The Consequences of Model Misspecification for the Estimation of Non-Linear Interaction Effects

Supplementary Materials

Janina Beiser-McGrath* Liam F. Beiser-McGrath†

February 9, 2022

1 *Issues with Graphical Diagnostics*

In addition to the Wald test, Hainmueller, Mummolo and Xu (2019) also suggest graphical tools to ascertain whether an interactive relationship is linear or non-linear. To show that graphic diagnostics also suffer in the presence of an omitted square term of a control variable that is related to the interaction terms, we use one of the datasets simulated under the data generating process described above. In the data generating process, the interaction between D and X is linear, but the correlated control variable Z exerts a quadratic effect on the outcome. X and Z have a population correlation of $\rho = 0.5$ whereas D and Z have a population correlation of $\rho = 0$. Both using the fully moderated and the unmoderated binning estimator, the Wald test performed on this particular dataset is highly statistically significant with $p \leq 0.001$.

When investigating the interaction between two continuous variables, Hainmueller, Mummolo and Xu (2019, 170) *"(...) recommend that researchers split the sample into three roughly equal sized groups based on the moderator: low X (first tercile), medium X (second tercile), and high X (third tercile). For each of the three groups*

*Lecturer (Assistant Professor) in Politics and International Relations (Quantitative), Royal Holloway, University of London. Contact e-mail: `janina.beiser-mcgrath@rhul.ac.uk`

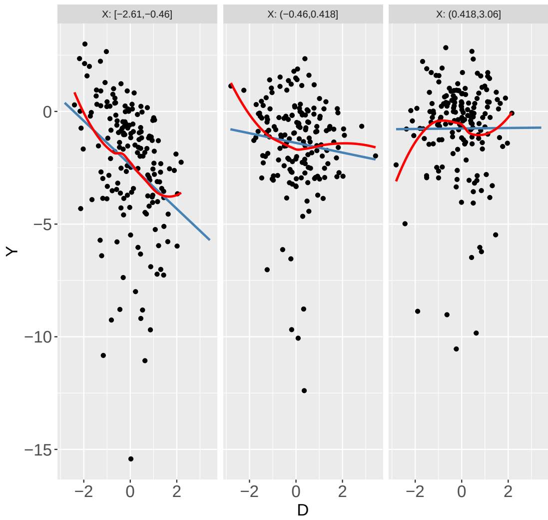
†Lecturer (Assistant Professor) in Politics and Director of the PECC Lab, Royal Holloway, University of London. Contact e-mail: `liam@liambeisermcgrath.com`, `liam.beiser-mcgrath@rhul.ac.uk`

we then plot Y against D while (...) overlaying both the linear and LOESS fits." In these graphs, Hainmueller, Mummolo and Xu (2019, 170) investigate whether *"the conditional expectation function of Y given D is well approximated by a linear model in all three samples of observations with low, medium, or high values on the moderator X."* In addition, as the LIE assumption pertains to both continuous interaction terms, the authors suggest that *"it is also useful to generate the LID plot in both directions to examine the conditional relationships of D | X and X | D" (...) and to use "a three-dimensional surface plot generated by a generalized additive model (GAM) (Hastie and Tibshirani, 1986)"* (Hainmueller, Mummolo and Xu, 2019, 170).

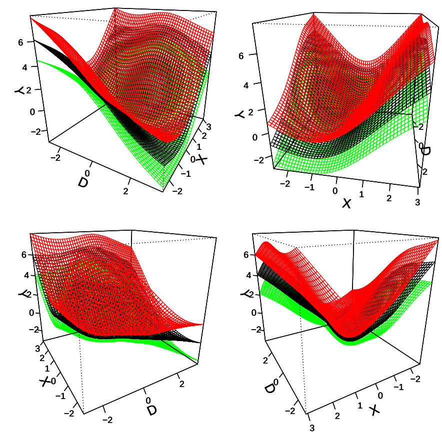
Panel a) in Figure 1 shows the Linear Interaction Diagnostic (LID) plots suggested in Hainmueller, Mummolo and Xu (2019), plotting Y against D for three intervals of X and including a linear and a LOESS fit. We partial the control variable Z out using a fully moderated specification.[1] As can be seen in panel a), the graph suggests considerable non-linearity in the effect of D on Y that changes with X. Thus, the graph, in combination with the statistically significant Wald test, would lead a researcher following the recommendations in Hainmueller, Mummolo and Xu (2019) to conclude that there is a non-linear interaction.

Panel b) shows a Linear Interaction Diagnostic plot based on a GAM model, that allows analysing the three-dimensional relationship between the two interaction terms and the outcome. Again, we partial out Z and use a fully moderated specification. As can be seen, the graph suggests that X exerts a non-linear effect on Y that changes in D. As can be seen in panel c), the GAM plot suggests less non-linearity in the not fully moderated specification, albeit even here some nonlinearity remains.

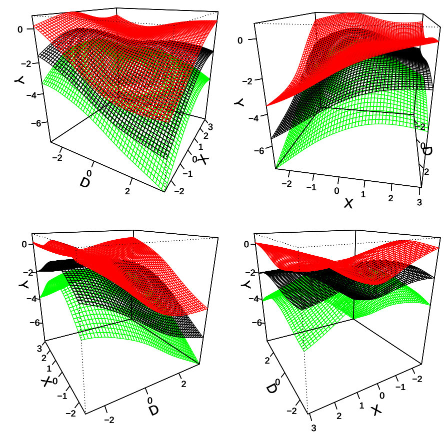
[1]The conclusion, however, is substantively similar irrespective of whether Z is partialed out or not.



(a) Plot of raw data, Z partialed out, fully moderated.



(b) GAM plot, Z partialed out, fully moderated.



(c) GAM plot, Z partialed out, not fully moderated.

Figure 1: Graphical diagnostic tools as suggested in Hainmueller, Mummolo and Xu (2019). Panel a) plots the raw data with linear and LOESS fit, panels b) and c) display GAM plots. Both plots partial out Z.

2 *Studies Included in Re-analysis*

Interaction code	Citation
adams_2006	Adams et al. (2006)
aklin_2013_1	Aklin and Urpelainen (2013)
aklin_2013_2	Aklin and Urpelainen (2013)
banks_2012_1	Banks and Valentino (2012)
bodea_2015_1	Bodea and Hicks (2015*a*)
bodea_2015_2	Bodea and Hicks (2015*a*)
bodea_io_2015_2	Bodea and Hicks (2015*b*)
bodea_io_2015_4	Bodea and Hicks (2015*b*)
chapman_2009	Chapman (2009)
clark_2014	Clark and Golder (2006)
hellwig_2007_1	Hellwig and Samuels (2007)
hellwig_2007_2	Hellwig and Samuels (2007)
hicken_2008	Hicken and Simmons (2008)
huddy_2015_1	Huddy, Mason and Aarøe (2015)
huddy_2015_2	Huddy, Mason and Aarøe (2015)
malesky_2012	Malesky, Schuler and Tran (2012)
neblo_2010	Neblo et al. (2010)
somer_2009	Somer-Topcu (2009)
tavits_2008	Tavits (2008)
vernby_2013_1	Vernby (2013)
vernby_2013_2	Vernby (2013)
williams_2011_1	Williams (2011)
williams_2011_2	Williams (2011)

Table 1: Articles from Hainmueller, Mummolo and Xu (2019)'s replication included in the reanalysis.

3 *Summary of Re-analysis Results*

	Binning, origil		Binning, Lasso-selected controls		Full Lasso		
Interaction code	Non-monot.	Vis. non-linear	Non-mot.	Vis. non-linear	Vis. non-linear	Lin. int.	No int.
adams_2006						X	
aklin_2013_1		X		X			X
aklin_2013_2		X	X				X
banks_2012_1							X
bodea_2015_1		X	X		X		
bodea_2015_2	X			X		X	
bodea_io_2015_2	X		X				X
bodea_io_2015_4	X		X			X	
chapman_2009						X	
clark_2014	X		X				X
hellwig_2007_1	X		X			X	
hellwig_2007_2	X		X				X
hicken_2008	X		X				X
huddy_2015_1						X	
huddy_2015_2						X	
malesky_2012	X		X				X
neblo_2010	X		X			X	
somer_2009	X		X			X	
tavits_2008		X					X
vernby_2013_1	X		X				X
vernby_2013_2	X		X				X
williams_2011_1	X		X				X
williams_2011_2		X		X			X

Table 2: Reanalysis results.

4 BART Analysis of Interaction Effects Estimated to be Zero by the Adaptive Lasso

In our re-analysis of 23 interaction effects from 17 studies (section 4 of the main text), we find that 14 of the 21 original interactions are classified as non-interactive by the adaptive Lasso. To ensure that this is not a consequence of this particular estimator, we re-analyse these 14 interaction effects using Bayesian Additive Regression Trees (BART) (**?**).

Figure 2. Replications including BART

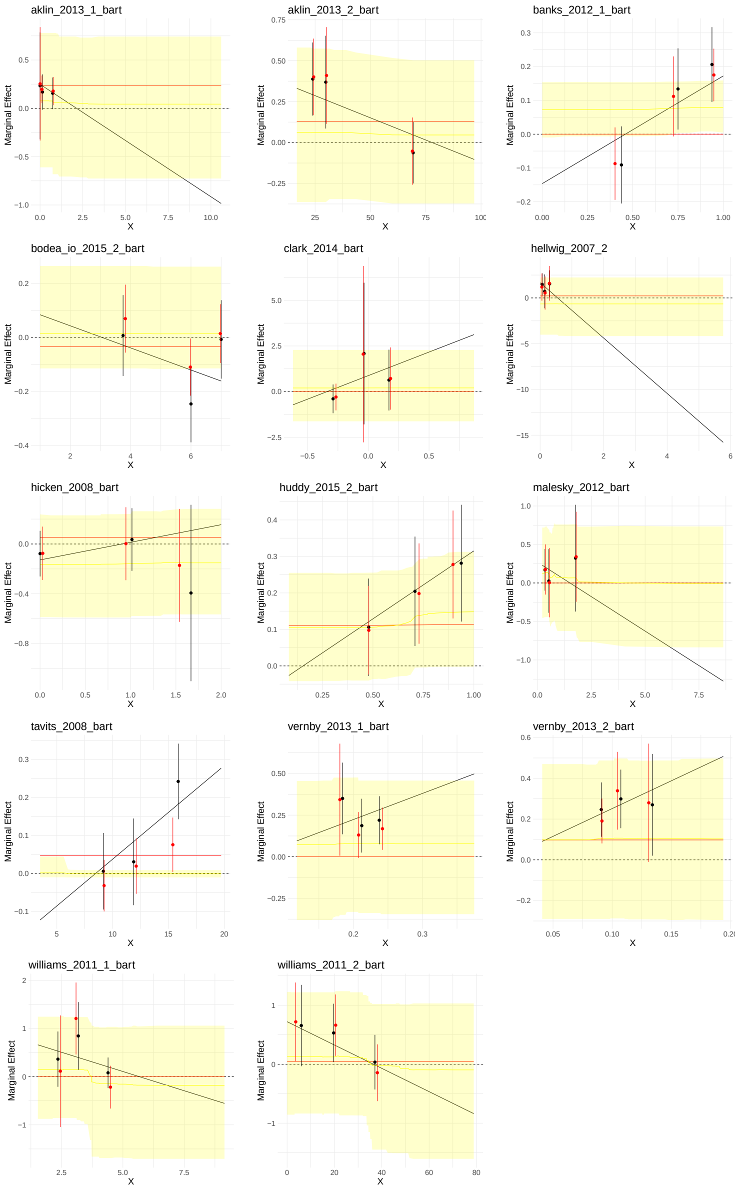


Figure 2 displays the results. For the vast majority of interactions, we also find constant marginal effects when using BART (marginal effect and 95% credible intervals displayed in yellow), further demonstrating the danger of misspecification bias for the estimation of non-linear interaction effects.

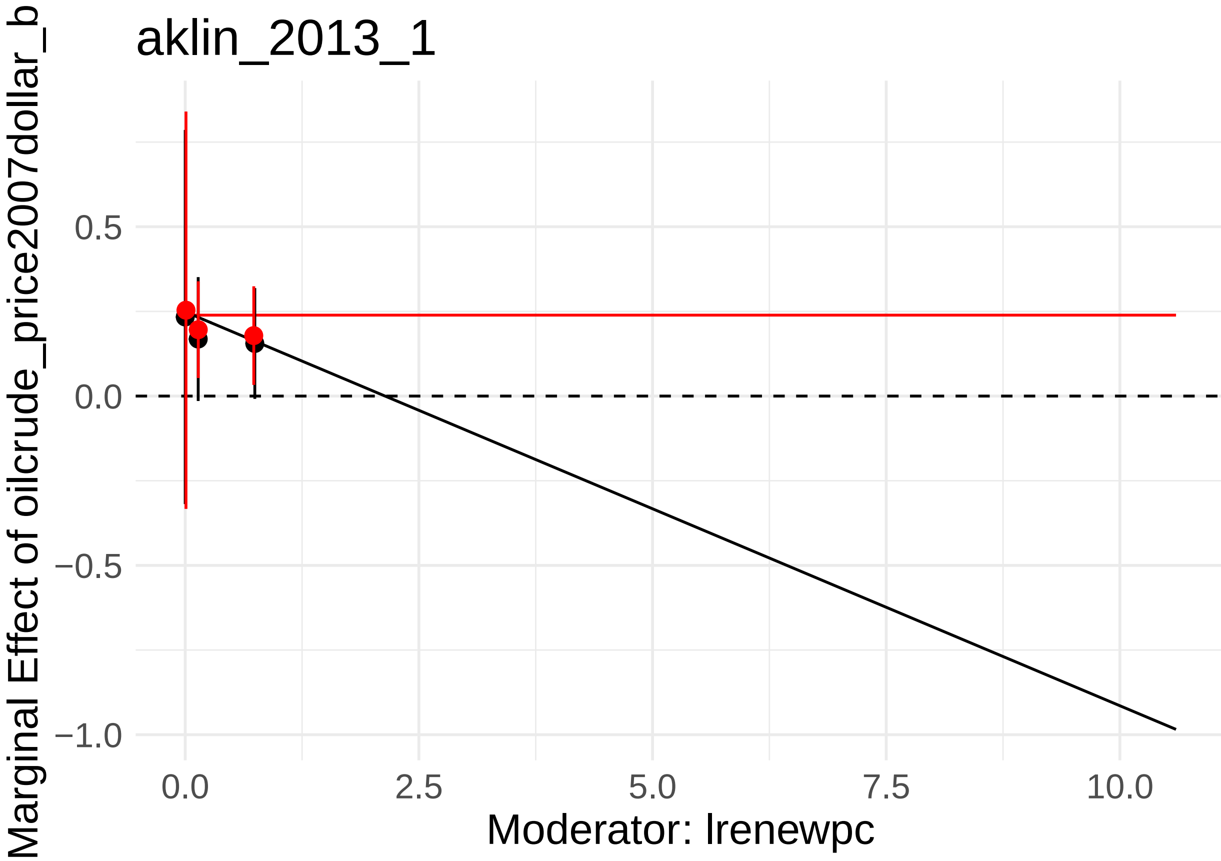
5 Full Results of Reanalysis

Detailed Results for Reanalysis of Previous Studies

Adams 2006

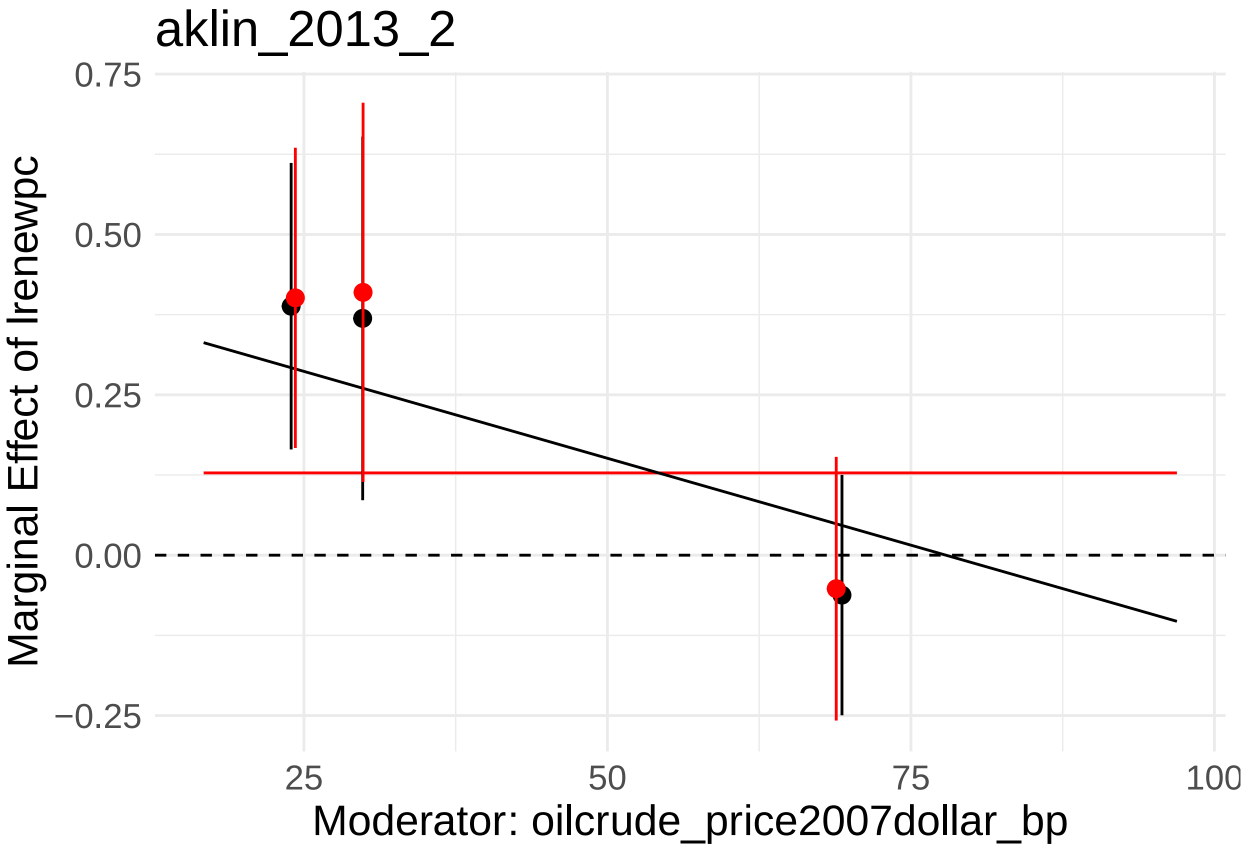
	Estimate
(Intercept)	0.1145629
X	0.2812707
pshiftt12	-0.4368104
D.X	-0.1192670
D.pshiftt12.pvoteshift	-0.0114397
X^2.votec1	**0.0000205**
X.pvoteshift^2	**0.0132353**
votec1^2.pvoteshift	**0.0000227**

Aklin 2013 1



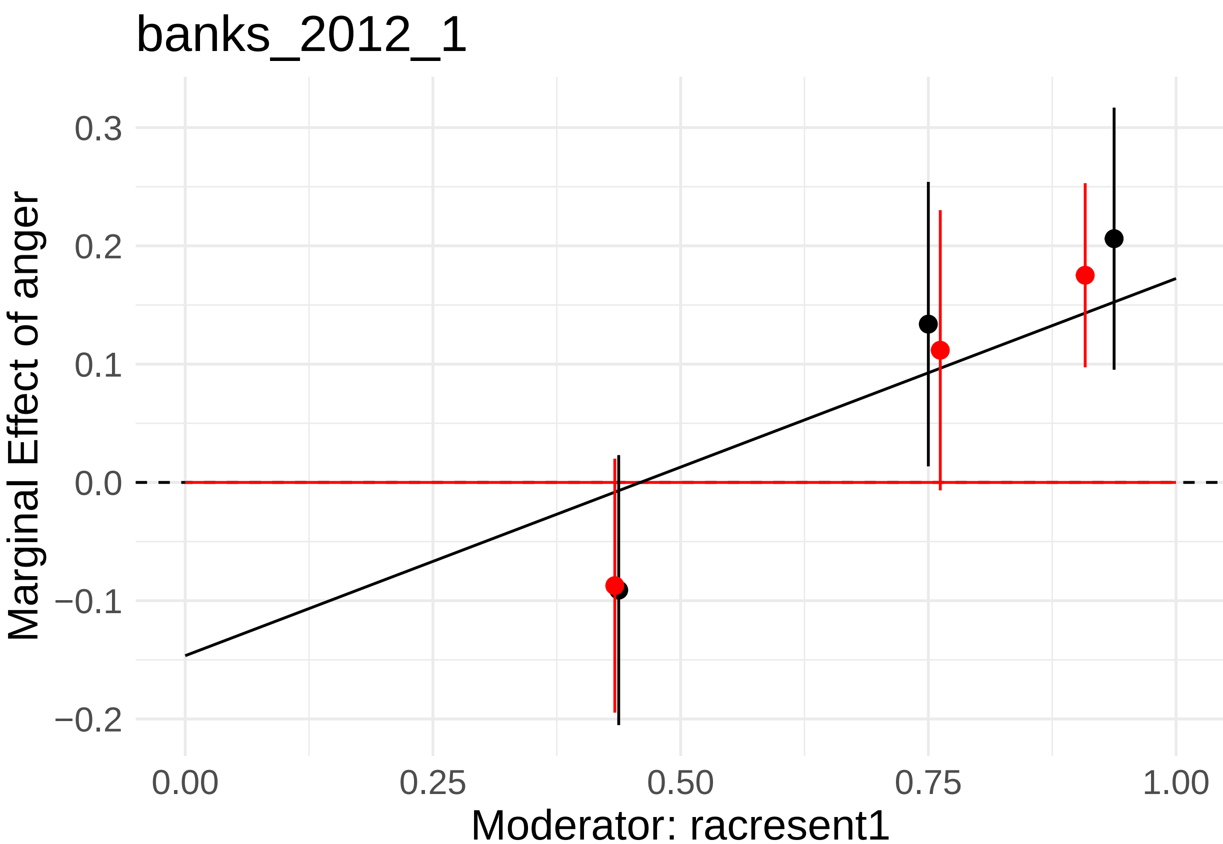
	Estimate
(Intercept)	0.4427206
D.year	0.0001195
X.year	0.0000670
right_executive.year	-0.0000487
renewablecapacity_3yr_average.year	0.0000076

aklin 2013 2



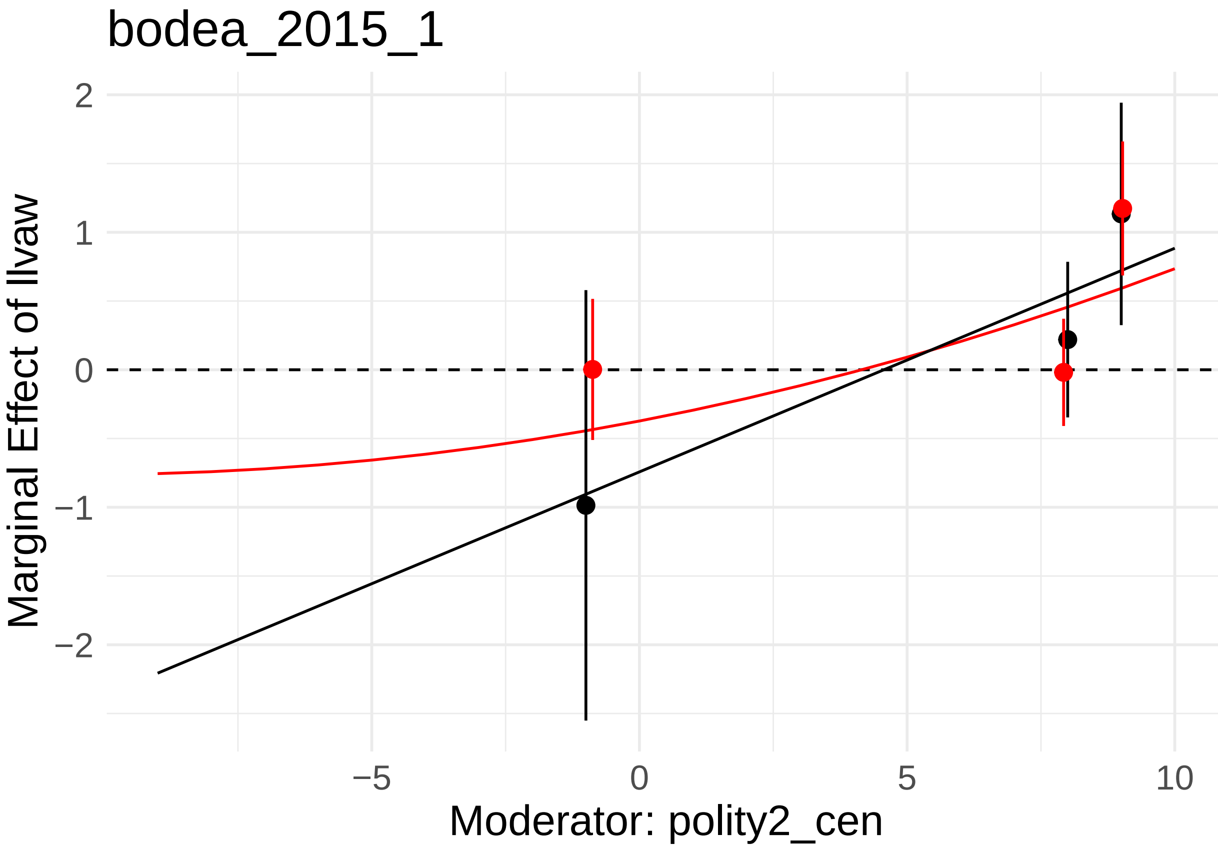
	Estimate
(Intercept)	0.0554325
D.year	0.0000642
X.year	0.0000058
right_executive.year	-0.0000471
renewablecapacity_3yr_average.year	0.0000075

banks 2012 1



	Estimate
(Intercept)	0.3853520
fearres1	0.4838278
fear	-0.0637551
ideology	0.1585302
openissuejo1	-1.2221235
X^2	**0.3083446**
fearres1.education	0.0621578
fearres1.income1	0.0143320
fearres1.openissuejo1	-0.1943822
fear.education	-0.1591989
disgusres1.openissuejo1	-0.8470430
age.openissuejo1	0.0092683
X^2.fearres1	**-0.2945805**
X.fearres1.openissuejo1	-0.4032511
X.age.openissuejo1	0.0281109
fearres1.income1.age	0.0003884
fearres1.age^2	**-0.0000549**

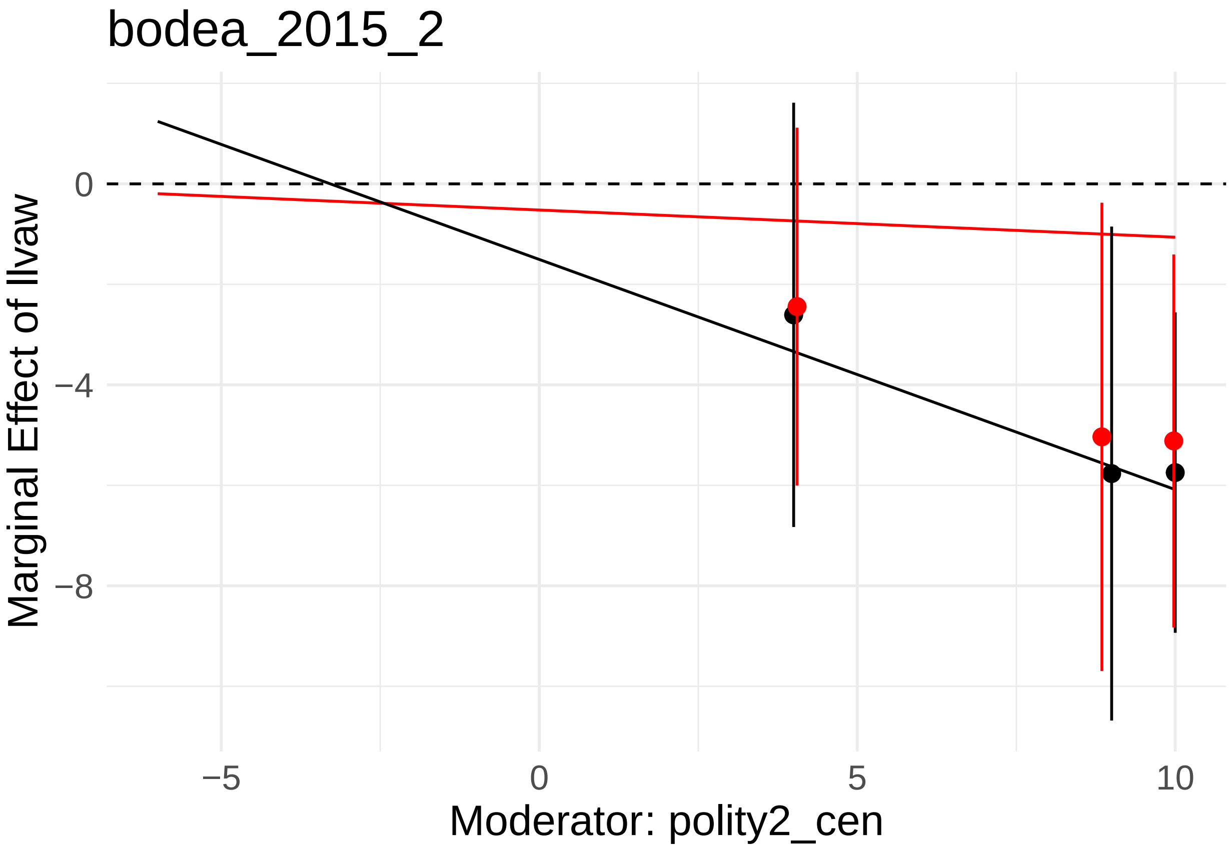
bodea 2015 1



bodea_2015_1
Marginal Effect of llvaw
2
1
0
−1
−2
−5
0
5
10
Moderator: polity2_cen

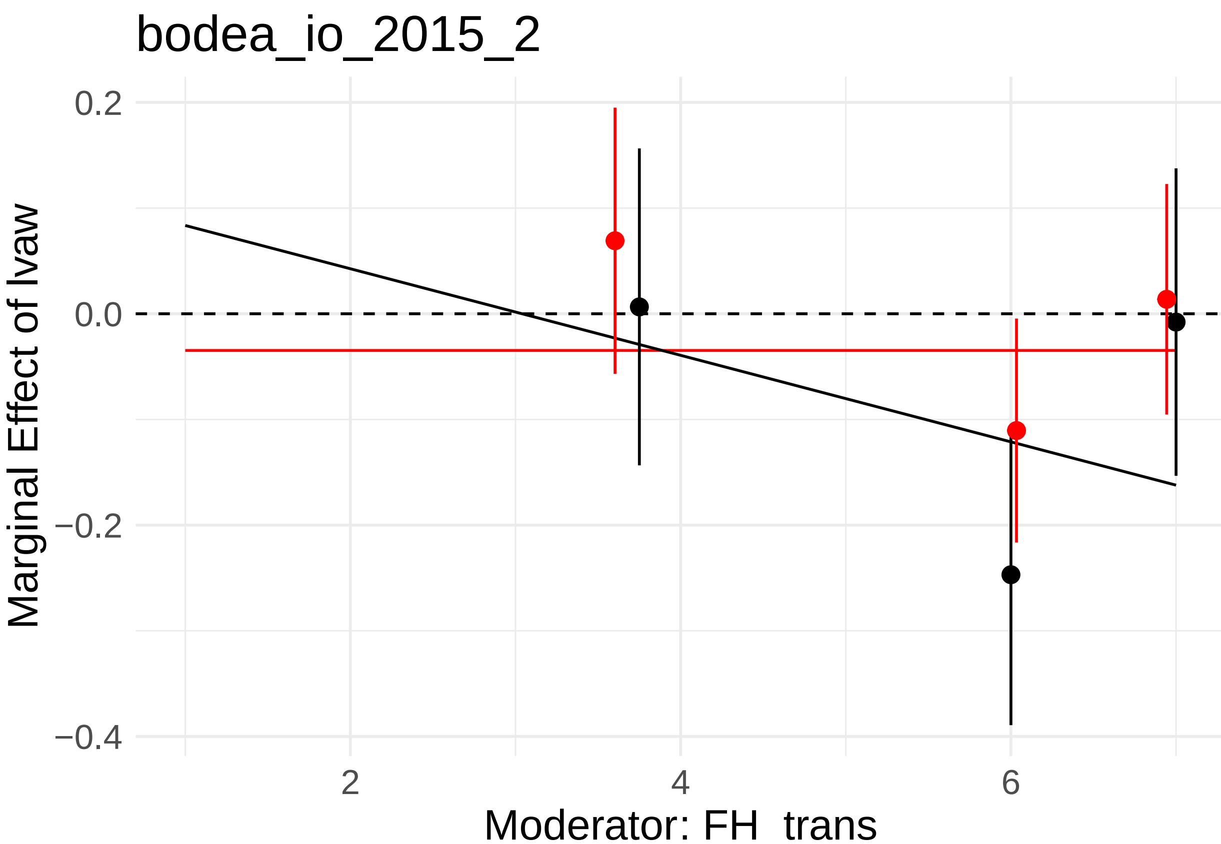
	Estimate
(Intercept)	-6.1027923
D	0.6860781
X	-0.1180723
l_dfxreserves	0.7428049
l_openness	0.0046312
l_dgdp_k	16.3482446
l_lngdppc	0.9589837
l_fiscal_balance	-0.4053528
l_capital_controls	-0.3395696
l_lninfl	-0.2880928
l_signyearfill	0.0016860
xrdum	-0.3435833
D.X	0.0741679
D.l_openness	-0.0209458
D.l_dgdp_k	0.9778514
D.l_lngdppc	0.0193536
D.l_fiscal_balance	-0.3655043
D.l_capital_controls	0.2846672
X^2	**0.0079038**
X.l_openness	0.0004226
X.l_dgdp_k	-0.3774470
X.l_lngdppc	0.0118497
X.l_fiscal_balance	-0.0038163
X.l_capital_controls	-0.0476400
X.l_lninfl	-0.0006577
X.l_signyearfill	-0.0154917
X.xrdum	0.2934915
l_dffrus^2	**0.3466969**
l_dffrus.xrdum	5.6699864
l_dfxreserves.l_lngdppc	0.0236517
l_dfxreserves.l_lninfl	-0.1705822
l_openness.l_lngdppc	0.0020650
l_openness.l_fiscal_balance	0.0043900
l_openness.l_capital_controls	-0.0031295
l_openness.l_lninfl	0.0031238
l_openness.xrdum	0.0111065
l_dgdp_k^2	**149.2173274**
l_dgdp_k.l_capital_controls	3.3285071
l_lngdppc^2	**-0.0265287**
l_lngdppc.l_capital_controls	0.0037445
l_lngdppc.l_lninfl	0.0000021
l_lngdppc.l_signyearfill	0.0186034
l_lngdppc.xrdum	-0.1651698
l_fiscal_balance.l_lninfl	0.0526587
l_fiscal_balance.l_signyearfill	0.0044739
l_fiscal_balance.xrdum	0.0384257
l_capital_controls.xrdum	0.0745739
l_lninfl.xrdum	-0.0137157
l_signyearfill^2	**-0.0008487**
l_signyearfill.xrdum	-0.0062333
D^2.l_lngdppc	**0.0271367**
D^2.l_fiscal_balance	**-0.0208472**
D.X^2	**0.0035925**
D.X.l_openness	0.0015600
D.X.l_lngdppc	-0.0222453
D.X.l_fiscal_balance	-0.0042467
D.X.l_capital_controls	0.0041903

bodea 2015 2



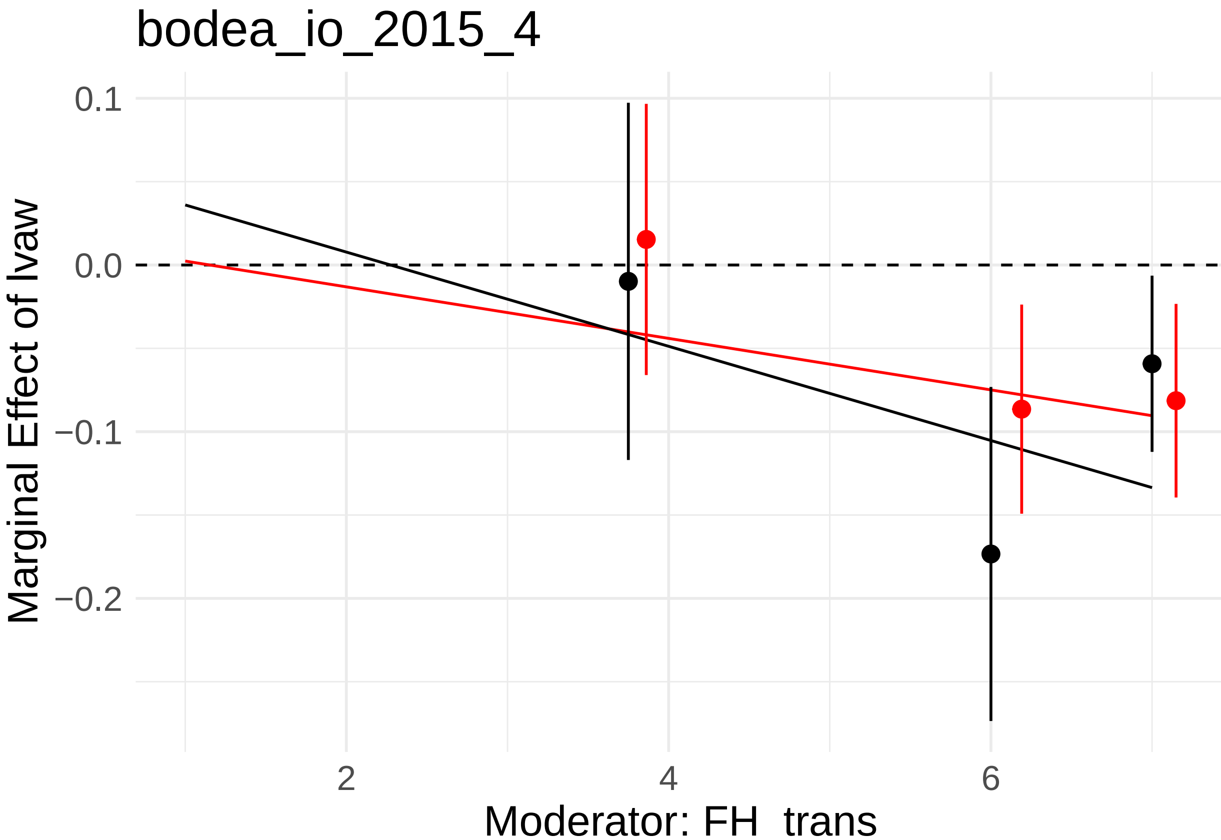
	Estimate
(Intercept)	7.5073392
X	0.1850638
l_fiscal_balance	-0.0041456
xrdum	0.6637987
D.X	-0.0541015
D.l_dffrus	-0.1530575
D.l_openness	-0.0031281
X^2	**-0.0140608**
l_dffrus.l_dgdp_k	-8.2448077
l_openness.l_lngdp	-0.0005314
l_wdgdpdefl.l_lngdp	-0.0283497
l_lngdppc.l_lngdp	-0.0099726
l_capital_controls.l_lngdp	0.0203498
l_lngdp.l_lninfl	-0.0018170
D^2.l_lngdp	**-0.0045698**
D.l_dgdp_k.l_lngdp	-0.1155659
X.l_openness.l_lngdp	-0.0000274
X.l_dgdp_k.l_lngdp	-0.0475129

bodea io 2015 2



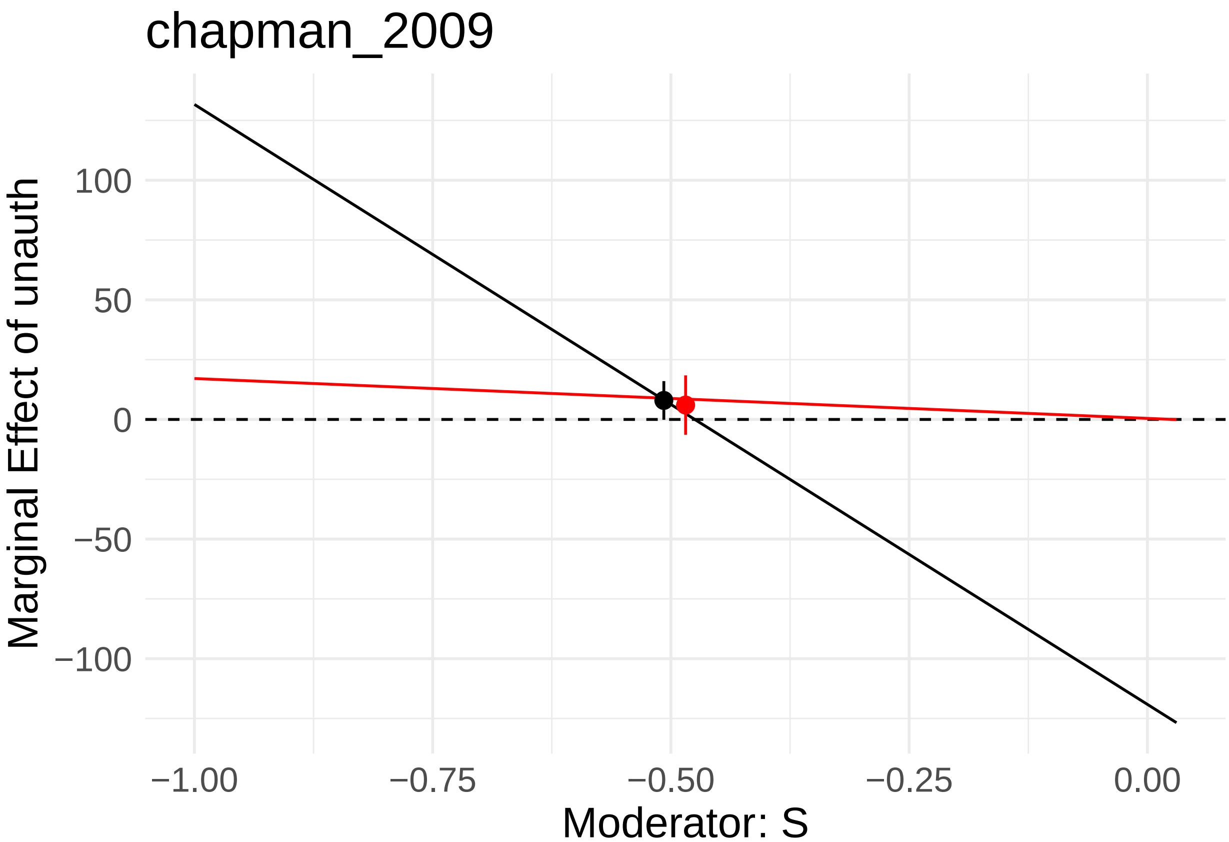
	Estimate
(Intercept)	2.5815795
X	0.0618474
L__fiscal__balance	-0.0022924
D.L__lngdp	-0.0049351
L__logdm2.L__lngdp	0.0010020
L__lngdp.xrdum	0.0001568
L__lngdp.pres__only	0.0040428
L__lngdp.leg__only	-0.0032826
L__lngdp.pres__leg	-0.0045289
L__dgdp__k^2	**2.0393336**
L__dgdp__k.xrdum	-0.7580030
L__fiscal__balance^2	**0.0004764**
D.L__lngdp.L__openness	0.0000520
X.L__lngdp^2	**-0.0002600**
X.L__lngdp.L__dgdp__k	0.0087183
L__logdm2^2.L__lngdp	**0.0033823**
L__lngdp^2.L__openness	**-0.0000034**
L__lngdp.L__fiscal__balance.pres__only	0.0013388

bodea io 2015 4



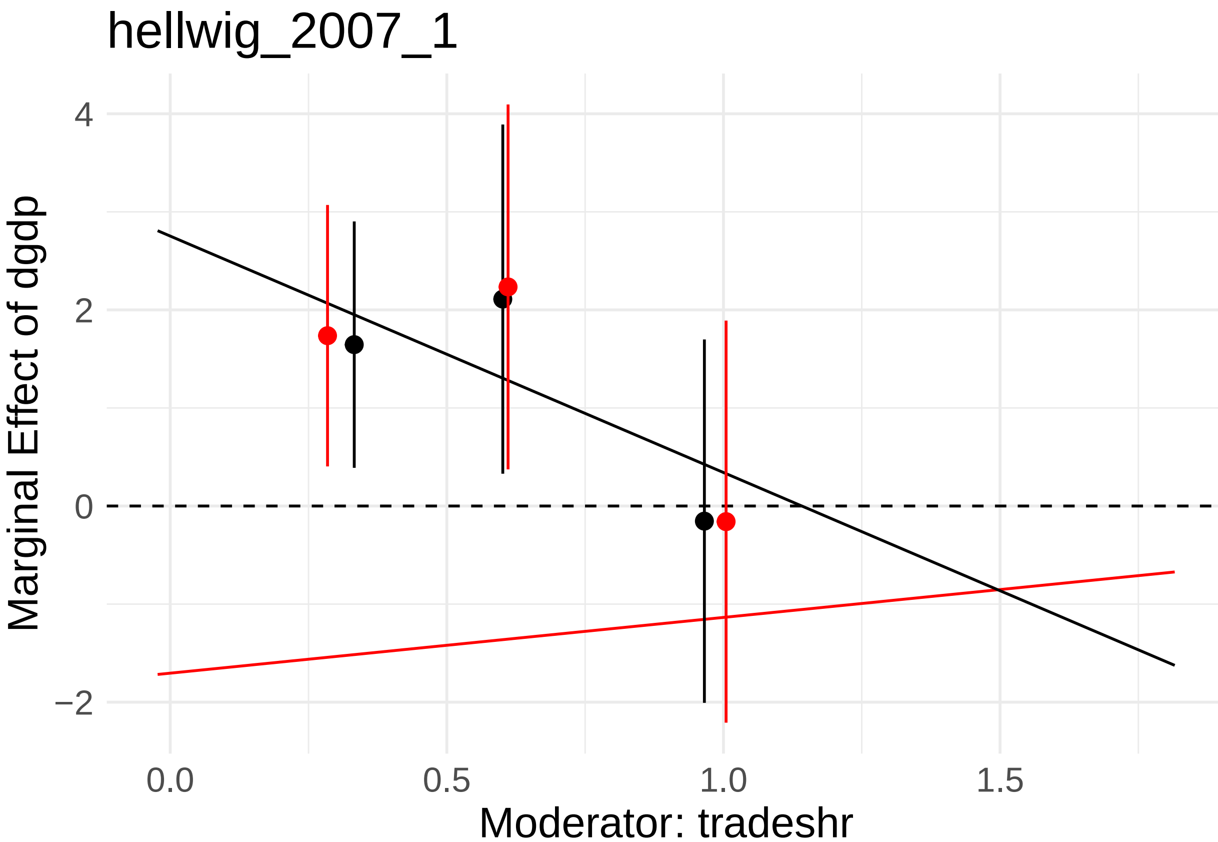
	Estimate
(Intercept)	1.1754952
L_lninfl	-0.0967048
L_dgdp_k	-2.3250930
L_fiscal_balance	0.0080642
pres_only	-0.5446888
D^2	**-0.0503567**
D.X	-0.0154567
D.L_logdm2	-0.0332074
D.L_lngdp	0.0013227
D.L_dgdp_k	-0.1244974
D.L_openness	0.0012770
D.xrdum	-0.0430435
X^2	**0.0045385**
X.L_fiscal_balance	0.0005739
X.pres_only	0.0201601
X.leg_only	0.0036623
X.L_wdgdpdefl	0.0075094
L_lninfl^2	**0.0897585**
L_lninfl.L_lngdp	0.0212997
L_lninfl.xrdum	0.0485303
L_lninfl.L_fiscal_balance	-0.0131349
L_lninfl.pres_only	0.0416361
L_lninfl.L_wdgdpdefl	-0.0031878
L_logdm2^2	**0.0367935**
L_logdm2.pres_only	-0.0365733
L_lngdp^2	**-0.0002207**
L_lngdp.L_openness	-0.0000138
L_lngdp.L_fiscal_balance	0.0004835
L_lngdp.leg_only	-0.0012447
L_lngdp.pres_leg	-0.0149317
L_dgdp_k.L_fiscal_balance	0.0094907
L_openness.L_wdgdpdefl	0.0000890
xrdum.L_wdgdpdefl	-0.0056703
D.L_lngdp^2	**0.0000960**
X.L_lninfl.pres_leg	0.0228650
X.L_logdm2.L_lngdp	0.0000080
X.L_lngdp^2	**-0.0002048**
X.L_lngdp.L_dgdp_k	0.0270200
X.L_lngdp.L_openness	-0.0000072
X.L_lngdp.xrdum	-0.0004432
L_lninfl^3	**-0.0120903**
L_lninfl^2.L_lngdp	**0.0000001**
L_lninfl.L_logdm2.L_lngdp	0.0001281
L_lninfl.L_logdm2.L_dgdp_k	-0.0373742
L_lninfl.L_lngdp^2	**0.0000647**
L_lninfl.L_lngdp.L_openness	-0.0000243
L_lninfl.L_lngdp.pres_leg	0.0020536
L_logdm2.L_fiscal_balance.pres_leg	0.0001538
L_lngdp^2.L_openness	**-0.0000007**
L_lngdp^2.pres_only	**0.0005125**
L_lngdp^2.pres_leg	**-0.0000007**
L_lngdp^2.L_wdgdpdefl	**-0.0000124**
L_lngdp.L_dgdp_k.pres_only	0.0586746
L_lngdp.L_openness^2	**0.0000001**
L_lngdp.L_openness.pres_only	0.0000213
L_lngdp.xrdum.L_wdgdpdefl	-0.0001272
L_lngdp.L_wdgdpdefl^2	**-0.0000197**

chapman 2009



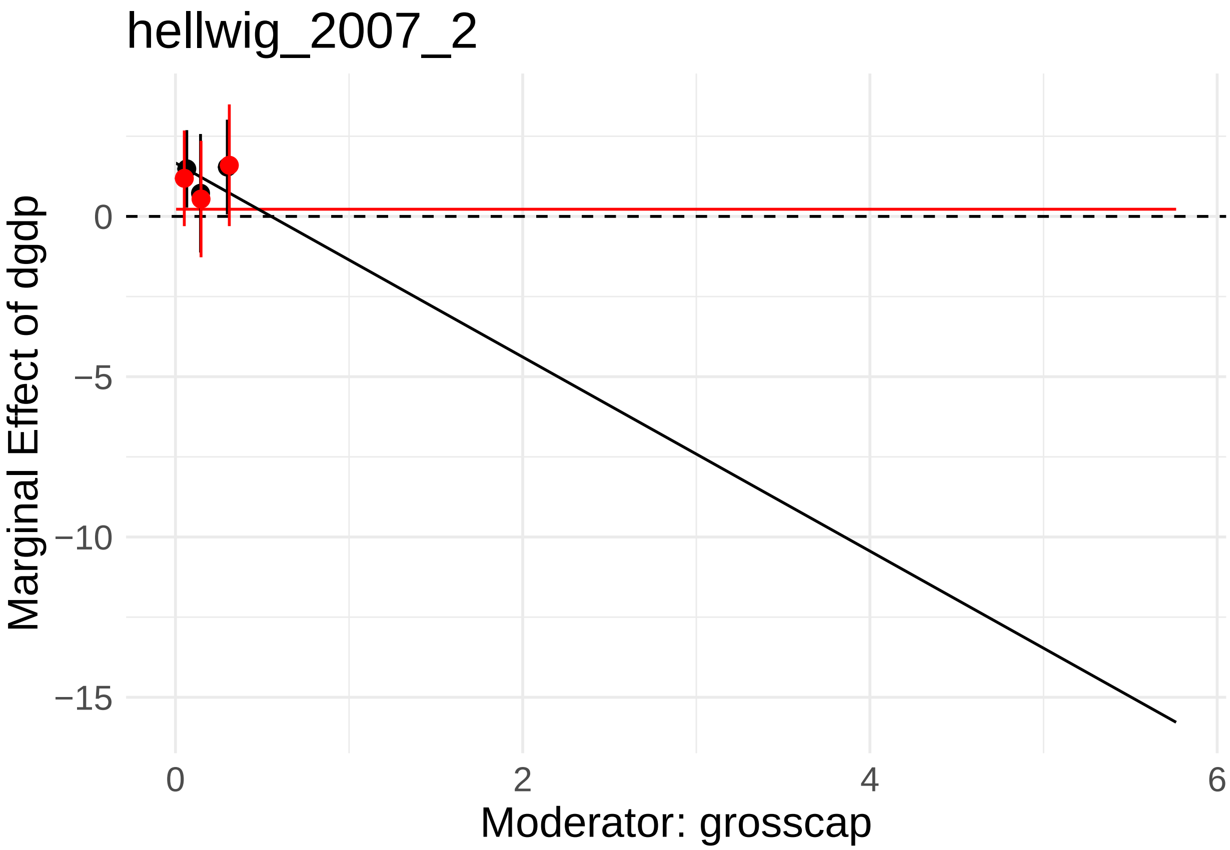
	Estimate
(Intercept)	-0.1239248
D.X	-16.6524684
D.bipart	4.5814339
X.bipart	-4.8180303
priorpop.SCappeal	0.0175291

hellwig 2007 1



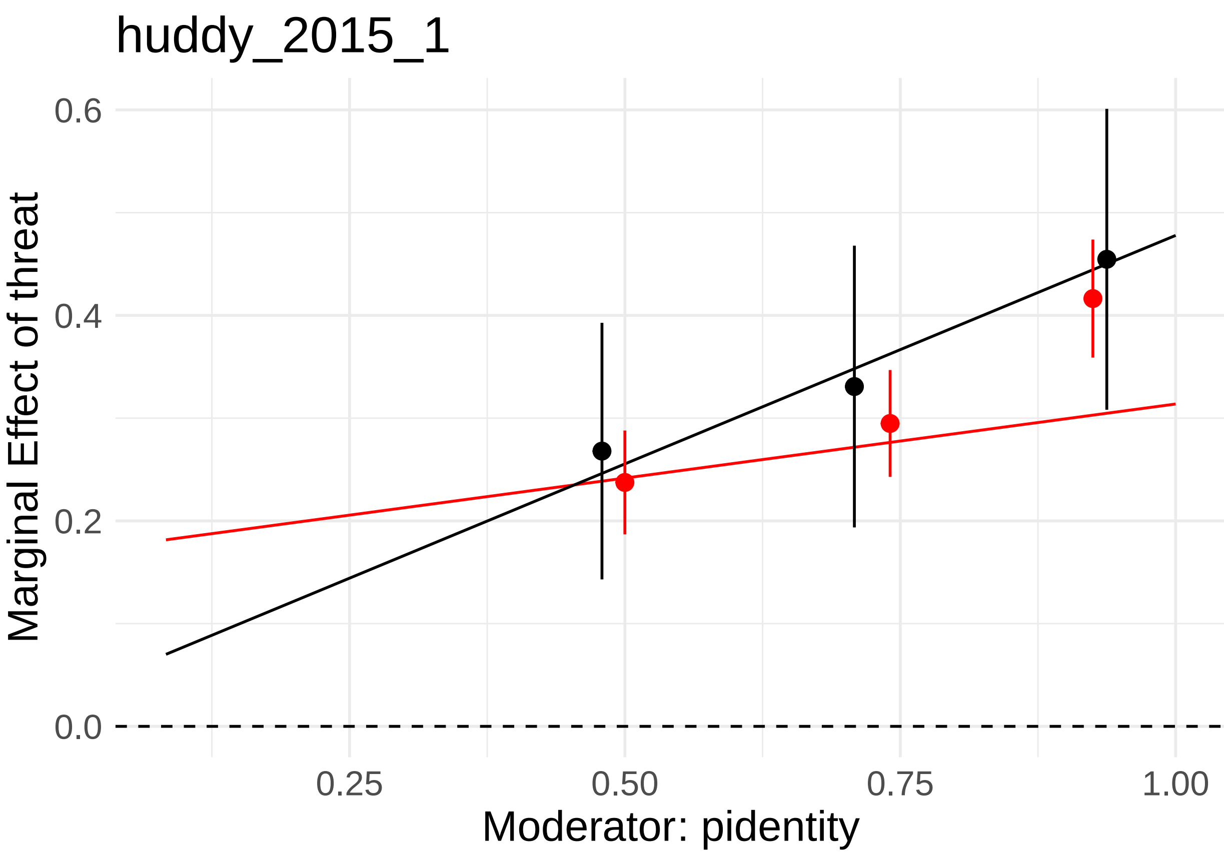
	Estimate
(Intercept)	29.0906019
incvotet1	0.1508821
electype	0.6724792
regafrica	-2.3043453
D.presrun	2.8413776
D.regafrica	9.5852387
X.regafrica	-7.4950726
incvotet1.regafrica	-0.0265541
electype.regafrica	-8.5405721
gdpxelectype.regafrica	1.9592934
gdpxelectype.regcee	0.0442093
presrun.regafrica	-0.4086690
enlp^2	**-0.1471997**
enlp.regafrica	2.3565735
income.regafrica	4.7749004
D^2.regafrica	**0.3831045**
D.X.regafrica	10.0421417
D.gdpxelectype.regafrica	-0.6456012
D.presrun.regafrica	-10.8659999
D.income.regafrica	-3.7987522
X^2.regafrica	**-20.6698779**
X.incvotet1.regafrica	0.4094045
X.electype.regcee	7.0349406
X.gdpxelectype.regafrica	-0.3089858
X.presrun.regafrica	25.4499288
X.presrun.regcee	-12.4844857
X.enlp.regafrica	-7.7211876
incvotet1^2.regafrica	**0.0020673**
incvotet1.electype.regcee	0.0015169
incvotet1.presrun.income	0.0087943
incvotet1.presrun.reglatam	0.1344129
electype.presrun.reglatam	1.1179026

hellwig 2007 2



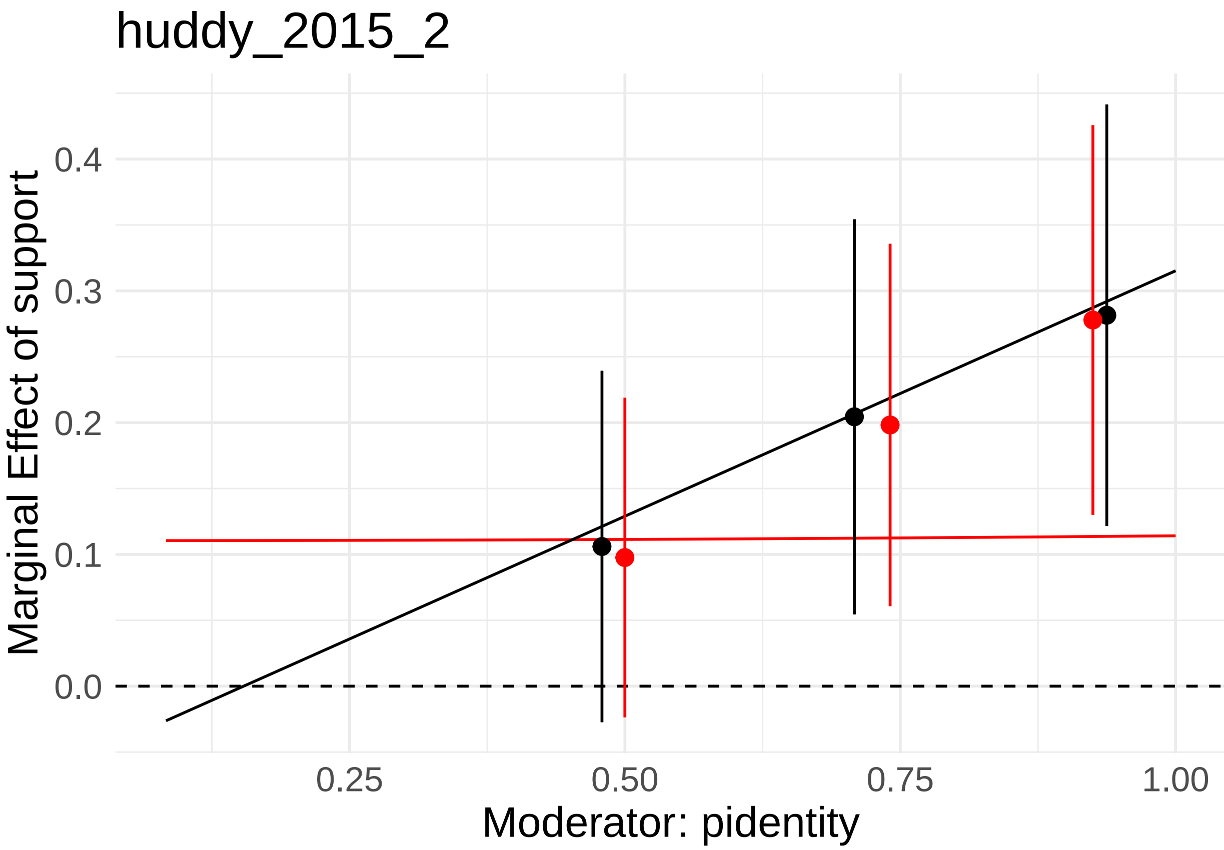
	Estimate
(Intercept)	32.9104172
regafrica	3.3819576
D.presrun	0.7358473
D.regafrica	4.2522033
X.regafrica	-31.2367085
presrun.regafrica	-2.5233866
presrun.regcee	-4.6768238
income.regafrica	1.9480169
D.gdpxelectype.regcee	0.1180165
D.presrun.income	0.0648862
D.income.regafrica	-0.3029350
X.incvotet1.regafrica	1.3602435
X.gdpxelectype.regafrica	-2.5392689
incvotet1^2.presrun	**0.0023180**
incvotet1.gdpxelectype.presrun	0.0204888
incvotet1.presrun.income	0.0029866
incvotet1.presrun.reglatam	0.0204331

huddy 2015 1



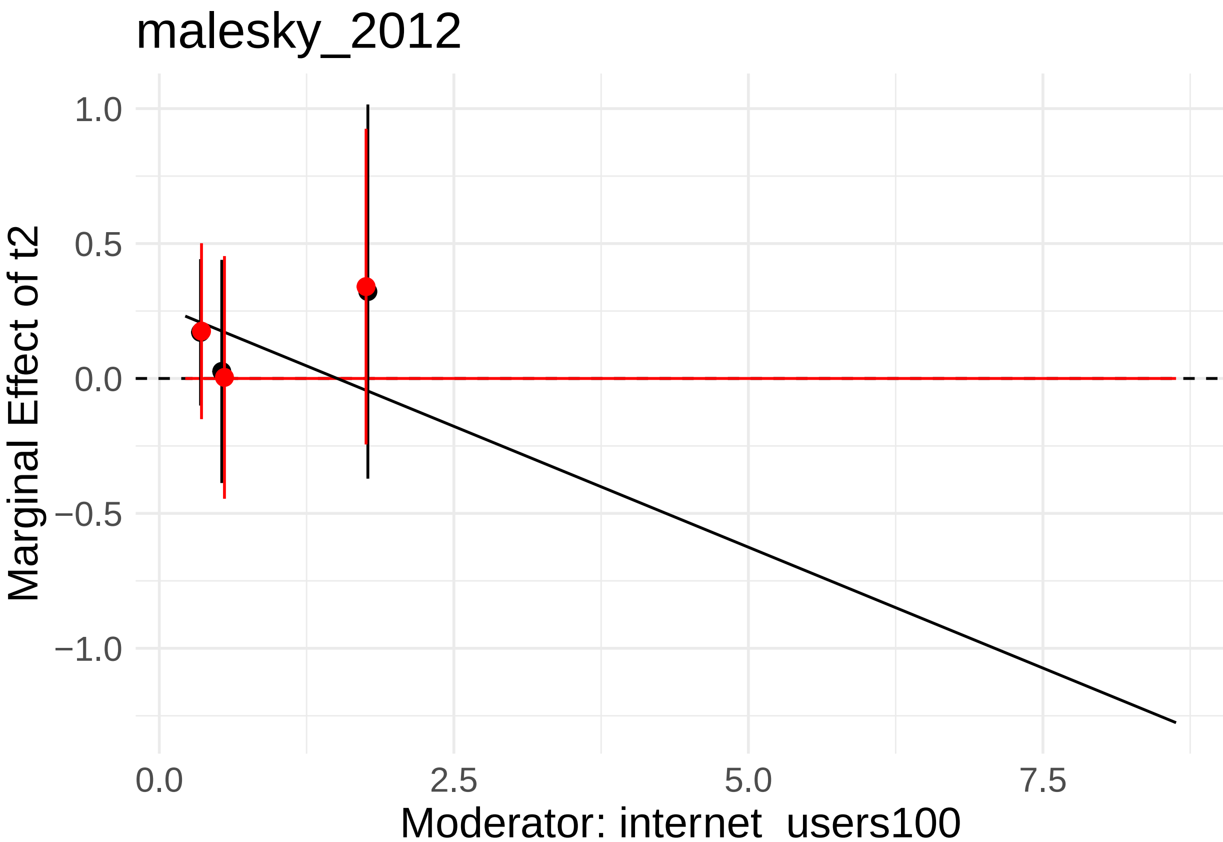
	Estimate
(Intercept)	0.4265036
D.knowledge	0.2030784
X.pidstr2_threat	0.0718860
issuestr2^2	**0.0315109**
pidstr2_threat.knowledge	-0.0135179
issuestr2_threat.male	-0.0515187
issuestr2_threat.age10	-0.0021112
knowledge^2	**-0.1963535**
knowledge.male	-0.0024745
knowledge.age10	-0.0014154
educ^2	**-0.0029797**
D.X.age10	0.0302686
D.educ.age10	-0.0081942
X^2.knowledge	**0.0075141**
X.issuestr2_threat.knowledge	0.0799113
X.knowledge.male	-0.0000898

huddy 2015 2



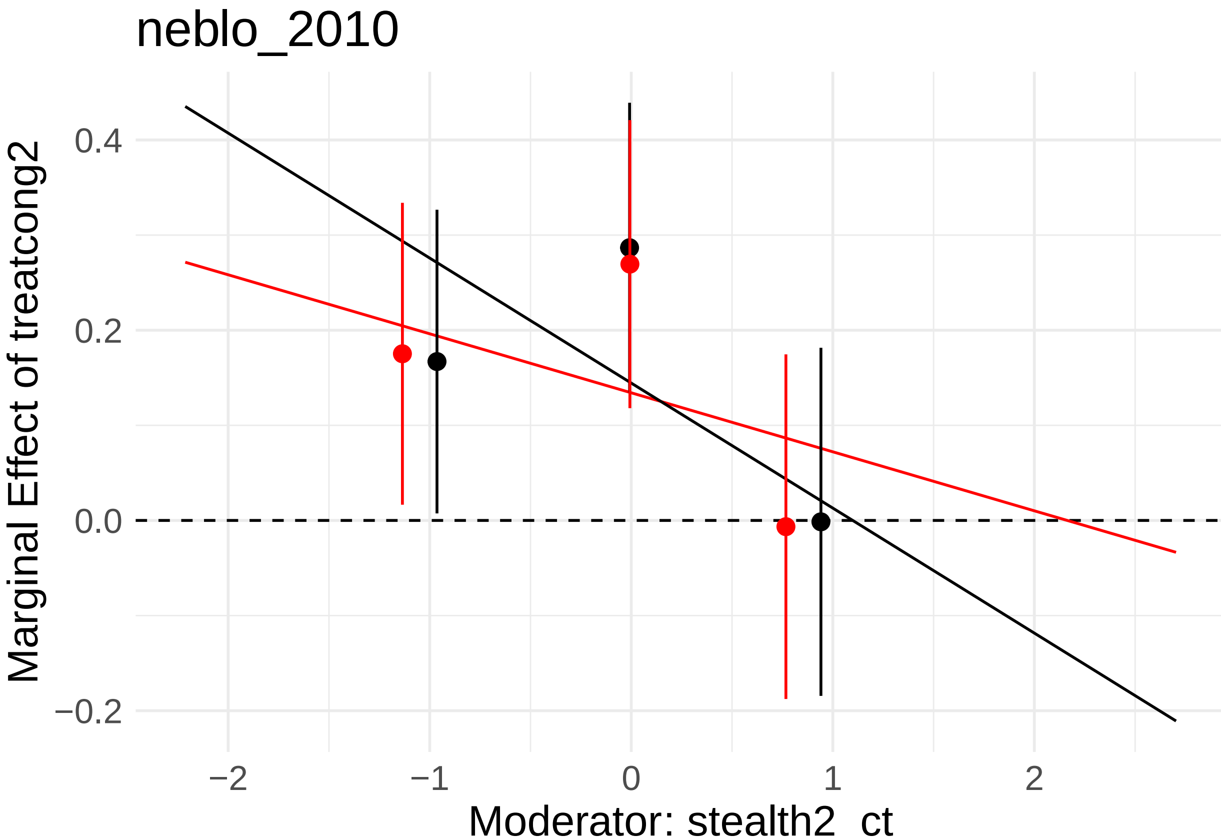
	Estimate
(Intercept)	0.1412464
issuestr2Xsupport	0.2185738
knowledge	-0.0100213
D.knowledge	0.1079544
D.male	0.0071810
pidstr2.knowledge	-0.0662391
issuestr2Xsupport.knowledge	0.0032026
issuestr2Xsupport.educ	-0.0797083
pidstr2Xsupport.male	0.0023088
knowledge.age10	0.0120440
D.X^2	**0.0037054**
X^2.pidstr2Xsupport	**0.2515000**
X.knowledge^2	**0.1019791**
pidstr2.knowledge.male	0.0389127
knowledge.educ^2	**-0.0349816**

malesky 2012



	Estimate
(Intercept)	0.0238612

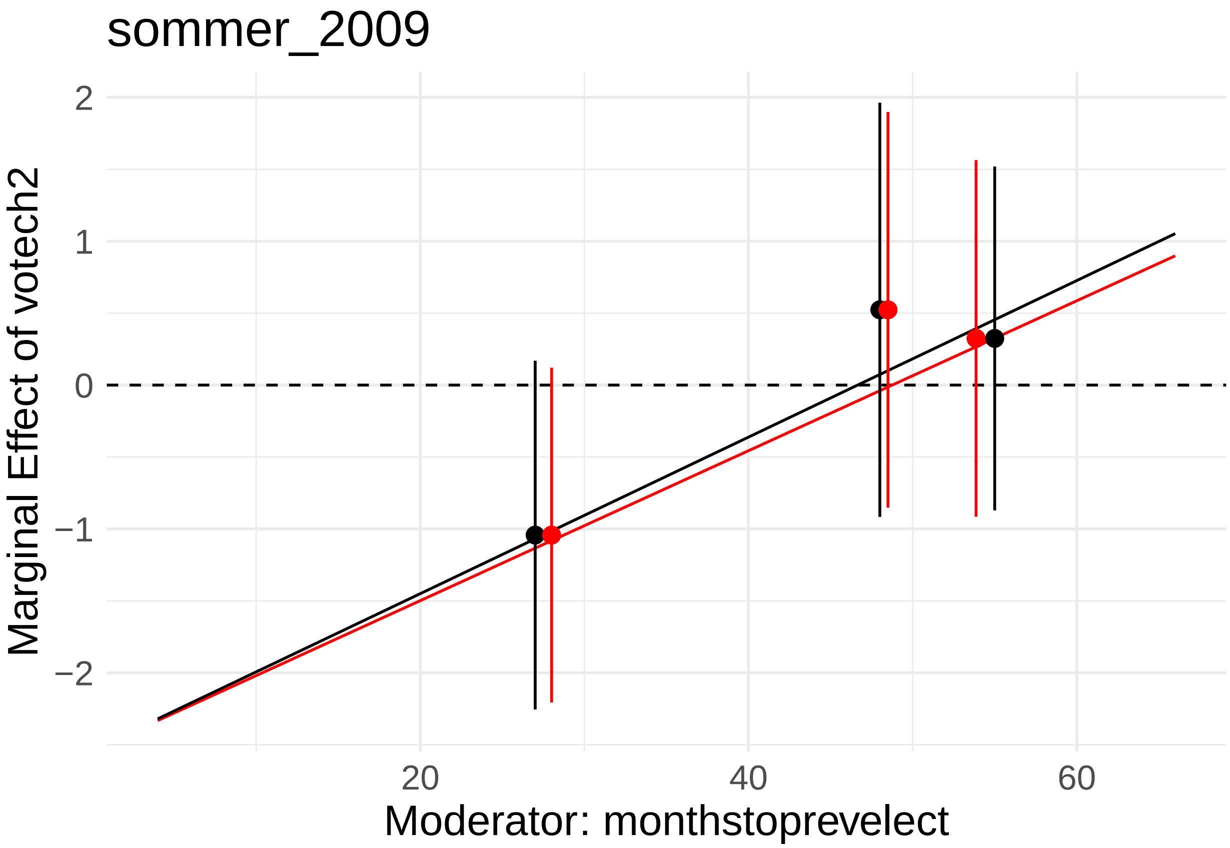
neblo 2010



neblo_2010
Marginal Effect of treatcong2
0.4
0.2
0.0
−0.2
−2
−1
0
1
2
Moderator: stealth2_ct

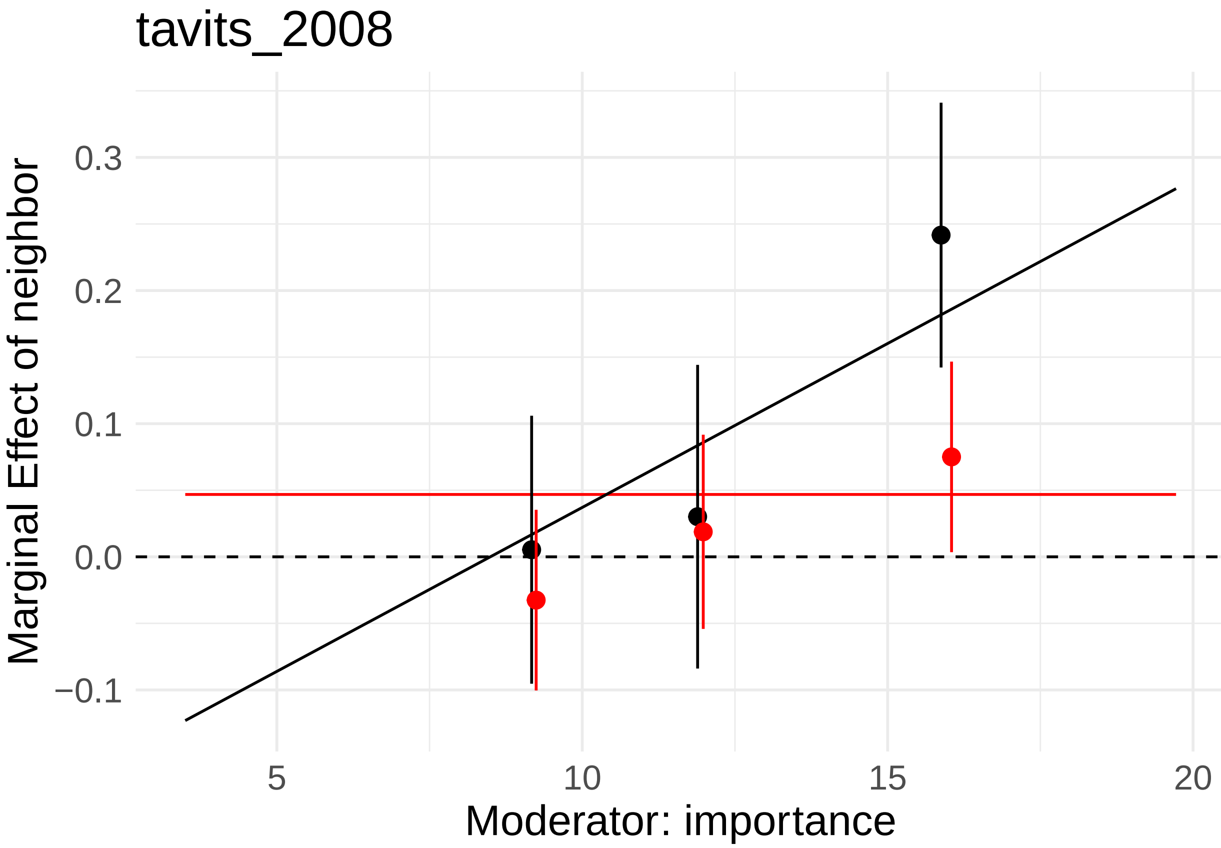
	Estimate
(Intercept)	3.6866309
D	0.1234916
needjud_ct	0.0338095
interest_ct	0.2093185
intxcon	-0.0256827
treatint	0.0705443
age_ct	-0.0029604
conflict_ct	-0.0266782
needcog_ct	0.1427560
empfull	0.0466358
D.X	-0.0376550
D.gender2	0.0187361
needjud_ct.interest_ct	-0.0221942
needjud_ct.intxcon	0.0031911
educ_ct.interest_ct	-0.0229469
educ_ct.chur_ct	0.0153788
income_ct.treatinc	-0.0019746
income_ct.needcog_ct	-0.0003920
interest_ct.intxcon	0.0000954
interest_ct.age_ct	-0.0000242
interest_ct.sunshine_ct	0.0303297
chur_ct.white	-0.0173697
intxcon.treatinc	0.0338141
intxcon.sunshine_ct	0.0098580
intxcon.white	-0.0197911
treattop.treatinc	0.0383028
treatinc.sunshine_ct	0.0416887
treatinc.empfull	0.0136961
needcog_ct.sunshine_ct	-0.0175005
D.X.white	-0.0274758
educ_ct^2.white	**-0.0168902**
educ_ct.sunshine_ct.white	0.0084419
income_ct.intxcon.white	-0.0018274
interest_ct.intxcon^2	**0.0121198**
interest_ct.pidcoll_ct.white	-0.0384826
gentrust_ct.intxcon.white	0.0052964
gentrust_ct.treatint.white	0.0186460
treatinc.sunshine_ct.white	0.0063191
gender2.age_ct.white	-0.0013993
age_ct.sunshine_ct.white	-0.0028725
sunshine_ct.efficacy_ct.white	0.0453695

sommer 2009



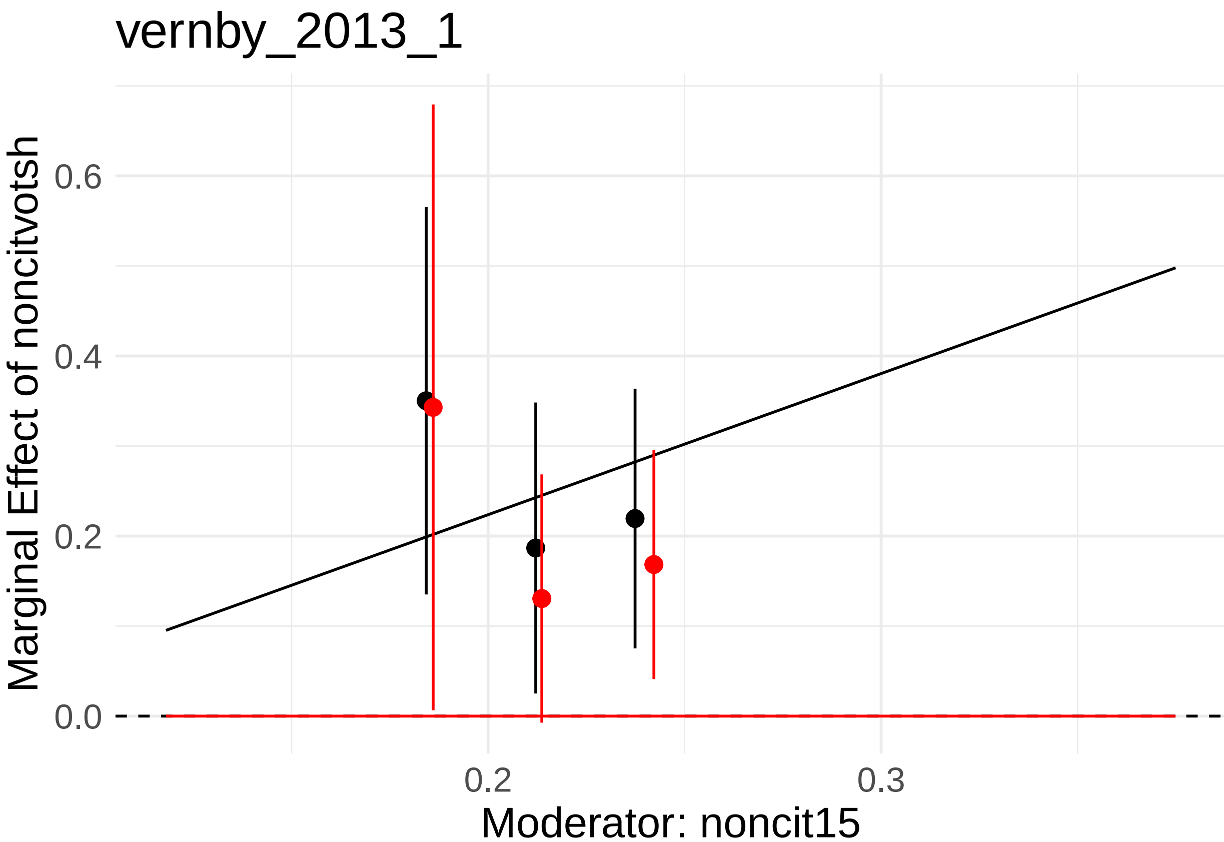
	Estimate
(Intercept)	1.6778671
D	-3.1373891
X	0.4252377
absch2	0.7395989
D^2	**0.1731444**
D.X	0.0530418
D.absch2	0.0293643
X^2	**-0.0090193**
absch2^2	**-0.0179161**
D^3	**0.0374727**
D^2.X	**-0.0009352**
X^3	**0.0000554**
X^2.absch2	**-0.0000855**
X.absch2^2	**0.0001941**
absch2^3	**0.0000900**

tavits 2008


tavits_2008
Marginal Effect of neighbor
0.3
0.2
0.1
0.0
−0.1
5
10
15
20
Moderator: importance

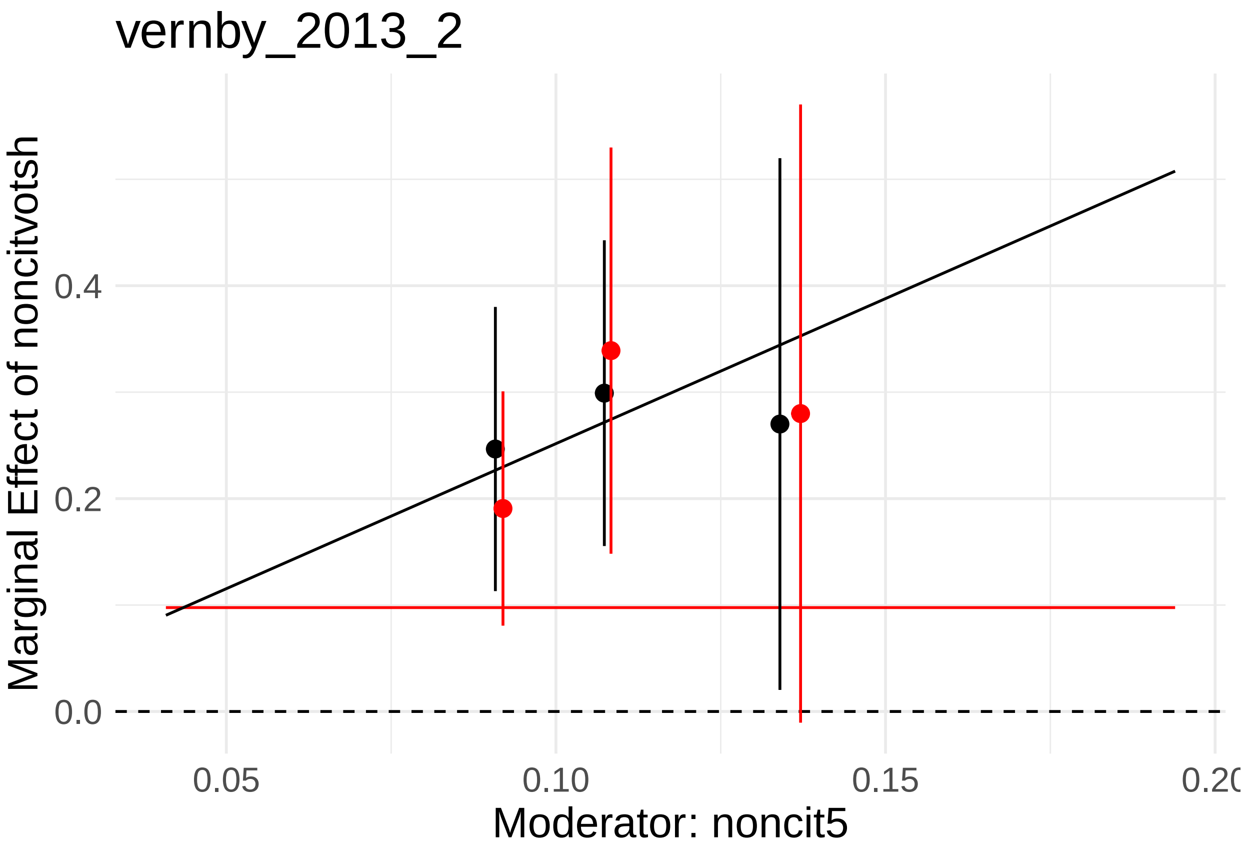
	Estimate
(Intercept)	0.2700753
D	0.1766222
yearsdem	-0.0998389
turn__ch	-0.0198666
govt	3.9404629
unemp	-0.0874265
gdp	0.0118429
ln__mdm	0.7588476
votes__last	-0.0758924
D.yearsdem	0.0423342
D.turn__ch	-0.0010601
D.unemp	-0.0117001
D.gdp	0.0186582
D.ln__mdm	-0.0115958
yearsdem^2	**-0.0002428**
yearsdem.turn__ch	0.0119558
yearsdem.unemp	0.0106564
yearsdem.gdp	0.0151459
yearsdem.ln__mdm	-0.1008215
yearsdem.ln__infl	0.0231050
yearsdem.votes__last	0.0000442
turn__ch^2	**0.0020862**
turn__ch.govt	0.2132663
turn__ch.unemp	-0.0030351
turn__ch.gdp	0.0029630
govt.unemp	-0.3375546
govt.ln__mdm	-1.4201780
govt.ln__infl	-0.3364957
govt.votes__last	-0.0358613
unemp.votes__last	0.0009932
gdp.votes__last	0.0026268
ln__mdm.votes__last	0.0394562
ln__infl.votes__last	-0.0520003
votes__last^2	**0.0089460**
D.yearsdem^2	**-0.0027045**
D.yearsdem.turn__ch	-0.0002185
D.yearsdem.unemp	0.0000489
D.yearsdem.ln__infl	-0.0076211
D.turn__ch.unemp	0.0000016
D.turn__ch.gdp	0.0003086
yearsdem^2.govt	**0.0000033**
yearsdem^2.votes__last	**0.0014183**
yearsdem.turn__ch.govt	0.0240910
yearsdem.turn__ch.votes__last	-0.0013188
yearsdem.govt.gdp	-0.0278381
yearsdem.govt.ln__mdm	0.0919522
yearsdem.govt.ln__infl	0.1022629
yearsdem.govt.votes__last	-0.0026017
yearsdem.unemp.votes__last	0.0003347
yearsdem.gdp.votes__last	-0.0023320
yearsdem.ln__mdm.votes__last	-0.0014576
yearsdem.ln__infl.votes__last	0.0027182
yearsdem.votes__last^2	**-0.0011694**
turn__ch^2.govt	**0.0124448**
turn__ch^2.votes__last	**-0.0003888**
turn__ch.govt.unemp	-0.0257909
turn__ch.unemp.votes__last	0.0004862
[illegible]	[illegible]

vernby 2013 1



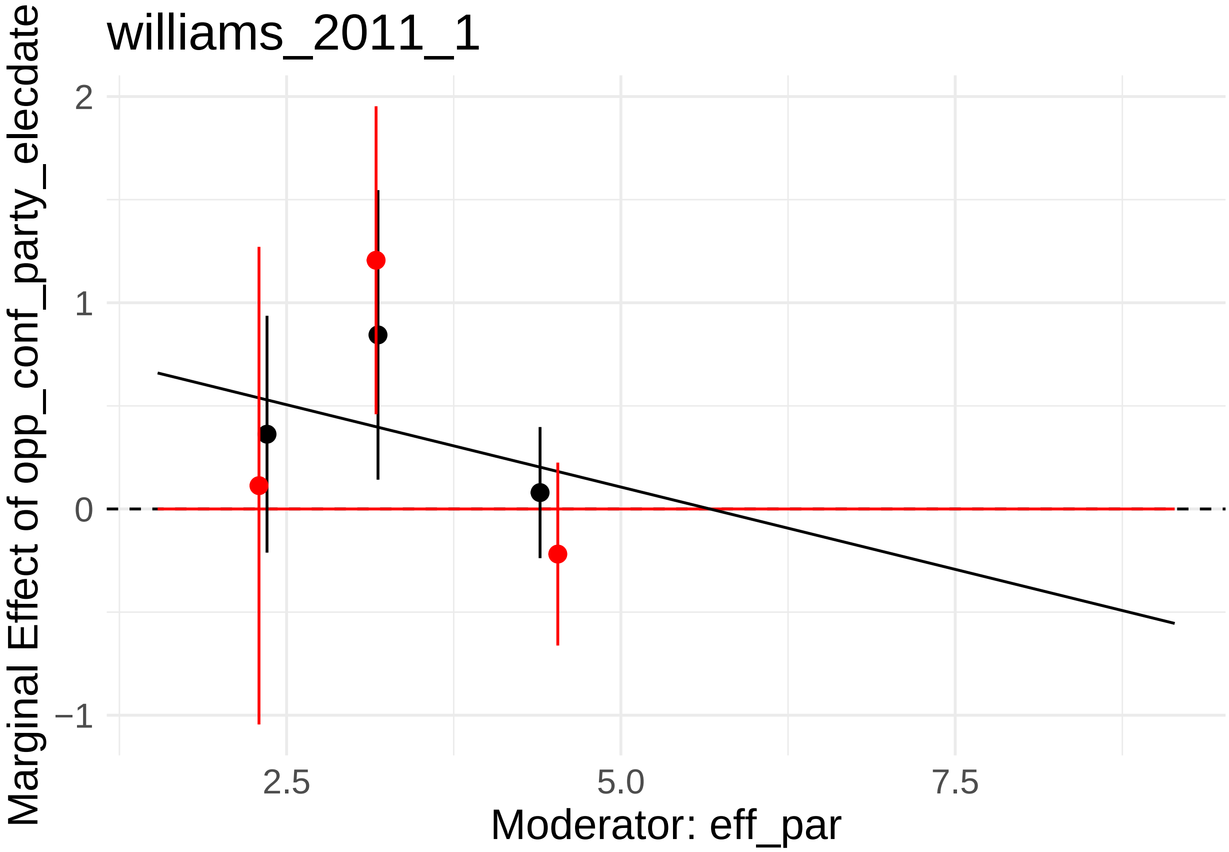
	Estimate
(Intercept)	0.8537362
Taxbase2.pop.pop.2	-0.0065910

vernby 2013 2



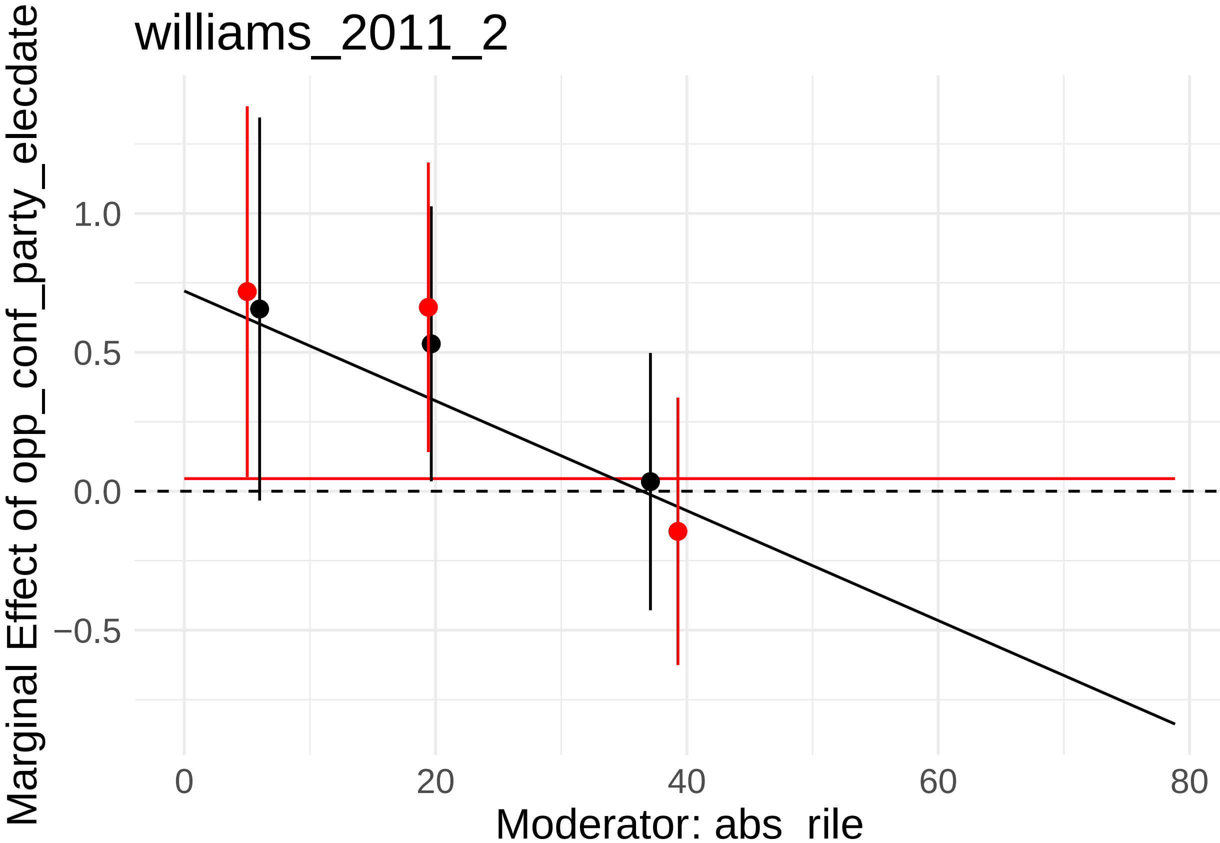
	Estimate
(Intercept)	0.7311059
pop.pop.2	0.1199289
D.Taxbase2.pop.2	0.3911241
Taxbase2.Taxbase2.2.pop.2	0.7566510
pop^2.manu	**-0.2472261**
pop.pop.2.manu	-3.2706236

williams 2011 1



	Estimate
(Intercept)	0.6013524
opp__conf__elecdate.gparties.eoc	-0.0005485
opp__conf__elecdate.eoc^2	**0.0000422**

williams 2011 2



	Estimate
(Intercept)	0.5088657
D^2	**0.0453551**
X.ncm_all_abs_rile	-0.0000289
majority.gparties	0.0549919
lag_pervote^2	**0.0005739**
lag_pervote.rgdppc_growth	-0.0033792
ncm_all_abs_rile^2	**-0.0000303**
X.lag_pervote.ncm_all_abs_rile	-0.0000028
majority.ncm_all_abs_rile^2	**0.0000021**
gparties^2.opp_conf_elecdate	**-0.0027006**
gparties.lag_pervote.opp_conf_elecdate	-0.0005275
gparties.ncm_all_abs_rile^2	**0.0000005**
gparties.ncm_all_abs_rile.opp_conf_elecdate	0.0000083
lag_pervote.opp_conf_elecdate^2	**-0.0002280**
ncm_all_abs_rile^3	**0.0000000**
ncm_all_abs_rile.opp_conf_elecdate^2	**0.0000705**

References

Adams, James, Michael Clark, Lawrence Ezrow and Garrett Glasgow. 2006. "Are Niche Parties Fundamentally Different from Mainstream Parties? The Causes and the Electoral Consequences of Western European Parties' Policy Shifts, 1976-1998." *American Journal of Political Science* 50(3):513–529.

Aklin, Michaël and Johannes Urpelainen. 2013. "Political Competition, Path Dependence, and the Strategy of Sustainable Energy Transitions: SUSTAINABLE ENERGY TRANSITIONS." *American Journal of Political Science* 57(3):643–658.

Banks, Antoine J. and Nicholas A. Valentino. 2012. "Emotional Substrates of White Racial Attitudes: EMOTIONAL SUBSTRATES OF WHITE RACIAL ATTITUDES." *American Journal of Political Science* 56(2):286–297.

Bodea, Cristina and Raymond Hicks. 2015*a*. "International Finance and Central Bank Independence: Institutional Diffusion and the Flow and Cost of Capital." *The Journal of Politics* 77(1):268–284.

Bodea, Cristina and Raymond Hicks. 2015*b*. "Price Stability and Central Bank Independence: Discipline, Credibility, and Democratic Institutions." *International Organization* 69(1):35–61.

Chapman, Terrence L. 2009. "Audience Beliefs and International Organization Legitimacy." *International Organization* 63(4):733–764.

Clark, William Roberts and Matt Golder. 2006. "Rehabilitating Duverger's Theory: Testing the Mechanical and Strategic Modifying Effects of Electoral Laws." *Comparative Political Studies* 39(6):679–708.

Hainmueller, Jens, Jonathan Mummolo and Yiqing Xu. 2019. "How Much Should We Trust Estimates from Multiplicative Interaction Models? Simple Tools to Improve Empirical Practice." *Political Analysis* 27(2):163–192.

Hastie, Trevor and Robert Tibshirani. 1986. "Generalized additive models." *Statistical Science* 1(3):297–318.

Hellwig, Timothy and David Samuels. 2007. "Voting in Open Economies: The Electoral Consequences of Globalization." *Comparative Political Studies* 40(3):283–306.

Hicken, Allen and Joel W. Simmons. 2008. "The Personal Vote and the Efficacy of Education Spending: PERSONAL VOTE AND EDUCATION SPENDING." *American Journal of Political Science* 52(1):109–124.

Huddy, Leonie, Lilliana Mason and Lene Aarøe. 2015. "Expressive Partisanship: Campaign Involvement, Political Emotion, and Partisan Identity." *American Political Science Review* 109(1):1–17.

Malesky, Edmund, Paul Schuler and Anh Tran. 2012. "The Adverse Effects of Sunshine: A Field Experiment on Legislative Transparency in an Authoritarian Assembly." *American Political Science Review* 106(4):762–786.

Neblo, Michael A., Kevin M. Esterling, Ryan P. Kennedy, David M.J. Lazer and Anand E. Sokhey. 2010. "Who Wants To Deliberate—And Why?" *American Political Science Review* 104(3):566–583.

Somer-Topcu, Zeynep. 2009. "Timely Decisions: The Effects of Past National Elections on Party Policy Change." *The Journal of Politics* 71(1):238–248.

Tavits, Margit. 2008. "Policy Positions, Issue Importance, and Party Competition in New Democracies." *Comparative Political Studies* 41(1):48–72.

Vernby, Køare. 2013. "Inclusion and Public Policy: Evidence from Sweden's Introduction of Noncitizen Suffrage: *INCLUSION AND PUBLIC POLICY*." *American Journal of Political Science* 57(1):15–29.

Williams, Laron K. 2011. "Unsuccessful Success? Failed No-Confidence Motions, Competence Signals, and Electoral Support." *Comparative Political Studies* 44(11):1474–1499.

Supplementary Information

Adaptive Lasso using glmnet

Abstract

This document includes code examples to fit models with non-linearities and inteactions using glmnet.


```
# Loading packages

library(glmnet)
```

```
## Loading required package: Matrix

## Loaded glmnet 4.1-2
```

```
library(inters)

library(margins)

library(MASS)

library(polywog)
```

```
## Loading required package: miscTools
```

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.0 --

## v ggplot2 3.3.3     v purrr   0.3.4

## v tibble  3.1.1     v dplyr   1.0.4

## v tidyr   1.1.2     v stringr 1.4.0

## v readr   1.4.0     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --

## x tidyr::expand() masks Matrix::expand()

## x dplyr::filter() masks stats::filter()

## x dplyr::lag()    masks stats::lag()
```

```
## x tidyr::pack()   masks Matrix::pack()
## x dplyr::select() masks MASS::select()
## x tidyr::unpack() masks Matrix::unpack()
```

We first create simulated data

```
set.seed(87654321)

# parameters
corr <- c(0.5)
b_a <- 0.5
b_b <- -0.5
b_c <- 0.5
b_ab <- -0.5
b_ac <- b_b2 <- c(0.5)
n <- c(1000)

vcov <- matrix(corr, nrow = 10, ncol = 10)
diag(vcov) <- 1

# X data generation
abc <- mvrnorm(n, c(0,0,0,0,0,0,0,0,0,0), vcov)
a <- abc[,1]
b <- abc[,2]
c <- abc[,3]
d <- abc[,4]
e <- abc[,5]
f <- abc[,6]
g <- abc[,7]
h <- abc[,8]
i <- abc[,9]
j <- abc[,10]
colnames(abc) <- c("a", "b", "c", "d", "e", "f", "g", "h", "i", "j")
X <- as.data.frame(abc)
```

```r
# y data generation
y <- 0.4 + 0.7 * a - 0.2 * b + 0.3 * b*b + 0.2 * a * c + rnorm(n, 0, 1)

df <- data.frame(y = y, a = a, b = b, c = c, d = d, e = e, f = f,
                 g = g, h = h, i = i, j = j)
```

The following function is used to create interpretable names for the products created

```r
polycolnames <- function(x, varnames) {
  info <- data.frame(indic = as.numeric(unlist(strsplit(x, split=".", fixed = TRUE))),
                     var = varnames)
  infosub <- info %>% filter(indic != 0)

  if (nrow(infosub) == 1) {
    out <- gsub("\\^1", "", paste0(infosub$var, "^", infosub$indic))
  }

  if (nrow(infosub > 1)) {
    out <- gsub("\\^1", "", paste0(infosub$var, "^", infosub$indic, collapse = "*"))
  }

  return(out)
}
```

The following function creates a matrix with all non-linear terms and interactions

```r
prodmatrix <- function(df, degree) {
  out_df <- poly(as.matrix(df), degree = degree, raw=TRUE)
  orig_varnames <- colnames(df)

  for (i in 1:length(colnames(out_df))) {
    colnames(out_df)[i] <- polycolnames(colnames(out_df)[i], orig_varnames)
  }
```

```
  return(out_df)
}
```

Using the function (degree = 3) creates a matrix with cubic polynomials and all possible interactions up to this degree (e.g. standard interactions, triple interactions, interactions with 2nd degree polynomials)

```
Xprod <- prodmatrix(X, degree = 3)
```

0.1 Estimation

To fit the adaptive Lasso we need to first use ridge regression to get the adaptive weights vector. In this example, we choose the lambda that gives the minimum mean cross-validated error.

```
# first use ridge regression to get the adaptive weights vector (w/ gamma = 1)
set.seed(87654321)
out_ridge <- cv.glmnet(Xprod, y, alpha = 0)
adap_w <- 1/abs(matrix(coef(out_ridge, s=out_ridge$lambda.min)
                   [, 1][2:(ncol(Xprod)+1)] ))^1
## Replacing values estimated as Infinite for 999999999
adap_w[adap_w[,1] == Inf] <- 999999999
```

We can then fit the adaptive Lasso by using these weights

```
set.seed(87654321)
out_adaplasso <- cv.glmnet(Xprod, y, alpha = 1, penalty.factor = adap_w)
```

We can then extract the (penalized) coefficient estimates, and return those that are not set to zero

```
betas_adaplasso <- data.frame(var = coef(out_adaplasso)@Dimnames[[1]],
                              beta = as.numeric(coef(out_adaplasso)))
betas_adaplasso %>%
  filter(beta != 0)
```

```
##              var      beta
## ...1 (Intercept) 0.4784787
## 2              a 0.5895886
## 9            b^2 0.2950390
```

```
## 18           a*c 0.0301004
```

If researchers wish to obtain confidence intervals for these estimates, they can embed this code within a bootstrap. Predictions can be made using the predict function.