Altruism, environmental externality and fertility

Makoto Hirazawa[1], Kimiyoshi Kamada[1*] and Takashi Sato[2]

[1] School of Economics, Chukyo University, Nagoya, Japan and [2] Faculty of Economics, Shimonoseki City University, Shimonoseki City, Japan

[*]Corresponding author. Email: kkamada@mecl.chukyo-u.ac.jp

ONLINE APPENDIX

A. The case of consumption externality

In this appendix, we examine the case of consumption externality where the pollution is generated by the consumption.

We assume that the consumption of childcare goods as well as consumption goods causes pollution, so the levels of pollution in periods 0 and 1 are given by

$$\pi_0 = \alpha N(c_0 + n\beta) = \alpha N[(1+r)b_0 - nb_1], \text{ and}$$

$$\pi_1 = \alpha Nnc_1 = \alpha Nn[(1+r)b_1],$$

respectively.

A.1 Competitive equilibrium

Given π_1 as well as π_0, the parents choose the number of children and the level of bequests so as to maximize the parental utility:

$$\underset{b,n}{Max}\, u_0[(1+r)b_0 - n(b_1+\beta)] - V_0(\pi_0) + n\delta(n)[u_1((1+r)b_1) - V_1(\pi_1)].$$

As in the text, we can see that the competitive equilibrium is given as a solution of the following equations:

$$F(k,\ n) \equiv -nu_0'(Ak_0 - n(k+\beta)) + n\delta(n)Au_1'(Ak) = 0,$$

$$G(k,\ n) \equiv -(k+\beta)u_0'(Ak_0 - n(k+\beta)) + [\delta(n) + n\delta'(n)]\left\{u_1(Ak) - V_1(\alpha ANnk)\right\} = 0.$$

A.2 Social optimum

When $\rho = 0$, the central planner chooses the number of children and the level of capital so as to maximize the welfare:

$$\underset{k,n}{Max}\, W = N\left\{u_0(Ak_0 - n(k+\beta)) - V_0(\alpha N(Ak_0 - nk)) + n\delta(n)[u_1(Ak) - V_1(\alpha NnAk)]\right\}.$$

The first-order conditions for the social optimum in this case are given by

$$\bar{F}^S(k,\ n) \equiv -nu_0'(Ak_0 - n(k+\beta)) + \alpha NnV_0'(\alpha N(Ak_0 - nk))$$
$$+ n\delta(n)[Au_1'(Ak) - \alpha ANnV_1'(\alpha ANnk)] = 0,$$

$$\bar{G}^S(k,\ n) \equiv -(k+\beta)u_0'(Ak_0 - n(k+\beta)) + \alpha NkV_0'(\alpha N(Ak_0 - nk))$$
$$+[\delta(n) + n\delta'(n)][u_1(Ak) - V_1(\alpha ANnk)] - n\delta(n)(\alpha ANk)V_1'(\alpha ANnk) = 0.$$

A.3 Comparing the competitive equilibrium to the social optimum

We define the following functions:

$$\bar{F}(k,\ n;\ \mu) \equiv F(k,\ n) + \mu\left\{\alpha NnV_0'(\alpha N(Ak_0 - nk)) - n\delta(n)\alpha ANnV_1'(\alpha ANnk)\right\},$$

$$\bar{G}(k,\ n;\ \mu) \equiv G(k,\ n) + \mu[\alpha NkV_0'(\alpha N(Ak_0 - nk)) - n\delta(n)\alpha ANkV_1'(\alpha NnAk)].$$

Differentiating $\bar{F}(k,\ n;\ \mu) = 0$ and $\bar{G}(k,\ n;\ \mu) = 0$ with respect to k, n and μ yields

$$\begin{pmatrix} \bar{F}_k & \bar{F}_n \\ \bar{G}_k & \bar{G}_n \end{pmatrix}\begin{pmatrix} dk \\ dn \end{pmatrix} = -\begin{pmatrix} \bar{F}_\mu \\ \bar{G}_\mu \end{pmatrix} d\mu, \tag{S1}$$

where

$$\bar{F}_k = n^2u_0'' + n\delta(n)A^2u_1'' + \mu[-(\alpha Nn)^2V_0'' - n\delta(n)(\alpha ANn)^2V_1''] < 0,$$

$$\bar{F}_n = n(k+\beta)u_0'' + n\delta'(n)Au_1'$$
$$+\mu[-(\alpha N)^2nkV_0'' - \left(\delta(n) + n\delta'(n)\right)(\alpha ANn)V_1' - n\delta(n)(\alpha AN)^2nkV_1''] < 0,$$

$$\bar{G}_k = n(k+\beta)u_0'' + n\delta'(n)Au_1' - (\delta(n) + n\delta'(n))(\alpha ANn)V_1'$$
$$+\mu\left[-(\alpha N)^2nkV_0'' - n\delta(n)(\alpha AN)^2nkV_1''\right] < 0$$

$$\bar{G}_n = (k+\beta)^2u_0'' + (2\delta'(n) + n\delta''(n))(u_1 - V_1) - (\delta(n) + n\delta'(n))(\alpha ANk)V_1'$$
$$+\mu\left[-(\alpha Nk)^2nkV_0'' - (\delta(n) + n\delta'(n))(\alpha ANk)V_1' - n\delta(n)(\alpha ANk)^2V_1''\right] < 0,$$

$$\bar{F}_\mu = \alpha NnV_0'(\alpha N(Ak_0 - nk)) - n\delta(n)(\alpha ANn)V_1'(\alpha NAnk), \tag{S2}$$

$$\bar{G}_\mu = \alpha\ N\ k_0 V'(\alpha\ (N\ A - k\ )n)k\ \delta\ (n\ )\alpha(n\ \ A_1')Nk(\ V \tag{S3}$$

Contrary to the case of production externality, the sign of $\bar{F}_\mu$ and $\bar{G}_\mu$ in this case could be positive or negative because changes in inheritance affects not only V_1', but also V_0' through the changes in parents' consumption and hence pollution in period 0 (the latter effect was absent in the case considered in the text). From (S2) and (S3), we have

$$\bar{G}_\mu = \bar{F}_\mu(\ k/\ n) \tag{S4}$$

From (S4), we derive the following *Lemma*.

Lemma: Sign $\bar{F}_\mu =$ Sign $\bar{G}_\mu$

From (S1) and (S4), we have

$$\frac{dn}{d\mu} = \frac{1}{\bar{D}(\mu)}\left[-\bar{G}_\mu \bar{F}_k + \bar{F}_\mu \bar{G}_k\right] = \frac{-F_k}{\bar{D}(\mu)}\left[\bar{G}_\mu - G_k \frac{\bar{F}_\mu}{F_k}\right]. \tag{S5}$$

From $-\bar{F}_\mu / F_k = (\partial k / \partial \mu)_{\mu=0}$, we can rewrite (S5) as

$$\left.\frac{dn}{d\mu}\right|_{\mu=0} = -\frac{F_k(\bar{k}^*,\ \bar{n}^*)}{\bar{D}(\mu)}\left[\bar{G}_\mu^* + G_k(\bar{k}^*,\ \bar{n}^*)\left.\frac{\partial k}{\partial \mu}\right|_{\mu=0}\right] \tag{S6}$$

where $\bar{G}_\mu^* = \alpha N \bar{k}^* V_0'(\alpha N(Ak_0 - \bar{n}^* \bar{k}^*)) - \bar{n}^* \delta(\bar{n}^*)(\alpha A N \bar{k}^*) V_1'(\alpha A N \bar{n}^* \bar{k}^*)$. The sign of (S6) is positive if

$$\bar{G}_\mu^* + G_k(\bar{k}^*,\ \bar{n}^*)\left.\frac{\partial k}{\partial \mu}\right|_{\mu=0} > 0. \tag{S7}$$

The *Lemma* implies that the sign of the interaction effect of n and k ($G_k(\bar{k}^*,\ \bar{n}^*)(\partial k / \partial \mu)\big|_{\mu=0}$) is opposite to that of the environmental externality effect of n ($\bar{G}_\mu^*$). From (S7), we have the following proposition.

Proposition

If (S7) is satisfied, then the number of children in the competitive equilibrium is smaller than that in the social optimum.

When we have $\bar{G}_{\mu} < 0$, the result in the consumption-externality case is basically the same as that in the production-externality case. The interaction effect of n and k is attenuated in this case as $|\bar{F}_{\mu}| < |\hat{F}_{\mu}|$, and from (S4) $|\bar{G}_{\mu}|$ is also small. Hence, we cannot assess whether the case of insufficient fertility in the competitive equilibrium is more plausible or not in comparison to the case in the text.

When we have $\bar{G}_{\mu} > 0$, we also have $\bar{F}_{\mu} > 0$. If the interaction effect of n and k is attenuated, we can see that both the number of children and the level of capital are insufficient and hence the level of pollution becomes lower in the competitive equilibrium than in the social optimum. On the contrary, if the interaction effect of n and k is greater relative to the environmental externality effect of n, then the number of children in the competitive equilibrium is larger than that in the social optimum.