Online Supplemental Appendix

The market-clearing and spatial-arbitrage conditions for the United States, India, China, and Vietnam are

$$X_{UU} + X_{IU} + X_{CU} + X_{VU} = Q_U^D\left(P_U^D\right)$$

$$X_{UI} + X_{II} + X_{CI} + X_{VI} = Q_I^D\left(P_I^D\right)$$

$$X_{UC} + X_{IC} + X_{CC} + X_{VC} = Q_C^D\left(P_C^D\right)$$

$$X_{UV} + X_{IV} + X_{CV} + X_{VV} = Q_V^D\left(P_V^D\right)$$

$$Q_U^S\left(P_U^S\right) = X_{UU} + X_{UI} + X_{UC} + X_{UV}$$

$$Q_I^S\left(P_I^S\right) = X_{IU} + X_{II} + X_{IC} + X_{IV}$$

$$Q_C^S\left(P_C^S\right) = X_{CU} + X_{CI} + X_{CC} + X_{CV}$$

$$Q_V^S\left(P_V^S\right) = X_{VU} + X_{VI} + X_{VC} + X_{VV}$$

$$P_U^D = \left(P_U^S - S_U + T_{UU} + \tau_{UU}\right) * E_{UU}$$

$$P_U^D = \left(P_I^S - S_I + T_{IU} + \tau_{IU}\right) * E_{IU}$$

$$P_U^D = \left(P_C^S - S_C + T_{CU} + \tau_{CU}\right) * E_{CU}$$

$$P_U^D = \left(P_V^S - S_V + T_{VU} + \tau_{VU}\right) * E_{VU}$$

$$P_I^D = \left(P_U^S - S_U + T_{UI} + \tau_{UI}\right) * E_{UI}$$

$$P_I^D = \left(P_I^S - S_I + T_{II} + \tau_{II}\right) * E_{II}$$

$$P_I^D = \left(P_C^S - S_C + T_{CI} + \tau_{CI}\right) * E_{CI}$$

$$P_I^D = \left(P_V^S - S_V + T_{VI} + \tau_{VI}\right) * E_{VI}$$

$$P_C^D = \left(P_U^S - S_U + T_{UC} + \tau_{UC}\right) * E_{UC}$$

$$P_C^D = \left(P_I^S - S_I + T_{IC} + \tau_{IC}\right) * E_{IC}$$

$$P_C^D = \left(P_C^S - S_C + T_{CC} + \tau_{CC}\right) * E_{CC}$$

$$P_C^D = \left(P_V^S - S_V + T_{VC} + \tau_{VC}\right) * E_{VC}$$

$$P_V^D = \left(P_U^S - S_U + T_{UV} + \tau_{UV}\right) * E_{UV}$$

$$P_V^D = \left(P_I^S - S_I + T_{IV} + \tau_{IV}\right) * E_{IV}$$

$$P_V^D = \left(P_C^S - S_C + T_{CV} + \tau_{CV}\right) * E_{CV}$$

$$P_V^D = \left(P_V^S - S_V + T_{VV} + \tau_{VV}\right) * E_{VV}$$

Assuming that all internal transport costs, T_{ii}, are equal to 0, all tariff rates and subsidies are equal to 0 with the exception of the tariff between the United States and China, τ_{UC}, and all exchange rates are equal to 1 with the exception of the exchange rate between the United States and China, E_{UC}, we get

$$X_{UU} + X_{IU} + X_{CU} + X_{VU} = Q_U^D\left(P_U^D\right)$$

$$X_{UI} + X_{II} + X_{CI} + X_{VI} = Q_I^D\left(P_I^D\right)$$

$$X_{UC} + X_{IC} + X_{CC} + X_{VC} = Q_C^D\left(P_C^D\right)$$

$$X_{UV} + X_{IV} + X_{CV} + X_{VV} = Q_V^D\left(P_V^D\right)$$

$$Q_U^S\left(P_U^S\right) = X_{UU} + X_{UI} + X_{UC} + X_{UV}$$

$$Q_I^S\left(P_I^S\right) = X_{IU} + X_{II} + X_{IC} + X_{IV}$$

$$Q_C^S\left(P_C^S\right) = X_{CU} + X_{CI} + X_{CC} + X_{CV}$$

$$Q_V^S\left(P_V^S\right) = X_{VU} + X_{VI} + X_{VC} + X_{VV}$$

$$P_U^D = P_U^S$$

$$P_U^D = P_I^S + T_{IU}$$

$$P_U^D = P_C^S + T_{CU}$$

$$P_U^D = P_V^S + T_{VU}$$

$$P_I^D = P_U^S + T_{UI}$$

$$P_I^D = P_I^S$$

$$P_I^D = P_C^S + T_{CI}$$

$$P_I^D = P_V^S + T_{VI}$$

$$P_C^D = \left(P_U^S + T_{UC} + \tau_{UC}\right) * E_{UC}$$

$$P_C^D = P_I^S + T_{IC}$$

$$P_C^D = P_C^S$$

$$P_C^D = P_V^S + T_{VC}$$

$$P_V^D = P_U^S + T_{UV}$$

$$P_V^D = P_I^S + T_{IV}$$

$$P_V^D = P_C^S + T_{CV}$$

$$P_V^D = P_V^S$$

By inferring which bilateral trade shipments are likely to occur, we are able to write each countries' price as a function of the US producer price, P_U^S. Firstly, since $P_U^D = P_U^S$ we know that US producer and consumer prices are equal and that US producer price can simply be substituted for US consumer price. Similarly, because China imports from the United States we know that the Chinese consumer price is equal to US producer price plus transport and tariff costs all multiplied by the exchange rate, i.e., $P_C^D = \left(P_U^S + T_{UC} + \tau_{UC}\right) * E_{UC}$. Also, because $P_C^D = P_C^S$we know that $P_C^D = \left(P_U^S + T_{UC} + \tau_{UC}\right) * E_{UC}$. In the same manner we find that the producer and consumer prices in Vietnam are both equal to $P_U^S + T_{UV}$. Lastly, because India exports to China, we know that the producer price in India is equal to the consumer price in China minus transport costs from India to China, T_{IC}. Therefore, $P_I^S = \left(P_U^S + T_{UC} + \tau_{UC}\right) * E_{UC} - T_{AC}$ and $P_I^D = \left(P_U^S + T_{UC} + \tau_{UC}\right) * E_{UC} - T_{AC}$. Substituting these values for the prices in the market clearing conditions gives

$$\begin{gathered}
X_{UU} + X_{IU} + X_{CU} + X_{VU} = Q_U^D\left(P_U^S\right), \\
X_{UI} + X_{II} + X_{CI} + X_{VI} = Q_I^D\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC} - T_{IC}\right), \\
X_{UC} + X_{IC} + X_{CC} + X_{VC} = Q_C^D\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC}\right), \\
X_{UV} + X_{IV} + X_{CV} + X_{VV} = Q_V^D\left(P_U^S + T_{UV}\right), \\
Q_U^S\left(P_U^S\right) = X_{UU} + X_{UI} + X_{UC} + X_{UV}, \\
Q_I^S\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC} - T_{IC}\right) = X_{IU} + X_{II} + X_{IC} + X_{IV}, \\
Q_C^S\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC}\right) = X_{CU} + X_{CI} + X_{CC} + X_{CV},
\end{gathered}$$

and

$$Q_V^S\left(P_U^S + T_{UV}\right) = X_{VU} + X_{VI} + X_{VC} + X_{VV}.$$

This is a system of 8 equations and 17 endogenous variables. To convert this into a square system and further condense the model, several variables can be logically excluded. For example, importing countries do not export and we can exclude X_{CU}, X_{CI}, X_{CV},X_{VU},X_{VI}, and X_{VC}. In addition, exporting countries do not trade among themselves, and we can ignore X_{IU} and X_{UI}. Further, we assume India does not export to Vietnam, $X_{IV} = 0$.[1] These

[1] This assumption is only in the theoretical analysis to make the model square. However, in the empirical analysis we allow bilateral trade between any pair of countries, because any country i can potentially export to country j.

simplifications lead to $Q_U^D\left(P_U^S\right) = X_{UU}$, $Q_I^D\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC} - T_{IC}\right) = X_{II}$, $Q_C^S\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC}\right) = X_{CC}$, and $Q_V^S\left(P_U^S + T_{UV}\right) = X_{VV}$. That is, for an exporting country, total internal demand is met by exports to itself, and for an importing country, total supply is equal to exports to itself. By plugging these relationships into the remaining equations, the simplified version of the model is

$$X_{UC} + X_{IC} + Q_C^S\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC}\right) = Q_C^D\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC}\right), \tag{1}$$

$$X_{UV} + Q_V^S\left(P_U^S + T_{UV}\right) = Q_V^D\left(P_U^S + T_{UV}\right), \tag{2}$$

$$Q_U^S\left(P_U^S\right) = Q_U^D\left(P_U^S\right) + X_{UC} + X_{UV}, \tag{3}$$

and

$$Q_I^S\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC} - T_{IC}\right) = Q_I^D\left(\left[P_U^S + T_{UC} + \tau_{UC}\right] * E_{UC} - T_{IC}\right) + X_{IC}. \tag{4}$$

Totally differentiating equations (1) - (4) while treating transport costs and tariff rates as constant results in the following.

$$dX_{UC} + dX_{IC} + \frac{\partial Q_C^S}{\partial P_C^S}\frac{\partial P_C^S}{\partial P_U^S}dP_U^S + \frac{\partial Q_C^S}{\partial P_C^S}\frac{\partial P_C^S}{\partial E_{UC}}dE_{UC} = \frac{\partial Q_C^D}{\partial P_C^D}\frac{\partial P_C^D}{\partial P_U^S}dP_U^S + \frac{\partial Q_C^D}{\partial P_C^D}\frac{\partial P_C^D}{\partial E_{UC}}dE_{UC}$$

$$dX_{UV} + \frac{\partial Q_V^S}{\partial P_V^S}\frac{\partial P_V^S}{\partial P_U^S}dP_U^S = \frac{\partial Q_V^D}{\partial P_V^D}\frac{\partial P_V^D}{\partial P_U^S}dP_U^S$$

$$\frac{\partial Q_U^S}{\partial P_U^S}dP_U^S = \frac{\partial Q_U^D}{\partial P_U^D}\frac{\partial P_U^D}{\partial P_U^S}dP_U^S + dX_{UC} + dX_{UV}$$

$$\frac{\partial Q_I^S}{\partial P_I^S}\frac{\partial P_I^S}{\partial P_U^S}dP_U^S + \frac{\partial Q_I^S}{\partial P_I^S}\frac{\partial P_I^S}{\partial E_{UC}}dE_{UC} = \frac{\partial Q_I^D}{\partial P_I^D}\frac{\partial P_I^D}{\partial P_U^S}dP_U^S + \frac{\partial Q_I^D}{\partial P_I^D}\frac{\partial P_I^D}{\partial E_{UC}}dE_{UC} + dX_{IC}$$

Simplifying and combining like terms:

$$dX_{UC} + dX_{IC} + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC}dP_U^S = -\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right)dE_{UC}$$

$$dX_{UV} + \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)dP_U^S = 0$$

$$\left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)dP_U^S - dX_{UC} - dX_{UV} = 0$$

$$\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC}dP_U^S-dX_{IC}=-\left(\frac{\partial Q^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}$$

Rewriting this system of equations in matrix form $\boldsymbol{Ax}=\boldsymbol{b}$.

$$\begin{bmatrix}\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC} & 1 & 0 & 1\\ \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 & 0\\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1 & 0\\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC} & 0 & 0 & -1\end{bmatrix}\begin{bmatrix}dP_U^S\\ dX_{UC}\\ dX_{UV}\\ dX_{IC}\end{bmatrix}=\begin{bmatrix}\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\\ 0\\ 0\\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\end{bmatrix}$$

The determinant of the matrix $\boldsymbol{A}$ is calculated as follows,

$$|\boldsymbol{A}|=\begin{vmatrix}\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC} & 1 & 0 & 1\\ \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 & 0\\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1 & 0\\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC} & 0 & 0 & -1\end{vmatrix}=-1\begin{vmatrix}\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1\\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1\\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC} & 0 & 0\end{vmatrix}-1\begin{vmatrix}\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC} & 1 & 0\\ \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1\\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1\end{vmatrix}$$

$$=-1*\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC}\begin{vmatrix}0 & 1\\ -1 & -1\end{vmatrix}-\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC}\begin{vmatrix}0 & 1\\ -1 & -1\end{vmatrix}+1\begin{vmatrix}\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 1\\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1\end{vmatrix}$$

$$=-\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC}-\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC}-\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)-\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)$$

$$|A|=-\left[\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)+\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC}+\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC}+\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)\right]<0$$

Using Cramer's rule to solve for the endogenous variables:

$\underline{dP_U^S}$:

$$\begin{vmatrix}-\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & -1 & -1 & 0\\ -\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 & 0 & -1\end{vmatrix}=-1\begin{vmatrix}-\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 1 & 1\\ 0 & -1 & 0\\ -\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 & -1\end{vmatrix}$$

$$= -1\left[1\begin{vmatrix} 0 & -1 \\ -\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0\end{vmatrix} - 1\begin{vmatrix} -\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 1 \\ 0 & -1\end{vmatrix}\right]$$

$$= \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} + \left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}$$

$$dP_U^S = \frac{\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} + \left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}}{|\boldsymbol{A}|}$$

$$\frac{dP_U^S}{dE_{UC}} = \left(P_U^S+T_{UC}+\tau_{UC}\right)\frac{\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)+\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)}{|\boldsymbol{A}|} < 0$$

$\underline{dX_{UC}}$:

$$\begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC} & -\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 & 1 \\ \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & 0 & -1 & 0 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC} & -\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 & -1 \end{vmatrix}$$

$$= -1\begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & 0 & -1 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)E_{UC} & -\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 \end{vmatrix} - 1\begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)E_{UC} & -\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC} & 0 \\ \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & 0 & -1 \end{vmatrix}$$

$$= -1\left(\left[\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 \end{vmatrix}\right) - 1\left(\left[\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right) & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 \end{vmatrix}\right)$$

$$= -1\left[\left[\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\left(-\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)-\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)\right)\right]$$

$$-1\left[\left[\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\left(-\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)-\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)\right)\right]$$

$$= \left[\left(\frac{\partial Q_I^S}{\partial P_I^S}-\frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\left[\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)+\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)\right] + \left[\left(\frac{\partial Q_C^S}{\partial P_C^S}-\frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S+T_{UC}+\tau_{UC}\right)dE_{UC}\right]\left[\left(\frac{\partial Q_V^S}{\partial P_V^S}-\frac{\partial Q_V^D}{\partial P_V^D}\right)+\left(\frac{\partial Q_U^S}{\partial P_U^S}-\frac{\partial Q_U^D}{\partial P_U^D}\right)\right]$$

$$= \left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right] \left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]$$

$$dX_{UC} = \frac{\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right] \left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]}{|\boldsymbol{A}|}$$

$$\frac{dX_{UC}}{dE_{UC}} = \frac{\left(P_U^S + T_{UC} + \tau_{UC}\right) \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right] \left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]}{|\boldsymbol{A}|} < 0$$

$\underline{dX_{UV}}$:

$$\begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} & 1 & -\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} & 1 \\ \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 0 & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & 0 & 0 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} & 0 & -\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} & -1 \end{vmatrix}$$

$$= -1 \begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & 0 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} & 0 & -\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \end{vmatrix} - 1 \begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} & 1 & -\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \\ \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & 0 \end{vmatrix}$$

$$= -1 \left[\left(-\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right) \begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 \end{vmatrix}\right] - 1 \left[\left(-\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right) \begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 \end{vmatrix}\right]$$

$$= -1 \left[\left(-\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right)\left(-\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)\right)\right] - 1 \left[\left(-\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right)\left(-\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)\right)\right]$$

$$= -\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) - \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)$$

$$= -\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)\left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right]$$

$$dX_{UV} = \frac{-\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)\left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right]}{|\boldsymbol{A}|}$$

$$\frac{dX_{UV}}{dE_{UC}} = \frac{-\left(P_U^S + T_{UC} + \tau_{UC}\right)\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right)\left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) + \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\right]}{|\boldsymbol{A}|} > 0$$

$\underline{dX_{IC}}$:

$$\begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} & 1 & 0 & -\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \\ \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 & 0 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1 & 0 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} & 0 & 0 & -\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC} \end{vmatrix}$$

$$= \left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right] \begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1 \\ \left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} & 0 & 0 \end{vmatrix} - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right] \begin{vmatrix} \left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} & 1 & 0 \\ \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 0 & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 & -1 \end{vmatrix}$$

$$= \left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix}\right]$$

$$- \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} - 1 \begin{vmatrix} \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) & 1 \\ \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right) & -1 \end{vmatrix}\right]$$

$$= \left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC}$$

$$- \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC} + \left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]$$

$$= \left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC} - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC}$$

$$- \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)$$

$$= \left[\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right)\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right) E_{UC}\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right] - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(\frac{\partial Q_C^S}{\partial P_C^S} - \frac{\partial Q_C^D}{\partial P_C^D}\right) E_{UC}\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]$$

$$- \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)$$

$$= -\left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) - \left[\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\right]\left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)$$

$$= -\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]$$

$$dX_{IC} = \frac{-\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left(P_U^S + T_{UC} + \tau_{UC}\right) dE_{UC}\left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]}{|\boldsymbol{A}|}$$

$$\frac{dX_{IC}}{dE_{UC}} = \frac{-\left(P_U^S + T_{UC} + \tau_{UC}\right)\left(\frac{\partial Q_I^S}{\partial P_I^S} - \frac{\partial Q_I^D}{\partial P_I^D}\right)\left[\left(\frac{\partial Q_V^S}{\partial P_V^S} - \frac{\partial Q_V^D}{\partial P_V^D}\right) + \left(\frac{\partial Q_U^S}{\partial P_U^S} - \frac{\partial Q_U^D}{\partial P_U^D}\right)\right]}{|\boldsymbol{A}|} > 0$$