The impact of green preferences on the relevance of history versus expectations

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ONLINE APPENDIX

Mathematical Appendix

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Equilibrium

Without loss of generality we impose the standard assumptions of a symmetric equilibrium in the intermediate sector, such that $c_{x_j} = c_{x_j}(\omega_j)$, $p_{x_j} = p_{x_j}(\omega_j)$, and $x_j = x_j(\omega_j)\ \forall\ \omega_j$, while average energy productivity is B_j and average fixed costs ϕ_j. Free entry drives profits down to zero, such that $p_{x_j} x_j = c_{x_j}(x_j + \phi_j)$. Noting that $p_{x_j} = m_j\ c_{x_j}$, we obtain

$$x_{j,t} = \frac{\phi_j}{m_{j,t} - 1}. \tag{A1}$$

Denoting further aggregate quantities of a variable by upper-case letters, the available number of services compatible to technology j is obtained as

$$N_{j,t} = \frac{(m_{j,t} - 1)\, B_j E_{j,t}^{\beta} L_j^{1-\beta}}{\phi_j\ m_{j,t}}, \tag{A2}$$

such that $\mathcal{F}_{j,t}$ reads as[1]

$$\mathcal{F}_{j,t} = (K_{j,t})^{\alpha} \cdot \left(\frac{B_j E_{j,t}^{\beta} L_j^{1-\beta}}{m_{j,t}} \right)^{1-\alpha}. \tag{A3}$$

According to (A3), the output level grows with total factor productivity in the production of energy services and the inputs capital, energy, and labor, while a higher markup decreases final output. We normalize aggregate labor supply to unity, such that

$$L_d = 1, \qquad \text{if } j = d, \tag{A4}$$

$$L_c + L_{E,c} = 1, \qquad \text{if } j = c. \tag{A5}$$

In regime (d) profit maximizing behavior implies $p_{E_{d,t}} = \frac{\partial Y_{d,t}}{\partial E_{d,t}}$, such that aggregate demand for fossils is obtained as

$$E_{d,t} = \left(\frac{\psi}{(\tau_t)^z} \right)^{-\frac{\gamma}{\tilde{\gamma}}} \left(\frac{\tilde{\beta}}{\bar{p}_{Ed} + \tau_t} \right)^{\frac{1}{\tilde{\gamma}}} \left(\frac{B_d}{m_{d,t}} \right)^{\frac{1-\alpha}{\tilde{\gamma}}} \left(K_{d,t} \right)^{\frac{\alpha}{\tilde{\gamma}}} \tag{A6}$$

with $\tilde{\beta} = (1-\alpha)\beta$ and $\tilde{\gamma} = 1 - \tilde{\beta} + \gamma$.
In regime c, aggregate energy supply is produced domestically according to (13) in the main text. The labor market equilibrium implies

$$L_{E,c} = \beta \tag{A7}$$

$$L_c = 1 - \beta. \tag{A8}$$

[1] In order to ease the notation, we modified the production function of final output slightly, in the sense that now $\mathcal{F}_j = (K_j)^{\alpha} \left[N_j^{1-m_j} \left(\int_0^{N_j} x_j(\omega_j)^{\frac{1}{m_j}} d\omega_j \right)^{m_j} \right]^{1-\alpha} = (K_j)^{\alpha} \left[N_j^{1-m_j} N_j^{m_j} x_j \right]^{1-\alpha} = (K_j)^{\alpha} (N_j x_j)^{1-\alpha}$, which is a standard procedure in literature, see Jaimovich (2007).

The preferred tax rate

The objective function of the representative household reads as

$$V_{d,t} = (1+\rho)\ln(w_{d,t}) + \varepsilon \ln(Q_{d,t+1}) + \tilde{\rho}. \tag{A9}$$

Hence,

$$\frac{\partial V_{d,t}}{\partial \tau_t} = \frac{(1+\rho)}{w_{d,t}} \frac{\partial w_{d,t}}{\partial \tau_t} + \frac{\varepsilon}{Q_{t+1}} \frac{\partial Q_{t+1}}{\partial \tau_t}. \tag{A10}$$

(i) **No abatement ($z = 0$)**

a. $\delta_Q = 1$: The first order condition of the representative household reads

$$\frac{\partial V_{d,t}}{\partial \tau_t} = -\frac{(1+\rho)(\tilde{\beta}-\gamma)}{\tilde{\gamma}} + \frac{\kappa\psi\varepsilon\left(\frac{\psi^{-\gamma}\tilde{\beta}}{\bar{p}_E+\tau_t}\right)^{\frac{1}{\tilde{\gamma}}}\left(\frac{B}{m_t}\right)^{\frac{1-\alpha}{\tilde{\gamma}}}\left(K_{d,t}\right)^{\frac{\alpha}{\tilde{\gamma}}}}{\bar{Q} - \kappa\psi\left(\frac{\psi^{-\gamma}\tilde{\beta}}{\bar{p}_E+\tau_t}\right)^{\frac{1}{\tilde{\gamma}}}\left(\frac{B}{m_t}\right)^{\frac{1-\alpha}{\tilde{\gamma}}}\left(K_{d,t}\right)^{\frac{\alpha}{\tilde{\gamma}}}} = 0, \tag{A11}$$

such that the preferred tax rate is obtained as

$$\tau_t = \tilde{\beta}\psi^{-\gamma}\left(\frac{B}{m_t}\right)^{1-\alpha}\left(K_{d,t}\right)^{\alpha}\left[\frac{\kappa\psi[(\tilde{\beta}-\gamma)(1+\rho)+\varepsilon]}{\bar{Q}(1+\rho)(\tilde{\beta}-\gamma)}\right]^{\tilde{\gamma}} - \bar{p}_E, \tag{A12}$$

where $\tilde{\gamma} = 1 - \tilde{\beta} + \gamma$.

Hence: $\frac{\partial \tau_t}{\partial K_t}, \frac{\partial \tau_t}{\partial \varepsilon} > 0$.

b. $0 < \delta_Q < 1$: In this case $Q_{t+1} = \delta_Q \bar{Q} + (1-\delta)Q_t - P_t$ and

$$\begin{aligned}\tau_t = {} & \tilde{\beta}\psi^{-\gamma}\left(\frac{B}{m_t}\right)^{1-\alpha}\left(K_{d,t}\right)^{\alpha}\left[\frac{\kappa\psi[(\tilde{\beta}-\gamma)(1+\rho)+\varepsilon]}{[\delta_Q\bar{Q}+(1-\delta)Q_t](1+\rho)(\tilde{\beta}-\gamma)}\right]^{\tilde{\gamma}} \\ & -\bar{p}_E.\end{aligned} \tag{A13}$$

Since $\frac{\partial Q_{t+1}}{\partial \delta_Q} = \bar{Q} - Q_t > 0$ it follows that the denominator in the first line of the above equation increases such that $\frac{\partial \tau_t}{\partial \delta_Q} < 0$.

c. $\kappa_g > 0$ and $0 < \delta_Q < 1$: In this case $Q_{t+1} = \delta_Q \bar{Q}_t + (1-\delta)Q_t - P_t$, where $\bar{Q}_t = \bar{Q} - \frac{\kappa_g[S_{t+1}^{global}+S_t^{global}]}{\delta_Q} - \kappa_g S_t^{global}$ and

$$\tau_t = \tilde{\beta}\psi^{-\gamma}\left(\frac{B}{m_t}\right)^{1-\alpha}\left(K_{d,t}\right)^{\alpha}\left[\frac{\kappa\psi[(\tilde{\beta}-\gamma)(1+\rho)+\varepsilon]}{\bar{Q}_t(1+\rho)(\tilde{\beta}-\gamma)}\right]^{\tilde{\gamma}} - p^{-}{}_E. \tag{A14}$$

Higher exposure to global pollutants is reflected by an increase in κ_g. As $\frac{\partial \bar{Q}_t}{\partial \kappa_g} < 0$ it follows that $\frac{\partial \tau_t}{\partial \kappa_g} > 0$.

(ii) **With abatement** ($z \in (0,1)$)

Summary of the reasoning

Consider now the presence of abatement measures (M_t) financed by state. The level of pollutants is determined by

$$P_t = \begin{cases} \frac{\psi E_{d,t}}{M_t} = \frac{\psi E_{d,t}^{1-z}}{\tau_t^z}, \; z \in [0,1) & \text{if} \quad E_{d,t} > E_d^{crit} \\ \tilde{\psi}, & \text{if} \quad 0 \leq E_{d,t} \leq E_d^{crit} \text{ or } j = c. \end{cases} \tag{A15}$$

z reflects the productivity of abatement measures. If $z = 0$, the abatement technology is inactive, the case considered so far, and $z < 1$ excludes perfect abatement. Similarly, environmental quality is specified as follows

$$Q_{t+1} = \begin{cases} \bar{Q} - \kappa\psi \frac{E_{d,t}^{1-z}}{\tau_t^z}, \quad \text{with} \;\; z \in [0,1), & \text{if} \quad j = d, \\ \bar{Q} - \kappa\tilde{\psi} = Q^{max} & \text{if} \quad 0 \leq E_{d,t} \leq E_d^{crit} \text{ or } j = c. \end{cases} \tag{A16}$$

For $z \in (0,1)$, the equilibrium of the economy changes only in so far as $\tilde{\gamma}$ reads now

$$\tilde{\gamma} = 1 - \tilde{\beta} + \gamma(1-z). \tag{A17}$$

Moreover, $\tilde{A}_d$ is obtained as

$$\tilde{A}_{d,t} = \left(\frac{\tilde{\beta}}{\bar{p}_{Ed} + \tau_t}\right)^{\frac{\tilde{\beta}-\gamma(1-z)}{\tilde{\gamma}}} \left(\frac{\psi}{\tau_t^z}\right)^{-\frac{\gamma}{\tilde{\gamma}}}. \tag{A18}$$

In regime d, we obtain as the indirect utility function

$$V_{d,t} = (1+\rho)\ln w_{d,t} + \varepsilon \ln\left(\bar{Q} - \kappa\psi \frac{E_{d,t}^{1-z}}{\tau_t^z}\right) + \tilde{\rho}, \tag{A19}$$

while $V_{c,t}$ remains obviously unaffected. The subsequent proposition summarizes the reaction of τ_t in response to changes in the capital stock, shifts in the preferences for environmental quality, and the efficiency of abatement measures.

Proposition 1. *(i) The first-order condition of the government reads*

$$\frac{\partial V_{d,t}}{\partial \tau_t} = \frac{(1+\rho)}{w_{d,t}} \frac{\partial w_{d,t}}{\partial \tilde{A}_{d,t}} \frac{\partial \tilde{A}_{d,t}}{\partial \tau_t} - \frac{\varepsilon[\kappa\psi(1-z)E_{d,t}^{-z}\frac{\partial E_{d,t}}{\partial \tau_t} - E_{d,t}^{1-z} z \tau_t^{-1}]}{[\bar{Q} - \kappa\psi \frac{E_{d,t}^{1-z}}{\tau_t^z}]\tau_t^z} \leq 0, \tag{A20}$$

where $\frac{\partial \tilde{A}_{d,t}}{\partial \tau_t}, \frac{\partial E_{d,t}}{\partial \tau_t} < 0$ *and* $\frac{\partial w_{d,t}}{\partial \tilde{A}_{d,t}} > 0$.

(ii) If $z \in (0,1)$, the energy tax is increasing in the capital stock (K_t) and environmental preferences (ε), i.e. $\frac{\partial \tau_t}{K_t}, \frac{\partial \tau_t}{\varepsilon} > 0$.

So far, the response of the tax rate with respect to changes in the capital stock and environmental preferences is qualitatively the same for $0 < z < 1$ compared to $z = 0$. The second part of this sections deals with changes in τ in response to changes in z. Since z increases $\tilde{A}_d$ it increases the marginal cost of taxation. Clearly, a higher effectiveness of the abatement technology increases also the marginal benefit. The tax rate increases in response to increases in z, if the latter increase exceeds the increase in the marginal costs. Analytical results are only obtainable for $\bar{Q} = 0$, in the sense that $\frac{\partial \tau_t}{\partial z} > 0$. If $\tilde{Q} > 0$, the marginal benefit of taxation depends on the difference between $\bar{Q}$ and the level of pollutants. If environmental quality is sufficiently low, the marginal benefit from taxation is strong enough, such that again $\frac{\partial \tau_t}{\partial z} > 0$. Hence, a higher effectiveness of abatement measures support higher taxes, such that the presence of productive abatement measures works in the same direction as our previous discussion and not as one might think against the reasoning presented in the previous sections.

Table A1. The impact of increasing effectiveness of abatement measures (z) on energy taxes in steady state (τ) and the relative distance of the exterior steady states to the interior steady state under different degrees of environmental preferences (ε).

$\varepsilon = 1$					
	$z = 0.001$	$z = 0.005$	$z = 0.01$	$z = 0.05$	$z = 0.1$
K_*^{low}	0.0955	0.0934	0.0917	0.0814	0.0716
K_*^{high}	2.8583	2.8136	2.76548	2.4738	2.19355
τ_*^{low}	0.0039	0.019	0.0482	0.2001	0.40163
τ_*^{high}	0.4892	0.5412	0.6001	0.9551	1.30289
$\varepsilon = 0.75$					
	$z = 0.001$	$z = 0.005$	$z = 0.01$	$z = 0.05$	$z = 0.1$
K_*^{low}	0.09565	0.0935	0.0918	0.0816	0.0718
K_*^{high}	2.9175	2.8642	2.8108	2.5074	2.2226
τ_*^{low}	0.0039	0.019	0.03932	0.1970	0.3950
τ_*^{high}	0.1729	0.2523	0.3236	0.6884	1.02341

Table A1 presents the numerical results for the impact of an increase in the effectiveness of abatement measures z on the energy tax and the relative distance of the inferior and the superior steady states to the interior steady state. Obviously, an increase in z increases the marginal benefit of taxation by more than it raises the costs, such that τ increases. This reduces $\tilde{A}_d$ and therefore the relative distances of the exterior steady states to the interior steady state. Hence an increase in z does not change the results of our paper but reenforces the mechanisms described here.

Further details of the reasoning

For $z \in (0, 1)$ closed form solutions for τ_t are not obtainable. We thus make use of the Implicit function theorem. Note, that

$$\frac{\partial w_{d,t}}{\partial \tau_t} = (1-\alpha)(1-\beta)\Big(\frac{B}{m_t}\Big)^{\frac{1-\alpha}{\tilde{\gamma}}}\Big(K_{d,t}\Big)^{\frac{\alpha}{\tilde{\gamma}}}\frac{\partial \tilde{A}_{d,t}}{\partial \tau_t} \tag{A21}$$

$$\frac{\partial Q_{t+1}}{\partial \tau_t} = -\kappa\psi\frac{(1-z)E_{d,t}^{-z}\frac{\partial E_{d,t}}{\partial \tau_t} - z\Big(E_{d,t}\Big)^{1-z}\tau_t^{z-1}}{(\tau_t)^z}. \tag{A22}$$

An interior solution exists, if $\frac{\partial w_{d,t}}{\partial \tau_t} < 0$ and $\frac{\partial Q_{d,t}}{\partial \tau_t} > 0$. $\frac{\partial w_{d,t}}{\partial \tau_t} < 0$ requires that $\frac{\partial \tilde{A}_{d,t}}{\partial \tau_t} < 0$ which is the case if

$$(\gamma - \tilde{\beta})\tau_t + \gamma z \bar{p}_E < 0. \tag{A23}$$

$\frac{\partial Q_{d,t}}{\partial \tau_t} > 0$ if $\frac{\partial E_{d,t}}{\partial \tau_t} < 0$ which is the case if

$$(\gamma z - 1)\tau_t + \gamma z \bar{p}_E < 0. \tag{A24}$$

It can be shown that the violation of (A23) implies a violation of (A24).[2] In light of the Implicit function theorem it follows that

$$\frac{\partial \tau_t}{\partial K_t} = -\frac{\frac{\partial F}{\partial K_t}}{\frac{\partial F}{\partial \tau_t}}, \tag{A25}$$

with $F = \frac{\partial V_{d,t}}{\partial \tau_t}$. A maximum of $V_{d,t}$ requires that $\frac{\partial F}{\partial \tau_t} = \frac{\partial^2 V_{d,t}}{\partial (\tau_t)^2} < 0$, such that $sign\frac{\partial \tau_t}{\partial K_t} = sign\frac{\partial F}{\partial K_t}$. As $\frac{\partial F}{\partial K_t} > 0$ if (A24) holds, we obtain $\frac{\partial \tau_t}{\partial K_t} > 0$.
As regards the reaction of the preferred tax rate in response to a change in environmental preferences (ε), we obtain similarly

$$\frac{\partial \tau_t}{\partial \varepsilon} = -\frac{\frac{\partial F}{\partial \varepsilon}}{\frac{\partial F}{\partial \tau_t}}, \tag{A26}$$

such that $sign\frac{\partial \tau_t}{\partial \varepsilon} = sign\frac{\partial F}{\partial \varepsilon}$. Since, $\frac{\partial F}{\partial \varepsilon}$ is obtained as

$$\frac{\partial F}{\partial \varepsilon} = \frac{-\psi\kappa[(\gamma z - 1)\tau_t + \gamma z \bar{p}_E]E_{d,t}}{(\bar{p}_E + \tau_t)(\bar{Q} - \kappa\psi E_{d,t})\tilde{\gamma}\tau_t} \tag{A27}$$

and $(\gamma z - 1)\tau_t + \gamma z \bar{p}_E$ is in light of (A24) smaller than zero, it follows that $\frac{\partial F}{\partial \varepsilon} > 0$ and $\frac{\partial \tau_t}{\partial \varepsilon} > 0$.

[2] Assuming that $(\gamma z - 1)\tau_t + \gamma z \bar{p}_E = 0$ implies $\gamma z \bar{p}_E = \tau_t(1 - \gamma z)$. Thus, $(\gamma - \tilde{\beta})\tau_t + \gamma z \bar{p}_E = (\gamma - \tilde{\beta})\tau_t + \tau_t(1 - \gamma z) = \tau_t(1 - \tilde{\beta} + \gamma(1 - z)) = \tau_t\tilde{\gamma}$ which is positive and not negative, such that (A23) holds only if (A24) holds and vice versa.

The sign of $\frac{\partial \tau_t}{\partial z}$ equals the sign of $\frac{\partial F}{\partial z}$. Observing (A10), we obtain

$$\frac{\partial F}{\partial z} = -\frac{(1+\rho)}{(w_{d,t})^2} \underbrace{\frac{\partial w_{d,t}}{\partial z}}_{>0} \underbrace{\frac{\partial w_{d,t}}{\partial \tau_t}}_{<0} + \frac{(1+\rho)}{w_{d,t}} \frac{\partial w_{d,t}}{\partial \tau_t \partial z} - \frac{\varepsilon}{(Q_{t+1})^2} \underbrace{\frac{\partial Q_{d,t}}{\partial z}}_{>0} \underbrace{\frac{\partial Q_{d,t}}{\partial \tau}}_{>0} + \frac{\varepsilon}{Q_{t+1}} \underbrace{\frac{\partial Q_{d,t}}{\partial \tau_t \partial z}}_{>0}. \quad \text{(A28)}$$

Analytical results are obtainable for $\bar{Q} = 0$. In this case, the indirect utility function of the representative household is reads as

$$V_{d,t} = (1+\rho)\ln(w_{d,t}) - \varepsilon \ln(P_t) + \tilde{\rho}. \quad \text{(A29)}$$

$$\frac{\partial (1+\rho)\ln(w_{d,t})}{\partial \tau_t \partial z} = \frac{\gamma\left((\bar{p}_E + \tau_t)\gamma + \left(-\tilde{\beta}+1\right)\bar{p}_E - \tilde{\beta}\tau_t\right)(1+\rho)}{(\bar{p}_E + \tau_t)\left(\tilde{\beta} - 1 + (z-1)\gamma\right)^2 \tau_t} > 0 \quad \text{(A30)}$$

$$\frac{\partial - \varepsilon \ln(P_t)}{\partial \tau_t \partial z} = \frac{(1-\tilde{\beta})[(1+\gamma-\tilde{\beta})\bar{p}_E + (\gamma - \tilde{\beta})\tau_t]}{(\bar{p}_E + \tau_t)\left(\tilde{\beta} - 1 + (z-1)\gamma\right)^2 \tau_t} > 0, \quad \text{(A31)}$$

such that $\frac{\partial F}{\partial z} > 0$ for $\bar{Q} = 0$ and thus $\frac{\partial \tau_t}{\partial z} > 0$. From (28), we see that $\frac{\partial (1+\rho)\ln(w_{d,t})}{\partial \tau_t \partial z}$ is also positive. However, as $\bar{Q} > 0$, the marginal benefit is weighted by the environmental quality and increasing if the quality is sufficiently low. Analytically, the sign of $\frac{\partial F}{\partial z}$ is therefore ambiguous. However, a declining environmental quality during the transition to the steady state makes the emergence of $\frac{\partial F}{\partial z} > 0$ and thus $\frac{\partial \tau_t}{\partial z} > 0$ likely.

Multiple steady states

The necessary condition for the emergence of multiple steady states hinges on the compensating effect of $m_j(K_j)$ on the marginal product of capital.

(i) Exogenous mark-ups: if m_j is constant and independent from K_j, there exists a unique saddle-point stable steady state.

(ii) Endogenous mark-ups: if $m_j = m_j(K_j)$, and $\kappa > 1$

(a) $lim_{K \to 0} \frac{\partial Y_{c,t+1}}{\partial K_{c,t+1}} = \infty$ and $lim_{K \to \infty} \frac{\partial Y_{c,t+1}}{\partial K_{c,t+1}} = 0$.

(b) The necessary condition for multiple steady states in regime (d) is

$$\frac{\partial \tilde{A}_d}{\partial K_d} - \tilde{A}_d(1-\alpha)\tilde{\gamma}\frac{\partial m_d}{\partial K_d} m_d^{-1} + (\alpha\tilde{\gamma} - 1)\tilde{A}_d K_d^{-1} = 0, \quad \text{(A32)}$$

with $\tilde{\gamma} = \frac{1-\alpha\beta}{1+\beta+\gamma(1-z)}$ and

$$-\tilde{A}_c \frac{\partial m_c}{\partial K_c} m_c^{-1} - \tilde{A}_c K_c^{-1} = 0, \tag{A33}$$

if $j = c$.

Robustness

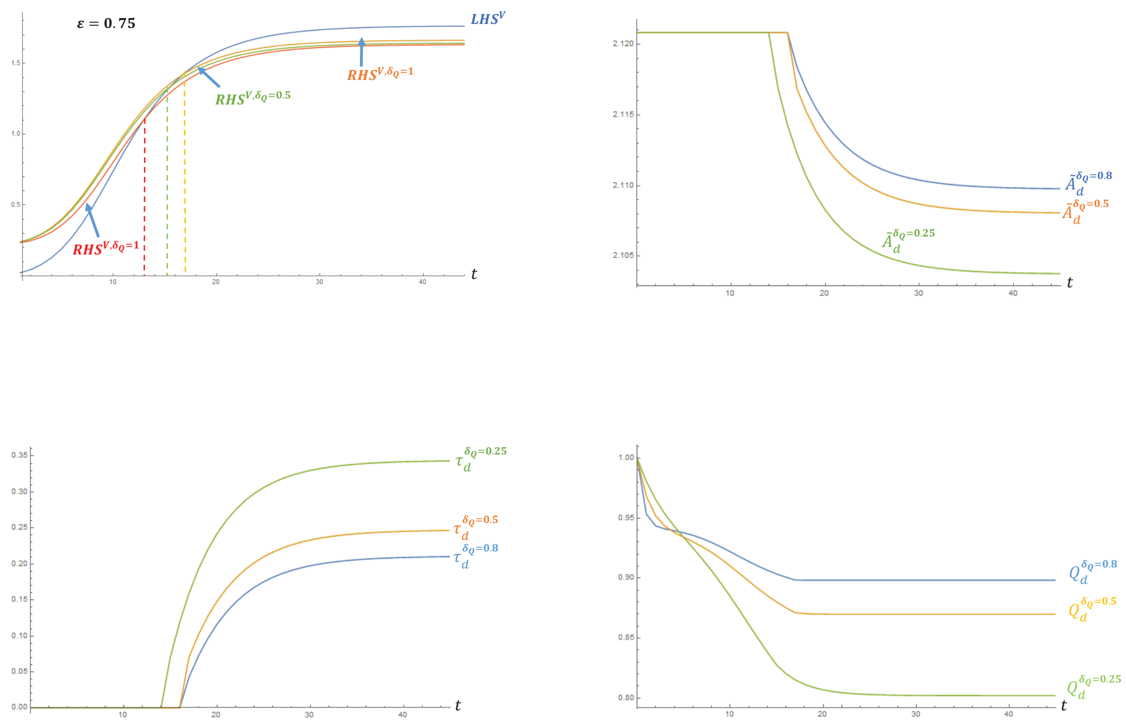

Figure A1. Regime switch to the green energy regime, transition paths for TFP ($\tilde{A}_d$), the energy tax (τ_d) and environmental quality (Q_d) for different regeneration capacities of the environment (δ_Q); ε =0.75.

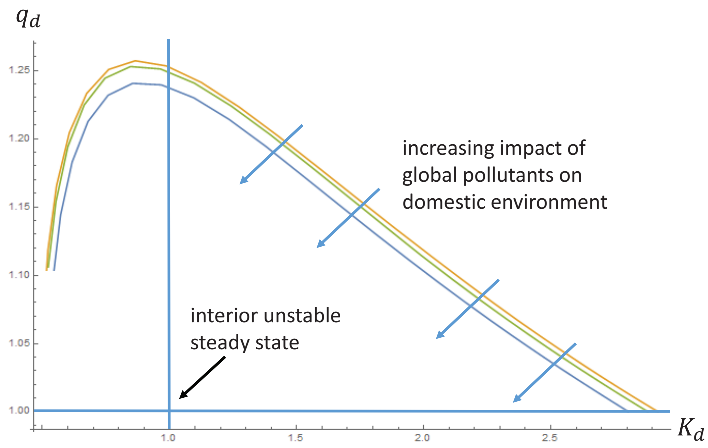

Figure A2. Declining role of expectations under increasing exposure to global climate risks (δ_Q =0.8). Global pollutants increase by 50% over the transition period, κ_g ={0; 0.1; 0.25}, ε =0.75.

Stability properties for $\delta_Q = 1$ and $0 < \delta_Q < 1$

(i) $\delta_Q = 1$: The dynamic system is given by (24) and (25) in the main text. The associated Jacobian reads as

$$\begin{bmatrix} \frac{\partial q_{j,t+1}}{\partial q_{j,t}} & \frac{\partial q_{j,t+1}}{\partial K_{j,t}} \\ \frac{\partial K_{j,t+1}}{\partial q_{j,t}} & \frac{\partial K_{j,t+1}}{\partial K_{j,t}} \end{bmatrix}, \tag{A34}$$

where the first line is obtained from the Implicit function theorem. It can be further shown that in steady state

$$\frac{\partial K_{j,t+1}}{\partial q_{j,t}} = \frac{\delta^{1-\eta} K_*}{\eta(1+\eta)\theta} > 0 \tag{A35}$$

$$\frac{\partial K_{j,t+1}}{\partial K_{j,t}} = 1. \tag{A36}$$

The eigenvalues are therefore obtained from

$$\left(\frac{\partial q_{j,t+1}}{\partial q_{j,t}} - \lambda\right)(1-\lambda) - \frac{\partial K_{j,t+1}}{\partial q_{j,t}} \frac{\partial q_{j,t+1}}{\partial K_{j,t}} = 0 \tag{A37}$$

and read as

$$\lambda_{1,2} = \frac{\frac{\partial q_{j,t+1}}{\partial q_{j,t}} + 1}{2} \pm \sqrt{\frac{[\frac{\partial q_{j,t+1}}{\partial q_{j,t}} + 1]^2}{4} + \frac{\partial K_{j,t+1}}{\partial q_{j,t}} \frac{\partial q_{j,t+1}}{\partial K_{j,t}} - \frac{\partial q_{j,t+1}}{\partial q_{j,t}}} \tag{A38}$$

(ii) $0 < \delta_Q < 1$: The dynamic system is given by (24), (25) and (17) in the main text, such that the associated Jacobian reads as

$$\begin{bmatrix} \frac{\partial q_{j,t+1}}{\partial q_{j,t}} & \frac{\partial q_{j,t+1}}{\partial K_{j,t}} & 0 \\ \frac{\partial K_{j,t+1}}{\partial q_{j,t}} & \frac{\partial K_{j,t+1}}{\partial K_{j,t}} & 0 \\ 0 & \frac{\partial Q_{j,t+1}}{\partial q_{j,t}} & \frac{\partial Q_{j,t+1}}{\partial Q_{j,t}} \end{bmatrix}. \tag{A39}$$

In this case the eigenvalues are obtained from

$$\Big(\frac{\partial q_{j,t+1}}{\partial q_{j,t}} - \lambda\Big)\Big[(1-\lambda)\Big(\frac{\partial Q_{j,t+1}}{\partial Q_{j,t}} - \lambda\Big)\Big] - \frac{\partial q_{j,t+1}}{\partial K_{j,t}}\frac{\partial K_{j,t+1}}{\partial q_{j,t}}\Big(\frac{\partial Q_{j,t+1}}{\partial Q_{j,t}} - \lambda\Big) = 0 \tag{A40}$$

and are given by (A38) and

$$\lambda_3 = \frac{\partial Q_{j,t+1}}{\partial Q_{j,t}} = (1-\delta_Q) < 1. \tag{A41}$$

Hence, the results of the stability analysis of the two dimensional system translate directly into the three-dimensional case. The emergence of an overlap region hinges on the existence of conjugate complex eigenvalues. Conjugate complex eigenvalues exist if

$$\Big[\frac{\partial q_{j,t+1}}{\partial q_{j,t}} - 1\Big]^2 < -4\frac{\partial K_{j,t+1}}{\partial q_{j,t}}\frac{\partial q_{j,t+1}}{\partial K_{j,t}}. \tag{A42}$$

The left-handside of the above inequality is positive. Moreover, $\frac{\partial K_{j,t+1}}{\partial q_{j,t}} = \frac{\delta^{1-\eta}K_{j,*}}{\eta(1+\eta)\theta}$ is positive. It can be shown (see below) that sign of $\frac{\partial q_{j,t+1}}{\partial K_{j,t}}$ is ambiguous. If $\frac{\partial q_{j,t+1}}{\partial K_{j,t}} > 0$ inequality (A42) is violated. Implying that for the two exterior steady states $\frac{\partial q_{j,t+1}}{\partial K_{j,t}} > 0$. If the interior unstable steady state exhibits complex conjugate eigenvalues it must thus hold that $\frac{\partial q_{j,t+1}}{\partial K_{j,t}} < 0$. $\frac{\partial q_{j,t+1}}{\partial K_{j,t}}$ can be obtained from applying the Implicit function theorem as

$$\frac{\partial q_{j,t+1}}{\partial K_{j,t}} = \frac{\frac{1}{1-\delta}\frac{\partial^2 Y_{j,t+1}}{\partial K_{j,t+1}^2}\frac{\partial K_{j,t+1}}{\partial K_{j,t}}}{-\frac{\delta}{1-\delta} - 1} \tag{A43}$$

Obviously, $\frac{\partial q_{j,t+1}}{\partial K_{j,t}} <$, if $\frac{\partial^2 Y_{j,t+1}}{\partial K_{j,t+1}^2} > 0$ which shows again that endogenous mark-ups being responsible for the non-monotonous behavior of the marginal product of capital are essential. As moreover, the marginal product of physical capital is increasing in total factor productivity ($\tilde{A}_{j,t}$), the emergence of complex conjugate eigenvalues in deed depends also on the level of $\tilde{A}_{j,t}$ given that $\frac{\partial^2 Y_{j,t+1}}{\partial K_{j,t+1}^2} > 0$.

Summary of parameters and variables

Table A2. Overview of all variables and parameters used in the model

Parameter / Variable	Explanation
Indices	
t	time index
j	regime index; d = dirty, c = clean
Households	
c	consumption
w	wage
s	savings
R, r	interest rate
V	life-time utility
ρ	discount factor for future consumption
ϵ	weight/importance of environmental quality
Environment	
Q	quality of the environment
S	pollution stock
P	pollution flow
δ_Q	regeneration rate of the environment
ψ	emission intensity per unit of energy
$\tilde{\psi}$	minimal level of pollution
κ_g	impact of global pollutant on domestic Q
Production	
Y	final output
$\mathcal{F}$	applied technology (d or c)
γ	adverse effect of pollution on productivity
A	total factor productivity of final good
α	output elasticity (final good)
K	input factor: physical capital
I	capital investment
δ	depreciation rate of capital
η	steers convexity of capital adjustment costs
θ	capital adjustment costs
ω	type of energy service
x	quantity of energy service (ω) used
N	range of energy services
s_j	elasticity of substitution
B	total factor productivity of intermediate good (energy services)
β	output elasticity (intermediate good)
ϕ	fixed costs for production of intermediate good
E, e	input factor: energy
l	input factor: labour
π	intermediates' profits
m	markup
p	price for intermediate good
c_x	marginal production costs intermediate good
Policy	
τ	tax on dirty energy
Ω	switching costs

References

Jaimovich N (2007) Firm Dynamics and Markup Variations: Implications for Sunspot Equilibria and Endogenous Economic Fluctuations. *Journal of Economic Theory* **137**, 300–325.