective differences and percentage changes between the annual average of 5 years prior to and after the adoption of IT for both groups of advanced and developing countries. In each group, countries are weighted by their real GDP.

Appendix B: Proofs and Mathematical Derivations

Household's Optimization. The current-value Hamiltonian associated with the household's optimization problem can be written as:

$$H(t) = \ln c(t)+\lambda(t)[w(t)+r(t)k(t)\text{-}P(t)c(t)\text{-}i(t)+TR(t)\text{-}\pi(t)m(t)]+q(t)[i(t)\text{-}\delta k(t)]+\varepsilon(t)[m(t)\text{-}P(t)c(t)].$$

The necessary conditions for this optimization problem are given by:

$$\frac{1}{c(t)} = P(t)[\lambda(t)+\varepsilon(t)], \tag{A1}$$

$$\dot{\lambda}(t) = \rho\lambda(t)-\varepsilon(t)+\lambda(t)\pi(t), \tag{A2}$$

$$\dot{q}(t) = \rho q(t)-\lambda(t)r(t)+q(t)\delta, \tag{A3}$$

$$q(t) = \lambda(t), \tag{A4}$$

while the transversality conditions are:

$$\lim_{t\to\infty}\lambda(t)m(t)e^{-\rho t}=0, \quad \lim_{t\to\infty}q(t)k(t)e^{-\rho t}=0.$$

While (A1) is the first-order condition for consumption, (A2) and (A3) govern the optimal accumulation of money and capital, respectively. Combining (A2)-(A4), we obtain the intertemporal no-arbitrage condition (11). ■

World Market Equilibrium. Given (5), the equilibrium conditions are summarized as follows:

$$q(t)=\lambda(t);\quad q^*(t)=\lambda^*(t), \tag{A5}$$

$$\varepsilon(t)=\lambda(t)[r\left(P(t)\right)+\tau(t)\pi^*-\delta];\quad \varepsilon^*(t)=\lambda^*(t)[r\left(P(t)\right)+\pi^*-\delta], \tag{A6}$$

$$\dot{\lambda}(t)=\lambda(t)[\rho+\delta-r\left(P(t)\right)];\quad \dot{\lambda}^*(t)=\lambda^*(t)[\rho+\delta-r\left(P(t)\right)], \tag{A7}$$

$$c(t)=\tfrac{1}{\lambda(t)P(t)[1+r(P(t))+\tau(t)\pi^*-\delta]};\quad c^*(t)=\tfrac{1}{\lambda^*(t)P(t)[1+r(P(t))+\pi^*-\delta]}, \tag{A8}$$

$$\dot{k}(t){=}w\left(P(t)\right){+}r\left(P(t)\right)k(t)\text{-}P(t)c(t)\text{-}\delta k(t);\quad \dot{k}^*(t){=}w\left(P(t)\right){+}r\left(P(t)\right)k^*(t)\text{-}P(t)c^*(t)\text{-}\delta k^*(t), \tag{A9}$$

$$m(t)=P(t)c(t);\quad m^*(t)=P(t)c^*(t). \tag{A10}$$

With $n(t)=\frac{\lambda^*(t)}{\lambda(t)}$ defined, the equilibrium can be reduced to the following one instantaneous relationship and four differential equations:

$$\frac{1}{\lambda(t)[1{+}r\left(P(t)\right){+}\tau(t)\pi^*\text{-}\delta]}+\frac{1}{\lambda^*(t)[1{+}r\left(P(t)\right){+}\pi^*\text{-}\delta]}=\frac{\left(1-\alpha_i\right)r\left(P(t)\right)K(t)-2\alpha_i w\left(P(t)\right)}{\alpha_c-\alpha_i}, \tag{A11}$$

$$\dot{\lambda}(t)=\lambda(t)[\rho+\delta-r\left(P(t)\right)], \tag{A12}$$

$$\dot{k}(t)=w\left(P(t)\right)+r\left(P(t)\right)k(t)-\frac{1}{\lambda(t)[1+r\left(P(t)\right)+\tau(t)\pi^*-\delta]}-\delta k(t), \tag{A13}$$

$$\dot{K}(t)=\frac{1}{\alpha_c-\alpha_i}\left[2\alpha_c w\left(P(t)\right)-\left(1-\alpha_c\right)r\left(P(t)\right)K(t)\right]-\delta K(t), \tag{A14}$$

$$\dot{\tau}(t)=a[1-\tau(t)], \tag{A15}$$

where we have used (A8) in deriving (A11) and (A13). ■

Proof of Theorem 1. In the steady state, the world economy is characterized by $\dot{\tau}(t)=0$, $\dot{\lambda}(t)=0$, $\dot{k}(t)=0$, and $\dot{K}(t)=0$. Equation (A15) with $\dot{\tau}(t)=0$ refers to $\widehat{\tau}=1$ in the steady state. Since $\frac{\dot{n}(t)}{n(t)}=\frac{\dot{\lambda}^*(t)}{\lambda(t)}-\frac{\dot{\lambda}(t)}{\lambda(t)}=0$ must hold for all t, $n(t)$ must jump to its long-run equilibrium value $\widehat{n}$ immediately. From (A12) with $\dot{\lambda}(t)=0$, we have the steady-state interest rate $\widehat{r}=\rho+\delta>0$. Accordingly, it follows from (5) that the equilibrium relative price and wage rate are given by:

$$\widehat{P}=\frac{[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}]^{\frac{1-\alpha_c}{1-\alpha_i}}}{\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}(\rho+\delta)^{\frac{\alpha_c-\alpha_i}{1-\alpha_i}}>0,$$

$$\widehat{w}=[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}(\rho+\delta)^{-\alpha_i}]^{\frac{1}{1-\alpha_i}}>0.$$

Moreover, we solve (A14) for the capital stock of the world as:

$$\widehat{K}=\frac{2\alpha_c[\alpha_i^{\alpha i}(1-\alpha_i)^{1-\alpha_i}(\rho+\delta)^{-\alpha_i}]^{\frac{1}{1-\alpha_i}}}{\left(1-\alpha_c\right)\rho+\left(1-\alpha_i\right)\delta}>0.$$

Obviously, the steady-state $\widehat{r}$, $\widehat{P}$, $\widehat{w}$, and $\widehat{K}$ are nondegenerate and unique.

From (19) and (A11), it is easy to derive:

$$\widehat{\lambda} = \frac{1+\frac{1}{\hat{n}}}{1+\rho+\pi^*} \cdot \frac{(1-\alpha_c)\,\rho+(1-\alpha_i)\,\delta}{2w(\widehat{P})\,[\rho+(1-\alpha_i)\,\delta]} > 0,$$

and, accordingly,

$$\widehat{c} = \frac{1}{\widehat{\lambda}\widehat{P}\,(1+\rho+\pi^*)} = \frac{1}{(1+\frac{1}{\hat{n}})\widehat{P}} \cdot \frac{2w(\widehat{P})\,[\rho+(1-\alpha_i)\,\delta]}{(1-\alpha_c)\,\rho+(1-\alpha_i)\,\delta} > 0.$$

Given $\widehat{c}$ and (A13) with $\dot{k}(t)=0$, we can obtain:

$$\widehat{k} = \frac{w(\widehat{P})}{\rho\,(1+\frac{1}{\hat{n}})} \left\{ \frac{(1+\alpha_c)\,\rho+(1-\alpha_i)\,\delta}{(1-\alpha_c)\,\rho+(1-\alpha_i)\,\delta} - \frac{1}{\hat{n}} \right\}. \tag{A16}$$

By analogy, the foreign capital stock is given by:

$$\widehat{k}^* = \frac{w(\widehat{P})}{\rho\,(1+\hat{n})} \left\{ \frac{(1+\alpha_c)\,\rho+(1-\alpha_i)\,\delta}{(1-\alpha_c)\,\rho+(1-\alpha_i)\,\delta} - \hat{n} \right\}. \tag{A17}$$

These two equations indicate that there exists a nondegenerate, unique equilibrium, provided that $\frac{(1+\alpha_c)\rho+(1-\alpha_i)\delta}{(1-\alpha_c)\rho+(1-\alpha_i)\delta} > \hat{n} > \frac{(1-\alpha_c)\rho+(1-\alpha_i)\delta}{(1+\alpha_c)\rho+(1-\alpha_i)\delta}$. Under this appropriate range, the foreign counterparts are also consistent with this unique equilibrium. ■

Proof of Proposition 1. It follows from (19) and (A8) that if $\hat{n}=1$, then $\widehat{\lambda}=\widehat{\lambda}^*$ and $\widehat{c}=\widehat{c}^*$. Under such a situation, from (6) and (A9), $\widehat{k}=\widehat{k}^*=\frac{\widehat{K}}{2}$ and $\widehat{y}_c=\widehat{y}_c^*$ are also true. As a result, the world market-clearing condition for the consumption good (17) indicates that $\widehat{c}=\widehat{y}_c=\widehat{c}^*=\widehat{y}_c^*$, implying that the economy's equilibrium is no trade. By focusing on the case with $\hat{n}>1$, we have: $\widehat{\lambda}<\widehat{\lambda}^*$ and $\widehat{c}>\widehat{c}^*$. Moreover, from (A16) and (A17), we have (23), implying that $\widehat{k}>\widehat{k}^*$ and hence the home country becomes capital-abundant. Thus, it follows from (6) that if $\alpha_c>\alpha_i$, then $\widehat{y}_c>\widehat{y}_c^*$, while if $\alpha_c<\alpha_i$, then $\widehat{y}_c<\widehat{y}_c^*$. With this understanding, we further obtain:

$$\frac{\triangle\left(\widehat{P}\widehat{c}-\widehat{P}\widehat{y}_c\right)}{\triangle\hat{n}} = -\underset{(+)}{\frac{\triangle\left(\widehat{P}\widehat{c}\right)}{\triangle\hat{n}}}\,\frac{(1-\alpha_c)\,\rho+(1-\alpha_i)\,\delta}{(\alpha_c-\alpha_i)\,\rho} \lesseqgtr 0 \quad \text{iff} \quad \alpha_c \gtreqless \alpha_i. \tag{A18}$$

Given that the home country is capital-abundant ($\widehat{k}>\widehat{k}^*$), it will export the consumption (investment) good if the consumption (investment) sector is capital-intensive, i.e., $\alpha_c>\alpha_i$ ($\alpha_c<\alpha_i$).

Under the case with $\hat{n}<1$, we have $\widehat{\lambda}>\widehat{\lambda}^*$, $\widehat{c}<\widehat{c}^*$, and $\widehat{k}<\widehat{k}^*$, implying that the home country is labor-abundant. The aggregate resource constraint (16) with $\dot{k}(t)=0$ indicates that $\widehat{P}(\widehat{y}_c-\widehat{c})=\widehat{i}-\widehat{y}_i$. With this relationship, we can conclude that the labor-abundant home country will export the investment (consumption) good, as the investment (consumption) sector is labor-intensive, i.e., $\alpha_c>\alpha_i$ ($\alpha_c<\alpha_i$). ■

Proof of Lemma 1. Substituting (19) into (A11) and totally differentiating, we obtain:

$$\begin{aligned}
p_\lambda &= \frac{-1}{\Omega\widehat{\lambda}^2}(1+\frac{1}{\hat{n}})(\frac{1}{1+\rho+\pi^*}) < 0, \\
p_\tau &= \frac{-1}{\Omega}[\frac{\pi^*}{\widehat{\lambda}\,(1+\rho+\pi^*)^2}] < 0, \\
p_n &= \frac{-1}{\Omega}[\frac{1}{\widehat{\lambda}\hat{n}^2\,(1+\rho+\pi^*)}] < 0, \\
p_K &= \frac{-(\rho+\delta)}{\Omega}(\frac{1-\alpha_i}{\alpha_c-\alpha_i}) \lesseqgtr 0, \quad \text{iff} \quad \alpha_c \gtreqless \alpha_i,
\end{aligned}$$

where $\Omega = \frac{r'}{(1+\rho+\pi^*)}\left[\frac{1}{\widehat{\lambda}(1+\rho+\pi^*)} + \frac{1}{\widehat{\lambda}\widehat{n}(1+\rho+\pi^*)}\right] + \frac{1}{\alpha_c-\alpha_i}\left[(1-\alpha_i)\, r'\widehat{K} - 2\alpha_i w'\right]$. From (5), we learn that if $\alpha_c > \alpha_i$, $w' < 0$ and $r' > 0$ are true and hence $\Omega > 0$. In the case of $\alpha_c < \alpha_i$, we have $w' > 0$ and $r' < 0$. The sign of Ω becomes more complicated. We then use (A11) to rewrite Ω as:

$$\Omega = \frac{r'}{\alpha_c - \alpha_i}\frac{(1-\alpha_i)\,\widehat{K}\,(1+\rho+\pi^*+\widehat{r})}{1+\rho+\pi^*} + \frac{2\alpha_i\left[\alpha_i(1+\rho+\pi^*) - \widehat{r}\,(1-\alpha_i)\right]}{(1+\rho+\pi^*)\,(\alpha_c-\alpha_i)^2}\frac{\widehat{w}}{\widehat{P}}.$$

Given that $1-\alpha_i < 1+\rho+\pi^*$, we can see that $\Omega > 0$ is also true in the case of $\alpha_c < \alpha_i$, provided that Assumption 2 holds. ■

Proof of Proposition 2 and Corollary 1. By substituting (21) into (A12)-(A15) and linearizing the resulting equations around the steady state, we have:

$$\begin{pmatrix} \dot{\tau}(t) \\ \dot{\lambda}(t) \\ \dot{K}(t) \\ \dot{k}(t) \end{pmatrix} = \begin{pmatrix} -a & 0 & 0 & 0 \\ -\widehat{\lambda}r' p_\tau & -\widehat{\lambda}r' p_\lambda & -\widehat{\lambda}r' p_K & 0 \\ \eta p_\tau & \eta p_\lambda & \eta p_K - \delta - \frac{1-\alpha_c}{\alpha_c-\alpha_i}\widehat{r} & 0 \\ \xi p_\tau + \frac{\pi^*}{\widehat{\lambda}(1+\rho+\pi^*)^2} & \xi p_\lambda + \frac{1}{\widehat{\lambda}^2(1+\rho+\pi^*)} & \xi p_K & \rho \end{pmatrix}\begin{pmatrix} \tau(t)-1 \\ \lambda(t)-\widehat{\lambda} \\ K(t)-\widehat{K} \\ k(t)-\widehat{k} \end{pmatrix} + \begin{pmatrix} 0 \\ -\widehat{\lambda}r' p_n \\ \eta p_n \\ \xi p_n \end{pmatrix}(n(t)-\widehat{n}) \tag{A19}$$

where $\eta = \frac{1}{\alpha_c-\alpha_i}\left[2\alpha_c w' - (1-\alpha_c)\, r'\widehat{K}\right] < 0$ and $\xi = w' + r'\widehat{k} + \frac{r'}{\widehat{\lambda}(1+\rho+\pi^*)^2}$. Equation (A19) allows us to determine $\tau(t)$, $\lambda(t)$, $K(t)$, and $k(t)$.

It is easy from (A19) to learn that $\phi_1 = -a < 0$ and $\phi_4 = \rho > 0$. Moreover, we can also compute the Jacobian matrix of the dynamical system to obtain the following relationships:

$$\phi_2 + \phi_3 = \left[\eta p_K - \delta - \frac{1-\alpha_c}{\alpha_c-\alpha_i}\widehat{r}\right] - \widehat{\lambda}r' p_\lambda,$$

$$\begin{aligned} \phi_2 \cdot \phi_3 &= -\widehat{\lambda}r'\left[p_\lambda\left(\eta p_K - \delta - \frac{1-\alpha_c}{\alpha_c-\alpha_i}\widehat{r}\right) - p_K \eta p_\lambda\right] \\ &= \widehat{\lambda}\left(\frac{r'}{\alpha_c-\alpha_i}\right) p_\lambda\left[(1-\alpha_c)\,\rho + (1-\alpha_i)\,\delta\right] < 0. \end{aligned}$$

The condition $\phi_2 \cdot \phi_3 < 0$ indicates that these two characteristic roots are of opposite signs.

We next turn to deriving the general solution. Given that $\phi_1 = -a$ and $\phi_4 = \rho$, the general solution to (A19) is given by:

$$\begin{aligned} \tau(t) &= 1 + h_{11}A_1 e^{-at}, \\ \lambda(t) &= \widehat{\lambda} + h_{21}A_1 e^{-at} + h_{22}A_2 e^{\phi_2 t} + h_{23}A_3 e^{\phi_3 t}, \\ K(t) &= \widehat{K} + h_{31}A_1 e^{-at} + h_{32}A_2 e^{\phi_2 t} + h_{33}A_3 e^{\phi_3 t}, \\ k(t) &= \widehat{k} + A_1 e^{-at} + A_2 e^{\phi_2 t} + A_3 e^{\phi_3 t} + A_4 e^{\rho t}, \end{aligned} \tag{A20}$$

where $h_{11}=\frac{a+\rho}{p_\tau\Psi}\psi$, $h_{21}=\frac{\widehat{\lambda}r'(a+\rho)\left(a+\rho+\Omega\ p_K\right)}{\Psi}$, $h_{22}=\frac{\widehat{\lambda}r'(\phi_2-\rho)}{\widehat{\lambda}r'[\xi p_\lambda+\frac{1}{\widehat{\lambda}^2(1+\rho+\pi^*)}]\text{-}\xi\left(\widehat{\lambda}r'p_\lambda+\phi_2\right)}$,
$h_{23}=\frac{\widehat{\lambda}r'(\phi_3-\rho)}{\widehat{\lambda}r'[\xi p_\lambda+\frac{1}{\widehat{\lambda}^2(1+\rho+\pi^*)}]\text{-}\xi\left(\widehat{\lambda}r'p_\lambda+\phi_3\right)}$, $h_{31}=\frac{-(a+\rho)a\eta}{\Psi}$, $h_{32}=\frac{-(\phi_2-\rho)\left(\widehat{\lambda}r'p_\lambda+\phi_2\right)}{p_K\{\widehat{\lambda}r'[\xi p_\lambda+\frac{1}{\widehat{\lambda}^2(1+\rho+\pi^*)}]\text{-}\xi\left(\widehat{\lambda}r'p_\lambda+\phi_2\right)\}}=2$,
$h_{33}=\frac{-(\phi_3-\rho)\left(\widehat{\lambda}r'p_\lambda+\phi_3\right)}{p_K\{\widehat{\lambda}r'[\xi p_\lambda+\frac{1}{\widehat{\lambda}^2(1+\rho+\pi^*)}]\text{-}\xi\left(\widehat{\lambda}r'p_\lambda+\phi_3\right)\}}$, $\psi=(\eta p_K-\delta-\frac{1-\alpha_c}{\alpha_c-\alpha_i}\widehat{r}+a)(a-\widehat{\lambda}r'p_\lambda)+\widehat{\lambda}r'p_K\eta p_\lambda$
$=(a+\phi_2)(a+\phi_3)$, and $\Psi=a\eta\Omega p_K-(a+\rho+\Omega p_K)\left[a(\xi-\Omega)+\frac{1}{2}\widehat{\lambda}r'p_\lambda\Omega\right]$.

The terminal conditions $A_3=A_4=0$ ensure that the dynamical system remains bounded as $t\to\infty$. The remaining A_1, A_2, and $\widehat{n}$ can be obtained by imposing the following initial conditions: $\tau_0=1+h_{11}A_1$, $K_0=\widehat{K}+h_{31}A_1+h_{32}A_2$, and $k_0=\widehat{k}+A_1+A_2$, which yield:

$$\begin{aligned}A_1&=\frac{\tau_0-1}{h_{11}},\\ A_2&=\frac{1}{h_{32}}\left(K_0-\widehat{K}+\frac{h_{31}}{h_{11}}(1-\tau_0)\right),\end{aligned}$$

and the expression for $\widehat{n}$ in (22). ■

Proof of Proposition 3. The instantaneous adjustments of $(\lambda(t),K(t),k(t))$ are given by:

$$\begin{aligned}\lambda(t)&=\begin{cases}\widehat{\lambda}\left(\widehat{n}=1\right), & t=0^-\\ \widehat{\lambda}\left(\hat{n}'\right)+\frac{h_{21}}{h_{11}}\left(\tau_0'-1\right)e^{-at}+\frac{h_{22}}{h_{32}}\frac{h_{31}}{h_{11}}\left(1-\tau_0'\right)e^{\phi_2 t}, & t\geq 0^+\end{cases},\\ K(t)&=\begin{cases}\widehat{K}, & t=0^-\\ \widehat{K}+\frac{h_{31}}{h_{11}}\left(\tau_0'-1\right)e^{-at}+\frac{h_{31}}{h_{11}}\left(1-\tau_0'\right)e^{\phi_2 t}, & t\geq 0^+\end{cases},\\ k(t)&=\begin{cases}\widehat{k}\left(\widehat{n}=1\right), & t=0^-\\ \widehat{k}\left(\hat{n}'\right)+\frac{\tau_0'-1}{h_{11}}e^{-at}+\frac{h_{31}}{h_{32}}\frac{1}{h_{11}}\left(1-\tau_0'\right)e^{\phi_2 t}, & t\geq 0^+\end{cases}.\end{aligned}\tag{A21}$$

Since the home capital stock evolves continuously from its given level of endowment, we have the condition $k(t=0^-)=k(t=0^+)$ at time 0, that is:

$$\widehat{k}\left(\widehat{n}=1\right)=\widehat{k}\left(\hat{n}'\right)+\frac{\tau_0'-1}{h_{11}}+\frac{h_{31}}{2h_{11}}\left(1-\tau_0'\right).$$

This condition allows us to derive:

$$\frac{\triangle\widehat{n}}{\triangle\tau_0}=-\frac{1}{h_{11}\cdot\frac{\triangle\widehat{k}}{\triangle\widehat{n}}}\left(1-\frac{h_{31}}{2}\right),\tag{A22}$$

where $\frac{\triangle\widehat{k}}{\triangle\widehat{n}}=\frac{1}{2\rho\widehat{\lambda}(1+\rho+\pi^*)}>0$. Given that the economy starts out in the steady state with $\widehat{n}=1$ and $\tau_0=1$, we can obtain the relationship: $1-\frac{h_{31}}{2}=\frac{p_\tau\Omega}{2(a+\rho)}h_{11}=\frac{\psi\Omega}{2\Psi}$. Substituting this relationship into (A22) yields:

$$\begin{aligned}\frac{\triangle\widehat{n}}{\triangle\tau_0}&=\frac{\pi^*}{2\left(a+\rho\right)\widehat{\lambda}\left(1+\rho+\pi^*\right)^2\frac{\triangle\widehat{k}}{\triangle\widehat{n}}}>0,\\ \frac{\triangle\widehat{k}}{\triangle\tau_0}&=\frac{\pi^*}{2\left(a+\rho\right)\widehat{\lambda}\left(1+\rho+\pi^*\right)^2}>0.\end{aligned}$$

In addition, it is easy to obtain: $\frac{\triangle \widehat{K}}{\triangle \tau_0} = 0$ and $\frac{\triangle \widehat{P}}{\triangle \tau_0} = 0$. ■

Proof of Proposition 4. From (A21), we can derive:

$$K(t) - \widehat{K} = -\left(\frac{h_{31}}{h_{11}}\right)\left(\tau_0^{'} - 1\right) e^{-at}\left(e^{(a+\phi_2)t} - 1\right) = \frac{a\eta p_\tau \left(\tau_0^{'} - 1\right)}{\psi} e^{-at}\left(e^{(a+\phi_2)t} - 1\right) > 0,$$

$$\dot{K}(t)(t = 0^+) = -\frac{h_{31}}{h_{11}}\left(\tau_0^{'} - 1\right)(a + \phi_2) = \frac{a\eta p_\tau}{\psi}\left(\tau_0^{'} - 1\right)(a + \phi_2) = \frac{a\eta p_\tau}{a + \phi_3}\left(\tau_0^{'} - 1\right) > 0.$$

Notice that $sgn(\psi) = sgn\,(a + \phi_2) = sgn(e^{(a+\phi_2)t} - 1)$.

If the economy starts out in the steady state where $\hat{n} = 1$, we can re-write,

$$\Psi(a) = \frac{1}{2}\left[\psi\Omega - (a + \rho)a\eta\right] = \frac{1}{2}\left\{(\Omega - \eta)\, a^2 + \left[\Omega\,(\phi_2 + \phi_3) - \eta\rho\right] a + \Omega\phi_2\phi_3\right\}.$$

Under Assumption 2, $\Psi(0) = \Omega\phi_2\phi_3/2 < 0$ and $\lim_{a\to\infty} \Psi\,(a) > 0$. We can thus depict $\Psi(a)$ in the following figure:

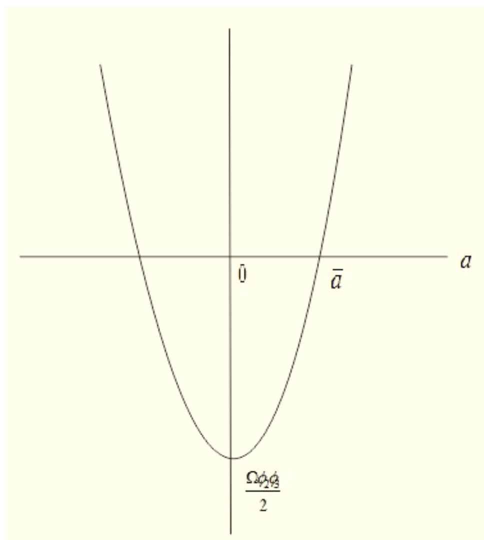

Figure A-3: Function of $\Psi(a)$

By focusing on the home capital stock, it follows from (A21) that if $a < \overline{a}$ (hence, $h_{31} < 0$, $h_{11} < 0$, $\Psi < 0$, and $\psi < 0$),

$$\dot{k}(t) = -\frac{\tau_0^{'} - 1}{h_{11}}\left(ae^{-at} + \frac{h_{31}}{2}\phi_2 e^{\phi_2 t}\right) > 0,$$

whereas if $a > \overline{a}$ (hence, $h_{31} > 0$ and $\Psi > 0$), then

$$\dot{k}(t)(t = 0^+) = -\frac{\tau_0^{'} - 1}{h_{11}}\left(a + \frac{h_{31}}{2}\phi_2\right) = \left(\tau_0^{'} - 1\right)\left\{-\frac{a + \phi_2}{h_{11}} + \phi_2 \frac{1 - \frac{h_{31}}{2}}{h_{11}}\right\} > 0,$$

$$\dot{k}(t) = -\frac{\tau_0^{'} - 1}{h_{11}}\left(ae^{-at} + \frac{h_{31}}{2}\phi_2 e^{\phi_2 t}\right) \lesseqgtr 0,$$

where $\frac{a+\phi_2}{h_{11}} = \frac{p_\tau \Psi}{(a+\rho)(a+\phi_3)} < 0$ and $\frac{1-\frac{h_{31}}{2}}{h_{11}} = \frac{p_\tau \Omega}{2(a+\rho)} < 0$.

From the definition of $K(t) = k(t) + k^*(t)$, (A21) allows us to further obtain:

$$k^*(t) = K(t) - k(t) = \left(\widehat{K} - \widehat{k}\left(\hat{n}'\right)\right) + \frac{\tau_0' - 1}{h_{11}}\left(h_{31} - 1\right)e^{-at} - \frac{h_{31}}{h_{11}}\left(\tau_0' - 1\right)\left(1 - \frac{1}{h_{32}}\right)e^{\phi_2 t},$$

$$\dot{k}^*(t) = \left(-\frac{\tau_0' - 1}{h_{11}}\right)\left[a\left(h_{31} - 1\right)e^{-at} + h_{31}\phi_2\left(1 - \frac{1}{h_{32}}\right)e^{\phi_2 t}\right].$$

With these relationships, we can also obtain:

$$\dot{k}^*(t)(t = 0^+) = \frac{a p_\tau \left(\tau_0' - 1\right)}{2\left(a + \rho\right)\left(a + \phi_3\right)}\left[\Omega\left(a + \phi_3\right) + \eta\left(a + \rho\right)\right] \gtrless 0. \blacksquare$$

Proof of Proposition 5. In order to have the dynamic adjustments of $P(t)$, we need to first investigate the associated dynamics of $\lambda(t)$. From (A21), we have

$$\dot{\lambda}(t) = \left(-\frac{h_{21}}{h_{11}}\right)\left(\tau_0' - 1\right)\left(ae^{-at} + \frac{h_{22}}{h_{32}} \cdot \frac{h_{31}}{h_{21}}\phi_2 e^{\phi_2 t}\right).$$

By this equation, we can further obtain the following two relationships:

$$\dot{\lambda}(t)(t = 0^+) = -\left(\frac{h_{21}}{h_{11}}\right)\left(\tau_0' - 1\right)\left(a + \frac{h_{22}}{h_{32}} \cdot \frac{h_{31}}{h_{21}}\phi_2\right) = -\frac{\widehat{\lambda} r' p_\tau a}{a + \phi_3}(\tau_0' - 1) \lesseqgtr 0 \;\; \text{iff} \quad \alpha_c \lesseqgtr \alpha_i.$$

$$\lim_{t\to\infty}\dot{\lambda}(t) = \begin{cases} -\left(\tau_0' - 1\right)\frac{h_{31}}{h_{11}}\frac{h_{22}}{h_{32}}\phi_2 \lim\limits_{t\to\infty} e^{\phi_2 t} & \text{if } a > -\phi_2 \\ -\left(\tau_0' - 1\right)\frac{h_{21}}{h_{11}} a \lim\limits_{t\to\infty} e^{-at} & \text{if } a < -\phi_2 \end{cases},$$

The first relationship will help us to determine the impact effect of the monetary policy change on the relative price $P(t)$. If $\alpha_c > \alpha_i$ (hence $r' > 0$), then $\dot{\lambda}(t)(t = 0^+) > 0$. Under such a situation, (A12) shows that $\rho + \delta > r\left(P(t)(t = 0^+)\right)$, implying that, on impact, the terms of trade falls below the steady-state $\widehat{P}$. By analogy, if $\alpha_c < \alpha_i$ (hence $r' < 0$), then $\dot{\lambda}(t)(t = 0^+) < 0$. It turns out that $\rho + \delta < r\left(P(t)(t = 0^+)\right)$, implying that, on impact, the terms of trade also falls below the steady-state $\widehat{P}$.

The second relationship will help us to describe the transition of the terms of trade. By focusing on the case with $\alpha_c > \alpha_i$, the second relationship turns out to be:

$$\lim_{t\to\infty}\dot{\lambda}(t) = \begin{cases} -\left(\tau_0' - 1\right)\dfrac{\overset{(+)}{a\eta p_\tau}}{\underset{(+)}{\psi}}\dfrac{\overset{(-)}{\widehat{\lambda} r' p_K}}{\underset{(-)}{\widehat{\lambda} r' p_\lambda + \phi_2}}\phi_2 \lim\limits_{t\to\infty} e^{\phi_2 t} > 0 & \text{if } a > -\phi_2 \\ -\left(\tau_0' - 1\right)\dfrac{\widehat{\lambda}\overset{(-)}{p_\tau}\overset{(-)}{\left[r'\left(a + \rho + \Omega p_K\right)\right]}}{\underset{(-)}{\psi}} a \lim\limits_{t\to\infty} e^{-at} > 0 & \text{if } a < -\phi_2 \end{cases},$$

Note that $a > -\phi_2$ ($a < -\phi_2$) implies that $\psi > 0$ ($\psi < 0$) due to $sgn(\psi) = sgn\left(a + \phi_2\right)$. Accordingly, $\lim\limits_{t\to\infty}\dot{\lambda}(t) > 0$ implies that $\rho + \delta > r\left(P(t)(t \to \infty)\right)$ is true. Since $r' > 0$ when $\alpha_c > \alpha_i$, $P(t)$ follows either Transition Path 1 or Transition Path 2, as shown in Figure 6. In addition, under the case with $\alpha_c < \alpha_i$, we can also easily prove that $\lim_{t\to\infty} \dot{\lambda}(t) > 0$ regardless of $a > -\phi_2$ or $a < -\phi_2$. Since $r' < 0$ when $\alpha_c < \alpha_i$, $P(t)$ follows Transition Path 3, as shown in Figure 6.

Recalling $E(t) = \frac{P_i(t)}{P_i^*(t)}$, a simple manipulation gives:

$$\frac{\dot{E}(t)}{E(t)} = \pi(t) - \pi^*(t) = (\tau(t) - 1)\,\pi^*.$$

This equation tells us that in transition the foreign exchange rate will monotonically increase (the home country's currency will depreciate) until $\tau(t)$ returns to the steady-state level (i.e., 1). In addition, given that $M(t)(t = 0^-) = M(t)(t = 0^+)$ and $M^*(t)(t = 0^-) = M^*(t)(t = 0^+)$, we can obtain:

$$E(t)(t = 0^+) - E(t)(t = 0^-) = \frac{-M_0 \cdot \Delta z}{M_0^* \cdot z(t)(t = 0^-) \cdot z(t)(t = 0^+)},$$

where $\Delta z = z(t)(t = 0^+) - z(t)(t = 0^-)$. Since $z(t)(t = 0^-) = \frac{\widehat{c}}{\widehat{c}^*} = 1$ $(n(t)(t = 0^-) = 1)$ and $z(t)(t = 0^+) = \frac{n(t)(t=0^+)(1+r(t)+\pi^*-\delta)}{1+r(t)+\tau\pi^*-\delta}$ $(n(t)(t = 0^+) > 1)$, we further derive:

$$\frac{\Delta z}{\Delta \tau_0} = \frac{1}{(1+\rho+\pi^*)}\left\{(1+\rho+\pi^*)\,\frac{\Delta \widehat{n}}{\Delta \tau_0} - \pi^*\right\} = \frac{-\pi^*}{1+\rho+\pi^*}\frac{a}{a+\rho} < 0.$$

The two equations above indicate that in response to an increase in the level of inflation targeting, the foreign exchange rate increases on impact and then continuously rises to reach its new steady state, as shown in Figure 7. ■

Proof of Section 5.1. In the presence of an endogenous labor-leisure choice, the dynamic equilibrium can be summarized as follows:

$$\lambda^*(t) = n(t)\lambda(t),$$

$$\frac{1}{\lambda(t)[1+r\left(P(t)\right)+\tau(t)\pi^*\text{-}\delta]} + \frac{1}{\lambda^*(t)[1+r\left(P(t)\right)+\pi^*\text{-}\delta]} = \frac{(1-\alpha_i)\,r\left(P(t)\right)K(t) - \alpha_i w\left(P(t)\right)L(t)}{\alpha_c - \alpha_i},$$

$$\dot{\lambda}(t) = \lambda(t)[\rho + \delta - r\left(P(t)\right)],$$

$$\dot{k}(t) = w\left(P(t)\right)\ell(t) + r\left(P(t)\right)k(t) - \frac{1}{\lambda(t)[1 + r\left(P(t)\right) + \tau(t)\pi^* - \delta]} - \delta k(t),$$

$$\dot{K}(t) = \frac{1}{\alpha_c - \alpha_i}\left[\alpha_c w\left(P(t)\right)L(t) - (1-\alpha_c)\,r\left(P(t)\right)K(t)\right] - \delta K(t),$$

$$\dot{\tau}(t) = a[1 - \tau(t)],$$

where $\ell(t) = [\lambda(t)w\left(P(t)\right)]^{\frac{1}{\chi}}$ and $L(t) = \ell(t) + \ell^*(t) = [\lambda(t)w\left(P(t)\right)]^{\frac{1}{\chi}}[1 + n(t)^{\frac{1}{\chi}}]$. With these equilibrium conditions, we can easily derive the results in Section 5.1. ■

Proof of Section 5.3 and Proposition 6. Repeating the same procedure, we can obtain the following equilibrium conditions:

$$q^*(t) = n(t)q(t), \tag{A23}$$

$$u(t) = x^*(t) - x(t), \tag{A24}$$

$$\frac{\theta}{q^*(t)[1 - \frac{1-\theta}{x^*(t)}]} + \frac{\theta n(t)}{q^*(t)[1 - \frac{1-\theta}{x^*(t)-u(t)}]} = \frac{(1-\alpha_i)\,r\left(P(t)\right)K(t) - 2\alpha_i w\left(P(t)\right)}{\alpha_c - \alpha_i}, \tag{A25}$$

$$\dot{u}(t) = (2\rho + \frac{1}{\theta} + \pi^* + \delta)u(t) + \pi^*[1 + \theta(\rho + \pi^*)][\tau(t) - \hat{\tau}], \tag{A26}$$

$$\dot{n}(t) = \frac{\rho + \delta}{1 + \theta(\rho + \pi^*)} u(t), \tag{A27}$$

$$\dot{q}^*(t) = q^*(t)[\rho + \delta - \frac{r\,(P(t))}{x^*(t)}], \tag{A28}$$

$$\dot{x}^*(t) = (2\rho + \frac{1}{\theta} + \pi^* + \delta)[x^*(t) - \hat{x}^*] - r^{'}(P(t) - \hat{P}), \tag{A29}$$

$$\dot{k}(t) = w\,(P(t)) + r\,(P(t))\,k(t) - \frac{\theta n(t)}{q^*(t)[1 - \frac{1-\theta}{x^*(t)-u(t)}]} - \delta k(t), \tag{A30}$$

together with the world market-clearing condition for the investment good (A14) and the home country's policy rule (A15).

Given the modified CIA constraint (10'), we can utilize (A25) to express the terms of trade as:

$$P(t) = \tilde{p}(q^*(t), n(t), x^*(t), u(t), K(t)),$$

where $\tilde{p}_{q^*} = \frac{-2}{\tilde{\Omega}(\hat{q}^*)^2}\frac{1+\theta(\rho+\pi^*)}{1+\rho+\pi^*} < 0$, $\tilde{p}_n = \frac{1}{\tilde{\Omega}\hat{q}^*}\frac{1+\theta(\rho+\pi^*)}{1+\rho+\pi^*} > 0$, $\tilde{p}_{x^*} = \frac{-1}{\tilde{\Omega}\hat{q}^*}\frac{2(1-\theta)}{\theta(1+\rho+\pi^*)^2} \leq 0$, $\tilde{p}_u = \frac{1}{\tilde{\Omega}\hat{q}^*}\frac{(1-\theta)}{\theta(1+\rho+\pi^*)^2} \geq 0$, $\tilde{p}_K = \frac{-(1-\alpha_i)r(\hat{P})}{\tilde{\Omega}(\alpha_c-\alpha_i)} \gtrless 0$, and $\tilde{\Omega} = \frac{(1-\alpha_i)r^{'}\hat{K}-2\alpha_i w^{'}}{\alpha_c-\alpha_i} > 0$. With this relationship and the definitions of $q^*(t) = n(t)q(t)$ and $u(t) = x^* - x(t)$, the world economy of the two-country model can be constructed by the following 7×7 dynamical system:

$$\begin{pmatrix} \dot{\tau}(t) \\ \dot{u}(t) \\ \dot{n}(t) \\ \dot{q}^*(t) \\ \dot{x}^*(t) \\ \dot{K}(t) \\ \dot{k}(t) \end{pmatrix} = \begin{pmatrix} -a & 0 & 0 & 0 & 0 & 0 & 0 \\ [1+\theta(\rho+\pi^*)]\pi^* & 2\rho+\frac{1}{\theta}+\pi^*+\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\rho+\delta}{1+\theta(\rho+\pi^*)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\hat{q}^*}{\hat{x}^*}r^{'}\tilde{p}_u & \frac{-\hat{q}^*}{\hat{x}^*}r^{'}\tilde{p}_n & \frac{-\hat{q}^*}{\hat{x}^*}r^{'}\tilde{p}_{q^*} & j_{45} & \frac{-\hat{q}^*}{\hat{x}^*}r^{'}\tilde{p}_K & 0 \\ 0 & -r^{'}\tilde{p}_u & -r^{'}\tilde{p}_n & -r^{'}\tilde{p}_{q^*} & j_{55} & -r^{'}\tilde{p}_K & 0 \\ 0 & j_{62} & j_{63} & j_{64} & j_{65} & j_{66} & 0 \\ 0 & j_{72} & j_{73} & j_{74} & j_{75} & j_{76} & r\text{-}\delta \end{pmatrix} \begin{pmatrix} \tau(t)\text{-}1 \\ u(t)\text{-}0 \\ n(t)\text{-}n' \\ q^*(t)\text{-}\hat{q}^* \\ x^*(t)\text{-}\hat{x}^* \\ K(t)\text{-}\hat{K} \\ k(t)\text{-}\hat{k} \end{pmatrix}$$

where $j_{45} = \frac{-\hat{q}^*}{\hat{x}^*}[r^{'}\tilde{p}_{x^*} - \frac{r(\hat{P})}{\hat{x}^*}]$, $j_{55} = 2\rho + \frac{1}{\theta} + \pi^* + \delta - r^{'}\tilde{p}_{x^*}$, $j_{62} = \eta\tilde{p}_u$, $j_{63} = \eta\tilde{p}_n$, $j_{64} = \eta\tilde{p}_{q^*}$, $j_{65} = \eta\tilde{p}_{x^*}$, $j_{66} = \eta\tilde{p}_K - [\frac{(1-\alpha_c)r(\hat{P})}{\alpha_c-\alpha_i} + \delta]$, $j_{72} = (\nu - \tilde{\Omega})\tilde{p}_u$, $j_{73} = (\nu - \tilde{\Omega})\tilde{p}_n$, $j_{74} = \frac{\eta}{2}\tilde{p}_{q^*}$, $j_{75} = \frac{\eta}{2}\tilde{p}_{x^*}$, $j_{76} = \nu\tilde{p}_K$, $\nu = w^{'} + r^{'}\hat{K}$ (recalling that $\eta = \frac{1}{\alpha_c-\alpha_i}\left[2\alpha_c w^{'} - (1-\alpha_c)\,r^{'}\widehat{K}\right]$).

From this 7×7 dynamical system, we learn that the first characteristic root is $\varphi_1 = -a < 0$, the second one is $\varphi_2 = 2\rho + \frac{1}{\theta} + \pi^* + \delta > 0$, the third one is $\varphi_3 = 0$, and the seventh one is $\varphi_7 = r(t) - \delta > 0$. The remaining three characteristic roots (φ_4, φ_5, and φ_6) satisfy the following three relationships:

$$\varphi_4 \cdot \varphi_5 \cdot \varphi_6 = \frac{\hat{q}^*}{\hat{x}^*}r^{'}\tilde{p}_{q^*}(\rho + \pi^* + \frac{1}{\theta})\digamma < 0,$$

$$\begin{aligned} \varphi_4 + \varphi_5 + \varphi_6 &= (2\rho + \frac{1}{\theta} + \pi^* + \delta) - r^{'}(\frac{\hat{q}^*}{\hat{x}^*}\tilde{p}_{q^*} + \tilde{p}_{x^*}) + j_{66} \\ &= (2\rho + \frac{1}{\theta} + \pi^*) - r^{'}[\frac{\hat{q}^*}{\hat{x}^*}\tilde{p}_{q^*} + \tilde{p}_{x^*}] + \Upsilon, \end{aligned}$$

$$\varphi_4 \cdot \varphi_5 + \varphi_4 \cdot \varphi_6 + \varphi_5 \cdot \varphi_6 = G_1 + G_2 + G_3,$$

where $F = \frac{1-\alpha_c}{\alpha_c-\alpha_i} r(\hat{P}) + \delta = \frac{(1-\alpha_c)[r(\hat{P})-\delta]+(1-\alpha_i)\delta}{\alpha_c-\alpha_i}$, $\Upsilon = \eta\tilde{p}_K - \frac{1-\alpha_c}{\alpha_c-\alpha_i} r(\hat{P}) = \frac{-2r(\hat{P})w'}{(\alpha_c-\alpha_i)\tilde{\Omega}} > 0$, $G_1 = -\frac{\hat{q}^*}{\hat{x}^*} r' \tilde{p}_{q^*}(\rho + \pi^* + \frac{1}{\theta})$, $G_2 = r'\tilde{p}_{x^*}F + (2\rho + \frac{1}{\theta} + \pi^* + \delta)j_{66}$, $G_3 = \frac{\hat{q}^*}{\hat{x}^*} r' \tilde{p}_{q^*} F$. The first relationship shows that due to $sgn(r') = sgn(F)$ and hence $\varphi_4 \cdot \varphi_5 \cdot \varphi_6 < 0$, two possible cases emerge: (i) there exist one root with a negative real part (say, $\varphi_4 < 0$) and two roots with positive real parts (say, $\varphi_5, \varphi_6 > 0$) or (ii) all three roots are negative. If $\alpha_c > \alpha_i$ (hence $r' > 0$ and $F > 0$), the second relationship indicates that $\varphi_4 + \varphi_5 + \varphi_6 > 0$ is true and the possibility of Case (ii) can be eliminated. If $\alpha_c < \alpha_i$ (hence $r' < 0$ and $F < 0$), we can see from the third relationship that both $G_1 < 0$ and $G_3 < 0$ as well as $G_2 < 0$, provided that $j_{66} < 0$. Under such a situation, we have: $\varphi_4 \cdot \varphi_5 + \varphi_4 \cdot \varphi_6 + \varphi_5 \cdot \varphi_6 < 0$ and accordingly, the case where three roots are negative can be also eliminated from our consideration. By focusing on the situation where $j_{66} > 0$, we further rewrite the third relationship as follows:

$$\varphi_4 \cdot \varphi_5 + \varphi_4 \cdot \varphi_6 + \varphi_5 \cdot \varphi_6 = G_1 + (2\rho + \frac{1}{\theta} + \pi^* + \delta)\eta\tilde{p}_K + [r'(\frac{\hat{q}^*}{\hat{x}^*}\tilde{p}_{q^*} + \tilde{p}_{x^*}) - (2\rho + \frac{1}{\theta} + \pi^* + \delta)]F.$$

This equation together with the second relationship indicates that $\varphi_4 \cdot \varphi_5 + \varphi_4 \cdot \varphi_6 + \varphi_5 \cdot \varphi_6 < 0$ holds true if $(2\rho + \frac{1}{\theta} + \pi^* + \delta) < r'(\frac{\hat{q}^*}{\hat{x}^*}\tilde{p}_{q^*} + \tilde{p}_{x^*})$, while $\varphi_4 + \varphi_5 + \varphi_6 > 0$ holds true if $(2\rho + \frac{1}{\theta} + \pi^* + \delta) > r'(\frac{\hat{q}^*}{\hat{x}^*}\tilde{p}_{q^*} + \tilde{p}_{x^*})$. As is evident, for generality, we should rule out the case with all three roots having negative real parts.

With these dynamical properties, the first two equations of this dynamical system allow us to have the instantaneous adjustments of $\tau(t)$ and $u(t)$, recursively:

$$\tau(t) = \begin{cases} 1, & t = 0^- \\ 1 + \left(\tau_0' - 1\right) e^{-at}, & t \geq 0^+ \end{cases} \quad \text{and } u(t) = \begin{cases} 0, & t = 0^- \\ -\frac{[1+\theta(\rho+\pi^*)]\pi^*}{a+2\rho+\frac{1}{\theta}+\pi^*+\delta} \cdot \left(\tau_0' - 1\right) e^{-at}, & t \geq 0^+ \end{cases}, \tag{A31}$$

indicating that in response to a rise in the home country's inflation target from 1 to $\tau_0' > 1$, $u(t)$ jumps down on impact and then gradually returns to the original level $\hat{u} = 0$ along the saddle path. In addition, substituting (A31) into (A27) with the initial value $\hat{n} = 1$ yields:

$$\dot{n}(t) = -\frac{\pi^*(\rho + \delta)}{a + 2\rho + \frac{1}{\theta} + \pi^* + \delta} \left(\tau_0' - 1\right) e^{-at}. \tag{A32}$$

By integrating the above equation, we can further obtain:

$$n(t) = \hat{n}' + \sigma \left(\tau_0' - 1\right) e^{-at}, \tag{A33}$$

where $\sigma = \frac{\pi^*(\rho+\delta)}{a(a+2\rho+\frac{1}{\theta}+\pi^*+\delta)} > 0$. With (A33), we can rewrite (A32) as:

$$\dot{n}(t) = -a[n(t) - \hat{n}']. \tag{A34}$$

Meanwhile, given the initial value $\hat{u} = 0$, (A31) and (A33) yield:

$$u(t) - \hat{u} = -\frac{a[1 + \theta(\rho + \pi^*)]}{(\rho + \delta)}[n(t) - \hat{n}']. \tag{A35}$$

Using (A34) and (A35), we can thus reduce the 7×7 dynamical system above to a 5×5 one, given by:

$$\begin{pmatrix} \dot{n}(t) \\ \dot{q}^*(t) \\ \dot{x}^*(t) \\ \dot{K}(t) \\ \dot{k}(t) \end{pmatrix} = \begin{pmatrix} -a & 0 & 0 & 0 & 0 \\ \frac{-\hat{q}^*}{\hat{x}^*}r'\Theta & \frac{-\hat{q}^*}{\hat{x}^*}r'\tilde{p}_{q^*} & j_{45} & \frac{-\hat{q}^*}{\hat{x}^*}r'\tilde{p}_K & 0 \\ -r'\Theta & -r'\tilde{p}_{q^*} & j_{55} & -r'\tilde{p}_K & 0 \\ \eta\Theta & j_{64} & j_{65} & j_{66} & 0 \\ (\nu-\tilde{\Omega})\Theta & j_{74} & j_{75} & j_{76} & r-\delta \end{pmatrix} \begin{pmatrix} n(t)-\hat{n}' \\ q^*(t)-\hat{q}^* \\ x^*(t)-\hat{x}^* \\ K(t)-\hat{K} \\ k(t)-\hat{k} \end{pmatrix},$$

where $\Theta = \tilde{p}_n - \frac{a[1+\theta(\rho+\pi^*)]^2}{r(\hat{P})}\tilde{p}_u = \frac{\hat{x}^*}{\tilde{\Omega}\hat{q}^*(1+\rho+\pi^*)}[1-\frac{(1-\theta)a}{\theta(1+\rho+\pi^*)(\rho+\delta)}]$. Accordingly, we can establish the instantaneous adjustments of $(n(t), q^*(t), x^*(t), K(t), k(t))$ as follows:

$$\begin{aligned} n(t) &= \begin{cases} \widehat{n}=1 & t=0^- \\ \widehat{n}' + g_{11}B_1e^{-at} & t\geq 0^+ \end{cases}, \\ q^*(t) &= \begin{cases} \widehat{q}^*(\widehat{n}=1) & t=0^- \\ q^*(\hat{n}') + g_{21}B_1e^{-at} + g_{24}B_4e^{\varphi_4 t} & t\geq 0^+ \end{cases}, \\ x^*(t) &= \begin{cases} 1 & t=0^- \\ 1+\theta(\rho+\pi^*) + g_{31}B_1e^{-at} + g_{34}B_4e^{\varphi_4 t} & t\geq 0^+ \end{cases}, \\ K(t) &= \begin{cases} \widehat{K} & t=0^- \\ \widehat{K} + g_{41}B_1e^{-at} + g_{44}B_4e^{\varphi_4 t} & t\geq 0^+ \end{cases}, \\ k(t) &= \begin{cases} \widehat{k}(\widehat{n}=1) & t=0^- \\ \widehat{k}(\widehat{n}') + B_1e^{-at} + B_4e^{\varphi_4 t} & t\geq 0^+ \end{cases}, \end{aligned} \tag{A36}$$

where B_i and g_{ij} are as yet undetermined coefficients. By letting

$$\Gamma = \begin{vmatrix} \frac{-\hat{q}^*}{\hat{x}^*}r'(\hat{P})\tilde{p}_{q^*}+a & j_{45} & \frac{-\hat{q}^*}{\hat{x}^*}r'(\hat{P})\tilde{p}_K \\ -r'(\hat{P})\tilde{p}_{q^*} & j_{55}+a & -r'(\hat{P})\tilde{p}_K \\ \eta\tilde{p}_{q^*} & \eta\tilde{p}_{x^*} & j_{66}+a \end{vmatrix},$$

we can obtain: $g_{11} = \frac{-[\delta-a-r(\hat{P})]\Gamma}{\frac{\Theta}{2}[\frac{a\eta}{2}(a+2\rho+\frac{1}{\theta}+\pi^*+\delta)(\delta-r(\hat{P})-a)+\tilde{\Omega}\Gamma]}$, $g_{41} = \frac{a\eta\Theta(a+2\rho+\frac{1}{\theta}+\pi^*+\delta)(\delta-r(\hat{P})-a)}{\frac{\Theta}{2}[\frac{a\eta}{2}(a+2\rho+\frac{1}{\theta}+\pi^*+\delta)(\delta-r(\hat{P})-a)+\tilde{\Omega}\Gamma]}$, and $g_{44}=2$, that are needed for deriving the steady-state effect of inflation targeting on the relative ratio of the shadow price $\hat{n}'$ and home capital $\hat{k}$. While B_i will be determined later, we do not report the other g_{ij} in order to save space.

It follows from (A33) and (A36) that at time 0, $B_1 = \frac{\sigma}{g_{11}}\left(\tau_0'-1\right)$. Given this, (A36) shows that the initial condition of the world capital stock is $g_{41}B_1 + g_{44}B_4 = 0$, indicating that:

$$B_4 = -\frac{g_{41}}{g_{44}} \cdot \frac{\sigma}{g_{11}}\left(\tau_0'-1\right).$$

Substituting B_1 and B_4, together with g_{11}, g_{41}, and g_{44}, into the initial condition of the home capital $\widehat{k}(\widehat{n}=1)=\widehat{k}(\hat{n}^{'})+B_1+B_4$ further yields:

$$\begin{aligned}
\hat{n}^{'} &= 1-\frac{\sigma(\tau_0^{'}-1)}{\frac{\Delta\widehat{k}}{\Delta\widehat{n}}}\frac{1}{g_{11}}(1-\frac{g_{41}}{g_{44}}) \\
&= 1-\frac{\sigma(\tau_0^{'}-1)}{\frac{\Delta\widehat{k}}{\Delta\widehat{n}}}\frac{\tilde{\Omega}\Theta}{2[a+r(\hat{P})-\delta]} \lesseqgtr 1, \text{ iff } \theta \gtreqless \bar{\theta}=\frac{a}{a+(\rho+\delta)(1+\rho+\pi^*)},
\end{aligned} \tag{A37}$$

recalling that $\Theta=\frac{\hat{x}^*}{\tilde{\Omega}\hat{q}^*(1+\rho+\pi^*)}[1-\frac{(1-\theta)a}{\theta(1+\rho+\pi^*)(\rho+\delta)}]$. In response to an increase in the home inflation from 1 to $\tau_0^{'}>1$, (A37) indicates that if $\theta<\bar{\theta}$ (hence $\Theta<0$), we then have $\hat{n}^{'}>1$, implying that $\hat{n}$ increases in the steady state. As a result, (26) implies a positive steady-state effect on the home capital stock $\hat{k}$. Under such a situation, the result of the benchmark model holds: the home country exports the investment good and imports the consumption good if $\alpha_c<\alpha_i$. By contrast, if $\theta>\bar{\theta}$, in the steady state $n(t)$ decreases ($\hat{n}^{'}<1$) in response to a rise in the home inflation. Nevertheless, $n(t)$ must jump up on impact. From (A33) and (A37), at the instant of the policy change $n(t)$ follows:

$$n(t)(t=0^+)-1=\sigma(\tau_0^{'}-1)\left[1-\frac{1}{\frac{\Delta\widehat{k}}{\Delta\widehat{n}}}\cdot\frac{1}{g_{11}}(1-\frac{g_{41}}{g_{44}})\right]=\sigma(\tau_0^{'}-1)\left(1-\frac{r(\hat{P})-\delta}{a+r(\hat{P})-\delta}\cdot\Xi\right)>0,$$

where $\Xi=1-\frac{(1-\theta)a}{\theta(1+\rho+\pi^*)(\rho+\delta)}$. Since both $\frac{r(\hat{P})-\delta}{a+r(\hat{P})-\delta}$ and Ξ are less than one, $n(t)(t=0^+)$ must be larger than one. ■