A corporate-crime perspective on fisheries: liability rules and non-compliance

FRANK JENSEN, Corresponding author
University of Copenhagen, Department of Food and Resource Economics, Rolighedsvej 23, 1958 Frederiksberg C., Copenhagen, Denmark. Tel: Phone: 0045 35336898. Email: fje@ifro.ku.dk

LINDA NØSTBAKKEN
Norwegian School of Economics, Department of Economics, Bergen, Norway. Email: Linda.Nostbakken@nhh.no

ONLINE APPENDIX

Appendix A: The basic model

A.1. Reaction functions

In this section, we derive and characterize the reaction functions presented and analyzed in the paper.

From the main text, the cost function satisfies the following properties:

$$\frac{\partial c}{\partial h_{Lt}} > 0\,, \frac{\partial^2 c}{\partial h_{Lt}{}^2} > 0\,, \frac{\partial c}{\partial h_{It}} > 0\,, \frac{\partial^2 c}{\partial h_{It}{}^2} > 0\,, \frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}} > 0\,, \frac{\partial c}{\partial x_t} < 0\,, \tag{A1}$$

$$\frac{\partial^2 c}{\partial h_{It}\partial x_t} < 0\,, \frac{\partial^2 c}{\partial h_{Lt}\partial x_t} < 0\,.$$

In addition, the properties of the penalty functions are:

$$G'(h_{It}) > 0,\ G''(h_{It}) > 0,\ F'(h_{It}) > 0\,, F''(h_{It}) > 0\,. \tag{A2}$$

From section 2.1, we have the following first-order conditions for the private optimum:

$$p - \frac{\partial c}{\partial h_{Lt}} - \varepsilon_t = 0 \tag{A3}$$

$$p - \frac{\partial c}{\partial h_{It}} - \gamma\left[F'(h_{It}) + G'(h_{It})\right] = 0 \tag{A4}$$

$$h_{Lt} = Q_t\,. \tag{A5}$$

We can express the equation system (A3)-(A5) as the reaction function presented in the main text:

$$h_{It} = h_{It}(Q_t, x_t, \gamma)\,. \tag{A6}$$

Total differentiating (A3)-(A5) yields the following:

$$\frac{\partial^2 c}{\partial h_{Lt}{}^2} dh_{Lt} + \frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}} dh_{It} + d\varepsilon = -\frac{\partial^2 c}{\partial h_{Lt}\partial x_t} dx_t \tag{A7}$$

$$\frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}} dh_{Lt} + [\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(F''(h_{It}) + G''(h_{It}))]dh_{It} = - \frac{\partial^2 c}{\partial h_{It}\partial x_t} dx_t - [F'(h_{It}) + G'(h_{It})]d\gamma \tag{A8}$$

$$dh_{Lt} = dQ_t \ . \tag{A9}$$

Inserting equation (A9) into (A7) and (A8) yields:

$$\frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}} dh_{It} + d\varepsilon = -\frac{\partial^2 c}{\partial h_{Lt}\partial x_t} dx_t - \frac{\partial^2 c}{\partial h_{Lt}^{\ 2}} dQ_t \tag{A10}$$

$$[\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(F''(h_{It}) + G''(h_{It}))]dh_{It} = -\frac{\partial^2 c}{\partial h_{It}\partial x_t} dx_t - [F'(h_{It}) + G'(h_{It})]d\gamma - \frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}} dQ_t \ . \tag{A11}$$

Note that equation (A11) only depends on dh_{It}. Using this equation, we can now find $\frac{dh_{It}}{dx_t}$ by setting $d\gamma = dQ_t = 0$:

$$\frac{dh_{It}}{dx_t} = -\frac{\frac{\partial^2 c}{\partial h_{It}\partial x_t}}{\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(F''(h_{It}) + G''(h_{It}))} \ . \tag{A12}$$

In (A12), the denominator is positive because $0 < \gamma < 1$, $\frac{\partial^2 c}{\partial h_{It}^{\ 2}} > 0$, $F''(h_{It}) > 0$, and $G''(h_{It})$ (cf. equations (A1) and (A2)). From (A1) we also have that $\frac{\partial^2 c}{\partial h_{It}\partial x_t} < 0$, which implies that $\frac{dh_{It}}{dx_t} > 0$.

Turning to $\frac{dh_{It}}{d\gamma}$, we set $dx_t = dQ_t = 0$ in (A11) and obtain:

$$\frac{dh_{It}}{d\gamma} = -\frac{[F'(h_{It}) + G'(h_{It})]}{\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(F''(h_{It}) + G''(h_{It}))} . \qquad \text{(A13)}$$

The denominator is identical to the one in equation (A12), and is thus positive, and from (A2) we have that $G'(h_{It}) > 0$ and $F'(h_{It})$ >0. Consequently, we find that $\frac{dh_{It}}{d\gamma} < 0$.

Let us finally determine the effect of quota on illegal harvest. We let $d\gamma = dx_t = 0$ in (A11), and arrive at:

$$\frac{dh_{It}}{dQ_t} = -\frac{\frac{\partial^2 c}{\partial h_{Lt} \partial h_{It}}}{\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(F''(h_{It}) + G''(h_{It}))} . \qquad \text{(A14)}$$

From before, we know that both the denominator and the numerator are positive, since $\frac{\partial^2 c}{\partial h_{Lt} \partial h_{It}} > 0$. This implies that $\frac{dh_{It}}{dQ_t} < 0$.

A.2 Enforcement costs

In this section, we derive and characterize the enforcement cost function used in the paper. We start out by inverting the reaction function in (A6), which yields:

$$\gamma = \gamma(Q_t, x_t, h_{It}) . \qquad \text{(A15)}$$

Total differentiating (A15) produces:

$$\frac{\partial \gamma}{\partial Q} dQ_t + \frac{\partial \gamma}{\partial x_t} dx_t + \frac{\partial \gamma}{\partial h_{It}} dh_{It} = 0 . \qquad \text{(A16)}$$

Next, we define the probability of being detected as a function of enforcement effort, $\gamma(e_t)$, and we assume that:

$$\frac{\partial \gamma}{\partial e_t} > 0 . \qquad \text{(A17)}$$

Note that we can invert $\gamma(e_t)$ to yield $e_t(\gamma)$, and because of (A17) we obtain the following:

$$\frac{\partial e_t}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e_t}} > 0 \ . \qquad \text{(A18)}$$

Substituting the inverted reaction function into $e_t(\gamma)$ gives $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$. Now, we want to find the sign of the derivatives of $\alpha(Q_t, x_t, h_{It})$. First, we investigate the sign of $\frac{\partial \alpha}{\partial h_{It}}$ by using:

$$\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}} . \qquad \text{(A19)}$$

From (A18), we have that $\frac{\partial e_t}{\partial \gamma} > 0$ and we note that:

$$\frac{\partial \gamma}{\partial h_{It}} = \frac{1}{\frac{\partial h_{It}}{\partial \gamma}} . \qquad \text{(A20)}$$

From (A13) we have that $\frac{\partial h_{It}}{\partial \gamma} < 0$, and consequently, from (A20) we get that $\frac{\partial \gamma}{\partial h_{It}} < 0$. Using (A19) now implies that $\frac{\partial \alpha}{\partial h_{It}} < 0$.

Concerning the sign of $\frac{\partial \alpha}{\partial x_t}$ we have that:

$$\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t} . \qquad \text{(A21)}$$

As above $\frac{\partial e_t}{\partial \gamma} > 0$. By setting *$dQ_t$ = 0* in (A16) and solving for $\frac{\partial \gamma}{\partial x_t}$ we reach:

$$\frac{\partial \gamma}{\partial x_t} = -\frac{\frac{\partial h_{It}}{\partial x_t}}{\frac{\partial h_{It}}{\partial \gamma}} . \qquad \text{(A22)}$$

From (A12), we know that $\frac{\partial h_{It}}{\partial x_t} > 0$, and from (A20) that $\frac{\partial h_{It}}{\partial \gamma} < 0$. Combining this with (A22) gives us that $\frac{\partial \gamma}{\partial x_t} > 0$, which in turn implies that $\frac{\partial \alpha}{\partial x_t} > 0$.

Finally, we find the sign of $\frac{\partial \alpha}{\partial Q_t}$ by using that:

$$\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t}. \qquad \text{(A23)}$$

Setting $dx_t = 0$ in (A16) and solving for $\frac{\partial \gamma}{\partial Q_t}$ we get:

$$\frac{\partial \gamma}{\partial Q_t} = -\frac{\frac{\partial h_{It}}{\partial Q_t}}{\frac{\partial h_{It}}{\partial \gamma}}. \qquad \text{(A24)}$$

We have established that $\frac{\partial h_{It}}{\partial \gamma} < 0$, and from (A14) we learned that $\frac{\partial h_{It}}{\partial Q_t} < 0$. Therefore, (A24) implies that $\frac{\partial \gamma}{\partial Q_t} < 0$, and using this in (A23) gives us that $\frac{\partial \alpha}{\partial Q_t} < 0$.

Let us next turn to the enforcement cost function, $K(e_t)$. We assume that:

$$\frac{\partial K}{\partial e_t} > 0 \text{ and } \frac{\partial^2 K}{\partial e_t^{\,2}} > 0. \qquad \text{(A25)}$$

From above $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$ and inserting this into the enforcement cost function gives $K(e_t(\gamma(Q_t, x_t, h_{It}))) = F(\alpha(Q_t, x_t, h_{It})) = E(Q_t, x_t, h_{It})$. We now want to determine the signs of the derivatives of the enforcement cost function, and we start by considering $\frac{\partial E}{\partial Q_t}$:

$$\frac{\partial E}{\partial Q_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial Q_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t}. \qquad \text{(A26)}$$

From (A23), $\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t} < 0$, and from (A25) we have that $\frac{\partial K}{\partial e_t} > 0$, which implies that $\frac{\partial E}{\partial Q_t} < 0$.

Next, for $\frac{\partial E}{\partial x_t}$ we have that:

$$\frac{\partial E}{\partial x_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial x_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t}\,. \tag{A27}$$

Using (A21), we have that $\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t} > 0$, and according to (A25), $\frac{\partial K}{\partial e_t} > 0$, from which it follows that $\frac{\partial E}{\partial x_t} > 0$.

Finally, for $\frac{\partial E}{\partial h_{It}}$ we get:

$$\frac{\partial E}{\partial h_{It}} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}}\,. \tag{A28}$$

From (A19) we know that $\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}} < 0$, and using (A23) we have that $\frac{\partial K}{\partial e_t} > 0$. This implies that $\frac{\partial E}{\partial h_{It}} < 0$.

Appendix B: Share of profit

B.1. Reaction functions

From section 3 we have the following first-order conditions:

$$\frac{\partial W}{\partial h_{Lt}} - \alpha \frac{\partial c}{\partial h_{Lt}} - u_t = 0 \tag{B1}$$

$$\frac{\partial W}{\partial h_{It}} - \alpha \frac{\partial c}{\partial h_{It}} - \gamma G'\left(h_{It}\right) = 0 \tag{B2}$$

$$h_{Lt} = Q_t . \tag{B3}$$

We also have the following wage scheme from section 3:

$$W\left(h_{Lt}, h_{It}\right) = \beta\left[p_t\left(h_{Lt} + h_{It}\right) - \left(1-\alpha\right)c\left(h_{Lt}, h_{It}, x_t\right) - \gamma F\left(h_{It}\right)\right]. \tag{B4}$$

From the wage scheme in (B4) we may obtain:

$$\frac{\partial W}{\partial h_{Lt}} = \beta(p - (1-\alpha)\frac{\partial c}{\partial h_{Lt}}) \tag{B5}$$

$$\frac{\partial W}{\partial h_{It}} = \beta(p - (1-\alpha)\frac{\partial c}{\partial h_{It}} - \gamma F'(h_{It})) . \tag{B6}$$

(B5) can be substituted into (B1) and (B6) into (B2). This gives the following rewritten first-order conditions:

$$\beta(p - \frac{\partial c}{\partial h_{Lt}}) - (1-\beta)\alpha \frac{\partial c}{\partial h_{Lt}} - u_t = 0 \tag{B7}$$

$$\beta(p - \frac{\partial c}{\partial h_{It}}) - (1-\beta)\alpha \frac{\partial c}{\partial h_{It}} - \gamma(\beta F'\left(h_{It}\right) + G'\left(h_{It}\right)) = 0 \tag{B8}$$

$$h_{Lt} = Q_t . \tag{B9}$$

(B7) - (B9) may be total differentiated which gives:

$$[\beta \frac{\partial^2 c}{\partial h_{Lt}^{\ 2}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{Lt}^{\ 2}}]dh_{Lt} + [\beta \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}]dh_{It} + du_t = -[\beta \frac{\partial^2 c}{\partial h_{Lt}\partial x_t} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{Lt}\partial x_t}]dx_t \qquad \text{(B10)}$$

$$[\beta \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}]dh_{Lt} + [\beta \frac{\partial^2 c}{\partial h_{It}^{\ 2}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(\beta F''(h_{It}) + G''(h_{It}))]dh_{It} = -(\beta F'(h_{It}) + G'(h_{It}))d\gamma - [\beta \frac{\partial^2 c}{\partial h_{It}\partial x_t} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial x_t}]dx_t \qquad \text{(B11)}$$

$$dh_{Lt} = dQ_t\ . \qquad \text{(B12)}$$

(B12) can be substituted into (B10) and (B11) which gives:

$$+[\beta \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}]dh_{It} + du_t = -[\beta \frac{\partial^2 c}{\partial h_{Lt}\partial x_t} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{Lt}\partial x_t}]dx_t - [\beta \frac{\partial^2 c}{\partial h_{Lt}^{\ 2}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{Lt}^{\ 2}}]dQ_t \qquad \text{(B13)}$$

$$+[\beta \frac{\partial^2 c}{\partial h_{It}^{\ 2}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(\beta F''(h_{It}) + G''(h_{It}))]dh_{It} = -(\beta F'(h_{It}) + G'(h_{It}))d\gamma - [\beta \frac{\partial^2 c}{\partial h_{It}\partial x_t} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial x_t}]dx_t - [\beta \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}} + (1-\beta)\alpha \frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}]dQ_t \qquad \text{(B14)}$$

dh_{It} is the only variable that enters in (B14) and, therefore, (B14) can be used to characterize the reaction function.

In (B14) we may set $d\gamma = dx_t = 0$ and reach:

$$\frac{dh_{It}}{dQ_t} = -\frac{(\beta + (1-\beta)\alpha)\frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}}{(\beta + (1-\beta)\alpha)\frac{\partial^2 c}{\partial h_{It}^{\ 2}} + \gamma(\beta F''(h_{It}) + G''(h_{It}))}\ . \qquad \text{(B15)}$$

We have that $0<\beta<1$, $0<\alpha<1, 0<\gamma<1, \frac{\partial^2 c}{\partial h_{It}^{\;2}}>0$, $F''(h_{It})>0$ and $G''(h_{It})>0$ and this imply that the denominator in (B15) is positive. With respect to the nominator $\frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}>0$ so the nominator is also positive. In total, we, therefore, reach the conclusion that $\frac{dh_{It}}{dQ_t}<0$.

Concerning $\frac{dh_{It}}{d\gamma}$ we set $dQ_t = dx_t = 0$ in (B14) and arrive at:

$$\frac{dh_{It}}{d\gamma} = -\frac{\beta F'(h_{It}) + G'(h_{It})}{(\beta + (1-\beta)\alpha)\frac{\partial^2 c}{\partial h_{It}^{\;2}} + \gamma(\beta F''(h_{It}) + G''(h_{It}))} . \qquad \text{(B16)}$$

From (B15) we have that the denominator is positive and, in addition, the nominator in (B16) is positive because $G'(h_{It})>0$ and $F'(h_{It})>0$. Therefore, we obtain that $\frac{dh_{It}}{d\gamma}<0$.

Last, by setting $dQ_t = d\gamma = 0$ we reach:

$$\frac{\partial h_{It}}{\partial x_t} = -\frac{(\beta + (1-\beta)\alpha)\frac{\partial^2 c}{\partial h_{It}\partial x_t}}{(\beta + (1-\beta)\alpha)\frac{\partial^2 c}{\partial h_{It}^{\;2}} + \gamma(\beta F''(h_{It}) + G''(h_{It}))} . \qquad \text{(B17)}$$

From above the denominator is positive. In addition, we have that $\frac{\partial^2 c}{\partial h_{It}\partial x_t}<0$ so the nominator is negative. In total, this implies that $\frac{\partial h_{It}}{\partial x_t}>0$.

B.2. Enforcement costs

The inverted reaction function is:

$$\gamma = \gamma(Q_t, x_t, h_{It}) . \qquad \text{(B18)}$$

From (B18) we get:

$$\frac{\partial \gamma}{\partial Q} dQ_t + \frac{\partial \gamma}{\partial x_t} dx_t + \frac{\partial \gamma}{\partial h_{It}} dh_{It} = 0 \ . \tag{B19}$$

Now $\gamma(e_t)$ is the probability of being detected as a function of enforcement effort and we have:

$$\frac{\partial \gamma}{\partial e_t} > 0 \, . \tag{B20}$$

We invert $\gamma(e_t)$ to get $e_t(\gamma)$ and due to (B20) we obtain:

$$\frac{\partial e_t}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e_t}} > 0 \ . \tag{B21}$$

$\gamma = \gamma(Q_t, x_t, h_{It})$ can be used in $e_t(\gamma)$ and this gives $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$. Now we can find the sign of $\frac{\partial \alpha}{\partial h_{It}}$ by using:

$$\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial h_{It}} \, . \tag{B22}$$

From (B21) $\frac{\partial e_t}{\partial \gamma} > 0$ and furthermore we have that:

$$\frac{\partial \gamma}{\partial h_{It}} = \frac{1}{\frac{\partial h_{It}}{\partial \gamma}} \, . \tag{B23}$$

From (B16) $\frac{\partial h_{It}}{\partial \gamma} < 0$ and by using this in (B23) we obtain $\frac{\partial \gamma}{\partial h_{It}} < 0$. Now (B22) now imply that $\frac{\partial \alpha}{\partial h_{It}} < 0$.

For the sign of $\frac{\partial \alpha}{\partial x_t}$ we have:

$$\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial x_t} \, . \tag{B24}$$

In (B21) it was stated that $\frac{\partial e_t}{\partial \gamma} > 0$ and using by $dQ_t = 0$ in (B19) it is obtained that:

$$\frac{\partial \gamma}{\partial x_t} = -\frac{\frac{\partial h_{It}}{\partial x_t}}{\frac{\partial h_{It}}{\partial \gamma}} \ . \qquad \text{(B25)}$$

From (B17) $\frac{\partial h_{It}}{\partial x_t} > 0$, and in (B16) we reached that $\frac{\partial h_{It}}{\partial \gamma} < 0$. Combining this in (B25) $\frac{\partial \gamma}{\partial x_t} > 0$, which by using (B24) gives $\frac{\partial \alpha}{\partial x_t} > 0$.

Lastly, we turn attention to the sign of $\frac{\partial \alpha}{\partial Q_t}$ where we have:

$$\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t} \ . \qquad \text{(B26)}$$

Using $dx_t = 0$ in (B19) and solving for $\frac{\partial \gamma}{\partial Q_t}$ we get:

$$\frac{\partial \gamma}{\partial Q_t} = -\frac{\frac{\partial h_{It}}{\partial Q_t}}{\frac{\partial h_{It}}{\partial \gamma}} \ . \qquad \text{(B27)}$$

From (B16) $\frac{\partial h_{It}}{\partial \gamma} < 0$ and using (B15) implies that $\frac{\partial h_{It}}{\partial Q_t} < 0$. Therefore, $\frac{\partial \gamma}{\partial Q_t} < 0$ and by using this in (B26) it follows that $\frac{\partial \alpha}{\partial Q_t} < 0$.

Now the enforcement cost function is given as $K(e_t)$ and we assume that:

$$\frac{\partial K}{\partial e_t} > 0 \text{ and } \frac{\partial^2 K}{\partial e_t^{\,2}} > 0 \ . \qquad \text{(B28)}$$

From before $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$ and inserting this in the enforcement cost function gives $K(e_t(\gamma(Q_t, x_t, h_{It}))) = F(\alpha(Q_t, x_t, h_{It})) = E(Q_t, x_t, h_{It})$. Now we can find the sign of the derivatives and we start by $\frac{\partial E}{\partial Q_t}$ where we have:

$$\frac{\partial E}{\partial Q_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial Q_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t}. \qquad \text{(B29)}$$

In (B26) $\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t} < 0$ and from (B28) $\frac{\partial K}{\partial e_t} > 0$, implying that $\frac{\partial E}{\partial Q_t} < 0$.

Next for the sign of $\frac{\partial E}{\partial x_t}$ we have that:

$$\frac{\partial E}{\partial x_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial x_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t}. \qquad \text{(B30)}$$

Using (B24) we have that $\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t} > 0$ and from (B28) $\frac{\partial K}{\partial e_t} > 0$ which implies that $\frac{\partial E}{\partial x_t} > 0$.

Last for the sign of $\frac{\partial E}{\partial h_{It}}$ we get:

$$\frac{\partial E}{\partial h_{It}} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}}. \qquad \text{(B31)}$$

From (B22) $\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}} < 0$ and using (B28) $\frac{\partial K}{\partial e_t} > 0$. In total, this implies that $\frac{\partial E}{\partial h_{It}} < 0$.

Appendix C: Share of revenue

C.1. Reaction functions

With the share of revenue rule the wage function is:

$$W\left(h_{Lt},h_{It}\right)=\beta\left(p_t\left(h_{Lt}+h_{It}\right)\right). \tag{C1}$$

The general first-order conditions for the employee are given by (B1)-(B3) in appendix B.1.

Inserting the derivatives of (C1) in the first-order conditions gives:

$$p-\alpha\frac{\partial c}{\partial h_{Lt}}-u_t=0 \tag{C2}$$

$$p-\alpha\frac{\partial c}{\partial h_{It}}-\gamma G'\left(h_{It}\right)=0 \tag{C3}$$

$$h_{Lt}=Q_t. \tag{C4}$$

By total differentiating (C2) - (C4) we get that:

$$\alpha\frac{\partial^2 c}{\partial h_{Lt}{}^2}dh_{Lt}+\alpha\frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}}dh_{It}+du_t=-\alpha\frac{\partial^2 c}{\partial h_{Lt}\partial x_t}dx_t \tag{C5}$$

$$\alpha\frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}dh_{Lt}+[\alpha\frac{\partial^2 c}{\partial h_{It}{}^2}+\gamma G''(h_{It})]dh_{It}=$$

$$-\alpha\frac{\partial^2 c}{\partial h_{It}\partial x_t}dx_t-G'(h_{It})d\gamma \tag{C6}$$

$$dh_{Lt}=dQ_t. \tag{C7}$$

(C7) can be inserted into (C5) and (C6) which yields:

$$\alpha\frac{\partial^2 c}{\partial h_{Lt}\partial h_{It}}dh_{It}+du_t=-\alpha\frac{\partial^2 c}{\partial h_{Lt}\partial x_t}dx_t-\alpha\frac{\partial^2 c}{\partial h_{Lt}{}^2}dQ_t \tag{C8}$$

$$[\alpha\frac{\partial^2 c}{\partial h_{It}{}^2}+\gamma G''(h_{It})]dh_{It}=-\alpha\frac{\partial^2 c}{\partial h_{It}\partial x_t}dx_t-G'(h_{It})d\gamma-\alpha\frac{\partial^2 c}{\partial h_{It}\partial h_{Lt}}dQ_t. \tag{C9}$$

Since (C9) only depends on dh_{It}, this equation is the one we will consider to derive the properties of the reaction function.

First, we investigate the sign of $\frac{\partial h_{It}}{dQ_t}$ and by setting $d\gamma = dx_t = 0$ in (C9) we reach:

$$\frac{\partial h_{It}}{dQ_t} = -\frac{\alpha \frac{\partial^2 c}{\partial h_{It} \partial h_{Lt}}}{\alpha \frac{\partial^2 c}{\partial h_{It}{}^2} + \gamma G''(h_{It})}. \tag{C10}$$

Concerning (C10) $0 < \alpha < 1$ $0 < \gamma < 1$ $\alpha \frac{\partial^2 c}{\partial h_{It}{}^2} > 0$ and $G''(h_{It}) > 0$ so the denominator is positive. The nominator is also positive because $\frac{\partial^2 c}{\partial h_{It} \partial h_{Lt}} > 0$. In total, (C10) therefore imply that $\frac{\partial h_{It}}{dQ_t} < 0$.

Setting $d\gamma = dQ_t = 0$ in (C9) gives:

$$\frac{\partial h_{It}}{dx_t} = -\frac{\alpha \frac{\partial^2 c}{\partial h_{It} \partial x_t}}{\alpha \frac{\partial^2 c}{\partial h_{It}{}^2} + \gamma G''(h_{It})}. \tag{C11}$$

As in (C10) the denominator is positive. However, now $\frac{\partial^2 c}{\partial h_{It} \partial x_t} < 0$ so the nominator is negative and this imply that $\frac{\partial h_{It}}{dx_t} > 0$.

Last, we evaluate the sign of $\frac{\partial h_{It}}{d\gamma}$ by setting $dx_t = dQ_t = 0$ in (C9). This gives:

$$\frac{\partial h_{It}}{d\gamma} = -\frac{G'(h_{It})}{\alpha \frac{\partial^2 c}{\partial h_{It}{}^2} + \gamma G''(h_{It})}. \tag{C12}$$

The denominator is positive from (C10) and the nominator is also positive because $G'(h_{It}) > 0$. This implies that $\frac{\partial h_{It}}{d\gamma} < 0$.

C.2. Enforcement costs

As before we have an inverted the reaction function given by:

$$\gamma = \gamma(Q_t, x_t, h_{It}) . \quad \text{(C13)}$$

(C13) can be total differentiating:

$$\frac{\partial \gamma}{\partial Q} dQ_t + \frac{\partial \gamma}{\partial x_t} dx_t + \frac{\partial \gamma}{\partial h_{It}} dh_{It} = 0 . \quad \text{(C14)}$$

Now the probability of being detected is defined as $\gamma(e_t)$ and we have that:

$$\frac{\partial \gamma}{\partial e_t} > 0 . \quad \text{(C15)}$$

From $\gamma(e_t)$ we get $e_t(\gamma)$ and because of (C15) we have that:

$$\frac{\partial e_t}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e_t}} > 0 \ . \quad \text{(C16)}$$

(C13) can be substituted into $e_t(\gamma)$ to obtain $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$. Now we want to find the sign of the derivatives of $\alpha(Q_t, x_t, h_{It})$. First, we consider the sign of $\frac{\partial \alpha}{\partial h_{It}}$:

$$\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial h_{It}} . \quad \text{(C17)}$$

From (C16) it is obtained that $\frac{\partial e_t}{\partial \gamma} > 0$. Furthermore, we have:

$$\frac{\partial \gamma}{\partial h_{It}} = \frac{1}{\frac{\partial h_{It}}{\partial \gamma}} . \quad \text{(C18)}$$

(C12) imply that $\frac{\partial h_{It}}{\partial \gamma} < 0$, and therefore we have that $\frac{\partial \gamma}{\partial h_{It}} < 0$ by using (C18). Now (C17) implies that $\frac{\partial \alpha}{\partial h_{It}} < 0$. Concerning the sign of $\frac{\partial \alpha}{\partial x_t}$ we get that:

$$\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial x_t} . \tag{C19}$$

(C16) express that $\frac{\partial e_t}{\partial \gamma} > 0$ and by using $dQ_t = 0$ in (C14) we reach:

$$\frac{\partial \gamma}{\partial x_t} = -\frac{\frac{\partial h_{It}}{\partial x_t}}{\frac{\partial h_{It}}{\partial \gamma}} . \tag{C20}$$

In (C11) we have that $\frac{\partial h_{It}}{\partial x_t} > 0$, and from (C18) we reached that $\frac{\partial h_{It}}{\partial \gamma} < 0$. Combining this information implies that $\frac{\partial \gamma}{\partial x_t} > 0$ and using (C19) gives $\frac{\partial \alpha}{\partial x_t} > 0$.

Lastly, we find the sign of $\frac{\partial \alpha}{\partial Q_t}$ by using that:

$$\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t} . \tag{C21}$$

By setting $dx_t = 0$ in (C14) we get:

$$\frac{\partial \gamma}{\partial Q_t} = -\frac{\frac{\partial h_{It}}{\partial Q_t}}{\frac{\partial h_{It}}{\partial \gamma}} . \tag{C22}$$

From (C18) $\frac{\partial h_{It}}{\partial \gamma} < 0$ and furthermore we have that $\frac{\partial h_{It}}{\partial Q_t} < 0$ in (C10). Therefore, (C22) implies that $\frac{\partial \gamma}{\partial Q_t} < 0$ and using this in (C21) gives $\frac{\partial \alpha}{\partial Q_t} < 0$.

Now the enforcement cost function is given as $K(e_t)$ and we assume that:

$$\frac{\partial K}{\partial e_t} > 0 \text{ and } \frac{\partial^2 K}{\partial e_t^{\,2}} > 0 . \quad \text{(C23)}$$

Now we have that $e_t = e_t(\gamma(Q_t, x_t, h_{It})) = \alpha(Q_t, x_t, h_{It})$ and inserting this in the enforcement cost function gives $K(e_t(\gamma(Q_t, x_t, h_{It}))) = F(\alpha(Q_t, x_t, h_{It})) = E(Q_t, x_t, h_{It})$. Now we can find the sign of the derivatives of the enforcement cost function and we start by the sign of $\frac{\partial E}{\partial Q_t}$ where we have:

$$\frac{\partial E}{\partial Q_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial Q_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t} . \quad \text{(C24)}$$

From (C21) we get that $\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial Q_t} < 0$ and from (C23) $\frac{\partial K}{\partial e_t} > 0$, implying that $\frac{\partial E}{\partial Q_t} < 0$.

Next for the sign of $\frac{\partial E}{\partial x_t}$ we have that:

$$\frac{\partial E}{\partial x_t} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial x_t} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t} . \quad \text{(C25)}$$

Using (C19) we have that $\frac{\partial \alpha}{\partial x_t} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial x_t} > 0$ and using that $\frac{\partial K}{\partial e_t} > 0$ in (C23) this implies that $\frac{\partial E}{\partial x_t} > 0$.

Last for $\frac{\partial E}{\partial h_{It}}$ we get:

$$\frac{\partial E}{\partial h_{It}} = \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial K}{\partial e_t}\frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}} . \quad \text{(C26)}$$

(C17) gives $\frac{\partial \alpha}{\partial h_{It}} = \frac{\partial e_t}{\partial \gamma}\frac{\partial \gamma}{\partial h_{It}} < 0$ and using (C23) we have that $\frac{\partial K}{\partial e_t} > 0$. In total, this implies that $\frac{\partial E}{\partial h_{It}} < 0$.