Cross-sectoral pollution externalities and multiple equilibria

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ONLINE APPENDIX

A general model

This appendix provides graphical intuition for the formal result, shown in the main text, that the "likelihood" of multiplicity of equilibria is non-monotonically related to the friction associated with a mobile factor. The analysis in the text may leave the reader with the impression that the conclusion requires a very special setting, and therefore is not robust. In addition, the analysis of the simple model requires some tedious calculation, which obscures intuition. To offset these disadvantages, we sketch here a general model that, under mild assumptions, reproduces the non-monotonicity result shown formally for the special model.

In the interests of brevity, we do not describe all of the assumptions that lead to the model presented here, or all of its implications. However, it is worth pointing out that here we assume that *the steady states are interior and are approached asymptotically.* These assumptions are easily satisfied, although neither the continuous time linear model in Karp and Paul (2007) nor the discrete time model in the text satisfy them.

There are two state variables: L_t is the fraction of labor in Manufacturing; K_t, the pollution stock generated by production in Manufacturing. The Manufacturing-Agriculture wage differential, $\omega(K, L)$, is an increasing function of K, because more pollution lowers productivity in Agriculture, and a weakly decreasing function of L, thus incorporating decreasing returns to scale in either or both sectors. Denote Ω_t as the trajectory over $(t, \infty]$ of $\omega_\tau \equiv \omega(K_\tau, L_\tau)$ and denote $\vec{0}_t$ as the trajectory where $\omega(K_\tau, L_\tau) = 0$ for $\tau \geq t$. The dynamics of the state variables are given by

$$\dot{L} = \frac{dL}{dt} = \gamma h(\Omega_t), \text{ with } h(\Omega_t) = 0 \text{ iff } \Omega_t = \vec{0}_t, \tag{A1}$$

$$\frac{dK}{dt} = gf(K, L), \tag{A2}$$

with f increasing in L (because more labor in Manufacturing increases the pollution flow) and decreasing in K (to account for natural depreciation). The function $f(\cdot)$ is given exogenously, and $g > 0$.

Equation (A1) implies that agents' intersectoral migration decisions depend on their beliefs about future wage differentials, as captured by the trajectory Ω_t. In a deterministic rational expectations equilibrium, agents' beliefs are correct in equilibrium. The endogenous functional $h(\cdot)$ is determined by the equilibrium condition to agents' problems (unlike the exogenous function f).[1] The parameter $\gamma > 0$ is inversely related to the amount of friction (e.g. the costs of migration). The restriction on $h(\cdot)$ states that migration stops if and only if the future trajectory of the wage differential is identically 0.

At any steady state, $\omega(K, L) = 0 = f(K, L)$. The graph of the curve $\omega(K, L) = 0$ in the (L, K) plane has a positive slope:

$$\frac{dK}{dL}_{\omega=0} = \frac{-\omega_L}{\omega_K} \geq 0.$$

The graph of $f(K, L) = 0$ also has a positive slope:

$$\frac{dK}{dL}_{f=0} = \frac{-f_L}{f_K} \geq 0.$$

Denote the K intercept of the graph of $\omega(K, L) = 0$, i.e. the solution to $\omega(K, 0) = 0$, as K_ω. Denote the K intercept of the graph of $f(K, L) = 0$ as K_f. We assume that $K_f < K_\omega$. This assumption means that at the first intersection of the two graphs (i.e. the intersection with the smallest L coordinate) the graph of $f(K, L) = 0$ intersects the graph of $\omega(K, L) = 0$ from below. Because both graphs of increasing, there are an odd number of steady states. Suppose, for concreteness, that there are exactly three steady states; the intermediate steady state is unstable and the other two are stable. By construction, all of the steady states are independent of the speed of adjustment parameters γ and g. There may or may not be multiple equilibria; that is, the ROM may have positive or 0 measure.

In the limit, as $\gamma \to \infty$ and $g \to \infty$, we obtain a static model for which the two stable steady states of the dynamic model are stable equilibria. In this static model there is a coordination problem (multiple equilibria). The ROM here is (trivially) the entire "state space", because the equilibrium does not depend on initial values of K and L; those initial values have no meaning in the static model.

[1] For example, $h(\cdot)$ may be a function of the present discounted value of the future stream of wage differentials, denoted q_t. Let $p\left(\dot{L}_t\right)$ be the price that an individual pays to migrate at time t. The equilibrium condition is $p\left(\dot{L}_t\right) = q$ or $\dot{L}_t = p^{-1}(q) \equiv \gamma h(q)$. The complete dynamical system of the model consists of equations (A1) and (A2) and $\dot{q} = rq_t - \omega_t$, where r is the constant discount rate.

For finite γ with $g = \infty$ we obtain a model that has many of the same features as the Krugman's one-state model. We adopt

Assumption 1 *For the one-state model (with $g = \infty$), the ROM is non-empty if and only if γ is sufficiently large.*

This assumption can be shown to hold if, for example, the model is closed using the equilibrium condition discussed in footnote 1.

Assumption 2 *The equilibrium correspondence (mapping initial conditions and parameter values into trajectories) is continuous in γ and g for all positive values.*

If Assumption 2 was not satisfied, then the comparative statics question addressed in this paper would be rather artificial. (However, proving that the Assumption holds is non-trivial.) The two assumptions may appear to suggest that the one-state model should provide a good approximation to the two-state model if the omitted state adjusts rapidly. In an important respect, however, the one-state model can be misleading precisely when the omitted state adjusts rapidly. In order to understand why, consider two limiting cases, in each of which the state is one-dimensional.

> Case i) $\gamma = \infty$ and $g < \infty$, so that the single state variable is K. In this case, unless K begins at the unstable steady state, all labor moves immediately to the high wage sector and the system then moves toward one of the two steady states. The equilibrium is unique; here the measure of the ROM is 0.
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> Case ii) $g = \infty$ and $\gamma < \infty$, so that the single state variable is L. In this case, by Assumption 1, the ROM has positive measure if and only if γ is sufficiently large.

The more interesting case occurs where g is large but finite and $\gamma < \infty$. If γ is large, the ROM has positive measure, by virtue of the two Assumptions. For large γ it is difficult for agents to predict what other agents will do in the future, because migration is cheap; this inability is important because the wage differential adjusts quickly to migration (g is large). Therefore the measure of the ROM is positive. However, as γ approaches ∞, we move toward Case i, where the measure of the ROM is 0. Given Assumption 2, the measure of the ROM must be decreasing in γ for γ large. For small γ, migration is slow in any equilibrium, so the value of being in a particular sector depends mostly on the predetermined variable K. For sufficiently small γ, expectations have negligible effect on the equilibrium, so the measure of the ROM is 0. For this model, and for $g < \infty$ but large, the measure of the ROM is therefore non-monotonic in γ.

It is worth emphasizing that this non-monotonicity arises in the situation where the state variable K adjusts quickly, precisely the situation where it might seem that little insight is lost by treating it as adjusting instantaneously.

References

Karp, L and Paul T (2007) Indeterminacy with environmental and labor dynamics. *Structural Change and Economic Dynamics* **18**(1), 100–119.