Environmental protection, public finance requirements and the timing of emission reductions*

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Abstract

The effects of four environmental policy options for the reduction of pollution emissions, i.e. taxes, emission standards, auctioned permits and freely allocated permits, are analyzed. The setup is a real option model where the amount of emissions is determined by solving the firm's profit maximization problem under each policy instrument. The regulator solves an optimal stopping problem in order to find the critical threshold for policy adoptions taking into account revenues from taxes and auctioned permits and government spending. In this framework we find the ranking of the alternative policy options in terms of their adoption lag and social welfare. We show that when the output demand is elastic, emission standards are preferred to freely allocated permits. Taxes and auctioned permits are always equivalent in terms of their adoption lag and social welfare, and also equivalent to emission standards when the regulator redistributes revenues.

Key words: Environmental policies; Taxes; Emission standards; Permits; Public abatement spending; Optimal implementation time; Real options

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Appendix A

Derivation of the critical threshold $\hat{\theta}_v$, T, S, P^{Au} and P^{Fr}

In this Appendix we derive the optimal timing for the environmental policy v. Let $W_v^N = W_v^N(\theta_t, M_t)$ denote the value function for the "no-adopt" region $0 \leq t < \tau_v$, in which $E_t = E^N$. The corresponding Hamilton-Jacobi-Bellman equation is:

$$rW_v^N = B^N(\theta_t, M_t) + \left(E^N - \delta M_t\right)\frac{\partial W_v^N}{\partial M_t} + \alpha\theta_t\frac{\partial W_v^N}{\partial \theta_t} + \frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 W_v^N}{\partial \theta_t^2}. \quad (19)$$

It has the following general solution:

$$W_v^N(\theta, M) = \Phi_{1,v}\theta^{\phi_1} + \Phi_{2,v}\theta^{\phi_2} + \left[-\frac{1}{r}\int_0^{Q^N} p'(q)q\,dq + \frac{\Pi^N}{r} - \frac{\theta M}{\kappa_2} - \frac{\theta E^N}{\kappa_1\kappa_2}\right] \quad (20)$$

where $\Phi_{1,v}$ and $\Phi_{2,v}$ are unknowns to be determined, $\kappa_1 = r - \alpha$ and $\kappa_2 = r + \delta - \alpha$. Here, ϕ_1 and ϕ_2 are the solution to the following characteristic equation:

$$\frac{1}{2}\sigma^2\phi(\phi - 1) + \phi\alpha - r = 0$$

and are given by:

$$\phi_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1,$$

$$\phi_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0.$$

The term between the squared parentheses in (20) is a particular solution, which captures the expected net benefit from emissions in the case where the environmental regulator has not adopted the policy and is calculated as:

$$\mathbb{E}\left\{\int_0^\infty e^{-rt}B^N(\theta_t, M_t)\,dt\right\} =$$

$$= \int_0^\infty e^{-rt}\left(CS^N + \Pi^N\right)dt - \int_0^\infty e^{-rt}\theta e^{\alpha t}\left[\frac{E^N}{\delta} + \left(M - \frac{E^N}{\delta}\right)e^{-\delta t}\right]dt$$

$$= -\frac{1}{r}\int_0^{Q_N} p'(q)q\,dq + \frac{\Pi^N}{r} - \frac{\theta M}{\kappa_2} - \frac{\theta E^N}{\kappa_1\kappa_2},$$

where E^N is given by (5). Therefore, the parenthesis in (20) represents the fundamental term and the exponential terms account for the perpetual American option value.

Next, let $W_v^A(\theta, M)$ denote the value function for the adopt region $t \geq \tau_v$, in which $\left(RV^A\right)_v = \zeta E^A$ if $v = T, P^{Au}$ and $\left(RV^A\right)_v = 0$ if $v = S,\ P^{Fr}$. Since we consider environmental policies which involve a one-time reduction in E_t, there is no option term after pollutant emissions have been reduced to E^A. So in this case the solution to the Hamilton-Jacobi-Bellman equation is given by its particular solution which captures the regulator's expectation about net benefit from abated emissions under the policy instrument v and is calculated as follows:

$$\begin{aligned} W_v^A(\theta, M) &= \mathbb{E}\left\{\int_0^\infty e^{-rt} B_v^A(\theta_t, M_t) dt\right\} \\ &= -\frac{1}{r}\int_0^{Q_A} p'(q) q dq + \frac{\left(\Pi^A\right)_v}{r} + \frac{\zeta E^A}{r} - \frac{\theta M}{\kappa_2} - \frac{\theta E^A}{\kappa_1 \kappa_2}. \end{aligned}$$

We know that the solutions for $W_v^N(\theta, M)$ and $W_v^A(\theta, M)$ must satisfy the following set of boundary conditions (see Pindyck 2000):

$$W_v^N(0, M) = 0, \tag{21}$$

$$W_v^N(\hat{\theta}_v, M) = W_v^A\left(\hat{\theta}_v, M\right) - K_v, \tag{22}$$

$$\frac{\partial W_v^N(\hat{\theta}_v, M)}{\partial \theta} = \frac{\partial W_v^A\left(\hat{\theta}_v, M\right)}{\partial \theta}, \tag{23}$$

and:

$$\frac{\partial W_v^N(\hat{\theta}_v, M)}{\partial M} = \frac{\partial W_v^A\left(\hat{\theta}_v, M\right)}{\partial M}. \tag{24}$$

Here, $\hat{\theta}_v$ is a free boundary, which must be found as part of the solution, and which separates the adopt from the no-adopt regions. It is also the solution to the stopping problem (16):

$$\tau_v = \inf\left\{t > 0,\ \theta \geq \hat{\theta}_v\right\}$$

The policy v should be adopted the first time the process θ_t crosses the threshold $\hat{\theta}_v$ from below. Boundary condition (21) reflects the fact that if θ_t is ever zero, it will remain at zero thereafter. Condition (22) is the value matching condition which says that the value function at the time of adoption is equal to the payoff from policy adoption. In addition, to ensure that policy adoption occurs along the optimal path, the value of social welfare satisfies the smooth-pasting conditions (23) and (24) at the endogenous adoption threshold (see Dixit and Pindyck, 1994).

In our problem boundary condition (21) implies that $\Phi_{2,v} = 0$, leaving the solution:

$$W_v^N(\theta, M) = \Phi_{1,v}\theta^{\phi_1} - \frac{1}{r}\int_0^{QN} p'(q)qdq + \frac{\Pi^N}{r} - \frac{\theta M}{\kappa_2} - \frac{\theta E^N}{\kappa_1\kappa_2}. \tag{25}$$

The first term on the right-hand side of Eq. (25) is the value of the option to adopt policy v and reduce emissions to E^A, while the remaining terms represent the regulator's expectation about net benefit from emissions $B^N(\theta_t, M_t)$.

The value matching condition (22) can be rearranged in the following manner:

$$\Phi_{1,v}\left(\hat{\theta}_v\right)^{\phi_1} = \frac{\hat{\theta}_v\left(E^N - E^A\right)}{\kappa_1\kappa_2} + \frac{1}{r}\int_0^{Q_N} p'(q)qdq - \frac{1}{r}\int_0^{Q_A} p'(q)qdq - \frac{\Pi^N - \left(\Pi^A\right)_v - \left(RV^A\right)_v}{r} - K_v. \tag{26}$$

The smooth-pasting condition (23) yields:

$$\Phi_{1,v} = \frac{1}{\phi_1\left(\hat{\theta}_v\right)^{\phi_1-1}}\left(\frac{E^N - E^A}{\kappa_1\kappa_2}\right), \tag{27}$$

Plugging (27) into (26), we get:

$$\hat{\theta}_v = \left(\frac{\phi_1\kappa_1\kappa_2}{\left(E^N - E^A\right)\left(\phi_1 - 1\right)}\right)\left(\frac{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq}{r} + \frac{\Pi^N - \left(\Pi^A\right)_v - \left(RV^A\right)_v}{r} + K_v\right). \tag{28}$$

Finally, substituting (28) into (27), we get:

$$\Phi_{1,v} = \left(\frac{E^N - E^A}{\phi_1\kappa_1\kappa_2}\right)^{\phi_1}\left(\frac{r\left(\phi_1 - 1\right)}{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq + \Pi^N - \left(\Pi^A\right)_v - \left(RV^A\right)_v + rK_v}\right)^{\phi_1-1}.$$

Proof of Proposition 1

We want to show that when the cost of the policy implementation is the same among the policies, the optimal adoption thresholds under taxes, auctioned permits and emission standards are equivalent. First, we show that $\hat{\theta}_T = \hat{\theta}_S$. Replacing K_v with $K\left(E^N - E^A\right)$ in (28),

$$\left(\frac{\phi_1\kappa_1\kappa_2}{\left(E^N - E^A\right)\left(\phi_1 - 1\right)}\right)\left(\frac{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq}{r} + \frac{\Pi^N - \left(\Pi^A\right)_T - \zeta E^A}{r} + K\left(E^N - E^A\right)\right)$$

$$= \left(\frac{\phi_1\kappa_1\kappa_2}{\left(E^N - E^A\right)\left(\phi_1 - 1\right)}\right)\left(\frac{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq}{r} + \frac{\Pi^N - \left(\Pi^A\right)_S}{r} + K\left(E^N - E^A\right)\right).$$

Simplifying the above equation yields:

$$\left(\Pi^A\right)_T + \zeta E^A = (\Pi^A)_S. \tag{29}$$

Substituting the expressions for the aggregate profits and the level of emissions in the presence of intervention, it is immediate to show that the terms on the left and on the right side of the equation (29) are equivalent. Thus, the critical thresholds under taxes and under emission standards are equivalent. In the same way, we can show that $\hat{\theta}_S = \hat{\theta}_{P^{Au}}$. Since the profits under auctioned permits and under emission taxes are equivalent, repeating the same calculations as before yields the result in proposition 1.

Proof of Proposition 2

In order to prove proposition 2 we rewrite $\Phi_{1,T}$ and $\Phi_{1,S}$, as follows:

$$\Phi_{1,T} = \left(\frac{E^N - E^A}{\phi_1\kappa_1\kappa_2}\right)^{\phi_1}\left(\frac{r\left(\phi_1 - 1\right)}{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq + \Pi^N - \left(\Pi^A\right)_T - \zeta E^A + rK\left(E^N - E^A\right)}\right)^{\phi_1 - 1}$$

$$\Phi_{1,S} = \left(\frac{E^N - E^A}{\phi_1\kappa_1\kappa_2}\right)^{\phi_1}\left(\frac{r\left(\phi_1 - 1\right)}{-\int_0^{Q_N} p'(q)qdq + \int_0^{Q^A} p'(q)qdq + \Pi^N - \left(\Pi^A\right)_S + rK\left(E^N - E^A\right)}\right)^{\phi_1 - 1}.$$

It is easy to show that $\Phi_{1,T}$ equals $\Phi_{1,S}$. Let us consider the exponential term in (25) which accounts for the perpetual American option value. Then, substituting the expressions for $\Phi_{1,T}$ and $\Phi_{1,S}$ into the option term and comparing the two values, we find that the value of the option to reduce emissions under taxes is equal to the value of the option to reduce emission under emission standards. It should be noted that the objective function W (as of $t = 0$) should not be interpreted as the option value alone but as the social welfare function that includes the value arising from the welfare prior to the environmental policy adoption (capturing the profits earned by the non adopting firms plus the consumer surplus minus the social damage with no adoption) plus the option to implement an environmental policy (which would reduce social damage at a cost $K\left(E^N - E^A\right)$). By comparing the two value functions we see that the value of the welfare function W under taxes is equal to the value of the welfare function under emission standards. Repeating the same reasoning it is easy to show that the value of the welfare function W under emission standards is equivalent to the value of the welfare function W under auctioned permits. The result in proposition 2 follows.

Proof of Proposition 3

We want to show that when the cost of the policy implementation is the same among the policies, the optimal adoption threshold under freely allocated permits is larger than the adoption thresholds under emission standards for $0 < b < b^*$. Thus:

$$\begin{aligned} &\left(\frac{\phi_1 \kappa_1 \kappa_2}{\left(E^N - E^A\right)\left(\phi_1 - 1\right)}\right)\left(\frac{b\left[\left(Q^N\right)^2 - \left(Q^A\right)^2\right]}{2r} + \frac{\Pi^N - \left(\Pi^A\right)_{P^{Fr}}}{r} + K\left(E^N - E^A\right)\right) \\ = \quad &\left(\frac{\phi_1 \kappa_1 \kappa_2}{\left(E^N - E^A\right)\left(\phi_1 - 1\right)}\right)\left(\frac{b\left[\left(Q^N\right)^2 - \left(Q^A\right)^2\right]}{2r} + \frac{\Pi^N - \left(\Pi^A\right)_S}{r} + K\left(E^N - E^A\right)\right). \end{aligned}$$

By simplifying the expression above we get:

$$\left(\Pi^A\right)_{P^{Fr}} < \left(\Pi^A\right)_S .$$

Comparing the equilibrium profits yields:

$$b < \frac{4c_1c_2(a - \zeta)}{3(c_1(2c_2(\varepsilon_1 + \varepsilon_2) + \zeta) + c_2\zeta)},$$

where $\frac{4c_1c_2(a-\zeta)}{3(c_1(2c_2(\varepsilon_1+\varepsilon_2)+\zeta)+c_2\zeta)} > 0$. Since $b > 0$ by assumption, the result in proposition 3 follows.

Proof of Proposition 4

The fact that $\left(\Pi^A\right)_{P^{Fr}} < \left(\Pi^A\right)_S$ for $0 < b < b^*$ may also explain why the value of the welfare function W under freely allocated permits is smaller than

the value of the welfare function W under emission standards for $0 < b < b^*$.
Let us rewrite $\Phi_{1,P^{Fr}}$ and $\Phi_{1,S}$, as follows:

$$\Phi_{1,P^{Fr}} = \left(\frac{E^N - E^A}{\phi_1 \kappa_1 \kappa_2}\right)^{\phi_1} \left(\frac{2r(\phi_1 - 1)}{b\left[(Q^N)^2 - (Q^A)^2\right] + 2\left[\Pi^N - (\Pi^A)_{P^{Fr}}\right] + 2rK(E^N - E^A)}\right)^{\phi_1 - 1}$$

$$\Phi_{1,S} = \left(\frac{E^N - E^A}{\phi_1 \kappa_1 \kappa_2}\right)^{\phi_1} \left(\frac{2r(\phi_1 - 1)}{b\left[(Q^N)^2 - (Q^A)^2\right] + 2\left[\Pi^N - (\Pi^A)_S\right] + 2rK(E^N - E^A)}\right)^{\phi_1 - 1}.$$

It is immediate to show that $\Phi_{1,P^{Fr}} < \Phi_{1,S}$ if $0 < b < b^*$. Let us consider the exponential term in (25) which accounts for the perpetual American option value. Then, substituting the expressions for $\Phi_{1,P^{Fr}}$ and $\Phi_{1,S}$ into the option term and comparing the two values, we find that the value of the option to reduce emission under freely allocated permits is smaller than the value of the option to reduce emission under emission standards for $0 < b < b^*$. As before, the objective function W (as of $t = 0$) should not be interpreted as the option value alone but as the social welfare function that includes the value arising from the welfare prior to the environmental policy adoption plus the option to implement an environmental policy. By comparing the two value functions, the result in proposition 4 follows.