Supplementary Material: Why we should use the Gini coefficient to assess punctuated equilibrium theory

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1 Relevance of L-Kurtosis in the Literature

We conducted a thorough literature review to gauge the relevance of the different measures used in PET research. We used a list of PET articles provided by Kuhlmann and van der Heijden (2018) as a starting point and added other relevant or newer research papers to it. The final list includes 66 papers with a quantitative assessment of PET published from 2005 until 2020. While the list gives a good overview, we do not claim it to be exhaustive. All papers were screened in regards to their measurement approach. Of the 66 identified articles 49 use the (L-)Kurtosis in some way. Of those, 31 mainly rely on the (L-)Kurtosis to support their findings, 19 combine or discuss it with other measures of punctuation. Table 1 gives an aggregated overview of all measures used. Table 2 lists all articles in detail.

Measuerement approach	Number of Papers
(L-)Kurtosis	31
Multiple, including (L-)Kurtosis	19
Log-log & semi-log plots	1
Quantile regression	2
Split into categories, superimposed normal	8
Variance and degrees of freedom	1
Other	5
Total	67
Using (L-)Kurtosis	49

Table 1: Measurements used in PET research since 2005 counted by the number of papers

Thus, although there have been other measures (Breunig and Jones, 2011) and identification strategies (Fatke, 2020) proposed the dominating measure in PET research is still the (L-)Kurtosis. Another strand of the literature has moved to use multivariate analysis mostly by superimposing a normal distribution upon the change values and splitting the observations into different categories (e.g. Flink, 2017, Flink and Robinson, 2020). Instead of trying to identify general change patterns this strand of the literature is more interested in explaining what exactly leads to observed change events. Of the 67 papers identified, 30 at least once use a sample size below 250, where we identified significant divergence in the precision of G and τ_4 . Of this 30 at least 14 rely on sample sizes below 100.

	Title	Author(s)	Journal	Year	Measurement
1	A model of choice for	Jones,	Journal of Public Ad-	2005	(L-)Kurtosis
	public policy	Baumgart-	ministration Research		
		ner	and Theory		
2	Policy Punctuations in	Mortensen	Public Administration	2005	Multiple, including (L-
	Danish Local Budgeting)Kurtosis
3	Comparative studies of	Baumgartner,	Journal of European	2006	(L-)Kurtosis ¹
	policy agendas	Green-	Public Policy		
		Pedersen,			
		Jones			
4	Punctuated Equilibrium	Baumgartner,	Journal of European	2006	(L-)Kurtosis
	in French Budgeting	Foucault,	Public Policy		
	Processes	Francois			
5	The more things change,	Breunig	Journal of European	2006	(L-)Kurtosis
	the more they stay the	_	Public Policy		
	same: A comparative		-		
	analysis of budget punc-				
	tuations				
6	Explaining policy	John	Journal of European	2006	Multiple, including (L-
	change: the impact		Public Policy)Kurtosis
	of the media, public		0		,
	opinion and political vi-				
	olence on urban budgets				
	in England				
7	Public Expenditures in	Soroka,	Journal of the Royal Sta-	2006	(L-)Kurtosis
	the UK: How Measures	Wlezien.	tistical Society		
	Matter	McLean	· <i>v</i>		
8	Punctuated Equilibrium	Robinson.	Political Research Quar-	2006	(L-)Kurtosis
	and Congressional Bud-	Caver	terly		
	geting		•		
9	Patterns of Change in	Schneider	Political Research Quar-	2006	Multiple, including (L-
	the Use of Imprison-		terly)Kurtosis
	ment in the American		v		,
	States: An Integration				
	of Path Dependence.				
	Punctuated Equilib-				
	rium and Policy Design				
	Approaches				
10	Punctuated equilibria	Breunig.	Policy Studies Journal	2006	(L-)Kurtosis
10	and budgets in the	Koski			()
	American States				

 $^{1}Introduction article without empirial application but reference to (L-)Kurtosis (leptokurtic change patterns).$

11	Explaining policy punc- tuations: Bureaucratiza- tion and budget change	Robinson, Caver, Meier, O'Toole	American Journal of Po- litical Science	2007	Split into categories, su- perimposed normal
12	Noah and Joseph Effects in Government Budgets: Analyzing Long-Term Memory	Jones, Bre- unig	Policy Studies Journal	2007	Multiple, including (L-)Kurtosis
13	Political Attention in a coalition system: Analysing Queen's Speeches in the Nether- lands 1945-2007	Breeman, Lowery ,Poppleaars, Resodi- hardjo, Timmer- mans, de Vries	Acta Politica	2009	Multiple, including (L-)Kurtosis
14	Punctuated Equilibrium in Comparative Perspec- tive	Baumgartner, Breunig, Green- Pedersen, Jones, Mortensen, Nuytemans, Walgrave	American Journal of Po- litical Science	2009	Multiple, including (L-)Kurtosis
15	The dynamics of polit- ical attention: public opinion and the Queen's Spaach in the United Kingdom	Jennings, John	American Journal of Po- litical Science	2009	Multiple, including (L-)Kurtosis
16	A General Empirical Law of Public Budgets: A Comparative Analysis	Jones, Baum- gartner, Breunig, Wlezien, Soroka, Foucault, François, Green- Pedersen, Koski, John, Mortensen, Varone, Walgrave	American Journal of Po- litical Science	2009	Multiple, including (L-)Kurtosis

17	Punctuated budgets and governors' institutional	Breunig, Koski	American Politics Re- search	2009	(L-)Kurtosis
18	Friction and Party Manifesto Change in 25 countries (1945-1988)	Walgrave, Nuytemans	Journal of European Public Policy	2009	(L-)Kurtosis
19	Policy Punctuations in Mature Welfare States	Jensen	Journal of Public Policy	2009	(L-)Kurtosis
20	Political Attention and Public Spending in the United States	Mortensen	Policy Studies Journal	2009	Split into categories
21	Exploring the Factors for Budget Stability and Punctuations: A Preliminary Analysis of State Government Sub-Functional Expen- ditures	Ryu	Policy Studies Journal	2009	(L-)Kurtosis
22	Punctuations and Turn- ing Points in British Politics? The Policy Agenda of the Queen's Speech, 1940-2005	John, Jen- nings	British Journal of Polit- ical Science	2010	Multiple, including (L-)Kurtosis
23	Why are policy agendas punctauted? Friction and cascading in parlia- ment and mass media in Belgium	Walgrave, Vliegen- thart	Journal of European Public Policy	2010	(L-)Kurtosis
24	Stability and Punctua- tions in Public Spend- ing: A Comparative Study of Budget Func- tions	Breunig, Koski, Mortensen	Journal of Public Ad- ministration Research and Theory	2010	(L-)Kurtosis
25	Incrementalism in Ap- propriations: Small Ag- gregation, Big Changes	Anderson, Harbridge	Public Administration Review	2010	Other
26	Reduction, Stasis, and Expansion of Budgets in Advanced Democracies	Breunig	Comparative Political Studies	2011	Quantile regression
27	Stochastic Process Methods with an Ap- plication to Budgetary Data	Breunig, Jones	Political Analysis	2011	Multiple, including (L-)Kurtosis

28	Combining Incremental-	Caamano-	Public Finance Review	2011	Multiple, including (L-
	ism and Exogenous Fac-	Alegre,)Kurtosis
	tors in Analyzing Na-	Lagos-			
	tional Budgeting: An	Penas			
	Application to Spain				
29	From There to Here	Iones	Policy Studies Journal	2012	Discusses (L_)Kurtosis
20	Punctuated Equilibrium	Baumgart	i oney studies tournar	2012	Discusses (E) frantosis
	to the Conorol Dupotuo	Daumgart-			as one measures of
	to the General Functua-	ner			punctuations.
	tion Thesis to a Theory				
	of Government Informa-				
	tion Processing				
30	When do new issues ap-	Van Assche	Acta Politica	2012	Multiple, including (L-
	pear? Punctuations in)Kurtosis
	the Belgian Executive				
	Agenda				
31	Policy punctuations	Alexandrova	Policy Studies Journal	2012	Multiple, including (L-
	and issue diversity on	,Carammia,)Kurtosis
	the European Council	Timmer-			
	agenda	mans			
32	The tortoise or the hare?	Breunig.	Policy Studies Journal	2012	(L-)Kurtosis
	Incrementalism, punctu-	Koski	0		< ', '
	ations and their conse-				
	quences				
22	What are policy punctu	John Bowan	Policy Studios Journal	2012	Other
55	what are poncy punctu-	John, Bevan	Folicy Studies Journal	2012	Other
	the Areado of the UK				
	the Agenda of the UK				
~ .	Government	~			
34	Rural and regional pol-	Cockfield,	Australian Journal of	2013	Other
	icy: A case of punc-	Botterill	Public Administration		
	tauted incrementalism?				
35	Representation, Agen-	Bevan, Jen-	European Journal of Po-	2013	(L-)Kurtosis
	das and Institutions	nings	litical Research		
36	EU Budgetary Dynam-	Citi	Journal of European	2013	Multiple, including (L-
	ics: Incremental or		Public Policy)Kurtosis
	Punctuated Equilib-				
	rium?				
37	MP's Issue Attention in	Brouard	Journal of Legislative	2013	(L-)Kurtosis
	Parliament: Evidence of		Studies		
	a Stick-Slip Process of				
	Attention Allocation in				
	the French National As-				
	sembly				
38	Newspaper attention	Chaqués-	Journal of Public Policy	2013	Other
00	and policy activities in	Bonafont	obtained of a done i oney	2010	0 1101
	Spain	Boumgart			
	Spain.	Daumgai t-			
		ner			

39	Punctuated Equilibrium and the Supreme Court	Robinson	Policy Studies Journal	2013	Multiple, including (L-)Kurtosis
40	Effects of political insti- tutions on punctuated- equilibrium in local emergency management policy processes: Ex- amination of county governments in Florida	Kwon, Choi, Bai	U.S. Lex Localis	2013	(L-)Kurtosis
41	Stability and change in US city policymaking: evidence and a path for- ward	Sapotichne, Johnson, Park	Joshua Sapotichne, Megan Johnson & Young-Shin Park	2013	(L-)Kurtosis
42	Organizational History and Budgetary Punctua- tion	Robinson, Flink, King	Journal of Public Ad- ministration Research and Theory	2014	Split into categories, su- perimposed normal
43	How Authoritarianism Intensifies Punctu- ated Equilibrium: The Dynamics of Policy Attention in Hong Kong	Lam, Chan	Governance	2014	(L-)Kurtosis
44	Two faces of media at- tention: media storms vs. general coverage	Boydstun, Hardy, Walgrave	Political Communica- tion	2014	(L-)Kurtosis
45	Point Predictions and the Punctuated Equilib- rium Theory: A Data Mining Approach—U.S. Nuclear Policy as Proof of Concept	Hegelich, Fraune, Knollmann	Policy Studies Journal	2015	Other
46	Punctuated Equilibrium Theory: An Empirical Investigation of Its Rele- vance for Global Health Expenditure	Martin, Streams	Public Budgeting and Finance	2015	(L-)Kurtosis
47	Punctuated Equilibrium and the Information Dis- advantage of Authoritar- ianism: Evidence from the People's Republic of China	Chan, Zhao	Policy Studies Journal	2016	(L-)Kurtosis
48	Agenda instability in Pennsylvania politics: Lessons for future replication	Mallinson	Research & Politics	2016	(L-)Kurtosis

49	Is Morality Policy Dif- ferent? Testing Sectoral and Institutional Expla-	Hurka, Adam, Knill	Policy Studies Journal	2017	log-log and semi-log plots
50	Budgetary change in au- thoritarian and demo- cratic regimes	Baumgartner, Carammia, Epp, No- ble, Rey, Yildirim	Journal of European Public Policy	2017	(L-)Kurtosis
51	Complexity, Capacity, and Budget Punctua- tions	Epp, Baum- gartner	Policy Studies Journal	2017	Multiple, including (L-)Kurtosis
52	Representative systems and policy punctuations	Fagan, Jones, Wlezien	Journal of European Policy	2017	(L-)Kurtosis
53	Rethinking Punctuated Equilibrium Theory: A Public Administration Approach to Budgetary Changes	Flink	Policy Studies Journal	2017	Split into categories, su- perimposed normal
54	Ordering Chaos: The Performance Conse- quences of Budgetary Changes	Flink	Journal of Public Ad- ministration Research and Theory	2017	Split into categories, su- perimposed normal
55	Punctuated equilibrium in democracy and au- tocracy: an analysis of Hungarian budgeting between 1868 and 2013	Sebök, Berki	European Political Sci- ence Review	2018	(L-)Kurtosis
56	Stability and change in international policy- making: A punctuated equilibrium approach	Lundgren, Squatrito, Tallberg	Review of International Organzizations	2018	(L-)Kurtosis
57	Copping Off and Bot- toming Out: Set- ting Budget Priorities Through Executive Power	Breunig	Policy Studies Journal	2018	Quantile regression
58	Wars, presidents, and punctuated equilibriums in US defense spending	Sharp	Policy Sciences	2019	(L-)Kurtosis

59	Punctuated equilibrium or incrementalism in policymaking: What we can and cannot learn from the distribution of policy changes	Desmarais	Research & Politics	2019	(L-)Kurtosis
60	Nepalese Budgetary Dy- namics: Following In- crementalism or Punctu- ated Equilibrium	Guragain, Lim	Public Organization Re- view	2019	Multiple, including (L-)Kurtosis
61	A Comparative Test of the Punctuated Equilib- rium Theory: Policy Punctuations in Tobacco Control	Vannoni	Journal of Comparative Policy Analysis: Re- search and Practice	2019	Multiple, including (L-)Kurtosis
62	Talk is not cheap: Pol- icy agendas, information processing, and the un- usually proportional na- ture of European Cen- tral Bank communica- tions policy responses	Cross, Greene	Governance	2019	(L-)Kurtosis
63	Predicting budgetary change: The effect of performance gaps	Flink	Journal of Public Ad- ministration Research and Theory	2019	Split into categories, su- perimposed normal
64	Budgetary Punctua- tions: A Fiscal Manage- ment Perspective	Xiao, Wang, Liu	Policy Studies Journal	2020	(L-)Kurtosis
65	Punctuated Equilibrium and Bureaucratic Au- tonomy in American City Governments	Park, Sapotichne	Policy Studies Journal	2020	Split into categories, su- perimposed normal
66	Systemic Dynamics of Policy Change: Over- coming Some Blind Spots of Punctuated Equilibrium Theory	Fernández- i-Marín, Hurka, Knill, Steinebach	Policy Studies Journal	2020	Variance and degrees of freedom
67	Corrective policy reac- tions: positive and neg- ative budgetary punctu- ations	Flink, Robinson	Journal of Public Policy	2020	Split into categories, su- perimposed normal

Table 2: List of quantitative PET articles identified

2 L-Kurtosis and Gini: Empirical data

2.1 Bootstrap on US Budget Outlays

Additional to the simulated results, we also tested the measures on empirical data. For this we relied on the data on US budget outlays collected by the Comparative Agendas Project (Jones et al., 2009). We use the version provided by (Fatke et al., 2019). Instead of simulating different draws from the same underlying distribution, we rely on a bootstrap procedure to estimate the precision of the G and τ_4 . We drew 10,000 samples with replacement from the data and calculated G and τ_4 for each sample. The resulting density distributions of the measures can be seen in Figure A1. The lines mark the values for the full distribution which are G = 0.82 and $\tau_4 = 0.57$.



Figure A1: Density distributions of Gini coefficient (G) and L-kurtois (τ_4) retrieved from bootstrap procedure (n=10,000) on US buget outlays.

The results show that G is superior when it comes to precision in this empirical example. With a standard deviation of $SD_G = 0.02$ versus a $SD_{\tau_4} = 0.07$ and coefficients of variation of $CV_G = 0.03$ and $CV_{\tau_4} = 0.12^2$. Thus, in this example, the difference in precision between the two values is more severe than in our simulated example in the main text. We fixed the seeds between both simulations so we can calculate the correlation between the two measures which is roughly p = 0.9. One concern when using G is that it is calculated based on absolute values. We tested if this affects the results retrieved by splitting

 $^{^{2}}$ When taking the log outlays for calculating the change values the difference is slightly smaller, with G still being twice as precise.

the data into positive and negative values and calculating G separately. We included the zeros in the calculation of the negative values. This resulted in $G_{positive} = 0.75$ and $G_{negative} = 0.92$. When calculating the weighted average, and rounding it to two digits G = 0.82, which is identical to the result retrieved when taking the absolute values and calculating G directly.

2.2 Example for possible Type I error caused by imprecision

As shown in the main text, the imprecision of τ_4 can be detrimental when assessing hypotheses. This is also true for empirical data. We show it for a comparative case by replicating the results from Lundgren, Squarito, and Tallberg (2018). Lundgren and colleagues assess the hypotheses that higher levels of institutional friction result in higher degrees of punctuation by comparing five different IOs with varying levels of friction. They calculate τ_4 for the change rate values of policy attention to assess their hypothesis. The calculated values of τ_4 vary from 0.26 to 0.31.

Figure A2 shows the corresponding density distribution of the change rate values, the corresponding Lorenz curves, and the calculation for G and τ_4 against the level of friction as identified by Lundgren et al. (2018). While Lundgren and colleagues conclude that there is a connection between friction and punctuation, the results from G show a different picture. This has two reasons. First off, the underlying distributions of the five different IOs are all rather similar. This can be seen in the density distributions and especially in the Lorenz curves. The difference between the lowest and the highest value of τ_4 is only 0.05. Given the imprecision we identified, we would argue that differences of this magnitude in τ_4 are not suited to make statements about the degrees of punctuation between these distributions. Second, looking at the density distribution shows a clear bump between 1 and 2, therefore, in change events in the magnitude of 100% to 200% difference to the previous observation period. This is not captured by τ_4 . Fur-



Figure A2: Top: Density distribution of policy attention change rates of 5 IOs based on data from Lundgren et al. (2018). Middle: Lorenz Curve of cumulative share of Policy Change events against cumulative share of years. Bottom: Gini coefficient (G) and L-Kurtosis (τ_4) against levels of friction

thermore, we can see that the Lorenz curve of the EU is more concave than the one of the OAS and the UN. Given the higher precision we identified when comparing G to τ_4 , we would argue that the finding presented by Lundgren et al. could be a result of a Type I error caused by the lack of precision of τ_4 .

3 L-Kurtosis and Gini: Type II errors

Type II errors are hard to define in PET research since there is no defined bound when a distribution is *not* punctuated or how different distributions have to be to possess different levels of punctuation. Again we turn to the t-distribution to give an approximation of how prone the measures are to Type II errors. The t-distribution creates a density distribution similar to a normal distribution, but with more observations in the tails and thinner shoulders. How *punctuated* the t-distribution is, depends on the degrees of freedom (DF) of the distribution (Fernández-i-Marín et al., 2019). Lower DF form a more punctuated shape, with rising DF the t-distribution converges towards a normal distribution. A threshold of 30 is often implied as the point, where the distributions become practically identical. Therefore, to give an approximation of Type II error we employ a similar simulation strategy as before. Instead of varying the sample size, we vary the DF of a t-distribution. Again, we use the same rejection criterion as before – we would reject H_0 if the value of a distribution is 0.05 higher than the true value. We simulate 1,000 draws with a sample size of 250 from a t-distribution for each degree of freedom between 2 and 30 and calculate both measures. Figure A3 shows how often in percent a researcher would reject H_0 under the given rejection criterion.

Unsurprisingly the rejection rate of τ_4 is higher than for the G. This shows, that the decision between G and τ_4 is also dependent on what kind of error is worse given the research context. Although there are certain scenarios where Type II errors are preferred to Type I errors, for example in certain medical tests, we would argue that in a social science context it is advisable trying to reduce Type I errors.



Figure A3: Type II error rate in percent against degrees of freedom of tdistribution. Simulated data: $1,000 \text{ draws n}=250 \text{ from t-distribution with vary$ ing degrees of freedom. Line: LOESS with 95% CI

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