Does idiosyncratic risk matter for climate policy?

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ONLINE APPENDIX

A Appendix

A1 The saving rate: Proposition 1

Proof. Using the definition for the saving rate and the constant $\Phi(\phi_2, \alpha; \Psi)$ which fully characterizes the level of idiosyncratic risk in this economy, note that equation (18) can be rewritten as

$$1 = \alpha\beta(1-\tau_{t+1}^K)\left[\frac{\frac{K_{t+1}}{s_t} - K_{t+1}}{K_{t+1}}\right]\Phi(\phi_2, \alpha; \Psi) \tag{A.1}$$

$$= \alpha\beta(1-\tau_{t+1}^K)\left[\frac{1-s_t}{s_t}\right]\Phi(\phi_2, \alpha; \Psi) \ , \tag{A.2}$$

and solving for s_t yields the saving rate in general equilibrium as a function of capital income taxes and idiosyncratic risk. ■

A2 First-best allocations: Proposition 2

Proof. Here, as in Krueger *et al.* (2021), the government would like to maximize the following social welfare function using arbitrary welfare weights, δ_t,

$$\max_{\{C_{1,t}, C_{2,t}(\chi_t), K_{t+1}, E_t\}_{t=0}^{\infty}} \delta_{-1}\int \log C_{2,0}(\chi_0)d\Psi(\chi_0) \tag{A.3}$$

$$+ \sum_{t=0}^{\infty}\delta_t\left[\log C_{1,t} + \beta\int \log C_{2,t+1}(\chi_{t+1})d\Psi(\chi)\right],$$

subject to resource constraint and the initial capital K_0,

$$C_{1,t} + \int C_{2,t}(\chi_{t+1})d\Psi + K_{t+1} = Y_t \ . \tag{A.4}$$

Since the government is not constrained in the transfers it can implement, it is optimal to provide full insurance such that $C_{2,t} = C_{2,t}(\chi_t)$ for each realization of χ_t and for all t. Then, the problem reduces to

$$\max_{\{C_{1,t},C_{2,t},K_{t+1},E_t\}_{t=0}^{\infty}} \delta_{-1} \log C_{2,0} + \sum_{t=0}^{\infty} \delta_t \left[\log C_{1,t} + \beta \log C_{2,t+1}\right] , \tag{A.5}$$

subject to the resource constraint

$$C_{1,t} + C_{2,t} + K_{t+1} = Y_t , \tag{A.6}$$

and the initial capital as before. Let μ_t be the Lagrange multiplier related to the feasibility constraint. The first-order conditions associated with this problem are then

$$\frac{\delta_t}{C_{1,t}} = \mu_t \tag{A.7}$$

$$\frac{\delta_t \beta}{C_{2,t+1}} = \mu_{t+1} \tag{A.8}$$

$$\frac{1}{F_{K_{t+1}}} = \frac{\mu_{t+1}}{\mu_t} \tag{A.9}$$

$$\sum_{i=1}^{\infty} \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} = F_{E_t} . \tag{A.10}$$

However, in order to maintain the same assumption about welfare weights as in the benchmark model, let $\gamma = \frac{\delta_{t+1}}{\delta_t} < 1$.[1] Then, it is easy to check that consumption across generations and the Euler equation are given by

[1]See Gerlagh *et al.* (2017) for a complete characterization of climate policies with welfare weights.

$$C_{2,t} = \frac{\beta}{\gamma} C_{1,t} \tag{A.11}$$

$$C_{2,t+1} = \beta r_{t+1} C_{1,t} \ . \tag{A.12}$$

And then it follows that

$$C_{1,t} = \frac{\gamma}{\gamma+\beta}[Y_t - K_{t+1}] \tag{A.13}$$

$$C_{2,t} = \frac{\beta}{\gamma+\beta}[Y_t - K_{t+1}] \ . \tag{A.14}$$

By substituting the previous expression into the Euler equation, and using the fact that $r_{t+1} = \frac{\alpha Y_{t+1}}{K_{t+1}}$ one obtains,

$$1 - \frac{K_{t+2}}{Y_{t+1}} = \alpha\gamma \left[\frac{Y_t}{K_{t+1}} - 1 \right] . \tag{A.15}$$

It is then straightforward to show, using the definition for the saving rate, that the optimal saving rate is constant over time and given by

$$s^* = \frac{\alpha\gamma}{(1-\phi_2)(1-\alpha)} \tag{A.16}$$

as required. To get the optimal carbon price, we use the fact that the carbon tax uses the market interest rate to discount future marginal costs as mentioned before, and that the optimal saving rate is constant so that the investment share is $\frac{K_{t+1}}{Y_t} = \alpha\gamma$,

$$\tau_t^E = \sum_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} r_{t+j}} \theta_i Y_{t+i} \tag{A.17}$$
$$= \frac{Y_t}{Y_t} \sum_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} \frac{\alpha Y_{t+j}}{K_{t+j}}} \theta_i Y_{t+i}$$
$$= Y_t \sum_{i=1}^{\infty} \gamma^i \theta_i \ .$$

■

A3 Second-best allocations: Proposition 3

Proof. Similar to the primal approach, the government would choose the optimal saving rate in this context and it would use capital income taxes to decentralize it, which can be derived using the result in Proposition 1. Thus, the government maximizes lifetime utility of current and future generations,

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \gamma^t \left[\log C_{1,t} + \beta \int \log C_{2,t+1}(\chi_{t+1}) d\Psi(\chi) \right]. \tag{A.18}$$

By using the budget constraint for the households and the first-order conditions from the firms' problem, the previous problem can be written as

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \gamma^t \left[\log[(1-\phi_2)(1-\alpha)Y_t - K_{t+1}] + \beta \log Y_{t+1} + \text{“terms"} \right], \tag{A.19}$$

where “terms” are aggregate terms which do not depend on control variables. Using the definition for the saving rate, the first-order condition with respect to aggregate capital,

K_{t+1}, implies that

$$\frac{s_t}{1-s_t} = \alpha\beta + \frac{\alpha\gamma}{1-s_{t+1}} \; . \tag{A.20}$$

It follows that the optimal saving rate that satisfies this condition, as noted in (23), is constant and given by

$$s = \frac{\alpha(\beta+\gamma)}{1+\alpha\beta} \;\; \forall t \; . \tag{A.21}$$

The implementation of this saving rate relies on the availability of capital income taxes. From (19), it turns out that the optimal capital tax is constant over time and chosen such that

$$\frac{\alpha(\beta+\gamma)}{1+\alpha\beta} = \frac{1}{1+\frac{1}{\alpha\beta(1-\tau^K)\Phi(\phi_2,\alpha;\Psi)}} \; . \tag{A.22}$$

And solving for the capital tax, it follows

$$1-\tau^K = \frac{\beta+\gamma}{\beta(1-\alpha\gamma)\Phi(\phi_2,\alpha;\Psi)} \tag{A.23}$$

as desired. The last part of the proposition deals with optimal climate policy. Notice that the saving rate in this second-best scenario implies the following investment share:

$$\frac{K_{t+1}}{Y_t} = \frac{\alpha(\beta+\gamma)(1-\phi_2)(1-\alpha)}{1+\alpha\beta} \; . \tag{A.24}$$

So that the carbon price can be derived as

$$\begin{aligned} \tau_t^E &= \sum_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} r_{t+j}} \theta_i Y_{t+i} \\ &= \frac{Y_t}{Y_t} \sum_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} \frac{\alpha Y_{t+j}}{K_{t+j}}} \theta_i Y_{t+i} \\ &= Y_t \sum_{i=1}^{\infty} \hat{\gamma}^i \theta_i \end{aligned} \tag{A.25}$$

where the adjusted discount factor is, $\hat{\gamma} \equiv \frac{(\beta+\gamma)(1-\phi_2)(1-\alpha)}{1+\alpha\beta}$. ■

References

Gerlagh R, Jaimes R and Motavasseli A (2017) Global demographic change and climate policies. CESifo Working Paper No. 6617.

Krueger D, Ludwig A and Villalvazo S (2021) Optimal taxes on capital in the OLG model with uninsurable idiosyncratic income risk. *Journal of Public Economics* 201, 1–14.