Supplementary Appendix File

Productivity Gaps and Tax Policies Under Asymmetric Trade

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B Supplementary Material

Aggregate Constraints: derivation of (17)-(18). Equation (17) is derived as follows. Substituting $n_i \equiv (V_i M_i)/L_i$ and (A.5) in (A.12), we obtain

$$V_h \dot{M}_h = \Pi_h M_h + P_L^h L_h - E_h^c - F_h L_h.$$

Plugging $V_i \dot{M}_i = P_Y^i Z_i$ from (5)-(A.4), and $M_i \Pi_i = M_i X_i \left(P_X^i - \varsigma P_Y^i\right)$ from (A.2), in the above equation, we obtain

$$P_Y^h Z_h + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h + P_L^h L_h - F_h L_h,$$

where we substitute $F_i L_i = a_i P_Y^i Z_i - b_i M_i P_X^i X_i - \tau_i P_R R_i$ from (11) to get

$$P_Y^h Z_h (1 + a_h) + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h (1 + b_h) + P_L^h L_h + \tau_h P_R R_h.$$

From the final sectors' profit-maximizing conditions, we can substitute $P_L^i L_i = \beta P_Y^i Y_i$ and $M_i P_X^i X_i (1 + b_i) = \alpha P_Y^i Y_i$ in the above equation, obtaining

$$E_h^c + P_Y^h Z_h (1 + a_h) + P_Y^h \varsigma M_h X_h = (\alpha + \beta) P_Y^h Y_h + \tau_h P_R R_h,$$

where we can plug $\alpha + \beta = 1 - \gamma$, and condition (2), to obtain

$$E_h^c + P_Y^h Z_h (1 + a_h) + P_Y^h \varsigma M_h X_h = P_Y^h Y_h - P_R R_h. \tag{B.1}$$

Substituting $E_h^d \equiv P_Y^h Z_h (1 + a_h)$ and $E_h^x \equiv P_Y^h \varsigma M_h X_h$ we obtain (17). Repeating the above steps for the Foreign economy starting from constraint (A.13), and recalling that $R - R_f = R_h$, we obtain (18).

Derivation of (A.22)-(A.23). Consider Home. From (A.21), substitute $\bar{\sigma}_h^d = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^c - \bar{\sigma}_h^x$ in (A.20), and eliminate $\bar{\sigma}_h^x$ by (A.19), to obtain

$$\widehat{\bar{\sigma}_i^c}(t) = \varphi_h \bar{\sigma}_h^c(t) + \varphi_h \frac{\alpha(1-\alpha)(1+a_h)+\alpha^2}{1+b_h} - \varphi_h(1-\tilde{\gamma}_h) - \rho, \tag{B.2}$$

Since $\varphi_h > 0$, equation (B.2) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics in the consumption propensity, we have

$$\bar{\sigma}_h^c = (1-\tilde{\gamma}_h) - \frac{\varphi_h\left[\alpha(1-\alpha)(1+a_h)+\alpha^2\right] - \rho(1+b_h)}{\varphi_h(1+b_h)} \text{ in each } t. \tag{B.3}$$

From (A.19) and (B.3), constant values of $\bar{\sigma}_h^c$ and $\bar{\sigma}_h^x$ imply a constant $\bar{\sigma}_h^d$ which, from (A.21), equals

$$\bar{\sigma}_h^d = 1 - \tilde{\gamma}_h - \bar{\sigma}_h^c - \bar{\sigma}_h^x = \frac{\varphi_h \alpha(1-\alpha)(1+a_h) - \rho(1+b_h)}{\varphi_h(1+b_h)}. \tag{B.4}$$

Derivation of (A.42)-(A.45). Equation (A.3) and result (21) imply

$$Y_i(t) = \frac{(\alpha^2/\varsigma)^{\frac{\alpha}{1-\alpha}}}{1+b_i} \cdot M_i(0)\left(v_i(0)L_i\right)^{\frac{\beta}{1-\alpha}}\left(R_i(0)\right)^{\frac{\gamma}{1-\alpha}} \cdot e^{(\Omega_i-\rho)t}, \tag{B.5}$$

where $M_i(0)$ and $v_i(0)$ are exogenously given. Initial resource use $R_i(0)$ is determined by the solution of the optimal extraction problem:[26]

$$R_h(0) = \frac{\bar{\theta}}{1+\bar{\theta}}\rho Q_0 \text{ and } R_f(0) = \frac{1}{1+\bar{\theta}}\rho Q_0. \tag{B.6}$$

Substituting (B.6) in (B.5) for each $i = h, f$, we obtain (A.42) and (A.43). Taking the ratio between (A.42) and (A.43), and defining $\psi_0 \equiv \left[\frac{M_h(0)}{M_f(0)}\left(\frac{1+b_f}{1+b_h}\right)\left(\frac{v_h(0)L_h}{v_f(0)L_f}\right)^{\frac{\beta}{1-\alpha}}\right]$, we obtain (A.44). Re-writing (A.28) as

$$\frac{P_Y^h(t)}{P_Y^f(t)} = \theta(t) \cdot \frac{1+\tau_h}{1+\tau_f}\frac{Y_f(t)}{Y_h(t)},$$

and using (A.44) to eliminate $Y_h(t)/Y_f(t)$, we obtain (A.45).

[26] Since $R = R_h + R_f$ and $\theta = \bar{\theta}$, the intertemporal resource constraint (10) can be written as $Q_0 = \int_0^\infty R_f(t)\left(1+\bar{\theta}\right)dt$ and directly integrated to obtain $R_f(0)$ in (B.6), from which $R_h(0)$ can be obtained as $\bar{\theta}R_f(0)$.

Conditional efficiency: proof of result (28). The proof consists in three steps, characterizing (i) conditional efficiency in Home, (ii) conditional efficiency in Foreign, (iii) derivation of (28).

Step 1. Conditional efficiency in Home. By definition, the CE-allocation in Home solves

$$\max_{\left\{E_h^c,E_h^x,E_h^d,R_h\right\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E_h^c)dt \text{ subject to}$$

$$Y_h = M_h X_h^\alpha \left(v_h L_h\right)^\beta {R_h}^\gamma,$$

$$E_h^x = P_Y^h \varsigma M_h X_h,$$

$$P_Y^h Y_h = E_h^c + E_h^d + E_h^x + P_R R_h,$$

$$\dot{M}_h = M_h \varphi_h \cdot \left[E_h^d/(P_Y^h Y_h)\right],$$

where $\omega = \omega(P_Y^h, P_Y^f)$ is taken as given and symmetry across varieties is already imposed without any loss of generality. The first constraint is the final-good technology (1), the second is the intermediate-good technology with linear cost, the third is (17), the fourth is the R&D technology (7) with knowledge spillovers taken into account. Recalling that $\sigma_h^d \equiv E_h^d/(P_Y^h Y_h)$ and combining the first three constraints, the problem becomes $\max_{\left\{E_h^c,X_h,\sigma_h^d,R_h\right\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_h) \cdot E_h^c)dt$ subject to

$$P_Y^h M_h X_h^\alpha \left(v_h L_h\right)^\beta {R_h}^\gamma \left(1-\sigma_h^d\right) = E_h^c + P_Y^h \varsigma M_h X_h + P_R R_h, \tag{B.7}$$

$$\dot{M}_h = M_h \varphi_h \sigma_h^d, \tag{B.8}$$

where the controls are $\left\{E_h^c, X_h, \sigma_h^d, R_h\right\}$ and the only state variable is M_h. The current-value Hamiltonian is

$$\ln\left[(\omega_h/L_h) \cdot E_h^c\right] + \mu_h' \cdot M_h \varphi_h \sigma_h^d +$$
$$+ \mu_h'' \cdot \left[P_Y^h M_h X_h^\alpha \left(v_h L_h\right)^\beta {R_h}^\gamma \left(1-\sigma_h^d\right) - E_h^c - P_Y^h \varsigma M_h X_h - P_R R_h\right]$$

where μ_h' is the dynamic multiplier associated to (B.8) and μ_h'' is the static multiplier attached to (B.7). The optimality conditions read

$$\frac{\partial}{\partial E_h^c} = 0 \rightarrow \qquad \frac{1}{E_h^c} = \mu_h'', \tag{B.9}$$

$$\frac{\partial}{\partial X_h} = 0 \rightarrow \qquad \left(1-\sigma_h^d\right)\alpha P_Y^h Y_h = P_Y^h \varsigma M_h X_h, \tag{B.10}$$

$$\frac{\partial}{\partial \sigma_h^d} = 0 \rightarrow \qquad \mu_h' M_h \varphi_h = \mu_h'' P_Y^h Y_h, \tag{B.11}$$

$$\frac{\partial}{\partial R_h} = 0 \rightarrow \qquad \left(1-\sigma_h^d\right)\gamma P_Y^h Y_h = P_R R_h \tag{B.12}$$

$$\rho\mu_h' - \dot{\mu}_h' = \frac{\partial}{\partial M_h} \rightarrow \qquad \rho\mu_h' - \dot{\mu}_h' = \mu_h'\varphi_h\sigma_h^d + \mu_h'' P_Y^h \left[\frac{Y_h}{M_h}\left(1-\sigma_h^d\right) - \varsigma K_h\right], \tag{B.13}$$

and imply[27]

$$\tilde{E}_h = \left[1-\gamma\left(1-\sigma_h^d\right)\right]\cdot P_Y^h Y_h, \tag{B.14}$$

$$\tilde{E}_h^x = \alpha\left(1-\sigma_h^d\right)\cdot P_Y^h Y_h, \tag{B.15}$$

$$\tilde{E}_h^c = \beta\left(1-\sigma_h^d\right)\cdot P_Y^h Y_h, \tag{B.16}$$

$$E_h^d = \sigma_h^d \cdot P_Y^h Y_h. \tag{B.17}$$

Substituting (B.10) and (B.11) in (B.13) we have

$$\frac{\dot{\mu}_h'}{\mu_h'} = \rho - \varphi_h\left[1-\alpha\left(1-\sigma_h^d\right)\right]. \tag{B.18}$$

Time-differentiating (B.11) and using (B.18) we have

$$\frac{\dot{\mu}_h''}{\mu_h''} = \rho - \varphi_h(1-\alpha)\left(1-\sigma_h^d\right) - \frac{\dot{P}_Y^h Y_h}{P_Y^h Y_h},$$

where we can substitute $\mu_h'' = 1/E_h^c$ from (B.9) to obtain

$$\frac{\dot{E}_h^c}{E_h^c} - \frac{\dot{\overline{P_Y^h Y_h}}}{P_Y^h Y_h} = \varphi_h(1-\alpha)\left(1-\sigma_h^d\right) - \rho. \tag{B.19}$$

From (B.16) we have $\frac{\dot{E}_h^c}{E_h^c} - \frac{\dot{\overline{P_Y^h Y_h}}}{P_Y^h Y_h} = -\frac{\dot{\sigma}_h^d}{1-\sigma_h^d}$ which can be combined with (B.19) to get

$$\dot{\sigma}_h^d = \rho\left(1-\sigma_h^d\right) - \varphi_h(1-\alpha)\left(1-\sigma_h^d\right)^2. \tag{B.20}$$

[27] Plugging (B.12) in constraint (17) we have (B.14). Plugging (B.10) in technology $E_h^k = P_Y^h \varsigma M_h K_h$ yields (B.15). Plugging (B.10) and (B.12) in (B.7) we have (B.16). Equation (B.17) is determined residually by $\tilde{E}_h^d = \tilde{E}_h - \tilde{E}_h^k - \tilde{E}_h^c$.

Equation (B.20) is globally unstable around its unique steady state: ruling out explosive dynamics by standard arguments, the conditionally-efficient rate of investment in R&D is

$$\tilde{\sigma}_h^d = \frac{\varphi_h (1-\alpha) - \rho}{\varphi_h (1-\alpha)} \text{ and } 1 - \tilde{\sigma}_h^d = \frac{\rho}{\varphi_h (1-\alpha)} \tag{B.21}$$

in each point in time. Substituting (B.21) in (B.15)-(B.16) we obtain

$$\tilde{\sigma}_h^x = \frac{\alpha\rho}{\varphi_h (1-\alpha)} \text{ and } \tilde{\sigma}_h^c = \frac{\beta\rho}{\varphi_h (1-\alpha)}. \tag{B.22}$$

Step 2. Conditional efficiency in Foreign. Following the same preliminary steps of the Home problem, the CE-allocation in Foreign solves

$$\max_{\left\{E_f^c, X_f, \sigma_f^d, R_h, R_f\right\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega/L_f) \cdot E_f^c) dt \text{ subject to}$$

$$P_Y^f M_f X_f^\alpha (v_f L_f)^\beta R_f{}^\gamma \left(1 - \sigma_f^d\right) = E_f^c + P_Y^f \varsigma M_f X_f - P_R R_h, \tag{B.23}$$

$$\dot{M}_f = M_f \varphi_f \sigma_f^d, \tag{B.24}$$

$$\dot{Q} = -R_h - R_f \tag{B.25}$$

where (B.23) follows from (18) and, differently from Home, we have the resource constraint (B.25) and also exported resources R_h as an additional control. The state variables are M_f and the resource stock Q. The Hamiltonian is

$$\begin{aligned}
&\ln\left[(\omega/L_f) \cdot E_f^c\right] + \mu_f' \cdot M_f \varphi_f \sigma_f^d + \\
&+ \mu_f'' \cdot \left[P_Y^f M_f X_f^\alpha (v_f L_f)^\beta R_f{}^\gamma \left(1 - \sigma_f^d\right) - E_f^c - P_Y^f \varsigma M_f X_f + P_R R_h\right] + \\
&+ \mu_f''' \cdot (-R_h - R_f)
\end{aligned}$$

where μ_f' is the dynamic multiplier associated to (B.24), μ_h'' is the Lagrange multiplier attached to (B.23), and μ_f''' is the dynamic multiplier associated to (B.25). The first order conditions read

$$\frac{\partial}{\partial E_f^c} = 0 \rightarrow \qquad \frac{1}{E_f^c} = \mu_f'', \tag{B.26}$$

$$\frac{\partial}{\partial X_f} = 0 \rightarrow \qquad \left(1-\sigma_f^d\right)\alpha P_Y^f Y_f = P_Y^f \varsigma M_f X_f, \tag{B.27}$$

$$\frac{\partial}{\partial \sigma_f^d} = 0 \rightarrow \qquad \mu_f' M_f \varphi_f = \mu_f'' P_Y^f Y_f, \tag{B.28}$$

$$\frac{\partial}{\partial R_h} = 0 \rightarrow \qquad \mu_f'' \cdot P_R = \mu_f''' \tag{B.29}$$

$$\frac{\partial}{\partial R_f} = 0 \rightarrow \qquad \mu_f'' \cdot \left(1-\sigma_f^d\right)\gamma P_Y^f Y_f = \mu_f''' R_f, \tag{B.30}$$

$$\rho\mu_f' - \dot{\mu}_f' = \frac{\partial}{\partial M_f} \rightarrow \qquad \rho\mu_f' - \dot{\mu}_f' = \mu_f'\varphi_f\sigma_f^d + \mu_f'' P_Y^f \left[\frac{Y_f}{M_f}\left(1-\sigma_f^d\right) - \varsigma K_f\right], \tag{B.31}$$

$$\rho\mu_f''' - \dot{\mu}_f''' = \frac{\partial}{\partial Q} \rightarrow \qquad \rho\mu_f''' - \dot{\mu}_f''' = 0. \tag{B.32}$$

Notice that, from (B.29)-(B.30) and definition $R_h = \theta R_f$, we have

$$P_R \tilde{R}_f = \left(1-\sigma_f^d\right)\gamma P_Y^f \tilde{Y}_f, \tag{B.33}$$

$$P_R \tilde{R}_h = \left(1-\sigma_f^d\right)\gamma\tilde{\theta} \cdot P_Y^f \tilde{Y}_f, \tag{B.34}$$

so that expenditures equal[28]

$$\tilde{E}_f = \left[1 + \left(1-\tilde{\sigma}_f^d\right)\gamma\tilde{\theta}\right] \cdot P_Y^f \tilde{Y}_f, \tag{B.35}$$

$$\tilde{E}_f^x = \alpha\left(1-\tilde{\sigma}_f^d\right) \cdot P_Y^f \tilde{Y}_f, \tag{B.36}$$

$$\tilde{E}_f^c = \left(1-\alpha+\gamma\tilde{\theta}\right)\left(1-\tilde{\sigma}_f^d\right) \cdot P_Y^f \tilde{Y}_f, \tag{B.37}$$

$$\tilde{E}_f^d = \tilde{\sigma}_f^d \cdot P_Y^f \tilde{Y}_f. \tag{B.38}$$

Step 3. Derivation of result (28). The efficient relative resource use $\tilde{\theta}$ is obtained as follows. Assume a symmetric equilibrium in which both Home and Foreign exhibit a CE-allocation. From the balanced trade condition (A.26), we have $P_R R_h + (1-\epsilon) E_h^c =$

[28] Plugging (B.34) in (18) yields (B.35). Plugging (B.27) in technology $E_f^k = P_Y^f \varsigma M_f K_f$ yields (B.36). Plugging (B.27) and (B.34) in (B.23) we have (B.37). Equation (B.38) is determined residually by $\tilde{E}_f^d = \tilde{E}_f - \tilde{E}_f^k - \tilde{E}_f^c$.

ϵE_f^c where we can use (B.16) and (B.37) to eliminate E_h^c and E_f^c, respectively, and also use (B.12) to eliminate $P_R R_h$, obtaining

$$\frac{1-\tilde{\sigma}_h^d}{1-\tilde{\sigma}_f^d} \cdot \frac{P_Y^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f} = \frac{\epsilon \left(1-\alpha+\gamma\tilde{\theta}\right)}{\gamma+(1-\epsilon)\beta}, \tag{B.39}$$

where tildas denote conditionally-efficient values. Taking the ratio between (B.12) and (B.34) we have

$$\tilde{\theta} = \frac{1-\tilde{\sigma}_h^d}{1-\tilde{\sigma}_f^d} \cdot \frac{P_Y^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f}. \tag{B.40}$$

Combining (B.40) with (B.39) we obtain

$$\tilde{\theta} = \frac{\epsilon}{1-\epsilon} \cdot \frac{1-\alpha}{\gamma+\beta} = \frac{\epsilon}{1-\epsilon}, \tag{B.41}$$

which proves result (28) in the main text. Also note that since relative resource use $\tilde{\theta}$ is constant over time, combining systems (B.35)-(B.38) with (B.14)-(B.22) implies constant propensities to spend output among its competing uses within each country. As a consequence, the "efficient" policies that decentralize the symmetric CE-equilibrium are characterized by constant R&D subsidies and taxes in each country over time.