The benefits of international cooperation under climate uncertainty: a dynamic game analysis

Xiao-Bing Zhang[1*] and Magnus Hennlock[2]

[1] School of Economics, Renmin University of China, Beijing, China and [2] IVL Swedish Environmental Research Institute, Gothenburg and Department of Economics and Statistics, University of Gothenburg, Sweden

*Corresponding author. Email: xbzhmail@gmail.com and xiaobing.zhang@ruc.edu.cn

ONLINE APPENDIX

Appendix

A1.Proof of $\xi - \eta < 0$ and $\psi - \mu < 0$ in the case of symmetric players.

Proof. From table 1, we know that:

$$\xi - \eta = \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)^2 + 16\varepsilon}}{4} - \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)^2 + 12\varepsilon}}{6}.$$

It can be noticed that we would have $\xi - \eta = 0$ had it been the case that $\varepsilon = 0$, i.e., $(\xi - \eta)\,|_{\varepsilon=0} = 0$. Furthermore, it is easy to show that

$$\frac{\partial[\xi - \eta]}{\partial\varepsilon} = -\frac{2}{\sqrt{(r-\sigma^2)^2 + 16\varepsilon}} + \frac{1}{\sqrt{(r-\sigma^2)^2 + 12\varepsilon}} = -\frac{1}{\sqrt{\frac{1}{4}(r-\sigma^2)^2 + 4\varepsilon}} + \frac{1}{\sqrt{(r-\sigma^2)^2 + 12\varepsilon}} < 0$$

for any $\varepsilon > 0$. By the mean value theorem, it can be known that $(\xi - \eta)\,|_{\varepsilon>0} - (\xi - \eta)\,|_{\varepsilon=0} < 0$. That is, $\xi - \eta < 0$ will hold with our assumption $\varepsilon > 0$.

Also, from table 1 one can obtain $\psi - \mu = \frac{2ar(\xi-\eta) - 2a\xi\eta}{(r-2\xi)(r-3\eta)}$. Because we have shown above that $\xi - \eta < 0$, and we know that $\xi < 0$ and $\eta < 0$, we have $\psi - \mu < 0$. ■

A2. Proof of $\frac{\partial\eta}{\partial\sigma} < 0$, $\frac{\partial\mu}{\partial\sigma} < 0$, $\frac{\partial\kappa}{\partial\sigma} < 0$, $\frac{\partial\xi}{\partial\sigma} < 0$, $\frac{\partial\psi}{\partial\sigma} < 0$, and $\frac{\partial\zeta}{\partial\sigma} < 0$ for the symmetric case.

Proof. Based on the expressions for η, μ, and κ in table 1, and keeping in mind that $\sigma^2 < r$ (see section 2) (and thus $\sqrt{(r-\sigma^2)^2 + 12\varepsilon} > r - \sigma^2 > 0$), we can obtain the following derivatives:

$$\frac{\partial \eta}{\partial \sigma} = -\frac{1}{6} \underbrace{\left(2\sigma - \frac{2\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2 + 12\varepsilon}}\right)}_{>0} < 0 \tag{A.1}$$

$$\frac{\partial \mu}{\partial \sigma} = \frac{2ar}{(r-3\eta)^2} \underbrace{\frac{\partial \eta}{\partial \sigma}}_{<0} < 0 \tag{A.2}$$

$$\frac{\partial \kappa}{\partial \sigma} = \frac{1}{r}[2a + 3\mu] \underbrace{\frac{\partial \mu}{\partial \sigma}}_{<0}. \tag{A.3}$$

Since $\mu = \frac{2a}{3}[\frac{r}{r-3\eta} - 1]$ (see table 1), we have: $2a + 3\mu = \frac{2ar}{r-3\eta} > 0$, which implies $\frac{\partial \kappa}{\partial \sigma} < 0$.

Similarly, based on the expressions for ξ, ψ, and ζ in table 1, and since $\sqrt{(r-\sigma^2)^2 + 16\varepsilon} > r - \sigma^2 > 0$, we have:

$$\frac{\partial \xi}{\partial \sigma} = -\frac{1}{4} \underbrace{\left(2\sigma - \frac{2\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2 + 16\varepsilon}}\right)}_{>0} < 0 \tag{A.4}$$

$$\frac{\partial \psi}{\partial \sigma} = \frac{2ar}{(r-2\xi)^2} \underbrace{\frac{\partial \xi}{\partial \sigma}}_{<0} < 0 \tag{A.5}$$

$$\frac{\partial \zeta}{\partial \sigma} = \frac{1}{r}[2(a+\psi) \underbrace{\frac{\partial \psi}{\partial \sigma}}_{<0}]. \tag{A.6}$$

Given $\psi = a[\frac{r}{r-2\xi} - 1]$ (see table 1), we have $a + \psi = \frac{ar}{r-2\xi} > 0$, and thus, $\frac{\partial \zeta}{\partial \sigma} < 0$. ■

A3. Proof of $\frac{1}{2}\zeta - \kappa > 0$, $\frac{1}{2}\psi - \mu > 0$, and $\frac{1}{2}\xi - \eta > 0$ for the symmetric case.

Proof. From table 1 we have:

$$\frac{1}{2}\xi - \eta = \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)^2 + 16\varepsilon}}{8} - \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)^2 + 12\varepsilon}}{6}.$$

It can be noticed that we would have $\frac{1}{2}\xi - \eta = 0$ had it been the case that $\varepsilon = 0$, i.e.,$\left(\frac{1}{2}\xi - \eta\right)|_{\varepsilon=0} = 0$. Furthermore, it is straightforward to show that

$$\frac{\partial[\frac{1}{2}\xi - \eta]}{\partial\varepsilon} = -\frac{1}{\sqrt{(r-\sigma^2)^2+16\varepsilon}} + \frac{1}{\sqrt{(r-\sigma^2)^2+12\varepsilon}} > 0$$

for any $\varepsilon > 0$. By the mean value theorem, it can be known that $\left(\frac{1}{2}\xi - \eta\right)|_{\varepsilon>0} - \left(\frac{1}{2}\xi - \eta\right)|_{\varepsilon=0} > 0$. That is, $\frac{1}{2}\xi - \eta > 0$ will always hold with our assumption $\varepsilon > 0$. Also, from table 1 one can obtain $\frac{1}{2}\psi - \mu = \frac{a[r(\xi-2\eta)+\xi\eta]}{(r-2\xi)(r-3\eta)}$. Because it has been shown above that $\frac{1}{2}\xi - \eta > 0$, and we know that $\xi < 0$ and $\eta < 0$, we have $\frac{1}{2}\psi - \mu > 0$.

Furthermore, after some calculations, one can obtain from table 1 that $\frac{1}{2}\zeta - \kappa = \frac{1}{2r}[(\psi - 2\mu)(a + \psi + a + \frac{3}{2}\mu) + \frac{1}{2}\psi\mu]$. Since we have shown above that $\frac{1}{2}\psi - \mu > 0$ (i.e., $\psi - 2\mu > 0$) and we know that $a + \psi = \frac{ar}{r-2\xi} > 0$ and $a + \frac{3}{2}\mu = \frac{ar}{r-3\eta} > 0$, we have $\frac{1}{2}\zeta - \kappa > 0$. ■

A4. Proof of Proposition 2.

Proof. From (A.1) and (A.4) in Appendix A2, one can easily find that:

$$\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma} = -\frac{1}{8}\left(2\sigma - \frac{2\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2+16\varepsilon}}\right) + \frac{1}{6}\left(2\sigma - \frac{2\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2+12\varepsilon}}\right).$$

Clearly, $\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma} = 0$ if ε were equal to zero, i.e., $\left(\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma}\right)|_{\varepsilon=0} = 0$.

One can show $\frac{\partial^2[\frac{1}{2}\xi-\eta]}{\partial\sigma\partial\varepsilon} = 2\sigma(r-\sigma^2)\left\{\frac{1}{[(r-\sigma^2)^2+12\varepsilon]^{3/2}} - \frac{1}{[(r-\sigma^2)^2+16\varepsilon]^{3/2}}\right\} > 0$ for all $\varepsilon > 0$. By the mean value theorem, we know that $\left(\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma}\right)|_{\varepsilon>0} - \left(\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma}\right)|_{\varepsilon=0} > 0$ will hold. That is, under our assumption of non-zero damage (i.e., $\varepsilon > 0$), $\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma} > 0$ will hold. Also, we know from (A.2) and (A.5) in Appendix A2 that:

$$\frac{1}{2}\frac{\partial\psi}{\partial\sigma} - \frac{\partial\mu}{\partial\sigma} = \frac{1}{2}\frac{2ar}{(r-2\xi)^2}\frac{\partial\xi}{\partial\sigma} - \frac{2ar}{(r-3\eta)^2}\frac{\partial\eta}{\partial\sigma} > \left[\frac{2ar}{(r-2\xi)^2} - \frac{2ar}{(r-3\eta)^2}\right]\frac{\partial\eta}{\partial\sigma}.$$

The last inequality holds due to $\frac{1}{2}\frac{\partial\xi}{\partial\sigma} > \frac{\partial\eta}{\partial\sigma}$, which we have just shown above.
Since $0 < -\frac{1}{2}\left((r-\sigma^2) - \sqrt{(r-\sigma^2)^2+12\varepsilon}\right) < -\frac{1}{2}\left((r-\sigma^2) - \sqrt{(r-\sigma^2)^2+16\varepsilon}\right)$, i.e., $0 < -3\eta < -2\xi$, we know that $\frac{2ar}{(r-2\xi)^2} - \frac{2ar}{(r-3\eta)^2} < 0$. Together with $\frac{\partial\eta}{\partial\sigma} < 0$ (see Appendix A2), we have $\frac{1}{2}\frac{\partial\psi}{\partial\sigma} - \frac{\partial\mu}{\partial\sigma} > 0$.

Moreover, we can obtain from (A.3) and (A.6) (see Appendix A2) that:

$$\frac{1}{2}\frac{\partial\zeta}{\partial\sigma} - \frac{\partial\kappa}{\partial\sigma} = \frac{1}{2}\frac{1}{r}[2(a+\psi)\frac{\partial\psi}{\partial\sigma}] - \frac{1}{2r}[(4a+6\mu)\frac{\partial\mu}{\partial\sigma}] > \frac{1}{r}(2\psi - 3\mu)\frac{\partial\mu}{\partial\sigma}.$$

where the above-proven result $\frac{1}{2}\frac{\partial\psi}{\partial\sigma} - \frac{\partial\mu}{\partial\sigma} > 0$ was applied in the last inequality.
Since $\frac{1}{2}\left((r-\sigma^2) - \sqrt{(r-\sigma^2)^2+16\varepsilon}\right) < \frac{1}{2}\left((r-\sigma^2) - \sqrt{(r-\sigma^2)^2+12\varepsilon}\right)$, i.e., $2\psi < 3\mu$, and $\frac{\partial\mu}{\partial\sigma} < 0$ (see (A.2)), we have $\frac{1}{2}\frac{\partial\zeta}{\partial\sigma} - \frac{\partial\kappa}{\partial\sigma} > 0$.

Given that $\frac{1}{2}\frac{\partial\zeta}{\partial\sigma} - \frac{\partial\kappa}{\partial\sigma} > 0$, $\frac{1}{2}\frac{\partial\psi}{\partial\sigma} - \frac{\partial\mu}{\partial\sigma} > 0$ and $\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma} > 0$, one can know that $\frac{\partial WGIC}{\partial\sigma} = \left(\frac{1}{2}\frac{\partial\zeta}{\partial\sigma} - \frac{\partial\kappa}{\partial\sigma}\right) + \left(\frac{1}{2}\frac{\partial\psi}{\partial\sigma} - \frac{\partial\mu}{\partial\sigma}\right)T_0 + \frac{1}{2}\left(\frac{1}{2}\frac{\partial\xi}{\partial\sigma} - \frac{\partial\eta}{\partial\sigma}\right)[T_0]^2 > 0$. This implies that the greater the uncertainty about global warming, the more gain we can expect from international cooperation. ■

B1. Proof of $\xi - \eta_1 > 0$, $\psi - \mu_1 > 0$ for the particular case of asymmetric players.

Proof. From table 2, we have:

$$\xi - \eta_1 = \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)+8\varepsilon}}{4} - \frac{(r-\sigma^2) - \sqrt{(r-\sigma^2)^2+4\varepsilon}}{2}.$$

It can be noticed that we would have $\xi - \eta_1 = 0$ had it been the case that $\varepsilon = 0$, i.e.$(\xi - \eta_1)\,|_{\varepsilon=0} = 0$. Furthermore, it is easy to show that

$$\frac{\partial[\xi - \eta_1]}{\partial\varepsilon} = -\frac{1}{\sqrt{(r-\sigma^2)^2+8\varepsilon}} + \frac{1}{\sqrt{(r-\sigma^2)^2+4\varepsilon}} > 0$$

for any $\varepsilon > 0$. By the mean value theorem, it can be known that $(\xi - \eta_1)\,|_{\varepsilon>0} - (\xi - \eta_1)\,|_{\varepsilon=0} > 0$. That is, $\xi - \eta_1 > 0$ will hold with our assumption of $\varepsilon > 0$.

Furthermore, after some calculations one can show that:

$$\psi - \mu_1 = \frac{a(1+\varphi)\xi}{r-2\xi} - \frac{a(1+\varphi)\eta_1}{r-\eta_1} = \frac{a(1+\varphi)[r(\xi-\eta_1)+\xi\eta_1]}{(r-2\xi)(r-\eta_1)} > 0.$$

The last inequality holds due to $\xi < 0$, $\eta_1 < 0$, and $\xi - \eta_1 > 0$. ■

B2. Proof of $\frac{\partial\eta_1}{\partial\sigma} < 0$, $\frac{\partial\mu_1}{\partial\sigma} < 0$, $\frac{\partial\kappa_1}{\partial\sigma} < 0$, $\frac{\partial\xi}{\partial\sigma} < 0$, $\frac{\partial\psi}{\partial\sigma} < 0$, and $\frac{\partial\zeta}{\partial\sigma} < 0$ for the asymmetric case.

Proof. Based on the expressions for η_1, μ_1, and κ_1 in table 2, and keeping in mind that $\sigma^2 < r$ (see section 2) (and thus $\sqrt{(r-\sigma^2)^2+4\varepsilon} > r - \sigma^2 > 0$), we can obtain the following derivatives:

$$\frac{\partial\eta_1}{\partial\sigma} = -\underbrace{[\sigma - \frac{\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2+4\varepsilon}}]}_{>0} < 0 \tag{A.7}$$

$$\frac{\partial\mu_1}{\partial\sigma} = \frac{ar(1+\varphi)}{(r-\eta_1)^2}\underbrace{\frac{\partial\eta_1}{\partial\sigma}}_{<0} < 0 \tag{A.8}$$

$$\frac{\partial\kappa_1}{\partial\sigma} = \frac{1}{r}[\mu_1 + a]\frac{\partial\mu_1}{\partial\sigma} + \frac{1}{r}\varphi a\frac{\partial\mu_1}{\partial\sigma} = \frac{1}{r}[\mu_1 + (1+\varphi)a]\underbrace{\frac{\partial\mu_1}{\partial\sigma}}_{<0}. \tag{A.9}$$

Since $\mu_1 + (1+\varphi)a = \frac{ar(1+\varphi)}{r-\eta_1} > 0$, we have $\frac{\partial\kappa_1}{\partial\sigma} < 0$.
Based on the expressions of ξ, ψ, and ζ in table 2, and since $\sqrt{(r-\sigma^2)^2+8\varepsilon(1+\gamma)} > r - \sigma^2 > 0$, we have:

$$\frac{\partial \xi}{\partial \sigma} = -\frac{1}{2}[\underbrace{\sigma - \frac{\sigma(r-\sigma^2)}{\sqrt{(r-\sigma^2)^2 + 8\varepsilon(1+\gamma)}}}_{>0}] < 0 \tag{A.10}$$

$$\frac{\partial \psi}{\partial \sigma} = \frac{ar(1+\varphi)}{(r-2\xi)^2} \underbrace{\frac{\partial \xi}{\partial \sigma}}_{<0} < 0 \tag{A.11}$$

$$\frac{\partial \zeta}{\partial \sigma} = \frac{2}{r}[\psi + \frac{a(1+\varphi)}{2}] \underbrace{\frac{\partial \psi}{\partial \sigma}}_{<0} . \tag{A.12}$$

Being aware of $\psi + \frac{a(1+\varphi)}{2} = \frac{ar(1+\varphi)}{2(r-2\xi)}$, we have $\frac{\partial \zeta}{\partial \sigma} < 0$. ∎

B3. Proof of $\Delta = \zeta - \kappa_1 - \frac{1}{2r}[\varphi a]^2 > 0$ for any $\varphi > 0$.

Proof. Making use of the expression of ζ and κ_1 in table 2, we have:

$$\Delta = \zeta - \kappa_1 - \frac{1}{2r}[\varphi a]^2 = \frac{1}{r}[\psi + \frac{a(1+\varphi)}{2}]^2 + \frac{a^2(1-\varphi)^2}{4r} - \frac{1}{2r}[a+\mu_1]^2 - \frac{1}{r}[\varphi a]\mu_1 - \frac{1}{2r}[\varphi a]^2.$$

Since $\psi + \frac{a(1+\varphi)}{2} = \frac{ar(1+\varphi)}{2(r-2\xi)}$ and $\mu_1 = \frac{\eta_1[a(1+\varphi)+\mu_2]}{r-(\eta_1+\eta_2)} = \frac{\eta_1 a(1+\varphi)}{r-\eta_1}$, we know that Δ is a quadratic function of φ. Let us denote $\Delta = Z\varphi^2 + Y\varphi + X$, where the coefficient of φ^2 can be found as:

$$\begin{aligned} Z &= \frac{a^2 r^2}{4r(r-2\xi)^2} + \frac{a^2}{4r} - \frac{\eta_1^2 a^2}{2r(r-\eta_1)^2} - \frac{\eta_1 a^2}{r(r-\eta_1)} - \frac{a^2}{2r} \\ &= \frac{a^2}{4r}[1 + \frac{r^2}{(r-2\xi)^2} - \frac{2r^2}{(r-\eta_1)^2}]. \end{aligned}$$

If we denote $\Omega = 1 + \frac{r^2}{(r-2\xi)^2} - \frac{2r^2}{(r-\eta_1)^2}$, then we have $Z = \frac{a^2}{4r}\Omega$. It can be noticed that we would have $Z = \frac{a^2}{4r}[1 + \frac{r^2}{r^2} - \frac{2r^2}{r^2}] = 0$ had it been the case that $\varepsilon = 0$, i.e., $Z|_{\varepsilon=0} = \frac{a^2}{4r}[1 + \frac{r^2}{r^2} - \frac{2r^2}{r^2}] = 0$. Furthermore, it is not difficult to show after some tedious but straightforward calculations that $\frac{\partial Z}{\partial \varepsilon} = \frac{a^2}{4r}\frac{\partial \Omega}{\partial \varepsilon} = a^2 r[\frac{\partial \xi}{\partial \varepsilon}/(r-2\xi)^3 - \frac{\partial \eta_1}{\partial \varepsilon}/(r-\eta_1)^3]$.

Since we have $\frac{\partial \xi}{\partial \varepsilon} = \frac{-1}{\sqrt{(r-\sigma^2)+8\varepsilon}}$ and $\frac{\partial \eta_1}{\partial \varepsilon} = \frac{-1}{\sqrt{(r-\sigma^2)^2+4\varepsilon}}$, we know that:

$$\frac{\partial \xi}{\partial \varepsilon}/(r-2\xi)^3 - \frac{\partial \eta_1}{\partial \varepsilon}/(r-\eta_1)^3 = -\frac{1}{\left((r-2\xi)^3\sqrt{(r-\sigma^2)^2+8\varepsilon}\right)} + \frac{1}{\left((r-\eta_1)^3\sqrt{(r-\sigma^2)^2+4\varepsilon}\right)} > 0,$$

where we make use of $2\xi < \eta_1 < 0$ (see equation (23.1) and table 1 in the main text) for the last inequality. Therefore, we have $\frac{\partial Z}{\partial \varepsilon} = a^2 r[\frac{\partial \xi}{\partial \varepsilon}/(r-2\xi)^3 - \frac{\partial \eta_1}{\partial \varepsilon}/(r-\eta_1)^3] > 0$. By the mean value theorem, we know that $Z|_{\varepsilon>0} - Z|_{\varepsilon=0} > 0$, i.e., $Z > 0$ under our assumption of $\varepsilon > 0$.

The coefficient of φ in Δ can be calculated as:

$$\begin{aligned} Y &= \frac{2a^2r^2}{4r(r-2\xi)^2} - \frac{2a^2}{4r} - [\frac{2\eta_1^2 a^2}{2r(r-\eta_1)^2} + \frac{2\eta_1 a^2}{2r(r-\eta_1)}] - \frac{\eta_1 a^2}{r(r-\eta_1)} \\ &= \frac{a^2}{2r}[r^2/(r-2\xi)^2 - (r^2 + 2r\eta_1 - \eta_1^2)/(r-\eta_1)^2]. \end{aligned}$$

Denoting $\Gamma = r^2/(r-2\xi)^2 - (r^2+2r\eta_1-\eta_1^2)/(r-\eta_1)^2$, we have $Y = \frac{a^2}{2r}\Gamma$. Clearly, $Y = \frac{a^2}{2r}[\frac{r^2}{r^2} - \frac{r^2}{r^2}] = 0$ would hold had it been the case that $\varepsilon = 0$, i.e., $Y|_{\varepsilon=0} = \frac{a^2}{2r}[\frac{r^2}{r^2} - \frac{r^2}{r^2}] = 0$. Taking the derivative of Y with respect to ε and doing some further arrangements yields:

$$\frac{\partial Y}{\partial \varepsilon} = 2a^2 r[\frac{\partial \xi}{\partial \varepsilon}/(r-2\xi)^3 - \frac{\partial \eta_1}{\partial \varepsilon}/(r-\eta_1)^3].$$

We have shown above that $\frac{\partial \xi}{\partial \varepsilon}/(r-2\xi)^3 - \frac{\partial \eta_1}{\partial \varepsilon}/(r-\eta_1)^3 > 0$, which implies that $\frac{\partial Y}{\partial \varepsilon} > 0$. By the mean value theorem, one knows that $Y|_{\varepsilon>0} - Y|_{\varepsilon=0} > 0$, i.e., $Y > 0$ under our assumption of $\varepsilon > 0$.

Besides, when $\varphi = 0$, we have $\Delta = X = \frac{1}{r}[\psi^2 + a\psi] - \frac{1}{2r}[\mu_1^2 + 2a\mu_1]$, and thus, $rX = \psi^2 + a\psi - \frac{1}{2}\mu_1^2 - a\mu_1 = (\psi - \mu_1)(a + \psi + \frac{1}{2}\mu_1) + \psi\mu_1$. Also, we know that, with $\varphi = 0$, we have $\frac{1}{2}(\mu_1 + a) = \frac{ar}{2(r-\eta_1)} > 0$ and $\psi + \frac{a}{2} = \frac{ar}{2(r-2\xi)} > 0$, which implies that $a + \psi + \frac{1}{2}\mu_1 > 0$. Taking into account that $\psi - \mu_1 > 0$ (see Appendix B1), $\psi < 0$, and

$\mu_1 < 0$, we know that $rX > 0$ thus $X > 0$.

Since we have shown that $Z > 0, Y > 0$, and $X > 0$, we know that $\Delta = Z\varphi^2 + Y\varphi + X > 0$ would hold for any $\varphi > 0$. ■

B4. Proof of Proposition 3.

Proof. As stated in equation (25) in the main text, the total gain from cooperation for the two countries would be:

$$\begin{aligned} TWGIC &= W(T_0) - [V_1(T_0) + V_2(T_0)] \\ &= \zeta + \psi T_0 + \tfrac{1}{2}\xi[T_0]^2 - (\kappa_1 + \mu_1 T_0 + \tfrac{1}{2}\eta_1[T_0]^2 + \tfrac{1}{2r}[\varphi a]^2) \\ &= \Delta + (\psi - \mu_1)T_0 + \tfrac{1}{2}(\xi - \eta_1)[T_0]^2, \end{aligned}$$

where $\Delta = \zeta - \kappa_1 - \frac{1}{2r}[\varphi a]^2$. From (A.7) and (A.10) in Appendix B2, one can easily find:

$$\frac{\partial[\xi - \eta_1]}{\partial\sigma} = -\frac{1}{2}[\sigma - \frac{\sigma(r - \sigma^2)}{\sqrt{(r - \sigma^2)^2 + 8\varepsilon(1 + \gamma)}}] + [\sigma - \frac{\sigma(r - \sigma^2)}{\sqrt{(r - \sigma^2)^2 + 4\varepsilon}}].$$

Clearly, $\frac{\partial[\xi - \eta_1]}{\partial\sigma} = 0$ if ε were equal to zero, i.e., $\frac{\partial[\xi - \eta_1]}{\partial\sigma}|_{\varepsilon=0} = 0$. One can show that: $\frac{\partial^2[\xi - \eta_1]}{\partial\sigma\partial\varepsilon} = 2\sigma(r - \sigma^2)[\frac{1}{[(r-\sigma^2)^2+4\varepsilon]^{3/2}} - \frac{1}{[(r-\sigma^2)^2+8\varepsilon]^{3/2}}] > 0$ for all $\varepsilon > 0$. By the mean value theorem, we know that: $\frac{\partial[\xi - \eta_1]}{\partial\sigma}|_{\varepsilon>0} - \frac{\partial[\xi - \eta_1]}{\partial\sigma}|_{\varepsilon=0} > 0$, i.e., under our assumption of $\varepsilon > 0$, $\frac{\partial[\xi - \eta_1]}{\partial\sigma} > 0$ will always hold. Also, we know from (A.8) and (A.11) in Appendix B2 that:

$$\frac{\partial[\psi - \mu_1]}{\partial\sigma} = \frac{ar(1 + \varphi)}{(r - 2\xi)^2}\frac{\partial\xi}{\partial\sigma} - \frac{ar(1 + \varphi)}{(r - \eta_1)^2}\frac{\partial\eta_1}{\partial\sigma} > [\frac{ar(1 + \varphi)}{(r - 2\xi)^2} - \frac{ar(1 + \varphi)}{(r - \eta_1)^2}]\frac{\partial\eta_1}{\partial\sigma},$$

where the last inequality holds due to $\frac{\partial[\xi - \eta_1]}{\partial\sigma} > 0$ (which has been shown above). Since $2\xi - \eta_1 < 0$ (see equation (23.1)), one knows that $\frac{\partial[\psi - \mu_1]}{\partial\sigma} > 0$.

Similarly, after some calculations, one can find:

$$\frac{\partial \Delta}{\partial \sigma} = \frac{a(1+\varphi)}{r-2\xi}\frac{\partial \psi}{\partial \sigma} - \frac{a(1+\varphi)}{r-\eta_1}\frac{\partial \mu_1}{\partial \sigma} > [\frac{a(1+\varphi)}{r-2\xi} - \frac{a(1+\varphi)}{r-\eta_1}]\frac{\partial \mu_1}{\partial \sigma} > 0.$$

Given that $\frac{\partial \Delta}{\partial \sigma} > 0$, $\frac{\partial[\psi-\mu_1]}{\partial \sigma} > 0$, and $\frac{\partial[\xi-\eta_1]}{\partial \sigma} > 0$, which we have shown above, one can know from (25) that $\frac{\partial[W(T_0)-(V_1(T_0)+V_2(T_0))]}{\partial \sigma} = \frac{\partial \Delta}{\partial \sigma} + \frac{\partial[\psi-\mu_1]}{\partial \sigma}T_0 + \frac{\partial[\xi-\eta_1]}{\partial \sigma}[T_0]^2 > 0$, which implies that the greater the uncertainty about global warming, the more gain we can expect from international cooperation, which is consistent with the result in the case of symmetric players. ■