A Proofs of Propositions and Lemmas

Lemma 1 *The optimal maximum age must be greater than or equal to the maximum sustainable yield age, i.e* $n^* \geq n_{MSY}$*.*

Proof of Lemma 1.

From our assumptions on the age-yield function in section 2.1.2, for all yields less than the maximum yield, there are two ages that generate that yield. That is, for all $\bar{y} \in (0, y(n_{MSY}))$, there exist $n_{\bar{y}}^- < n_{MSY} < n_{\bar{y}}^+$, such that $y(n_{\bar{y}}^-) = y(n_{\bar{y}}^+) = \bar{y}$.

On the graph of the isoquant, these two n values generate the same area, $\bar{L}$, since $L = \frac{\bar{Q}}{y(n)}$ so $\frac{\bar{Q}}{y(n_{\bar{y}}^-)} = \frac{\bar{Q}}{y(n_{\bar{y}}^+)} = \bar{L}$.

Now compare the costs of these two n values.

$$\begin{aligned} C(n_{\bar{y}}^-, \bar{L}) - C(n_{\bar{y}}^+, \bar{L}) &= \left(C_f + \frac{C_n}{n_{\bar{y}}^-}\right)\bar{L} + C_D\, y(n_{\bar{y}}^-)\bar{L}^{1.5} - \left(C_f + \frac{C_n}{n_{\bar{y}}^+}\right)\bar{L} - C_D\, y(n_{\bar{y}}^+)\bar{L}^{1.5} \\ &= \left(C_f + \frac{C_n}{n_{\bar{y}}^-}\right)\bar{L} + C_D\, \bar{y}\bar{L}^{1.5} - \left(C_f + \frac{C_n}{n_{\bar{y}}^+}\right)\bar{L} - C_D\, \bar{y}\bar{L}^{1.5} \\ &= C_n\bar{L}\left(\frac{1}{n_{\bar{y}}^-} - \frac{1}{n_{\bar{y}}^+}\right)(> 0) \end{aligned}$$

Hence for any level of yield, the cost minimizing maximum age is greater than or equal to the maximum sustainable yield age, i.e. $n^* \geq n_{MSY}$. ■

Lemma 2 *The minimum of the isoquant is located at* n_{MSY}*.*

Proof of Lemma 2.

The isoquant is defined by $y(n)L = \bar{Q}$. This can be rewritten so that L is a function of n, i.e. for a particular level of feedstock production $L = \frac{\bar{Q}}{y(n)}$. The minimum of this function (i.e. the least quantity of land necessary to produce the desired quantity) occurs when the derivative of this function is set to zero.

$$\left.\frac{dL}{dn}\right|_{\text{isoquant}} = \frac{-\bar{Q}y'(n)}{[y(n)]^2} = 0 \Leftrightarrow y'(n) = 0$$

From the conditions imposed on the age-yield function in section 2.1.2 there is a unique maximum of the yield function located at n_{MSY}. Hence the unique minimum of the isoquant function occurs at n_{MSY}. ■

Lemma 3 *The minimum of the isocost curve is located at $n < n_{MSY}$.*

Proof of Lemma 3.

The isocost curve is defined by a level set of the cost function: $C(n, L) = \bar{C}$. We wish to locate the set of local extrema of the isocost curve, where L is expressed as a function of n. This set is a subset of the critical points of $\frac{dL}{dn}$.

Totally differentiate the cost function:

$$\left[\left(C_f + \frac{C_n}{n}\right) + 1.5\, C_D\, y(n)\, L^{0.5}\right] dL + \left[\frac{-C_n\, L}{n^2} + C_D\, y'(n)\, L^{1.5}\right] dn = 0$$

Thus

$$\left.\frac{dL}{dn}\right|_{\text{isocost}} = \frac{C_n\, L/n^2 - C_D\, y'(n)\, L^{1.5}}{\left(C_f + \frac{C_n}{n}\right) + 1.5\, C_D\, y(n)\, L^{0.5}} = 0 \Leftrightarrow C_n\, L/n^2 = C_D\, y'(n)\, L^{1.5}$$

since all the terms in the denominator are non-negative. The only term in this last equality that can change sign is $y'(n)$. All other terms are constrained to be non-negative. Hence the equality cannot be satisfied if $y'(n) < 0$, which occurs when $n > n_{MSY}$. Also, if $n = n_{MSY}$ it must be that $L = 0$ for the equality to be satisfied. If $L = 0$ we have $C(n_{MSY}, 0) = 0$, so for any positive level of cost $(n_{MSY}, 0)$ is not an element of the graph of the isocost function, and n_{MSY} cannot be a critical point.

Hence for any positive level of cost, any extrema of the isocost function must occur when $n < n_{MSY}$. ■

Lemma 4 *The isocost curve has a positive slope for all $n \geq n_{MSY}$.*

Proof of Lemma 4.

This follows immediately from the proof of lemma 3 since the expression for the slope of the isoquant curve is strictly positive for all $n > n_{MSY}$. ■

Proof of proposition 1.

The optimal n must be strictly greater than n_{MSY}, i.e. $n^* > n_{MSY}$.

The isocost curve has a positive slope for all $n \geq n_{MSY}$ (lemma 4). The isoquant curve has a zero slope at n_{MSY} (lemma 2). Hence the isocost and isoquant curves cannot be tangential at n_{MSY}, so $n^* \neq n_{MSY}$. Combining this with lemma 1 gives us the result.

■

Lemma 5 *Assumptions (1)-(4) imply that* $0 < \lim_{n\to\infty} \int_0^n f(a)\ da < \infty$

Proof of Lemma 5.

We can split $\lim_{n\to\infty} \int_0^n f(a)\ da$ in two by partitioning its domain:

$$\begin{aligned}\lim_{n\to\infty} \int_0^n f(a)\ da &= \lim_{n\to\infty} \int_0^k f(a)\ da + \lim_{n\to\infty} \int_k^n f(a)\ da \\ &= \int_0^k f(a)\ da + \lim_{n\to\infty} \int_k^n f(a)\ da\end{aligned}$$

Now consider $\int_0^k f(a)\ da$. The age-yield function is bounded below by 0 by construction ($f(a)$ represents a physical quantity). Assumptions (1)-(3) imply that $f(a)$ is bounded above. Hence $f(a)$ is bounded on the domain $[0, k]$ for all $k \in \mathbb{R}_{>0}$. Thus $0 \leq \int_0^k f(a)\ da < \infty$ since this is the integral of a bounded positive function on a finite domain.

We must consider two possibilities when analyzing $\lim_{n\to\infty} \int_k^n f(a)\ da$: either $f(a) > 0$ for all $a \in \mathbb{R}_{\geq 0}$, or there exists some $\hat{k} \in \mathbb{R}_{\geq 0}$ such that for all $a > \hat{k}$ $f(a) = 0$. In the first case, we must establish that $\lim_{n\to\infty} f(a)$ approaches zero fast enough that $\lim_{n\to\infty} \int_k^n f(a)\, da$ is not infinite. Assumption (4) implies that there exist $k \in \mathbb{R}_{\geq 0}$ and $p > 1$ such that for all $a > k$ $f(a) < \frac{1}{a^p}$ (if such k and p did not exist, $\lim_{n\to\infty} a\, f(a)$ would either be strictly positive, or infinite). Thus

$$\lim_{n\to\infty} \int_k^n f(a)\, da < \lim_{n\to\infty} \int_k^n \frac{1}{a^p}\, da < \infty$$

since integrals of the form $\int_k^\infty \frac{1}{x^p}\ dx$ are convergent if and only if $p > 1$. In the second case, $\lim_{n\to\infty} \int_{\hat{k}}^n f(a)\ da = 0$, and $\lim_{n\to\infty} \int_0^n f(a)\ da = \int_0^k f(a)\ da$. Thus $0 \leq \lim_{n\to\infty} \int_k^n f(a)\, da < \infty$

Assumption 3 implies that $f(a)$ is strictly positive on some subset of $\mathbb{R}_{\geq 0}$ with non-empty interior. Hence $\lim_{n\to\infty} \int_0^n f(a)\, da > 0$

Therefore $0 < \lim_{n\to\infty} \int_0^n f(a)\, da < \infty$ ■

Proof of proposition 2.

Given assumptions (1)-(4), a solution, n^*, to the cost minimization problem exists such that $n^* \in (n_{MSY}, \infty)$.

Sketch of the proof: We have already demonstrated that n^* must be greater than n_{MSY}. At n_{MSY} the slope of the isocost curve is strictly positive and the slope of the isoquant curve is zero. We show that as n approaches infinity, the slope of the isocost curve approaches zero, while the slope of the isoquant curve approaches a positive value. By continuity the slope functions must cross at least once, and hence there must exist at least one point where the isocost and isoquant curves are tangent to each other.

We begin by showing that the slope of the isocost curve approaches zero as n approaches infinity. The slope of the isocost function when L is written as a function of n (as derived in lemma 2)

$$\left.\frac{dL}{dn}\right|_{\text{isocost}} = \frac{C_n\, L(n)/n^2 - C_D\, y'(n)\, L(n)^{1.5}}{\left(C_f + \frac{C_n}{n}\right) + 1.5\, C_D\, y(n)\, L(n)^{0.5}}$$

To take the limit of this expression as n approaches infinity, we need to know how $L(n)$ on the isocost function behaves as n approaches infinity. The isocost function is defined as

$$C(n, L) = (C_f + \frac{C_n}{n})L + C_D\, y(n)\, L^{1.5} = \bar{C}$$

This implicitly defines L as a function of n.

$$C(n) = (C_f + \frac{C_n}{n})L(n) + C_D\, y(n)\, L(n)^{1.5} = \bar{C}$$

Now we take the limit of this expression as $n \to \infty$ and solve for the unknown value L_∞.

$$\begin{aligned}
&\lim_{n\to\infty} (C_f + \frac{C_n}{n})L(n) + C_D\, y(n)\, L(n)^{1.5} = \bar{C} \\
&\quad \Rightarrow C_f\, L_\infty = \bar{C} \\
&\quad \Rightarrow L_\infty = \frac{\bar{C}}{C_f} \qquad \text{A constant}
\end{aligned}$$

Returning to the derivative of the isocost function

$$\lim_{n\to\infty} \left.\frac{dL}{dn}\right|_{\text{isocost}} = \lim_{n\to\infty} \frac{C_n\, L(n)/n^2 - C_D\, y'(n)\, L^{1.5}}{\left(C_f + \frac{C_n}{n}\right) + 1.5\, C_D\, y(n)\, L(n)^{0.5}}$$

$$= \frac{0-0}{C_f + 0 + 0} \qquad \text{Since } y(n) \text{ and } y'(n) \text{ both approach 0, and } L(n) \text{ approaches a constant as } n \to \infty$$

$$= 0$$

Now we show that under a certain condition the slope of the isoquant function approaches a positive constant as $n \to \infty$. The isoquant function is given by $y(n)\, L = \bar{Q}$ and can be rewritten as

$$L = \frac{\bar{Q}}{\frac{1}{n}\int_0^n f(a)\, da} = \frac{\bar{Q}\, n}{\int_0^n f(a)\, da}$$

The slope of the isoquant function is given by

$$\left.\frac{dL}{dn}\right|_{\text{isoquant}} = \frac{\bar{Q}\, \left(\int_0^n f(a)\, da - n\, f(n)\right)}{\left[\int_0^n f(a)\, da\right]^2}$$

The limit of the slope as n approaches infinity is

$$\lim_{n\to\infty} \left.\frac{dL}{dn}\right|_{\text{isoquant}} = \lim_{n\to\infty} \frac{\bar{Q}\,\left(\int_0^n f(a)\,da - n\,f(n)\right)}{\left[\int_0^n f(a)\,da\right]^2}$$

$$= \bar{Q}\,\frac{\lim_{n\to\infty}\left(\int_0^n f(a)\,da - n\,f(n)\right)}{\lim_{n\to\infty}\left[\int_0^n f(a)\,da\right]^2} \qquad \text{since } \lim_{n\to\infty}\int_0^n f(a)\,da > 0 \text{ (Lemma 5)}$$

$$= \bar{Q}\,\frac{\overbrace{\lim_{n\to\infty}\int_0^n f(a)\,da - \lim_{n\to\infty} n\,f(n)}^{+}}{\underbrace{\lim_{n\to\infty}\left[\int_0^n f(a)\,da\right]^2}_{+}} \qquad \text{since } \lim_{n\to\infty}\int_0^n f(a)\,da > n\,f(n) = 0 \text{ and } 0 < \lim_{n\to\infty}\int_0^n f(a)\,da < \infty \text{ (Lemma 5)}$$

$$\implies 0 < \lim_{n\to\infty} \left.\frac{dL}{dn}\right|_{\text{isoquant}} < \infty$$

Now define a function that returns the difference in the slopes of the isocost and isoquant functions, $h(n) = \left.\frac{dL}{dn}\right|_{\text{isocost}} - \left.\frac{dL}{dn}\right|_{\text{isoquant}}$. Since both constituent functions are continuous on the the interval (n_{MSY}, ∞), $h(n)$ is also continuous on this interval. At the maximum yield age $h(n_{MSY}) > 0$ (from lemmas 2 and 4) and, as we have just shown, when n approaches infinity the limit of $h(n)$ is strictly less than zero. Hence by the intermediate value theorem, there must exist some $n^* \in (n_{MSY}, \infty)$ such that $h(n) = 0$, and the isocost and isoquant curves are tangent to one another. ■

A.1 Proofs of comparative static results

Proof of $\frac{dn^*}{d\bar{Q}} < 0$.

As processing facility size increases, the optimal age decreases, i.e. $\frac{dn^*}{d\bar{Q}} < 0$.

Totally differentiating $g(n, \bar{Q})$ (The derivative of the cost function when the constraint is used to eliminate L — derived in the proof of proposition 2) gives us an expression for the desired comparative static

$$\frac{dn^*}{d\bar{Q}} = \frac{-g_{\bar{Q}}}{g_n}$$

At an optimum the second order condition for a minimum must hold, so g_n must be positive. Hence

$$\text{sign}\left(\frac{dn^*}{d\bar{Q}}\right) = -\,\text{sign}(g_{\bar{Q}})$$

Differentiating $g(n, \bar{Q})$ with respect to $\bar{Q}$, and evaluating at the optimum yields

$$g_{\bar{Q}} = -0.25\,\underbrace{[y(n^*)]^{-1.5}}_{+}\,\underbrace{y'(n^*)}_{-}\,\underbrace{\bar{Q}^{-0.5}}_{+} \qquad (> 0)$$

Hence

$$\frac{dn^*}{d\bar{Q}} < 0$$

■

Proof of $y''(n*) < 0 \;\Rightarrow\; \frac{dL^*}{d\bar{Q}} > 0$.

The change in optimal growing region size with respect to a change in processing facility capacity is generally ambiguous, but if $y''(n^*) < 0$, then increased processing facility capacity leads to increase growing region size, i.e. $\frac{dL^*}{d\bar{Q}} > 0$.

To analyze this comparative static of the constrained cost minimization problem using the substitution method we need to define the inverse yield function, $g(y) = n$ $(g^{-1}(n) = y(n))$. Since the yield function is not surjective, we can only define and analyze the inverse yield on a subset of the domain. Fortunately, as shown by proposition 1, the optimal n is found in the subset $n > n_{MSY}$. On this subset the yield function is bijective, and we are guaranteed the existence of $g(y)$.

Using the constraint on processing facility capacity $(y(n)L = \bar{Q} \Rightarrow y(n) = \frac{\bar{Q}}{L}$ and $n = g(\frac{\bar{Q}}{L}))$ we can rewrite the cost function as a function of growing region only.

$$C(n(L), L) = \left(C_f + \frac{C_n}{g\left(\frac{\bar{Q}}{L}\right)} \right) L + C_D \, \bar{Q} L^{0.5}$$

The first order condition with respect to a minimum is

$$\frac{dC}{dL} = C_f + \frac{C_n}{g\left(\frac{\bar{Q}}{L}\right)} + \frac{\bar{Q}\, C_n \, g'\left(\frac{\bar{Q}}{L}\right)}{L\left[g\left(\frac{\bar{Q}}{L}\right)\right]^2} + (0.5)C_D \, \bar{Q} \, L^{-0.5} = 0$$

Cross multiply by $L\left[g\left(\frac{\bar{Q}}{L}\right)\right]^2$

$$h(L) = C_f \, L \left[g\left(\frac{\bar{Q}}{L}\right)\right]^2 + C_n \, L \, g\left(\frac{\bar{Q}}{L}\right) + C_n \, \bar{Q} \, g'\left(\frac{\bar{Q}}{L}\right) + (0.5)C_D \, \bar{Q} \, \left[g\left(\frac{\bar{Q}}{L}\right)\right]^2 L^{0.5} = 0$$

Totally differentiating $h(n, \bar{Q})$ gives us an expression for the desired comparative static

$$\frac{dL^*}{d\bar{Q}} = \frac{-h_{\bar{Q}}}{h_L}$$

At an optimum the second order condition for a minimum must hold, so g_n must be positive. Hence

$$\operatorname{sign}\left(\frac{dL^*}{d\bar{Q}}\right) = -\operatorname{sign}(h_{\bar{Q}})$$

$$\begin{aligned}
h_{\bar{Q}} = (0.5)C_D \left[g\left(\frac{\bar{Q}}{L}\right)\right]^2 L^{0.5} & \qquad (>) \qquad (16)\\
+ 2(0.5)C_D\,\bar{Q}\,g\left(\frac{\bar{Q}}{L}\right)g'\left(\frac{\bar{Q}}{L}\right)L^{-0.5} & \qquad (<) \qquad (17)\\
+ 2C_f\,g\left(\frac{\bar{Q}}{L}\right)g'\left(\frac{\bar{Q}}{L}\right) & \qquad (<) \qquad (18)\\
+ 2C_n g'\left(\frac{\bar{Q}}{L}\right) & \qquad (<) \qquad (19)\\
+ \frac{C_n\,\bar{Q}g''\left(\frac{\bar{Q}}{L}\right)}{L} & \qquad \text{(Ambiguous)} \qquad (20)
\end{aligned}$$

977 If $g''\left(\frac{\bar{Q}}{L}\right) < 0$ at L^*, the term 20 in $h_{\bar{Q}}$ is negative.

Aside: Rewriting this condition in terms of $y(n^*)$

This condition on the second derivative of the inverse yield function is not particularly intuitive. We can rewrite this condition in terms of $y(n^*)$ which makes it much easier to understand. To do this we must rewrite this condition on the second derivative of an inverse function in terms of the original function. The relationship between the second derivative of a function and its inverse is

$$(f^{-1})''(f(x)) = \frac{-f''(x)}{[f'(x)]^3}$$

For the inverse yield function this becomes

$$g''\left(\frac{\bar{Q}}{L}\right) = g''(y(n^*)) = \frac{-y''(n^*)}{[y'(n^*)]^3}$$

So

$$g''\left(\frac{\bar{Q}}{L}\right) < 0 \;\Leftrightarrow\; \frac{-y''(n^*)}{[y'(n^*)]^3} < 0$$

Since $y'(n^*) < 0$ and the cubing operation preserves sign, this inequality is satisfied if and only if $y''(n^*) < 0$.

Returning to the proof

Given that $y''(n^*) < 0$, we now show that term (16) plus term (17) is negative.

$$(16) + (17) = (0.5)C_D \left[g\left(\frac{\bar{Q}}{L}\right)\right]^2 L^{0.5} + 2(0.5)C_D\,\bar{Q}\,g\left(\frac{\bar{Q}}{L}\right)g'\left(\frac{\bar{Q}}{L}\right)L^{-0.5}$$

Extract common factors

$$(16) + (17) = \underbrace{(0.5)C_D\,g\left(\frac{\bar{Q}}{L}\right)L^{0.5}}_{>0}\left[g\left(\frac{\bar{Q}}{L}\right) + 2\bar{Q}\,g'\left(\frac{\bar{Q}}{L}\right)L^{-1}\right]$$

Therefore

$$\text{sign}\,((16) + (17)) = \text{sign}\left(g\left(\frac{\bar{Q}}{L}\right) + 2\bar{Q}\,g'\left(\frac{\bar{Q}}{L}\right)L^{-1}\right)$$

Substitute the definition of $\bar{Q} = y(n)\,L$

$$\begin{aligned}
&g\left(\frac{y(n)\,L}{L}\right) + 2y(n)\,L\,g'\left(\frac{y(n)\,L}{L}\right)L^{-1} \\
=&g(y(n)) + 2y(n)\,g'(y(n)) \\
=&n + \frac{2y(n)}{y'(n)} && \text{since } g(.) \text{ is inverse of } y(.)
\end{aligned}$$

Recall $y'(n) = \frac{f(n)-y(n)}{n}$, so

$$\begin{aligned} n + \frac{2y(n)}{y'(n)} =& n + \frac{2n\,y(n)}{f(n)-y(n)} \\ =& n\left(1 + \frac{2y(n)}{f(n)-y(n)}\right) \\ =& n\left(1 - \frac{2y(n)}{y(n)-f(n)}\right) \end{aligned}$$

For $n > n_{MSY}$, $y(n) > f(n) \geq 0$, hence $\frac{2y(n)}{y(n)-f(n)} > 1$, so $(16)-(17) < 0$, $h_{\bar{Q}} < 0$, and $\frac{dL^*}{d\bar{Q}} > 0$. ■

Proof of remaining comparative statics.

See table on page 20

As explained in the proofs the previous two comparative statics, the sign of the comparative static of n^* and L^* with respect to any exogenous variable x can be found by analyzing the sign of the relevant derivative of the first order condition, i.e.

$$\text{sign}\left(\frac{dn^*}{dx}\right) = -\,\text{sign}(g_x) \quad \text{and} \quad \text{sign}\left(\frac{dL^*}{dx}\right) = -\,\text{sign}(h_x)$$

We now present and sign the expressions of g_x for the parameters of interest.

$$g_{C_f} = -\frac{\bar{Q}\,\overbrace{y'(n^*)}^{-}}{y(n^*)^2} \qquad (>0)$$

$$g_{C_n^*} = \underbrace{\frac{-\bar{Q}}{n^*\,y(n^*)}}_{-}\underbrace{\left[\frac{1}{n^*} + \frac{y'(n^*)}{y(n^*)}\right]}_{+} \quad (<0) \qquad \text{Since } \varepsilon_{y(n^*)>-1} \Rightarrow \tfrac{1}{n^*} + \tfrac{y'(n^*)}{y(n^*)} > 0 \quad \text{(Prop 2)}$$

$$g_{C_D} = \underbrace{(-0.5)}_{-} \bar{Q}^{1.5}\, y(n^*)^{-1.5} \underbrace{y'(n^*)}_{-} \qquad (> 0)$$

We now present and sign the expressions of h_x for the parameters of interest.

$$h_{C_f} = L(n^*)^2 \qquad (> 0)$$

$$h_{C_n} = Ln^* + \frac{\bar{Q}}{y'(n^*)} \quad (< 0) \qquad \text{Since } \varepsilon_{y(n^*)>-1} \Rightarrow Ln^* + \frac{\bar{Q}}{y'(n^*)} < 0 \quad \text{(Prop 2)}$$

$$h_{C_D} = (0.5)\,(n^*)^2\, \bar{Q}\, L^{0.5} \qquad (> 0)$$

■

B Calibration of the Cost-Minimization Problem

A piecewise linear spline function is used to estimate the age-yield function. With this functional form, the cost minimization problem has 9 parameters for which values must be found.

B.1 Brazilian Sugarcane

B.1.1 Piece-wise Linear Age-Yield Function: t_1, t_{max}, t_T, f_{max}

The piece-wise linear age-yield function has four parameters. Parameter t_1 designates the age at which the yield first becomes positive, t_{max} is the age at which maximum yield is achieved, and t_T is the age at which yield returns to zero. During the increasing phase, between t_1 and t_{max} the function is a positive affine and during the decreasing phase, between t_{max} and t_T, the function is negative affine.

To estimate the parameters, we fit the linear-piecewise function to age-yield data obtained from Margarido and Santos (2012). Figure 6 shows the original data and the fitted age-yield function.

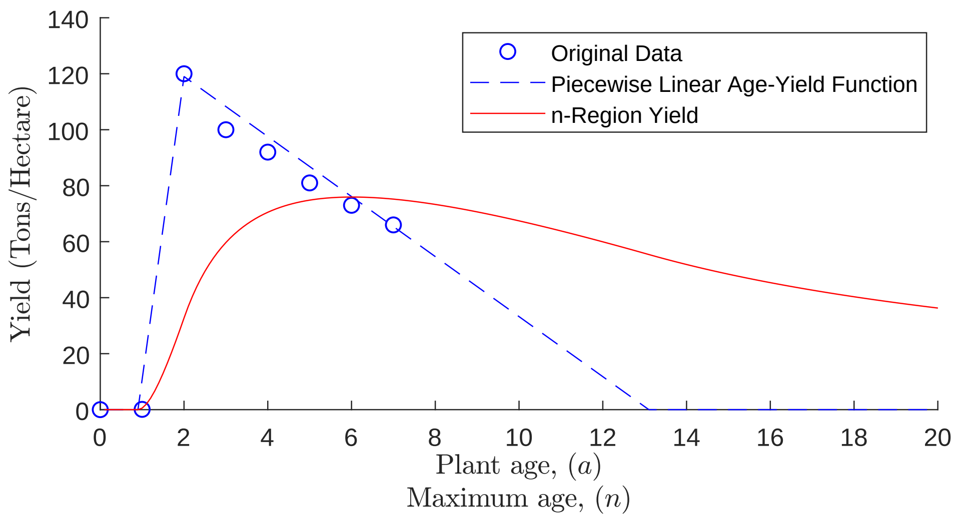

Figure 6: Fitting the piecewise-linear age-yield function to the Brazilian age-yield data from Margarido and Santos (2012).

The parameter values obtained are $t_1 = 1$, $t_{max} = 2$, $t_T = 13$, and $f_{max} = 120$.

B.1.2 Farm-gate Cost parameters: C_f, C_n

We derived the feedstock cost parameters, C_f and C_n, from Teixeira (2013), and the delivery cost parameter, C_D, from Crago et al. (2010).

Teixeira (2013) presents an example operating budget for a 5-cut (6-age-class) sugarcane operation in São Paulo state, where they assume that 80 percent of the cane is harvested burned, and 20 percent is harvested raw. Costs are divided into five categories, delivery costs, and four that account for farm gate feedstock costs: preparing the soil, planting, harvest, and maintenance of the ratoon. The total farm gate feedstock costs for a 6 hectare operation is given by

$$\begin{aligned}\substack{\text{Total Farm Gate}\\\text{Feedstock Costs}} =& \text{Soil Preparation} + \text{Planting}\\ &+ 5 \times \text{Harvest} + 4 \times \text{Ratoon maintenance}\end{aligned}$$

Since the total cost is given for 6 hectares, the total cost per hectare is

$$\begin{aligned}\substack{\text{Total Farm Gate}\\\text{Feedstock Costs}\\\text{(Per Hectare)}} =& \frac{1}{6} \times \text{Soil Preparation} + \frac{1}{6} \times \text{Planting}\\ &+ \frac{5}{6} \times \text{Harvest} + \frac{4}{6} \times \text{Ratoon Maintenance}\end{aligned}$$

Assuming that these cost parameters are constant with respect to the number of age-classes we can write the total farm gate feedstock per hectare as a function of the age structure

$$\begin{aligned}\substack{\text{Farm gate}\\\text{feedstock costs}}(n) =& \frac{1}{n} \times \text{Soil Preparation} + \frac{1}{n} \times \text{Planting}\\ &+ \frac{n-1}{n} \times \text{Harvest} + \frac{n-2}{n} \times \text{Ratoon maintenance}\end{aligned}$$

Substituting Teixeria's numbers (in Reals) from the example budget, the cost function becomes

$$\substack{\text{Farm gate}\\\text{feedstock costs}}(n) = \frac{656.07}{n} + \frac{4159.83}{n} + \frac{n-1}{n} \times 1273.13 + \frac{n-2}{n} \times 986.54$$

Which on rearranging becomes

$$\substack{\text{Farm gate}\\\text{feedstock costs}}(n) = 2259.67 + \frac{1569.69}{n}$$

Hence for the simulations we use a baseline of $C_f = 2259.67$ and $C_n = 1569.69$.

B.1.3 Delivery Cost parameter: C_D

While Teixeira (2013) does include estimates of delivery costs, he does not include the processing facility size that this example farm is feeding. We therefore turn to Crago et al. (2010) to derive the delivery cost parameter.

The total delivery cost from a growing region is given by

$$\begin{aligned} \substack{\text{Total} \\ \text{Delivery Costs}} = & \text{Average Cost Per Ton Kilometer} \\ & \times \text{Quantity Transported} \\ & \times \text{Average Delivery Distance} \end{aligned}$$

Let δ represent the average delivery cost per ton kilometer (i.e. the average cost to transport one ton of feedstock one kilometer). Crago et al. (2010) report an average transport cost of R\$6.7 to transport a ton of feedstock from the farm gate to the mill. The average delivery distance in this study was 22 kilometers so in this case $\delta = 0.3045$.

The average mill size in Crago et al. (2010) is 4.8 million tons. Given our assumption that the growing region produces the exact quantity required to feed the mill, this implied that the average quantity of feedstock transported was 4.8 million tons.

When calculating the average delivery distance, we must make a distinction between the area of land planted with sugarcane, L, and the area of the growing region, A. Although we are assuming that the growing region is circular, it is not necessarily the case that all the land is planted with sugarcane. In fact, relaxing the link between planted area and growing region area is necessary to correctly calibrate the model to the data in Crago et al. (2010).

Let d be the average density of sugarcane fields in the growing region, and A be the area of the growing region. Hence

$$\mathrm{L} = d \times A$$

The average delivery distance is given by the expression

$$r_{av} = \frac{2}{3} r_{max} = \frac{2}{3}\sqrt{\frac{A}{\pi}}$$

Since the average delivery distance, r_{av}, from Crago et al. (2010) is 22km, the size of the growing region is $A = 342\,119$ ha.

We calculate the density parameter from

$$\text{Total Quantity} = \text{Yield} \times \text{Density} \times \text{Growing Region Area}$$

Crago et al. (2010) reports an average yield of 75 tons per hectare. So we calculate the density as

$$4800000 = 75 \times d \times 342119 \Rightarrow d = 0.187$$

Hence the expression for the total delivery cost becomes

$$
\begin{aligned}
\text{Total Delivery Costs} &= \delta \times Q \times r_{av} \\
&= \delta \times Q \times \frac{2}{3}\sqrt{\frac{A}{\pi}} \\
&= \delta \times Q \times \frac{2}{3}\sqrt{\frac{L}{d \times \pi}} \\
&= \frac{2\delta}{3}\sqrt{\frac{1}{d \times \pi}} \times Q \times \sqrt{L} \\
&= \frac{2\delta}{3}\sqrt{\frac{1}{d \times \pi}} \times y(n)L\sqrt{L} \\
&= \frac{2\delta}{3}\sqrt{\frac{1}{d \times \pi}} \times y(n)L^{1.5} \\
&= C_D \times y(n)L^{1.5}
\end{aligned}
$$

For the d and δ derived from Crago et al. (2010), $C_D = 0.2649$.

B.2 Calibrated parameters and ranges used in simulations

	Parameter	**Min Value**	**Calibration**	**Max Value**
Yield	t_1	0	1	2
	t_{max}	$t_1 + 1$	2	$t_1 + 5$
	t_T	$t_{max} + 7$	13	$t_{max} + 13$
	f_{max}	60	120	180
Cost	C_f	1129.84	2259.67	3389.51
	C_n	784.85	1569.69	2354.54
	C_D	0.13	0.26	0.40
Capacity	$\bar{Q}$	1 000 000	19 000 000	36 000 000

Table 3: Support for random parameters used in cost minimization. The parameters are drawn from a uniform distribution centered on the Brazilian calibration