**Online Appendix**

**The Impact of an Increase to a Minimum Wage on the Egg Industry**

**Background on Egg Production**

Pullets are female chicks that are old enough to develop feathers but too young to lay full-sized eggs. During the “growing stage,” the newly-acquired chicks are kept for many weeks, maturing into pullets. At 17 to 20 weeks of age, pullets are sent to the egg-laying facilities. Here, the hens have an egg-laying life of about 75 weeks before being replaced by fresh pullets from the grower facilities (Bell, 2002). While many grocery stores offer different types of eggs (e.g., different colors, sizes, cage-free, organic, omega-3, etc.), each type’s production process is similar except for locally-sourced or home-grown eggs. Even cage-free eggs share the same style of the supply chain as traditional eggs, aside from the structure of the laying facilities. Because of the similarities between production processes, we aggregate and designate all these types of eggs as “table eggs.”

 At the egg-laying facility, and after the hens lay eggs, workers collect the eggs, wash them with sprays and brushes, candle and sort them, package them, and ship them to retail outlets.2 Some facilities further process their eggs via breaker machines, creating liquid, powdered, or frozen eggs. Specifically, breakers crack the eggs, and the liquid is gravity-fed into vats. Filters strain the egg liquid to remove shells, and the eggs are pumped through another filter to be cooled. Next, the egg liquid is pumped out and transported for pasteurization. Pasteurization requires a short period of high temperatures, followed by a longer period of relatively lower temperatures. At this point, the eggs are packaged in liquid form or can be frozen or dried. Frozen eggs have a longer shelf life than table eggs or liquid eggs, so long as they are kept at the correct temperature. Alternatively, the liquid eggs can be dried using a variety of methods to produce the form of eggs with the longest shelf life. Dried eggs can be further improved by introducing chemicals to extend shelf life, enhance flavor, and add vitamins, among other possibilities. We refer to these processed eggs as, “processed eggs” as opposed to the unprocessed “table eggs” and refers to both as “eggs.”

*Egg Marketing and Distribution*

The production of eggs from breeders to retailers involves the farm-level breeder/hatchery firm (“breeder”) and the wholesale level grower/layer/breaker firm (“egg producer”), as shown in Figure 1. The eggs are sold to institution/restaurant/retailer firms (“retailer”). The egg producer has contracts with the breeder to secure the female chicks, as well as contracts with retailers to sell its eggs. A third-party organization known as the Egg Clearinghouse (“the clearinghouse”) has emerged and facilitates the heavily-contracted egg industry (MacDonald, Hoppe, and Newton, 2018). Egg producers that have, for whatever reason, produced above what is required by their marketing contracts with retailers can offload their excess table eggs at the clearinghouse. Conversely, the clearinghouse provides egg producers whose production has fallen short of their contractual obligations a means to easily cover their shortcomings. While the clearinghouse is an important part of the egg industry, it is not considered in this research.

**Deriving the Basic Equilibrium Displacement Model**

The basic equilibrium displacement model (EDM) is comprised of four primary equations:

|  |  |  |
| --- | --- | --- |
| (1) |  | $$q=q^{D}\left(p\right)$$ |
| (2) |  | $x=x^{S}\left(w\right)$. |
| (3) |  | $$q=f\left(x\right)$$ |
| (4) |  | $$pf\_{x}-w=0$$ |

Where (1) represents the demand for the output good, and (2) represents the supply of the input, $q^{D}$ is the output quantity demanded, $p$ is the output price, $x$ is the input quantity, and $w$ is the input price. Equation (3) is the production function and (4) the optimum condition for the unconstrained profit maximization problem. The primary equations (1) – (4) can be mathematically manipulated into their final EDM forms of equations (5) – (8):

|  |  |  |
| --- | --- | --- |
| (5) |  | $$E\left(q\right)=η\_{D}E\left(p\right)$$ |
| (6) |  | $E\left(x\_{i}\right)=Σ\_{j}κ\_{j}ε\_{ij}E\left(w\_{j}\right)$. |
| (7) |  | $$E\left(p\right)=Σ\_{j}κ\_{j}E\left(w\_{j}\right)$$ |
| (8) |  | $$E\left(x\_{i}\right)=E\left(q\right)+Σ\_{j}κ\_{j}σ\_{ij}E\left(w\_{j}\right)$$ |

Equations (5) – (8) are the basis for the EDM used in this research. The main text demonstrates how these equations can be further modified to represent the egg industry. The following describes the derivation of the basic EDM from the primary equations.

**Deriving the Demand Equation**

Start with the demand function in equation (1):

|  |  |  |
| --- | --- | --- |
|  | $$q=q^{D}\left(p\right)$$ |  |

Totally differentiate the equation:

|  |  |  |
| --- | --- | --- |
|  | $dq=\left(\frac{dq^{D}}{dp}\right)dp.$  |  |

Divide both sides by $q$:

|  |  |  |
| --- | --- | --- |
|  | $\frac{dq}{q}=\left(\frac{1}{q}\right)\frac{dq^{D}}{dp}dp$.  |  |

Multiply the right-hand side by $p/p:$

|  |  |  |
| --- | --- | --- |
|  | $\frac{dq}{q}=\left(\frac{1}{q}\right)\left(\frac{p}{p}\right)\frac{dq^{D}}{dp}dp.$  |  |

Rearrange in order to put the equation into elasticity form:

|  |  |  |
| --- | --- | --- |
|  | $\frac{dq}{q}=\left(\frac{dq^{D}}{dp}\frac{p}{q}\right)\left(\frac{dp}{p}\right)$. |  |

Rewrite by inserting the appropriate elasticity symbols to get equation (5)

|  |  |
| --- | --- |
|  | $E\left(q\right)=η\_{D}E\left(p\right)$. |

**Deriving the Input Supply Equation**

A similar approach is taken to derive the input supply function in equation (6). Start with equation (2)

|  |  |  |
| --- | --- | --- |
|  | $x=x^{S}\left(w\right)$. |  |

By total differentiation:

|  |  |  |
| --- | --- | --- |
|  | $dx=\left(\frac{dx^{S}}{dw}\right)dw$.  |  |

Divide both sides by $x$ to get:

|  |  |  |
| --- | --- | --- |
|  | $\frac{dx}{x}=\left(\frac{1}{x}\right)\frac{dx^{S}}{dw}dw$  |  |

Multiply the right-hand side by $w/w$ to get:

|  |  |  |
| --- | --- | --- |
|  | $\frac{dx}{x}=\left(\frac{1}{x}\right)\left(\frac{w}{w}\right)\frac{dx^{S}}{dw}dw.$  |  |

Rearrange in order to put the equation into elasticity form:

|  |  |  |
| --- | --- | --- |
|  | $\frac{dx}{x}=\left(\frac{dx^{S}}{dw}\frac{w}{x}\right)\left(\frac{dw}{w}\right)$  |  |

Rewrite by inserting the appropriate elasticity symbols to get:

|  |  |  |
| --- | --- | --- |
|  | $$E\left(x\right)=ε^{S}E\left(w\right)$$ |  |

Finally, for the case of $n$ inputs, the elasticity of supply is replaced by the weighted sum of input supply elasticities, weighted by factor share $κ\_{j},$ so as to arrive at equation (B6):

|  |  |
| --- | --- |
|  | $E\left(x\_{i}\right)=\sum\_{j}^{}κ\_{j}ε\_{ij}E\left(w\_{j}\right) for i=1,2,…,n$. |

**Deriving the Optimum Condition and Production Function**

Starting with the production function (3) and the optimum condition (4), take the total differentials:

|  |  |  |
| --- | --- | --- |
|  | $$dq=f\_{x}dx$$ |  |
|  | $f\_{x}dp+pf\_{xx}dx-dw=0$. |  |

These two equations can be put into matrix form:

|  |  |  |
| --- | --- | --- |
| (9) |  | $\left[\begin{matrix}0&f\_{1}&f\_{2}&\cdots &f\_{n}\\f\_{1}&pf\_{11}&pf\_{12}&\cdots &pf\_{1n}\\f\_{2}&pf\_{21}&pf\_{22}&\cdots &pf\_{2n}\\\vdots &\vdots &\vdots &\ddots &\vdots \\f\_{n}&pf\_{n1}&pf\_{n2}&\cdots &pf\_{nn}\end{matrix}\right]\left[\begin{matrix}dp\\dx\_{1}\\dx\_{2}\\\vdots \\dx\_{n}\end{matrix}\right]=\left[\begin{matrix}1&0&0&\cdots &0\\0&1&0&\cdots &0\\0&0&1&\cdots &0\\\vdots &\vdots &\vdots &\ddots &\vdots \\0&0&0&\cdots &1\end{matrix}\right]\left[\begin{matrix}dq\\dw\_{1}\\dw\_{2}\\\vdots \\dw\_{n}\end{matrix}\right]$. |

The system in (9) has four matrices—two of which are vectors, and one is an identity matrix. From left to right, they are in the form $A ⋅B=I ⋅X$. Ignoring $I,$ the identity matrix (its presence does not affect the outcome), the explicit solutions for this system of equations can be found by solving for the $B $vector: $B=A^{-1} ⋅X$. The results are expressed in terms of $q$ and $w$:

|  |  |  |
| --- | --- | --- |
| (10) |  | $$p=p^{\*}\left(q,w\right)$$ |
| (11) |  | $x=x^{\*}\left(q,w\right)$. |

To solve the system, take the total differential of each solution, (10) and (11), to get

|  |  |  |
| --- | --- | --- |
| (12) |  | $dp=\left(\frac{∂p^{\*}}{∂q}\right)dq+Σ\_{j}\left(\frac{∂p^{\*}}{∂w\_{j}}\right)dw\_{j}$  |
| (13) |  | $dx\_{i}=\left(\frac{∂x\_{i}^{\*}}{∂q}\right)dq+Σ\_{j}\left(\frac{∂x\_{i}^{\*}}{∂w\_{j}}\right)dw\_{j}$. |

From these two equations, $\left(\frac{∂p^{\*}}{∂q}\right), \left(\frac{∂p^{\*}}{∂w\_{j}}\right)$,$ \left(\frac{∂x\_{i}^{\*}}{∂q}\right)$, and$ \left(\frac{∂x\_{i}^{\*}}{∂w\_{j}}\right)$ are of interest. With some algebra it can be shown that the values of the derivatives are:

|  |  |  |
| --- | --- | --- |
| (14) |  | $\left(\frac{∂p^{\*}}{∂q}\right)=0$, $\left(\frac{∂x\_{j}^{\*}}{∂q}\right)=\left(\frac{∂p^{\*}}{∂w\_{j}}\right)=\frac{x\_{j}}{q}$,and $\left(\frac{∂x\_{i}^{\*}}{∂w\_{j}}\right)=\frac{κ\_{j}σ\_{ij}x\_{i}}{w\_{j}}$. |

Hence,

|  |  |  |
| --- | --- | --- |
| (15) |  | $dp=\left(0\right)dq+Σ\_{j}\left(\frac{x\_{j}}{q}\right)dw\_{j}=Σ\_{j}\left(\frac{x\_{j}}{q}\right)dw\_{j}$  |
| (16) |  | $dx\_{i}=\left(\frac{x\_{j}}{q}\right)dq+Σ\_{j}\left(\frac{κ\_{j}σ\_{ij}x\_{i}}{w\_{j}}\right)dw\_{j}$. |

These expressions can be converted to EDM form:

|  |  |  |
| --- | --- | --- |
| (17) |  | $\frac{dp}{p}=Σ\_{j}\frac{w\_{j}x\_{j}}{pq}\frac{dw\_{j}}{w\_{j}}=Σ\_{j}\frac{w\_{j}x\_{j}}{w∙x}\frac{dw\_{j}}{w\_{j}}=Σ\_{j}κ\_{j}\frac{dw\_{j}}{w\_{j}}=Σ\_{j}κ\_{j}E\left(w\_{j}\right)$  |
| (18) |  | $\frac{dx\_{i}}{x\_{i}}=\frac{1}{q}dq+Σ\_{j}\frac{κ\_{j}σ\_{ij}}{w\_{j}}dw\_{j}=\frac{dq}{q}+Σ\_{j}κ\_{j}σ\_{ij}\frac{dw\_{j}}{w\_{j}}=E\left(q\right)+E\_{j}κ\_{j}σ\_{ij}E\left(w\_{j}\right)$. |

Equations (17) and (18) show the EDM versions of the production function and optimum conditions, respectively.

**References**

MacDonald, J.M., R.A. Hoppe, and D. Newton. 2018. *Three Decades of Consolidation in U.S. Agriculture,* EIB-189, U.S. Department of Agriculture, Economic Research Service.