The effects of migration and pollution on cognitive skills in Caribbean economies: a theoretical analysis

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ONLINE APPENDIX

Appendix A. Detailed comparative statics of household choices

The equations for savings and fertility are:

$$
\begin{aligned}
s_t^\star &= \frac{\beta\alpha(1-\rho)(1-\gamma)}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} w_t h_t \\
n_t^\star &= \frac{\beta\gamma(1-\rho+\rho\varepsilon)(1-\gamma)(1-\alpha)(1-\mu)}{\sigma(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]} N_t^{-\delta}
\end{aligned}
$$

The derivative of fertility with respect to the emigration rate, ρ, the net migration gain, ε, and the transfer rate, γ are, respectively:

$$
\begin{aligned}
\frac{\partial n_t^\star}{\partial \rho} &= \frac{\beta\gamma\alpha\varepsilon(1-\alpha)(1-\gamma)(1-\mu)}{\sigma N_t^\delta(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} > 0 \\
\frac{\partial n_t^\star}{\partial \varepsilon} &= \frac{\beta\gamma\alpha\rho(1-\alpha)(1-\gamma)(1-\mu)}{\sigma N_t^\delta(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} > 0 \\
\frac{\partial n_t^\star}{\partial \gamma} &= \frac{\beta(1-\alpha)(1-\mu)(1-\rho+\rho\varepsilon)}{\sigma N_t^\delta(1+\beta)} \times \frac{\alpha(1-\rho)(1-2\gamma)-\gamma^2(1-\alpha)(1-\rho+\rho\varepsilon)}{[\alpha(1-\rho)+\Lambda_h(1-\alpha)]^2}
\end{aligned}
$$

The sign of $\frac{\partial n_t^\star}{\partial \gamma}$ is given by the numerator of the second term of the derivative, which is a second-degree equation. It admits a unique positive solution (given below, $\bar{\gamma}$), and $\frac{\partial n_t^\star}{\partial \gamma}$ is negative if γ is smaller than this value:

$$
\bar{\gamma} = \frac{\sqrt{\alpha(1-\rho)}\left[\sqrt{\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)}-\sqrt{\alpha(1-\rho)}\right]}{(1-\alpha)(1-\rho+\rho\varepsilon)}
$$

Population size is proportional to fertility with a factor $(1-\rho)$. Thus, the parameters' impacts on population are the same as those on fertility, except for the migration rate, which has a negative effect on the number of adults staying in the domestic area. The derivative of the steady state adult generation size with respect to ρ is as follows:

$$\frac{\partial N^{\star}}{\partial \rho} = \frac{1}{\delta}\left[\frac{\beta\gamma(1-\mu)(1-\gamma)(1-\alpha)}{\sigma(1+\beta)}\right]^{\frac{1}{\delta}}\left[\frac{(1-\rho)(1-\rho+\rho\varepsilon)}{\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)}\right]^{\frac{1-\delta}{\delta}} \times \left[\frac{\alpha(1-\rho)(\varepsilon-1)(1-\rho)-\gamma(1-\rho+\rho\varepsilon)(1-\alpha)(1-\rho+\rho\varepsilon)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]$$

The sign of this derivative is given by the following condition:

$$\frac{\partial N^{\star}}{\partial \rho} > 0 \Leftrightarrow \frac{(1-\rho)}{(1-\rho+\rho\varepsilon)} > \sqrt{\frac{\gamma(1-\alpha)}{\alpha(\varepsilon-1)}}$$

Appendix B. Case 1: Equilibrium with the total abatement of pollution emissions

B.1 Proof of Proposition 2 and of the stability of the equilibrium

Proof of Proposition 2. The population size is independent of the evolution of the capital per unit of efficient labor. Therefore, when the labor force reaches its steady state level, the capital per efficient unit of labor is as follows:

$$k_{t+1} = k_t = k_{BGP} = \left(\frac{A\sigma(1-\tau)}{1-\mu}\right)^{1-\mu} \frac{\alpha}{\theta\Lambda_h[\mu(1-\alpha)]^{\mu}} k_{BGP}^{\alpha(1-\mu)} (N^{\star})^{\delta(1-\mu)} \qquad \textbf{(A1)}$$

Replacing $N^{\star}$ in this equation leads directly to the level of capital per unit of efficient labor on the BGP. □

Proof of the stability of the equilibrium. The population dynamics can be denoted by the function $g(N_t) = N_{t+1}$. It is concave, and there are two points such that $N_{t+1} = N_t$, which are $N_t = 0$ and $N_t = N^{\star}$ satisfying $0 < f'(N^{\star}) < 1$. Therefore, a unique nontrivial equilibrium is locally stable, and N shows regular convergence.

$$\lim_{N_t \to 0} g'(N_t) = +\infty$$

$$\lim_{N_t \to +\infty} g'(N_t) = 0$$

$$\lim_{N_t \to +\infty} g(N_t) = +\infty$$

When the population reaches its steady state value, the dynamics of the capital per unit of efficient labor are given by equation (A1). Thus, the function $f(k_t) = k_{t+1}$, which is concave, admits two points such that $k_{t+1} = k_t$, which are $k_t = 0$ and $k_t = k_{BGP}$ satisfying $0 < f'(k_{BGP}) < 1$. Therefore, a unique nontrivial equilibrium is locally stable, and the model shows regular convergence to k_{BGP}.

$$\lim_{k_t \to 0} f'(k_t) = +\infty$$

$$\lim_{k_t \to +\infty} f'(k_t) = 0$$

$$\lim_{k_t \to +\infty} f(k_t) = +\infty$$

□

B.2 Proof of Propositions 3 and 4

Proof of Proposition 3. Replacing Ψ and Λ_h with their values in equation (36), the following is obtained:

$$k_{BGP} = \left[\frac{\alpha}{\theta[\mu\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^{\mu}} \left[\frac{\beta A(1-\rho)(1-\alpha)(1-\gamma)(1-\tau)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]} \right]^{1-\mu} \right]^{\frac{1}{1-\alpha(1-\mu)}}$$

Directly, it appears that $\frac{\partial k_{BGP}}{\partial \theta} < 0$, $\frac{\partial k_{BGP}}{\partial \gamma} < 0$, $\frac{\partial k_{BGP}}{\partial \varepsilon} < 0$, $\frac{\partial k_{BGP}}{\partial A} > 0$, $\frac{\partial k_{BGP}}{\partial \beta} > 0$.

For the emigration rate, the following derivative is obtained:

$$\frac{\partial k_{BGP}}{\partial \rho} = -\frac{1}{1-\alpha(1-\mu)}\left[\left[\frac{\alpha}{\theta[\mu\gamma(1-\alpha)]^{\mu}}\right]\left[\frac{\beta A(1-\tau)(1-\alpha)(1-\gamma)}{(1+\beta)}\right]^{1-\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times\left[(1-\rho+\rho\varepsilon)^{-\mu}\left[\frac{(1-\rho)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{1-\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times\left[\frac{\mu(\varepsilon-1)}{1-\rho+\rho\varepsilon}+\frac{\gamma\varepsilon(1-\alpha)(1-\mu)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]<0$$

□

Proof of Proposition 4. Replacing Ψ and Λ_h with their values in equation (37), the growth factor is written as follows:

$$g_{BGP} = \left[[\theta(\mu\gamma(1-\rho+\rho\varepsilon)^{\mu}]^{1-\alpha}\left[\frac{\beta A(1-\tau)\alpha^{\alpha}(1-\alpha)^2(1-\gamma)(1-\rho)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$

Without calculations, it appears that $\frac{\partial g_{BGP}}{\partial \theta}>0$, $\frac{\partial g_{BGP}}{\partial \varepsilon}>0$, $\frac{\partial g_{BGP}}{\partial A}>0$, $\frac{\partial k_{BGP}}{\partial \beta}>0$.

The derivative with respect to the net gain from migration is written as follows:

$$\frac{\partial g_{BGP}}{\partial \varepsilon} = \left[\theta^{1-\alpha}\left[\frac{\beta A(1-\tau)\alpha^{\alpha}(1-\gamma)(1-\alpha)[\mu\gamma(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times\frac{\mu\alpha\rho(1-\alpha)}{1-\alpha(1-\mu)}\left[\frac{(1-\rho+\rho\varepsilon)^{1-\alpha}}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}(1-\rho+\rho\varepsilon)^{-\alpha}$$
$$\times\left[\frac{(1-\rho)+\Lambda_h}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]>0$$

For the emigration rate, the following derivative is obtained:

$$\frac{\partial g_{BGP}}{\partial \rho} = \left[\theta^{1-\alpha}\left[\frac{\beta A(1-\tau)\alpha^{\alpha}(1-\gamma)(1-\alpha)[\mu\gamma(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times\frac{\mu(1-\alpha)}{1-\alpha(1-\mu)}\left[\frac{(1-\rho+\rho\varepsilon)^{1-\alpha}(1-\rho)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}(1-\rho+\rho\varepsilon)^{-\alpha}$$
$$\times\left[\frac{\alpha(1-\rho)(\varepsilon-1)(1-\rho)-\Lambda_h[\varepsilon-(1-\alpha)(\varepsilon-1)(1-\rho)]}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]$$

Thus, this leads to the following condition:

$$\frac{\partial g_{BGP}}{\partial \rho} > 0 \Leftrightarrow \frac{(1-\rho)}{\Lambda_h} > \frac{[\varepsilon - (1-\alpha)(\varepsilon-1)(1-\rho)]}{\alpha(\varepsilon-1)(1-\rho)}$$

The derivative of the growth factor on the BGP with respect to γ is:

$$\begin{aligned}\frac{\partial g_{BGP}}{\partial \gamma} = & \left[\theta^{1-\alpha}\left[\frac{\beta A\alpha^{\alpha}(1-\rho)(1-\alpha)[\mu(1-\rho+\rho\varepsilon)(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}} \\ & \times \frac{\mu}{1-\alpha(1-\mu)}\left[\frac{\gamma^{1-\alpha}(1-\gamma)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}\gamma^{-\alpha} \\ & \times\left[\frac{\alpha(1-\rho)[(1-\alpha)(1-\gamma)-\gamma]-(1-\alpha)\Lambda_h[1-(1-\alpha)(1-\gamma)]}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]\end{aligned}$$

Hence, the following condition can be defined:

$$\frac{\partial g_{BGP}}{\partial \gamma} > 0 \Leftrightarrow \frac{\Lambda_h}{(1-\rho)} < \frac{\alpha}{1-\alpha}\frac{(1-\alpha)(1-\gamma)-\gamma}{1-(1-\alpha)(1-\gamma)}$$

However, note that if the intergenerational transfer rate is high, $(1-\alpha)(1-\gamma)-\gamma$ can be negative. □

B.3 BGP values of consumption

In the steady state, adult and old age consumption are given by the following equations:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}w_t h_t \\ d_{t+1} &= s_t R_{t+1} + n_t \Lambda_h w_{t+1} h_{t+1}\end{aligned}$$

Introducing household choices in terms of savings and fertility (s_t and n_t), factor prices

(w_{BGP} and R_{BGP}) and the labor force on the BGP ($N^{\star}$), the following is obtained:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}(1-\alpha)A(1-\tau){k_{BGP}}^{\alpha}h_t \\ d_{t+1} &= \frac{(1-\alpha)A(1-\tau){k_{BGP}}^{\alpha}}{\alpha\Psi A\alpha(1-\tau)}h_t\left[{k_{BGP}}^{1-\alpha}+\frac{(1-\alpha)\Lambda_h}{(1-\rho)\alpha\Psi A\alpha(1-\tau)}\frac{h_{t+1}}{h_t}\right] \end{aligned}$$

On the BGP, $\frac{h_{t+1}}{h_t}$ is given by the growth rate of the economy, g_{BGP}, which leads to:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}(1-\alpha)A{k_{BGP}}^{\alpha}h_t \\ d_{t+1} &= \frac{(1-\alpha)A{k_{BGP}}^{\alpha}}{\alpha\Psi A\alpha}h_t\left[{k_{BGP}}^{1-\alpha}+\frac{(1-\alpha)\Lambda_h}{(1-\rho)\alpha\Psi A\alpha(1-\tau)}g_{BGP}\right] \end{aligned}$$

Consequently, in the long run, the growth factors of consumption $\frac{c_{t+1}}{c_t}$ and $\frac{d_{t+2}}{d_{t+1}}$ depend solely on human capital dynamics. This means than on the BGP, it is equivalent to studying the growth rate of the economy and the utility per capita linked to consumption.

Appendix C. Structural parameter value estimation and calibration

Table A1 below reports the model's structural economic parameters, their respective economic interpretations, the support range for credible values and the calculation methods used in Ait Benhamou and Cassin (2020). The data were extracted from the World Bank (2018) World Development Indicators (WDI) and the University of Pennsylvania World Table (PWT). In this section, only the parameters obtained with a method different from that of Ait Benhamou and Cassin (2020) are described in detail. Indeed, in their paper, there is no congestion, and fertility is exogenous, but the parameters $\beta, \alpha, \rho, \gamma, \mu, \bar{\theta}, \varepsilon$ can be used directly. The parameters δ and σ are defined w ith t he m ethod described below.

The ability to obtain credible values for the structural parameters is contingent upon the available data. This is particularly the case for small emerging economies, such as the Caribbean islands. Kydland and Prescott (1991) provide a comprehensive framework for

Table A1. Model structural parameters

Economic Parameters	**Range**	Method	Data source
Preference factor for the future	$\beta \in [0, 1[$	Calibration	WDI
Capital intensity in production	$\alpha \in [0, 1]$	*idem*	WDI & PWT
Technology level	$A > 0$	*idem*	PWT
Emigration rate	$\rho \in [0, 1]$	*idem*	WDI
Net gain from migration	$\varepsilon > 1$	*idem*	*idem*
Share of income remitted	$\gamma \in [0, 1]$	*idem*	*idem*
Efficiency – education	$\mu \in [0, 1]$	Estimation	WDI & PWT
Efficiency – human capital accumulation	$\theta > 0$	*idem*	*idem*
Cost of child-rearing	$\sigma \in [0, 1]$	Calibration	*idem*
Congestion parameter	$\delta \in [0, 1]$	Calibration	*UN Data*

discussing calibration in general equilibrium models. While they insist on the method to choose the benchmark values for structural parameters, in the absence of panel studies on households and firms – which are optimal to compute agent behavior parameters – one should focus as much as possible on standard calibration. Consequently, here, the calibration relies on optimal household choices in the steady state and uses long-run averages of variables in the dataset built for the sample of SIDS countries. Most available data can be traced back to the 1970s, and a dataset for the time period 1970-2014 is built. Numerical simulations will then be computed with initial values corresponding to the year 1970. Due to their differences, two countries are scrutinized to illustrate the model: Barbados and Jamaica.

- $N^\star$ is the steady-state population value. Using data from the World Bank, the forecast of population levels (initially until 2050) is extended until demographic growth is close or equal to zero. This allows us to extrapolate steady state population $N^\star$ at the corresponding dates.

- δ is the congestion parameter and captures the speed of convergence to the steady-state population level. The value is estimated thanks to the following expression:

$$N_{t+1} = \Lambda_n N_t^{1-\delta}$$

where Λ_n is the collection of structural parameters in the model. The equation is rewritten in log terms and differentiated such that δ is estimated by regressing future demographic growth on logged present population, namely:

$$\Delta\% N_{t+1} = \ln \Lambda_n - \delta \ln N_t$$

A logistic transformation is introduced to ensure that estimated values for the parameter δ always belong to the interval $(0, 1)$.

- σ denotes the child-rearing cost per individual. Its value is calibrated to match the parameters of the model, as well as the estimated values of μ and θ in Ait Benhamou and Cassin (2020). The congestion component of the model is incorporated such that σ matches the following expression:

$$\sigma = \frac{\gamma\beta(1-\rho+\rho\varepsilon)(1-\gamma)(1-\alpha)(1-\rho)(1-\mu)}{N^{\star\delta}(1+\beta)\left[\alpha(1-\rho)+(1-\alpha)\gamma(1-\rho+\rho\varepsilon)\right]}$$

For the initial values, output is normalized to unity in 1970, and the capital stock is computed using the capital-to-output ratio for the same year. The figures in table A2 report harmonized initial values for physical capital for comparison purposes. The same calibration is computed for efficient units of labor, which are derived from normalized output and capital. Using the Cobb-Douglas equation (1), it is possible to deduce $N_0 h_0$ for a given K_0 and $y_0 = 1$. Finally, given that human capital is reported as an index in PWT, the 1970 value is directly used as the initial value.

Table A2 gives the values of the parameters for Barbados ans Jamaica.

Appendix D. Case 2: Equilibrium with partial abatement of pollution

D.1 Proof of proposition 5

Proof of Proposition 5. The population size dynamics remain the same as in case 1. Using equation (40) in the dynamics of capital, given by equation K_{t+1} in system (29), a relationship is obtained between the steady state values of the capital stock $K^\star$ and

Table A2. Calibrated values for structural parameters – Caribbean SIDS countries

Parameters		Barbados	Jamaica
Preference factor for the future	β	0.940	0.944
Capital intensity in production	α	0.340	0.312
Technology level	A	1.034	1.014
Education efficiency	μ	0.130	0.162
Efficiency of human capital accumulation	θ	5.025	4.898
Cost of rearing a child	σ	0.171	0.063
Emigration rate	ρ	0.370	0.490
Net gain from migration	ε	1.91	6.580
Share of income remitted	γ	0.121	0.200
Congestion parameter	δ	0.636	0.623
Capital stock	K_0	0.021	0.335
Human capital stock	h_0	1.367	1.083
Labor	N_0	0.006	0.051
Pollution Stock	Z_0	0	0

Note: Calibrated values for individual countries use available data points for the period 1961-2014. Initial values for capital stock and labor are given with a factor of 10^6

human capital level $h^\star$.

$$K^\star = \left[\frac{\alpha\beta A(1-\tau)(1-\alpha)(1-\gamma)(1-\rho)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]}\right]^{\frac{1}{1-\alpha}} N^\star h^\star \tag{A2}$$

When introduced into the equation for human capital dynamics, given by equation (9), equations (A2) and (40) ($N^\star$) lead to the following:

$$\left[\frac{\mu A\sigma(1-\alpha)(1-\tau)}{1-\mu}\right]^{\mu} h^\star = \theta(Z^\star)\,(K^\star)^{\alpha\mu}(N^\star)^{\mu(\delta-\alpha)}(h^\star)^{1-\alpha\mu} \tag{A3}$$

After some computations, the steady-state value of $\theta(Z^\star) \equiv \chi$ is:

$$\chi = [\gamma\mu(1-\rho+\rho\varepsilon)(1-\alpha)]^{-\mu}\left[\frac{\beta A\alpha^{\alpha}(1-\tau)(1-\rho)(1-\gamma)(1-\alpha)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]}\right]^{-\frac{\mu}{1-\alpha}} \tag{A4}$$

Note that the steady-state value of the efficiency of human capital accumulation is not linked to the level of emissions. The stock of pollution can be defined as the value of the inverse function of $\theta(Z_t)$ written as $\theta^{-1}(.)$. Thus, in the steady state, $Z^\star$ is defined as $Z^\star = \theta^{-1}(\chi)$.[12] Finally, using the dynamics of the pollution stock given by the last equation in the system (29), the steady-state human capital can be defined as:

$$h^\star = \left[\frac{aZ^\star}{(\Omega - \tau\xi)A} K^{\star -\alpha} N^{\star \alpha - 1}\right]^{\frac{1}{1-\alpha}} \tag{A5}$$

Replacing the steady-state values of population and pollution stock (given by (40) and (43), respectively) in the system of equations defined by (A2) and (A5), the values of $h^\star$ and $K^\star$ can be determined as:

$$\begin{aligned} h^\star &= \frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi}\left[\alpha A(1-\tau)\right]^{-\frac{\alpha}{1-\alpha}} \left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}\right]^{\frac{1}{\delta}} \left[\Psi(1-\rho)\right]^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}} \\ K^\star &= \Psi\alpha(1-\rho)(1-\tau)\frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi} \end{aligned}$$

where $\Psi \equiv \frac{\beta(1-\gamma)(1-\alpha)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}$ □

D.2 Stability of the steady state: further results

In figure A1, the second derivatives of the functions $\theta_1(Z_t)$ and $\theta_2(Z_t)$ are depicted to define whether these functions are convex or concave. According to the signs of these second derivatives, $\theta_2(Z_t)$ appears to be concave, which leads to an unstable equilibrium.

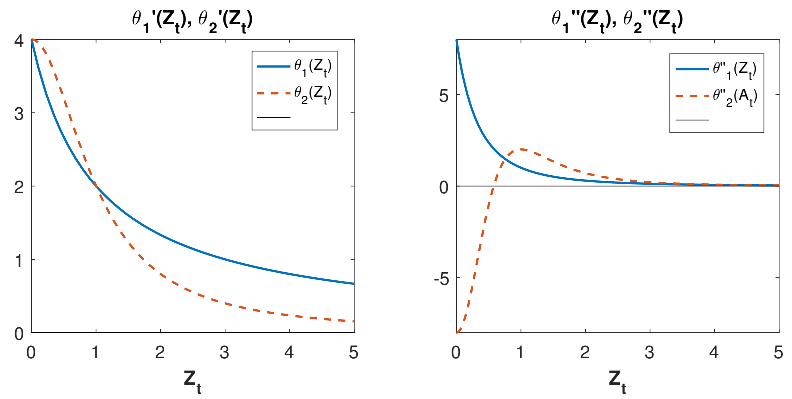

Figure A1. The functions $\theta_1(Zt)$ and $\theta_2(Z_t)$ and their second derivatives.

Second, the effects of Ω and a are tested in the absence of any environmental policy.

[12] The properties of the function $\theta(.)$ are studied in the numerical analysis.

In Figures A2 and A3, the production (Y_t), physical capital (K_t), pollution (Z_t) and human capital (h_t) of the benchmark economy are represented according to different values of the absorption rate – *i.e.,* $a = \{0.2, 0.8\}$ – and of the pollution intensity – *i.e.,* $\Omega = \{0.2, 0.5, 0.8\}$.

A comparison with the first case might help to understand these d ynamics. When the pollution externality is completely corrected, human capital increases monotonically on the BGP, as do production and the capital stock. In the presence of pollution emissions, human capital increases also lead to a rise in pollution. When pollution is high, the externality leads to an abrupt decrease in human capital and the capital stock (because of income loss). In this case, production is lessened and in return this reduces the pollution stock. When pollution is low, another cycle begins, with increasing human capital, production and physical capital. Those increases will be slower because future human capital depends on past values of human capital. Consequently, it is possible to reach a steady state with damped oscillations. This cyclical convergence has also been found by Varvarigos (2013) with similar mechanisms.

Before the steady state is reached, a high pollution intensity, Ω, or a low natural absorption of pollution, a, accelerates the accumulation of the pollution stock. This results in a larger cycle amplitude. However, although surprising, the steady-state pollution stock does not depend on Ω or a. This is because equilibrium is reached when the marginal increase in human capital is exactly compensated by the marginal loss due to the reduction in cognitive skills. This equality does not depend on pollution intensity but on parameters that change the rate of human capital accumulation. Consequently, the long-term pollution stock depends only on the level that will stop the accumulation of human capital. This is captured by χ. Here, only the time necessary to stop the fluctuations, as well as their extent, is impacted by Ω and a.

Therefore, the effects of environmental features depend on the time scale considered. In the short term, the pollution intensity has a strong impact on the pollution stock. This affects the human capital obtained in the short term, income and thus the amount saved. If a is small or Ω is high, the pollution stock is higher in the early periods. If the earlier level of human capital is low, the later level of human capital is also lower. This leads to a decrease in the steady-state values of production, $Y^\star$, and physical capital, $K^\star$. However, the pollution stock will converge to the same steady-state value – determined by χ – regardless of the values of the environmental features.

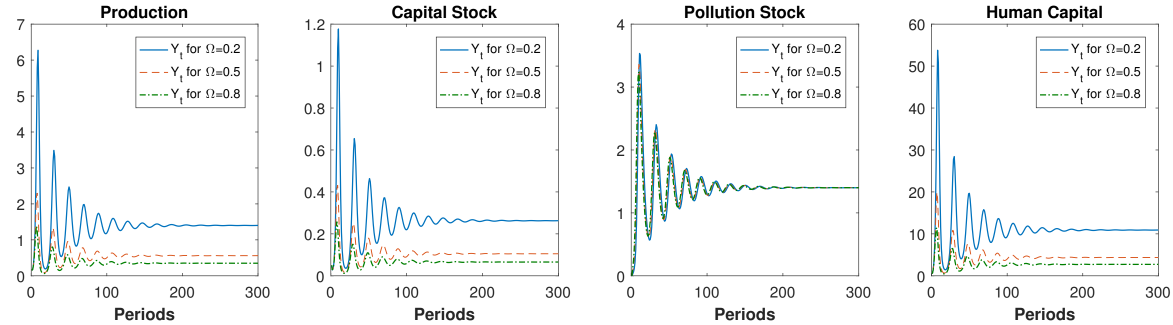

Figure A2. The effect of Ω on convergence for $a = 0.2$.

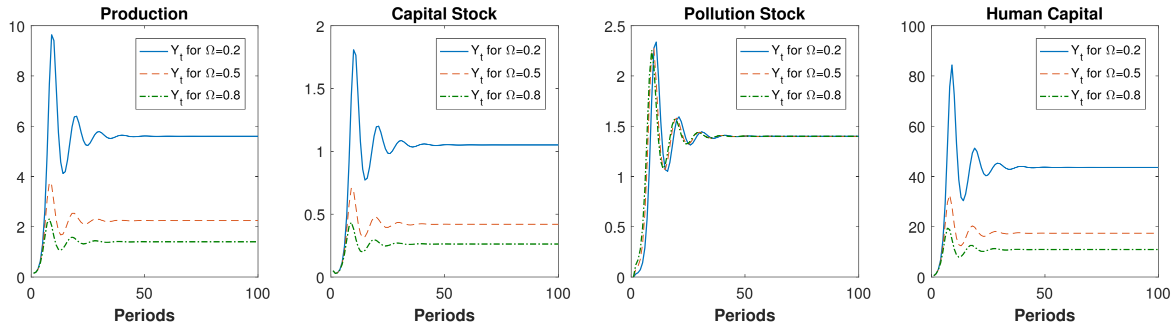

Figure A3. The effect of Ω on convergence for $a = 0.8$.

Figures A4 to A6 depict the same features as the figures displayed for Barbados.

D.3 Comparative statics

D.3.1 The efficiency of human capital accumulation, χ, and pollution, Z_t

The first step in studying the changes in pollution, human capital or capital with respect to the parameters is to define the effect of the different parameters on the long-term efficiency of human capital accumulation $\chi \equiv \theta(Z^\star)$. In the SS, the derivatives of χ

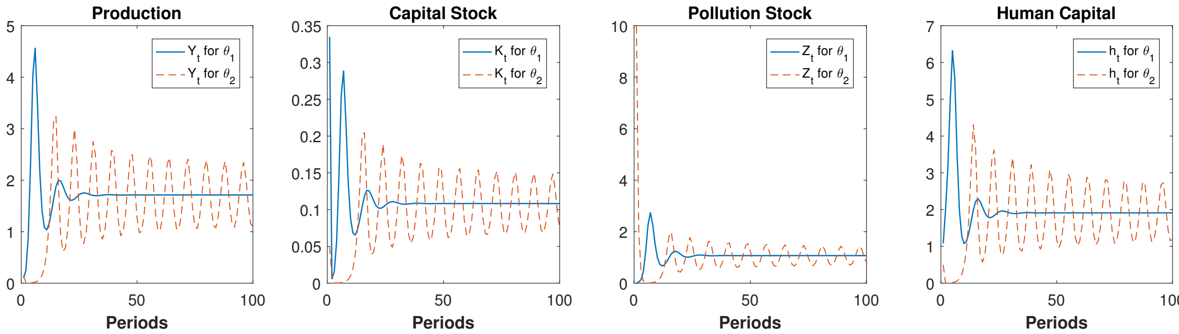

Figure A4. The effect of $\theta(Z_t)$ on the stability of the steady state.

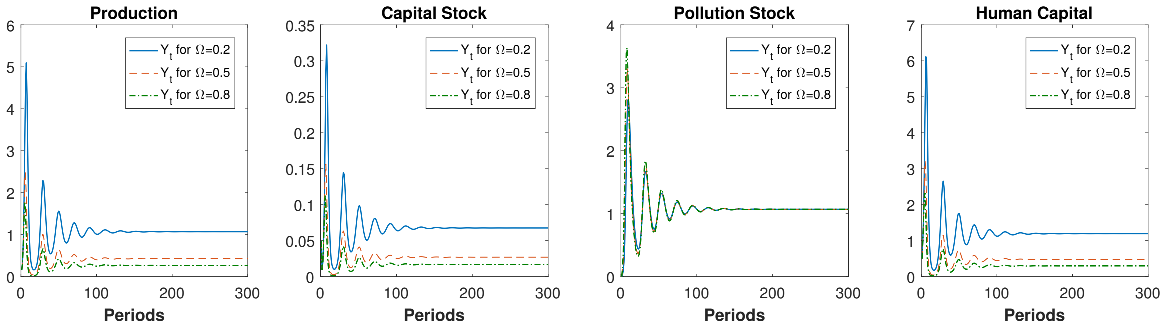

Figure A5. The effect of Ω on convergence for $a = 0.2$.

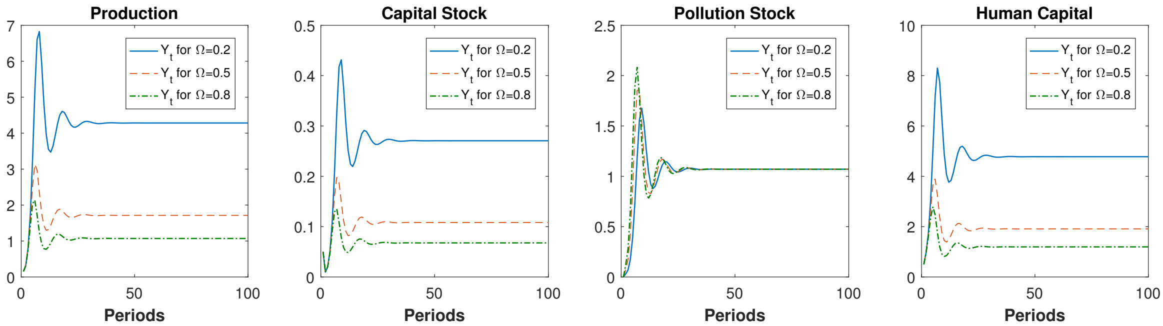

Figure A6. The effect of Ω on convergence for $a = 0.8$.

with respect to the parameters are given by the following equations:

$$\frac{\partial \chi}{\partial \beta} = -\frac{\mu}{1-\alpha}\frac{\chi}{\beta(1+\beta)} < 0 \tag{A6}$$

$$\frac{\partial \chi}{\partial A} = -\frac{\mu}{1-\alpha}\frac{\chi}{A} < 0 \tag{A7}$$

$$\frac{\partial \chi}{\partial \tau} = \frac{\mu}{1-\alpha}\frac{\chi}{\tau} > 0 \tag{A8}$$

Moreover, the conditions for the parameters linked to the impact of migration features can be computed. The derivative of χ with respect to ρ is given by the following equation:

$$\frac{\partial \chi}{\partial \rho} = -\mu\chi \left[\frac{\gamma\varepsilon(1-\alpha)}{(1-\alpha)(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{\varepsilon-1}{1-\rho+\rho\varepsilon} \right] \tag{A9}$$

Therefore, the sign of this derivative depends on the following condition:

$$\begin{aligned} \frac{\partial \chi}{\partial \rho} > 0 \quad &\Leftrightarrow \quad \left[\frac{\gamma\varepsilon(1-\alpha)}{(1-\alpha)(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{\varepsilon-1}{1-\rho+\rho\varepsilon} \right] > 0 \\ \frac{\partial \chi}{\partial \rho} > 0 \quad &\Leftrightarrow \quad \frac{\varepsilon}{(\varepsilon-1)} > \frac{(1-\rho)[\alpha(1-\rho)+\lambda_h(1-\alpha)]}{\Lambda_h} \end{aligned}$$

The derivative of χ with respect to the net gain from migration is as follows:

$$\begin{aligned} \frac{\partial \chi}{\partial \varepsilon} = \mu\rho &\left[\frac{\beta A\alpha^{\alpha}(1-\tau)(1-\rho)(1-\gamma)(1-\alpha)^{2-\alpha}}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} \right]^{\frac{-\mu}{(1-\alpha)}} (\gamma\Lambda_h)^{-\mu} \\ \times &\left[\frac{\gamma}{\alpha(1-\rho)+\Lambda_h(1-\alpha)} - \frac{1}{1-\rho+\rho\varepsilon} \right] \end{aligned} \tag{A10}$$

The sign of this derivative depends on the second term of this product:

$$\begin{aligned} \frac{\partial \chi}{\partial \varepsilon} \quad &\Leftrightarrow \quad \frac{\gamma}{\alpha(1-\rho)+\Lambda_h(1-\alpha)} - \frac{1}{1-\rho+\rho\varepsilon} > 0 \\ &\Leftrightarrow \quad \Lambda_h > (1-\rho) \end{aligned}$$

This can be rewritten to keep only ε on the left side. The larger γ and ρ are, the lower the threshold to have a positive impact from an increase in ε.

$$\varepsilon > \frac{(1-\rho)(1-\gamma)}{\gamma\rho}$$

The derivative of χ with respect to the intergenerational transfer rate is as follows:

$$\frac{\partial \chi}{\partial \gamma} = \mu\chi \left[\frac{\alpha(1-\rho) + (1-\alpha)(1-\rho+\rho\varepsilon)}{(1-\alpha)(1-\gamma)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} - \frac{1}{\gamma} \right] \tag{A11}$$

The sign of this derivative depends on the following condition, which is quite difficult to interpret:

$$\begin{aligned} \frac{\partial \chi}{\partial \gamma} > 0 \quad &\Leftrightarrow \quad \left[\frac{\alpha(1-\rho) + (1-\alpha)(1-\rho+\rho\varepsilon)}{(1-\alpha)(1-\gamma)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} - \frac{1}{\gamma} \right] > 0 \\ \frac{\partial \chi}{\partial \gamma} > 0 \quad &\Leftrightarrow \quad \frac{(1-\alpha)(1-\gamma)}{\gamma} < \frac{\alpha(1-\rho) + (1-\alpha)(1-\rho+\rho\varepsilon)}{\alpha(1-\rho) + \Lambda_h(1-\alpha)} \end{aligned}$$

Given the effects of the parameters on the steady-state value of the efficiency of human capital accumulation, it is now possible to study the other variables on the steady state. First, $Z^\star$ depends directly on the level attained by the efficiency of human capital accumulation, χ, knowing that $\theta(Z_t)$ and its inverse are decreasing and monotonic functions. The stock of pollution is positively correlated with the parameters A and β and is negatively correlated with τ. The effect of the other parameters depends on the opposite of the conditions given for χ.

D.3.2 The capital stock, $K^\star$

The equation of $K^\star$ is:

$$K^\star = \frac{\beta\alpha(1-\rho)(1-\tau)(1-\gamma)(1-\alpha)}{(1+\beta)\left[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]} \frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi}$$

Without calculations, it appears that $K^\star$ is negatively correlated with pollution emissions Ω and positively correlated with the absorption rate a and the efficiency of the abatement effort, ξ. The derivatives of this equation with respect to β and A are given by equations (A12) and (A13), respectively.

$$\frac{\partial K^\star}{\partial \beta} = K^\star\left[\frac{1}{\beta(1+\beta)} + \frac{\partial \theta^{-1}(\chi)}{\partial \beta}\frac{1}{\theta^{-1}(\chi)}\right] \tag{A12}$$
$$\frac{\partial K^\star}{\partial A} = K^\star\frac{\partial \theta^{-1}(\chi)}{\partial A} \tag{A13}$$

Knowing that the derivatives of $\theta^{-1}(\chi)$ with respect to β and A are positive, both derivatives (A12) and (A13) are positive.

The effects of the other parameters according to the natural capital stock depend on the conditions. First, let us give the derivative of $K^\star$ with respect to the tax on production, τ.

$$\frac{\partial K^\star}{\partial \tau} = K^\star\left[\frac{\partial \theta^{-1}(\chi)}{\partial \tau}\frac{\Omega-\tau\xi}{\theta^{-1}(\chi)} + \frac{\xi-\Omega}{1-\tau}\right] \tag{A14}$$

The elasticity of pollution with respect to the tax rate is denoted by:

$$\zeta_\tau = -\frac{\partial \theta^{-1}(\chi)}{\partial \tau}\frac{\tau}{\theta^{-1}(\chi)} > 0$$

Therefore, the condition to observe an increase in the long-term capital stock is:

$$\frac{\partial K^\star}{\partial \tau} > 0 \Leftrightarrow \zeta_\tau < \frac{\tau(\xi-\Omega)}{(1-\tau)(\Omega-\tau\xi)}$$

Second, the derivatives of $K^\star$ with respect to the emigration rate, ρ, the net gain from migration, ε, and the intergenerational transfer, γ, are given by the following equations:

$$\frac{\partial K^\star}{\partial \rho} = K^\star\left[\frac{\partial \theta^{-1}(\chi)}{\partial \rho}\frac{1}{\theta^{-1}(\chi)} - \frac{\gamma\varepsilon(1-\alpha)}{(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right] \tag{A15}$$
$$\frac{\partial K^\star}{\partial \varepsilon} = K^\star\left[\frac{\partial \theta^{-1}(\chi)}{\partial \varepsilon}\frac{1}{\theta^{-1}(\chi)} - \frac{\gamma\rho(1-\alpha)}{[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right] \tag{A16}$$
$$\frac{\partial K^\star}{\partial \gamma} = K^\star\left[\frac{\partial \theta^{-1}(\chi)}{\partial \gamma}\frac{1}{\theta^{-1}(\chi)} - \frac{(1-\rho+\rho\varepsilon)-\alpha\rho\varepsilon)}{(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right] \tag{A17}$$

The sign of these derivatives depends on the term within brackets. The elasticities of the pollution stock with respect to ρ, ε and γ are denoted by ζ_ρ, ζ_ε and ζ_γ, respectively.

$$\zeta_\rho = \frac{\partial \theta^{-1}(\chi)}{\partial \rho} \frac{\rho}{\theta^{-1}(\chi)}, \qquad \zeta_\varepsilon = \frac{\partial \theta^{-1}(\chi)}{\partial \varepsilon} \frac{\varepsilon}{\theta^{-1}(\chi)}, \qquad \zeta_\gamma = \frac{\partial \theta^{-1}(\chi)}{\partial \gamma} \frac{\gamma}{\theta^{-1}(\chi)}$$

However, note that the signs of ζ_ρ, ζ_ε and ζ_γ are not always positive because the sign of the derivatives of θ^{-1} with respect to these parameters depends on their level. Therefore, their impacts depend on the following conditions:

$$\begin{aligned}
\frac{\partial K^\star}{\partial \rho} > 0 \quad &\Leftrightarrow \quad \zeta_\rho > \frac{\rho}{1-\rho} \frac{\gamma\varepsilon(1-\alpha)}{[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} \\
\frac{\partial K^\star}{\partial \varepsilon} > 0 \quad &\Leftrightarrow \quad \zeta_\varepsilon > \frac{\varepsilon\gamma\rho(1-\alpha)}{\alpha(1-\rho) + \Lambda_h(1-\alpha)} \\
\frac{\partial K^\star}{\partial \gamma} > 0 \quad &\Leftrightarrow \quad \zeta_\gamma > \frac{\gamma(1-\rho+\rho\varepsilon) - \gamma\alpha\rho\varepsilon}{(1-\gamma)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]}
\end{aligned}$$

D.3.3 Human capital, $h^\star$

Finally, the impact of the parameters on steady-state human capital (given below) is studied:

$$\begin{aligned}
h^\star = &\frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi} [\alpha A(1-\tau)]^{-\frac{\alpha}{1-\alpha}} \left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}\right]^{\frac{1}{\delta}} \\
&\times \left[\frac{\beta(1-\gamma)(1-\alpha)(1-\rho)}{(1+\beta)\,[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}\right]^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}}
\end{aligned}$$

Without calculations, it appears that $h^\star$ is negatively correlated with pollution emissions Ω and positively correlated with the absorption rate a and the cost of rearing children. Following the same method as for the capital stock, the derivatives of $h^\star$ with respect to A (equation (A18)), β (equation (A19)), τ (equation (A20)), ρ (equation (A21)),

ε (equation (A22)) and γ (equation (A23)) are given by:

$$\begin{aligned}
\frac{\partial h^\star}{\partial A} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial A}\right] \frac{1}{\theta^{-1}(\chi)} - \frac{1}{A(1-\alpha)}] && \text{(A18)}\\
\frac{\partial h^\star}{\partial \beta} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \beta} \frac{1}{\theta^{-1}(\chi)} - \frac{1-\alpha(1-\delta)}{\delta\beta(1-\alpha)(1+\beta)}\right] && \text{(A19}\\
\frac{\partial h^\star}{\partial \tau} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \tau} \frac{1}{\theta^{-1}(\chi)} + \frac{\xi(1-\tau)-\alpha(\xi-\Omega)}{(1-\alpha)(1-\tau)(\omega-\xi\tau)}\right] && \text{(A20)}\\
\frac{\partial h^\star}{\partial \rho} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \rho} \frac{1}{\theta^{-1}(\chi)} - \left[\frac{\varepsilon-1}{\delta(1-\rho+\rho\varepsilon)} - \frac{\gamma\varepsilon(1-\alpha(1-\delta))}{\delta(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\right] && \text{(A21)}\\
\frac{\partial h^\star}{\partial \varepsilon} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \varepsilon} \frac{1}{\theta^{-1}(\chi)} - \left[\frac{\alpha\rho[(1-\rho)+\Lambda_h\delta]}{\delta(1-\rho+\rho\varepsilon)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\right] && \text{(A22)}\\
\frac{\partial h^\star}{\partial \gamma} &= h^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \gamma} \frac{1}{\theta^{-1}(\chi)} + \left[\frac{(1-\alpha(1-\delta))[(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)]}{\delta(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{1}{\delta\gamma}\right]\right] && \text{(A23)}
\end{aligned}$$

The signs of these derivatives depend on the terms within brackets, where ζ_A and ζ_β denote the elasticities of the pollution with respect to ρ, ε and γ, respectively.

$$\zeta_A = \frac{\partial \theta^{-1}(\chi)}{\partial A} \frac{A}{\theta^{-1}(\chi)}, \qquad \zeta_\beta = \frac{\partial \theta^{-1}(\chi)}{\partial \beta} \frac{\beta}{\theta^{-1}(\chi)}$$

In the SS, the human capital level $h^\star$ is negatively correlated with pollution emissions Ω. It is positively impacted by a and A. Under the following conditions, $h^\star$ is positively correlated with the other parameters:

$$\begin{aligned}
&\frac{\partial h^\star}{\partial A} > 0 \quad \Leftrightarrow \quad \zeta_A > \frac{1}{1-\alpha}\\
&\frac{\partial h^\star}{\partial \beta} > 0 \quad \Leftrightarrow \quad \zeta_\beta > \frac{1-\alpha(1-\delta)}{\delta(1-\alpha)(1+\beta)}\\
&\frac{\partial h^\star}{\partial \tau} > 0 \quad \Leftrightarrow \quad \zeta_\tau < \tau \left[\frac{\xi(1-\tau)-\alpha(1-\Omega)}{(1-\tau)(\xi-\alpha)(\Omega-\tau\xi)}\right]\\
&\frac{\partial h^\star}{\partial \rho} > 0 \quad \Leftrightarrow \quad \zeta_\rho > \left[\frac{\rho(\varepsilon-1)}{\delta(1-\rho+\rho\varepsilon)} - \frac{\rho\gamma\varepsilon(1-\alpha(1-\delta))}{\delta(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\\
&\frac{\partial h^\star}{\partial \varepsilon} > 0 \quad \Leftrightarrow \quad \zeta_\varepsilon > \left[\frac{\alpha\rho[(1-\rho)+\Lambda_h\delta]}{\delta(1-\rho+\rho\varepsilon)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\\
&\frac{\partial h^\star}{\partial \gamma} > 0 \quad \Leftrightarrow \quad \zeta_\gamma > \left[\frac{1}{\delta} - \frac{\gamma(1-\alpha(1-\delta))[(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)]}{\delta(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]
\end{aligned}$$

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