Zsuzsanna B. Magyar
What Makes Party Systems Different?
A Principal Component Analysis of 17 Advanced Democracies 1970-2013
Online Appendix

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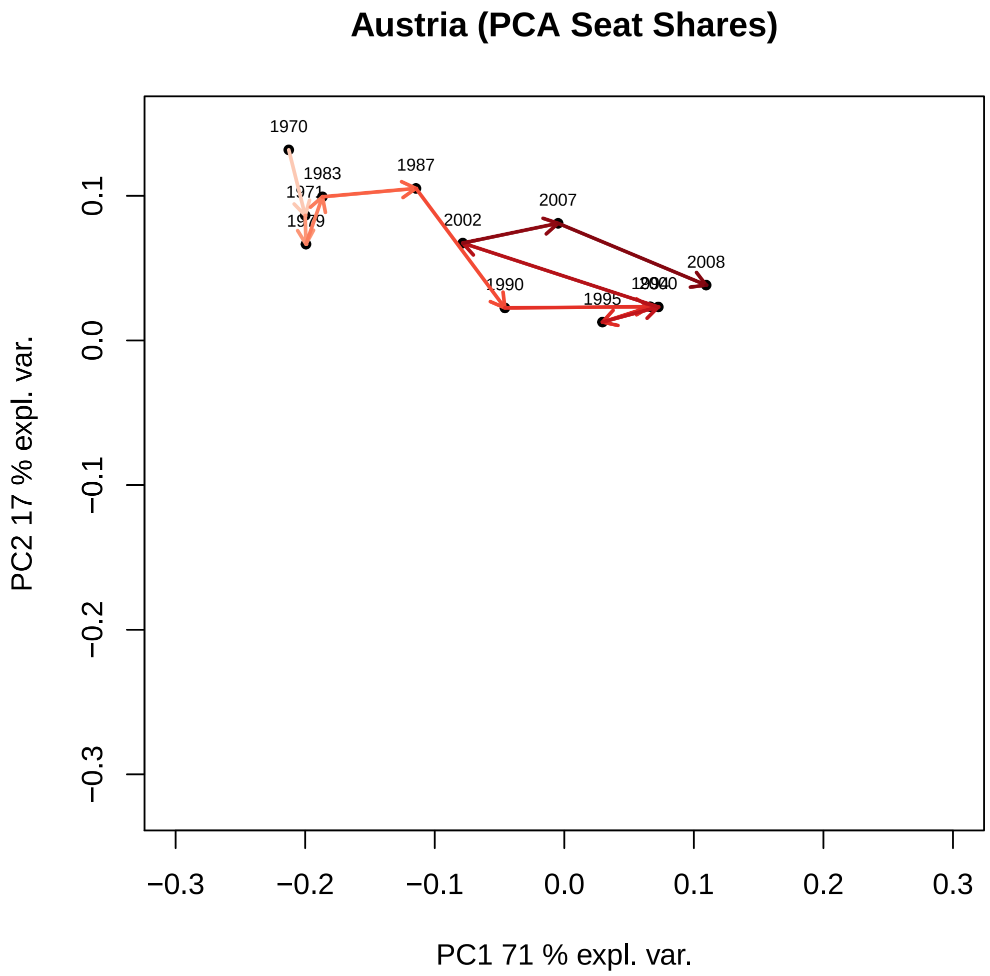
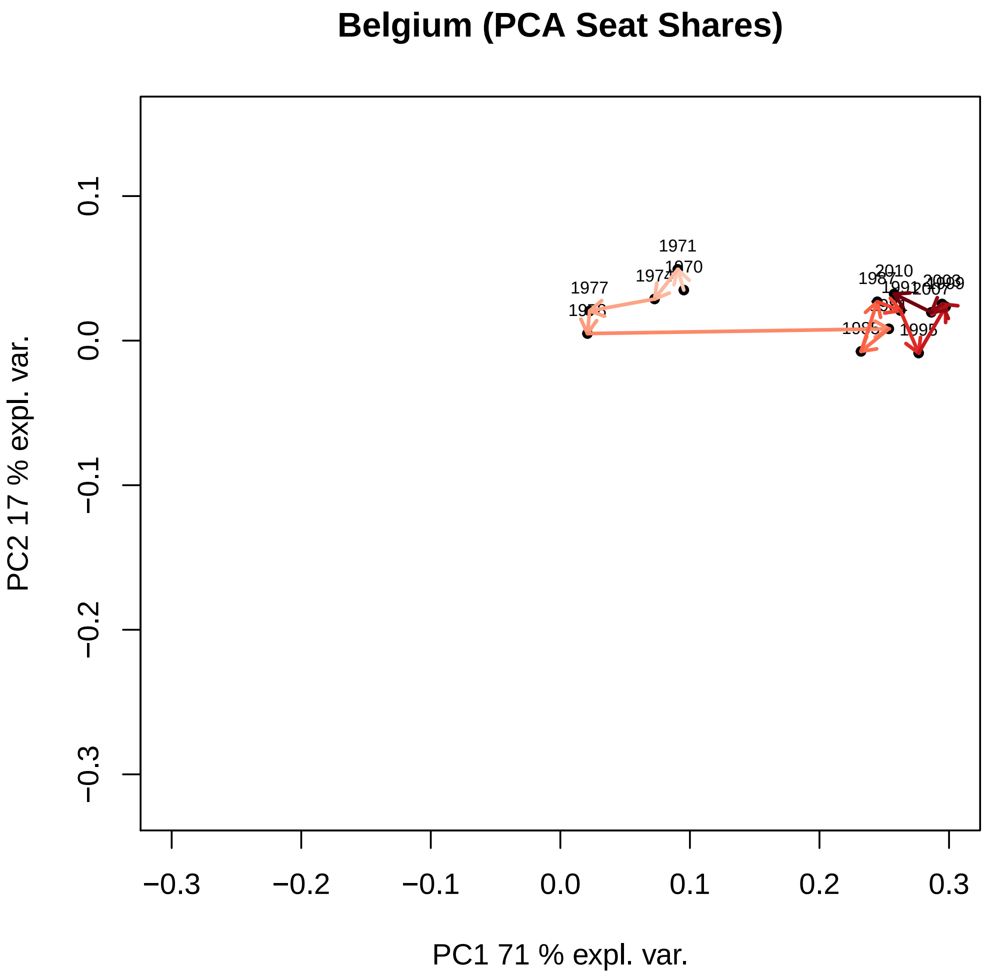


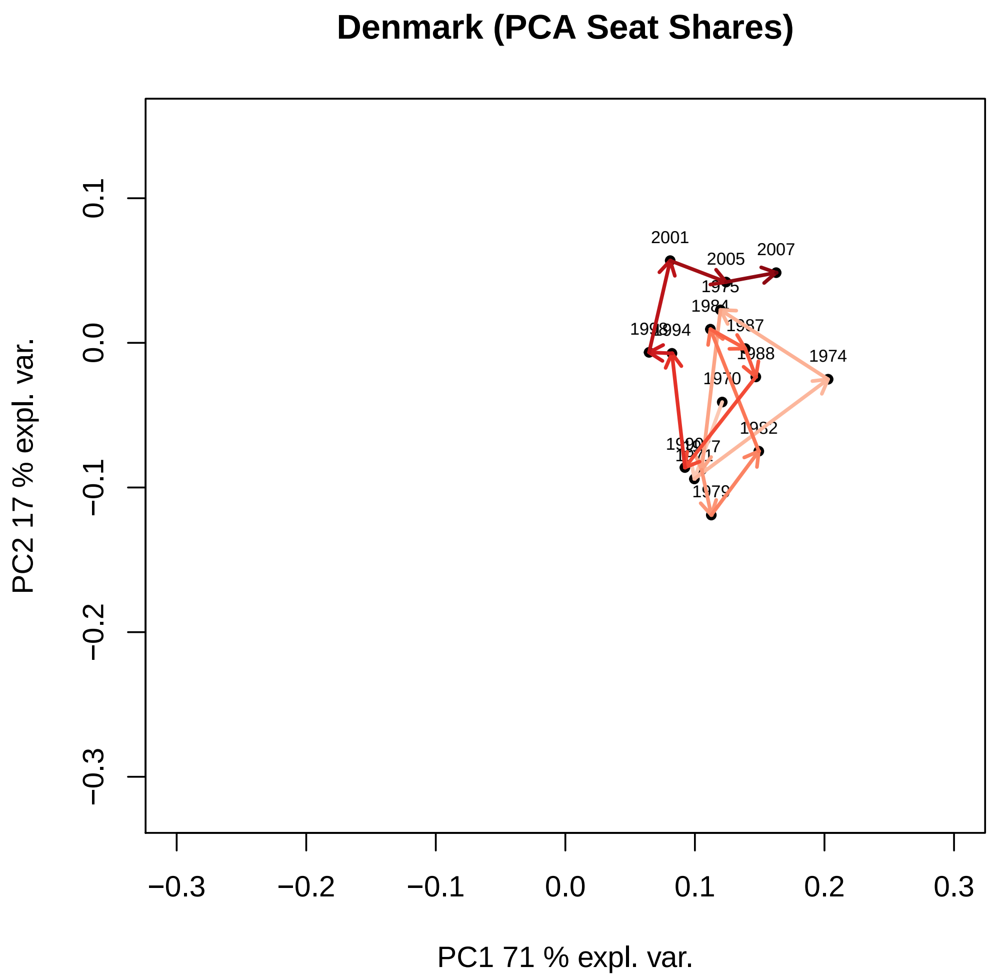
Notes: The plot shows the Austrian party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.1: Austria on the PCA Dimensions



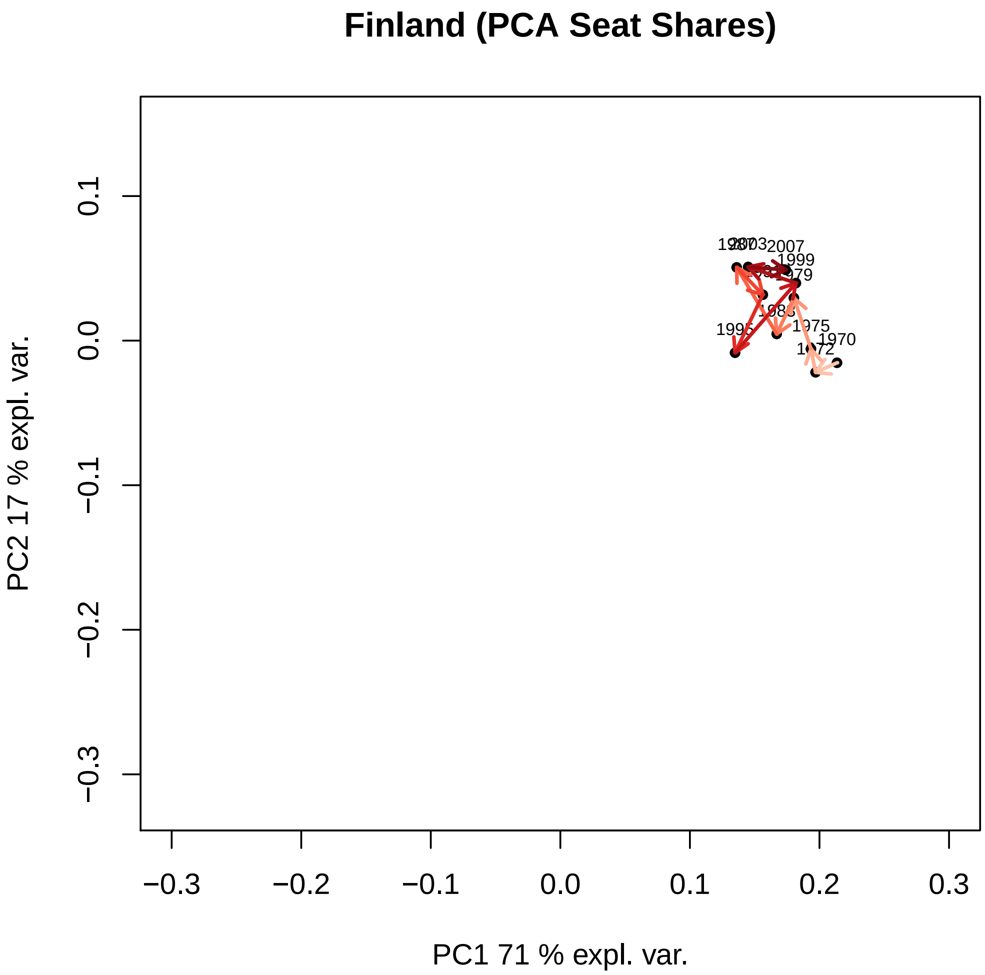
Notes: The plot shows the Belgian party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.2: Belgium on the PCA Dimensions



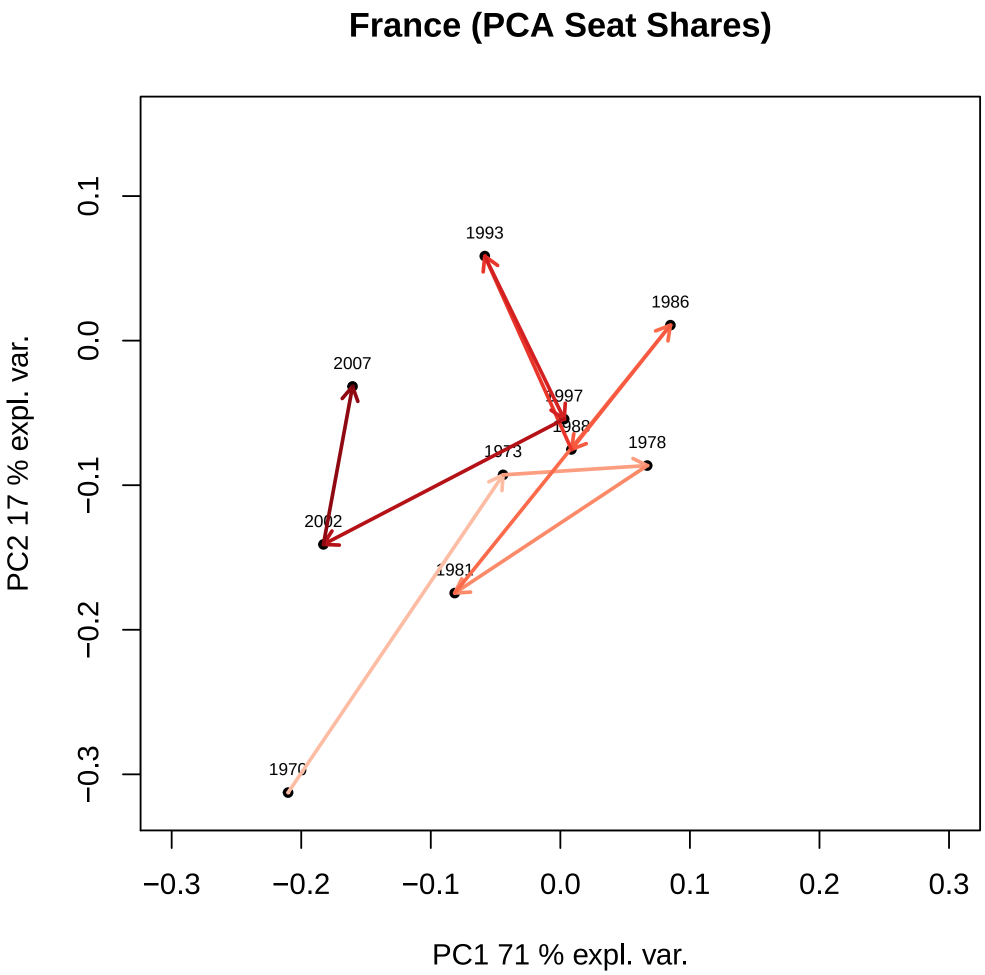
Notes: The plot shows the Danish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.3: Denmark on the PCA Dimensions



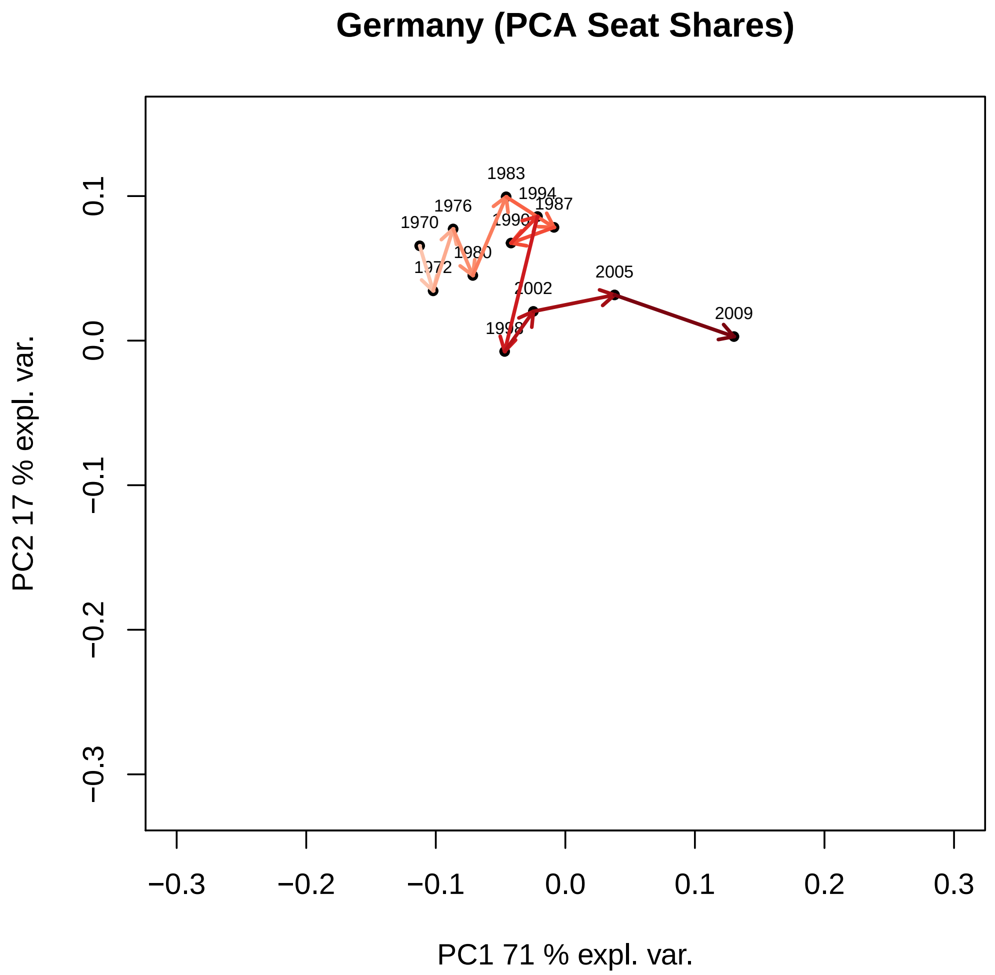
Notes: The plot shows the Finish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.4: Finland on the PCA Dimensions



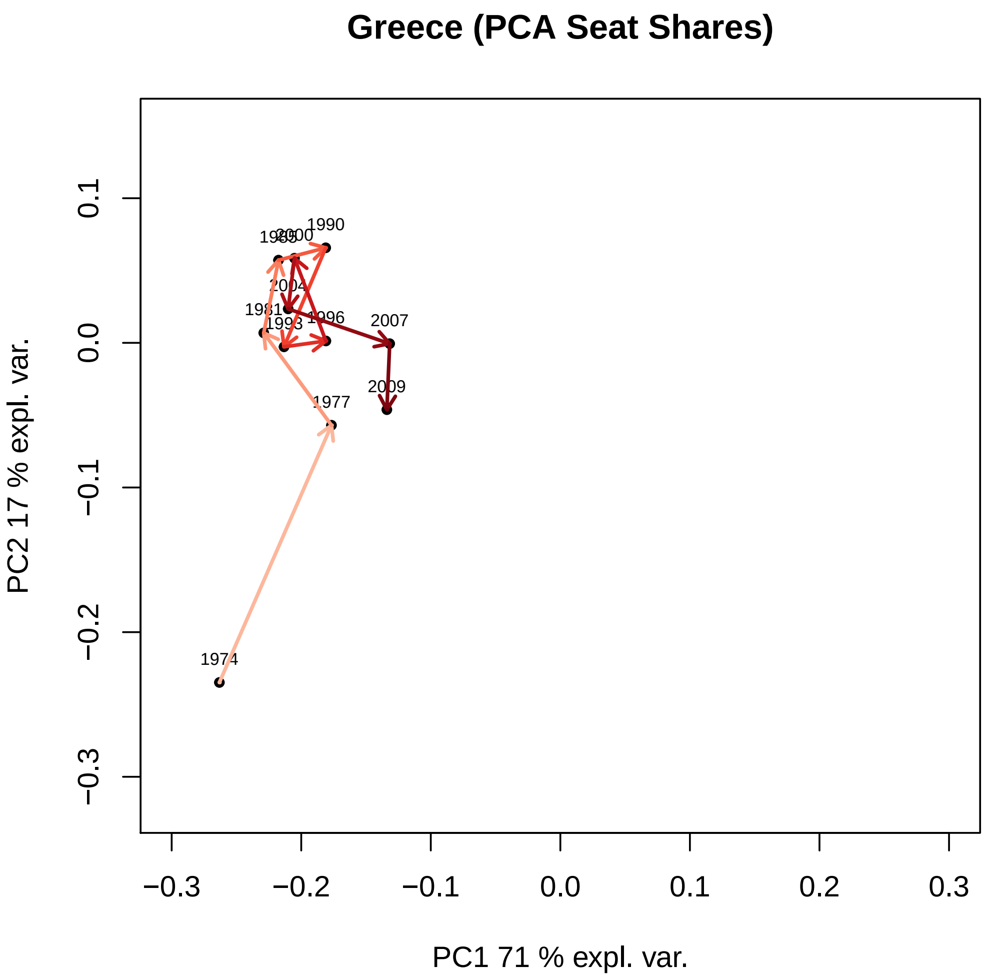
Notes: The plot shows the French party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.5: France on the PCA Dimensions



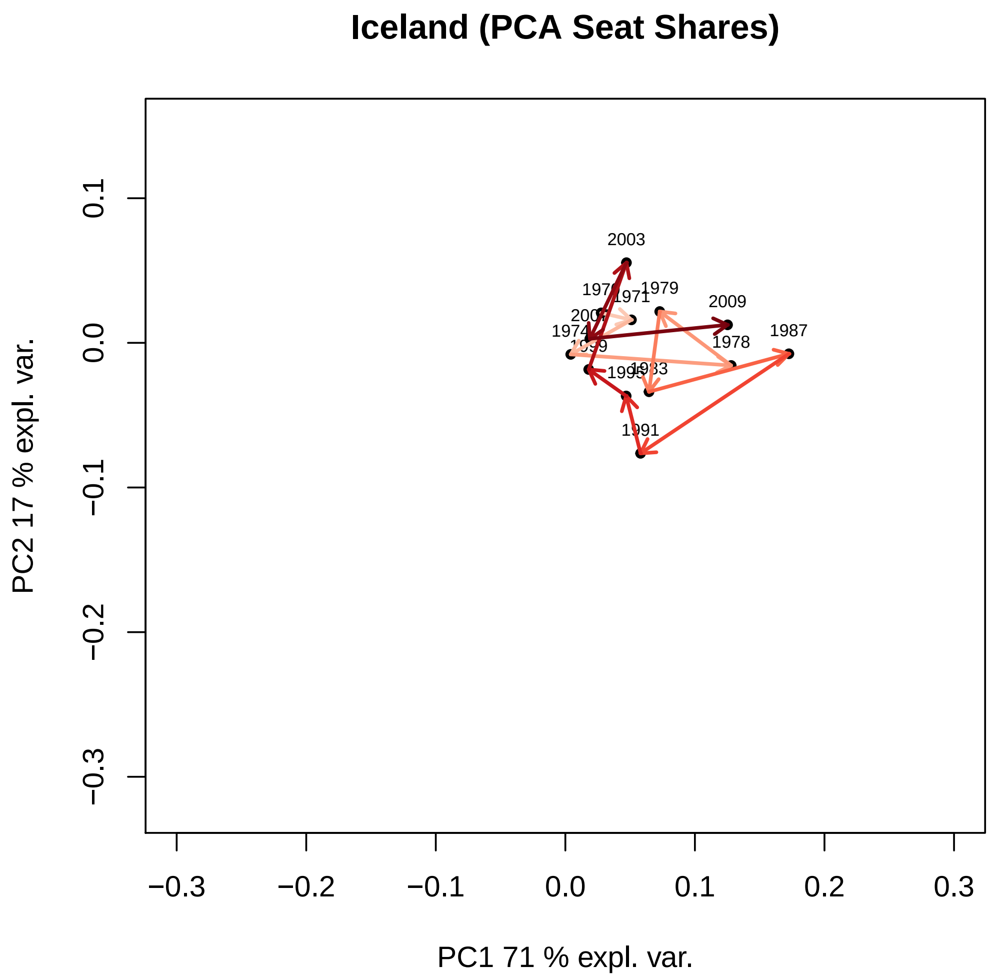
Notes: The plot shows the German party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.6: Germany on the PCA Dimensions



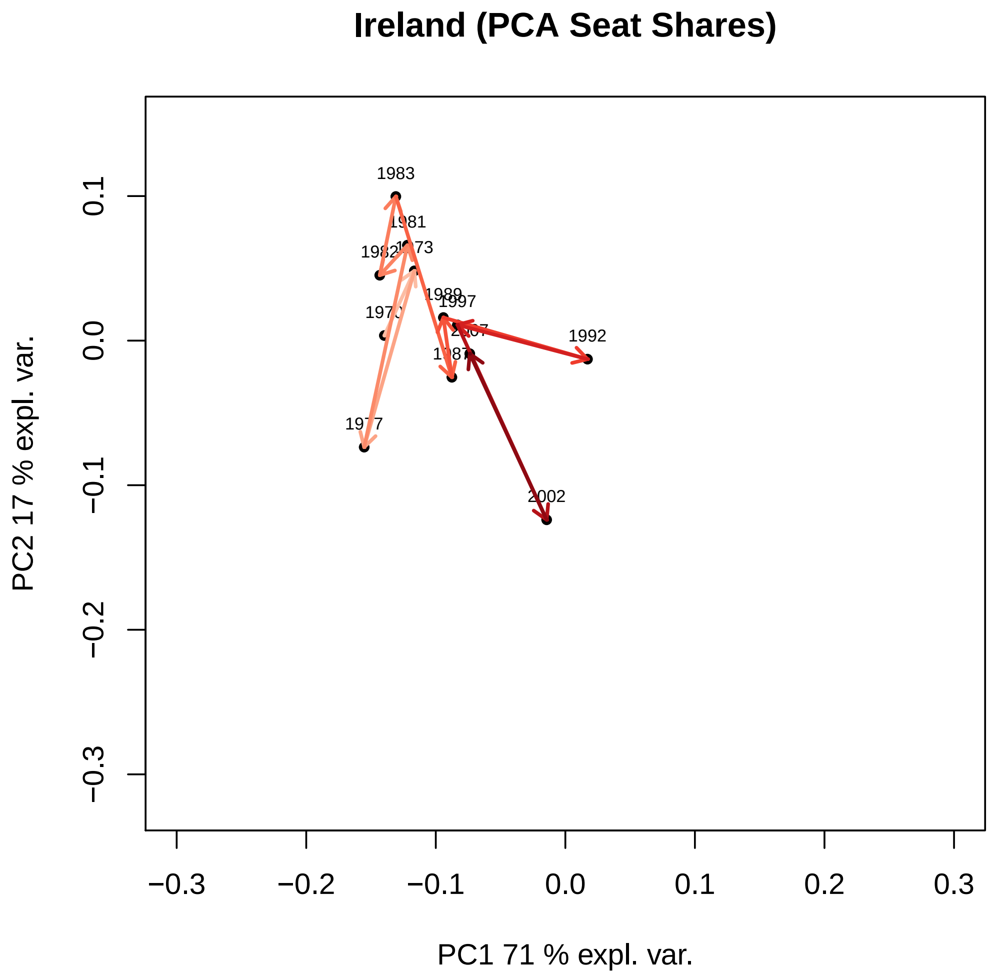
Notes: The plot shows the Greek party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.7: Greece on the PCA Dimensions



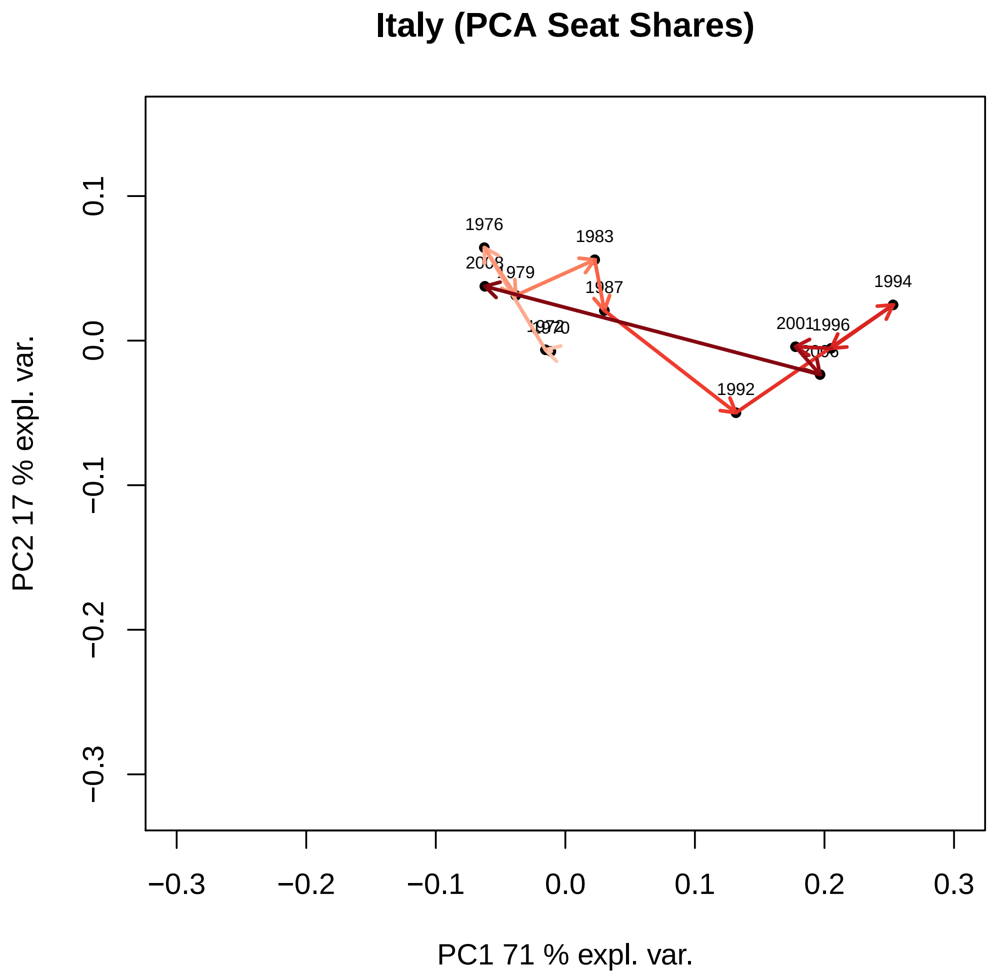
Notes: The plot shows the Icelandic party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.8: Iceland on the PCA Dimensions



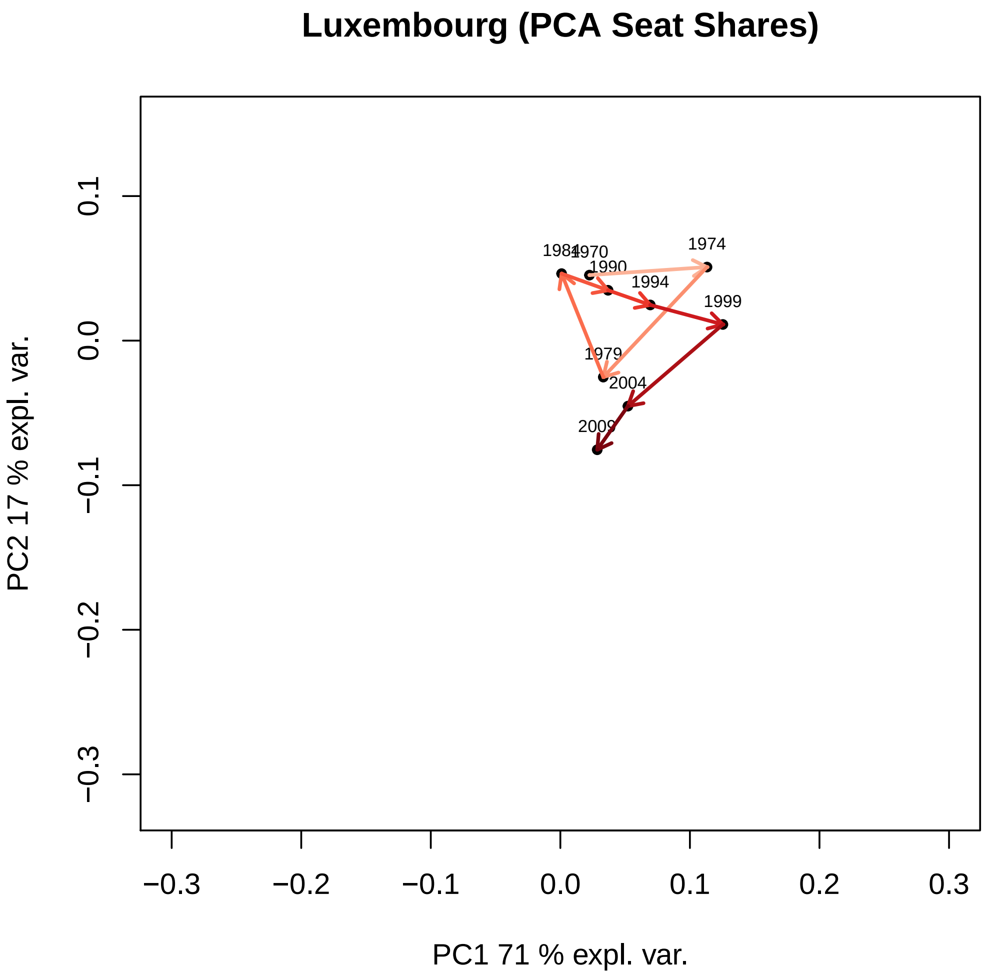
Notes: The plot shows the Irish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.9: Ireland on the PCA Dimensions



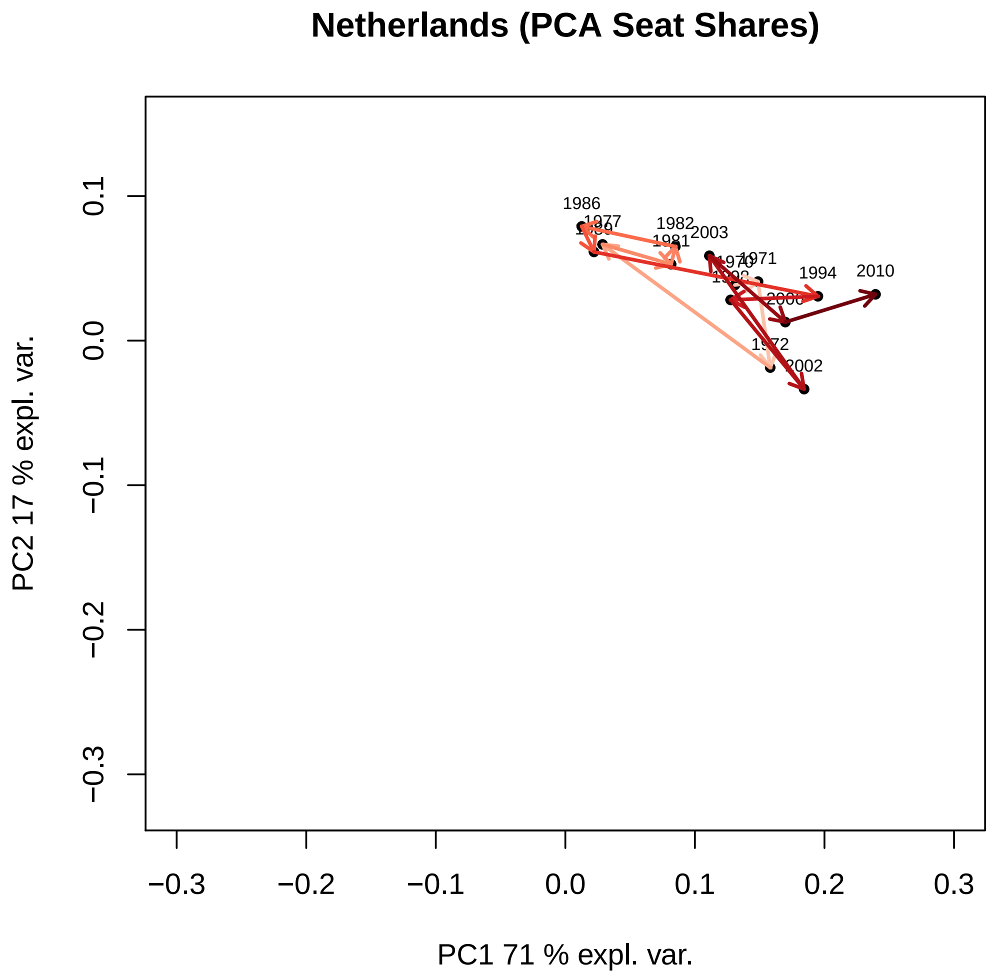
Notes: The plot shows the Italian party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.10: Italy on the PCA Dimensions



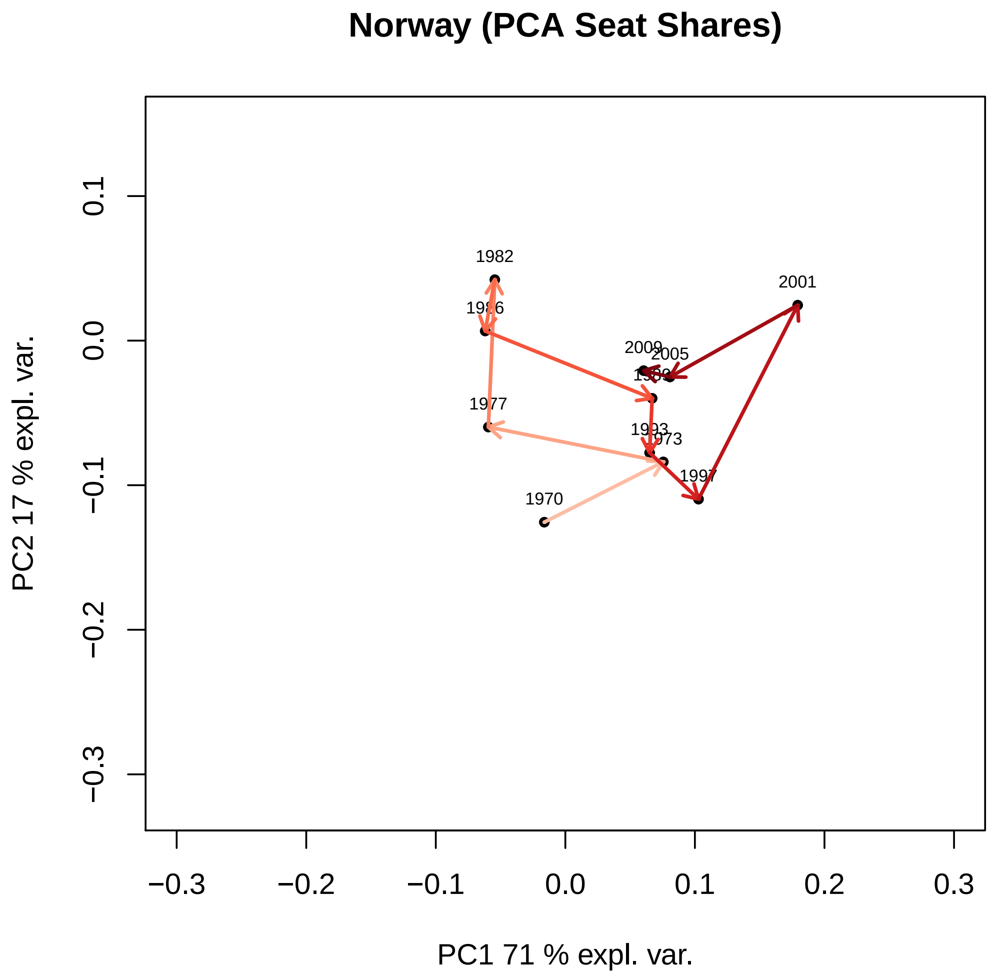
Notes: The plot shows the Luxembourgish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.11: Luxembourg on the PCA Dimensions



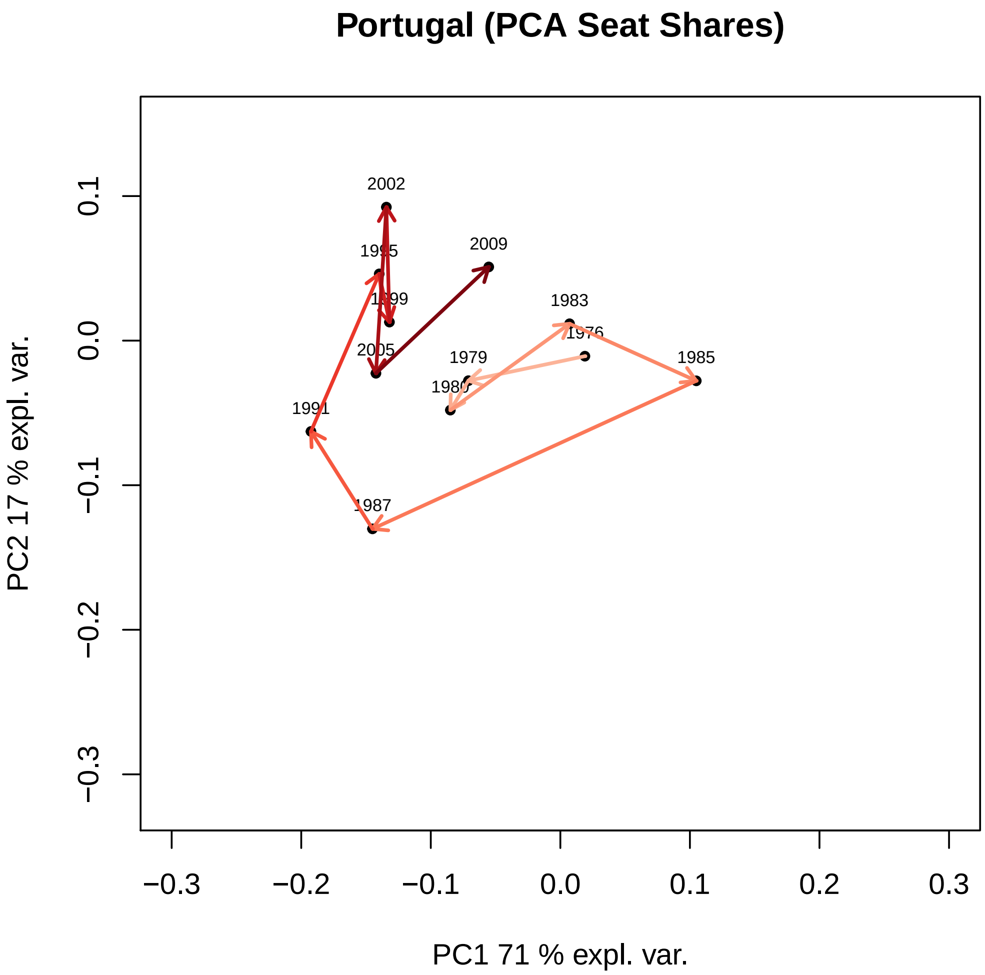
Notes: The plot shows the Dutch party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.12: the Netherlands on the PCA Dimensions



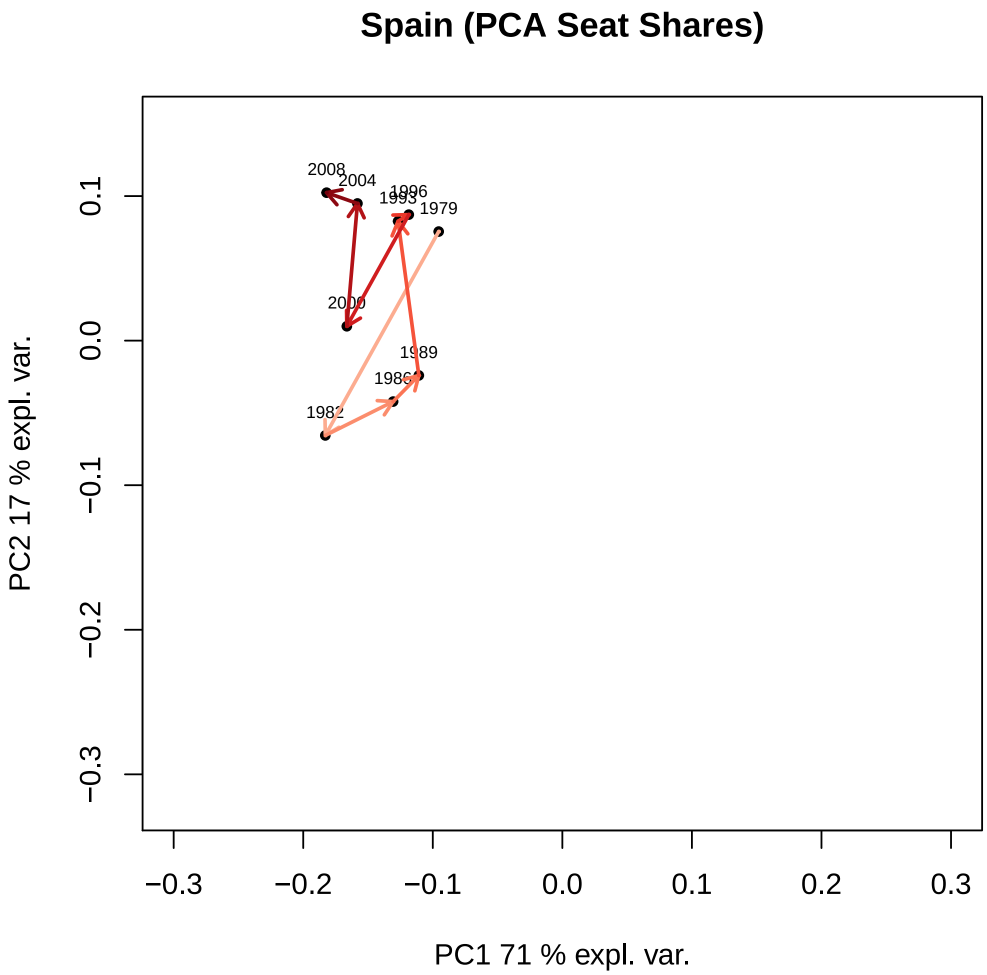
Notes: The plot shows the Norwegian party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.13: Norway on the PCA Dimensions



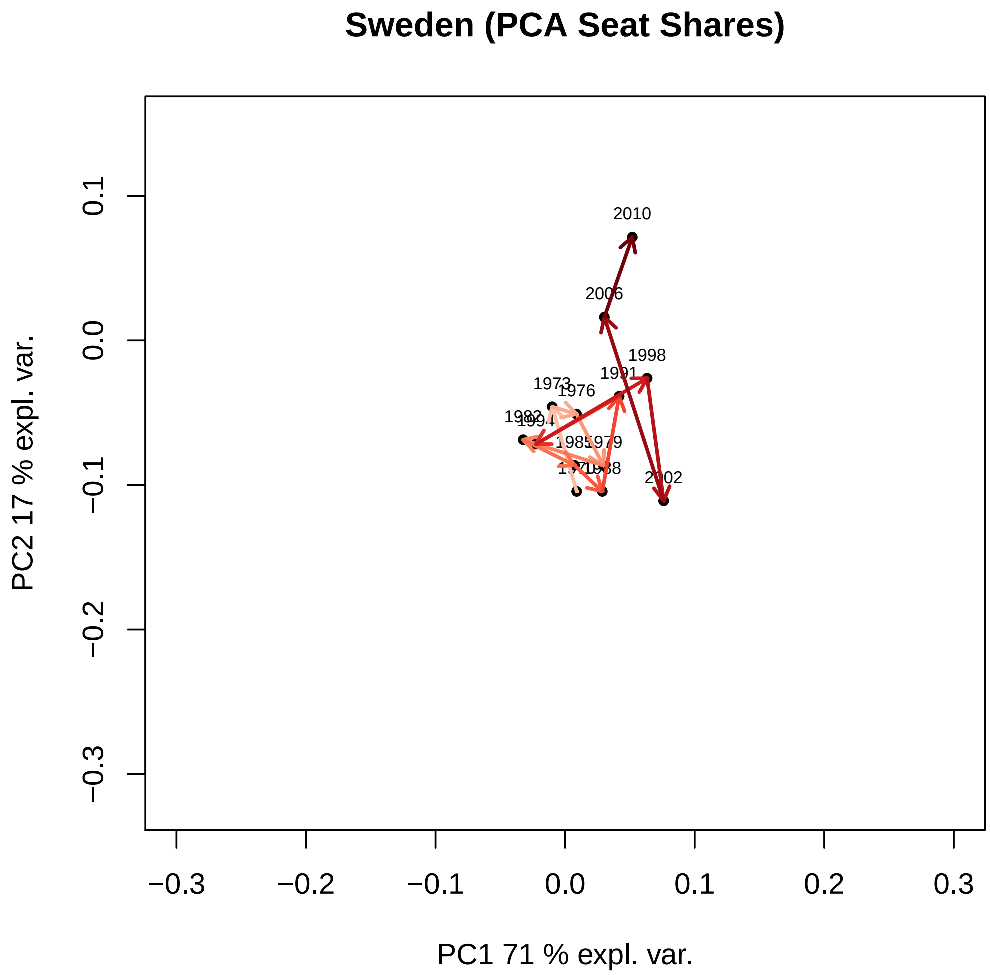
Notes: The plot shows the Portuguese party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.14: Portugal on the PCA Dimensions



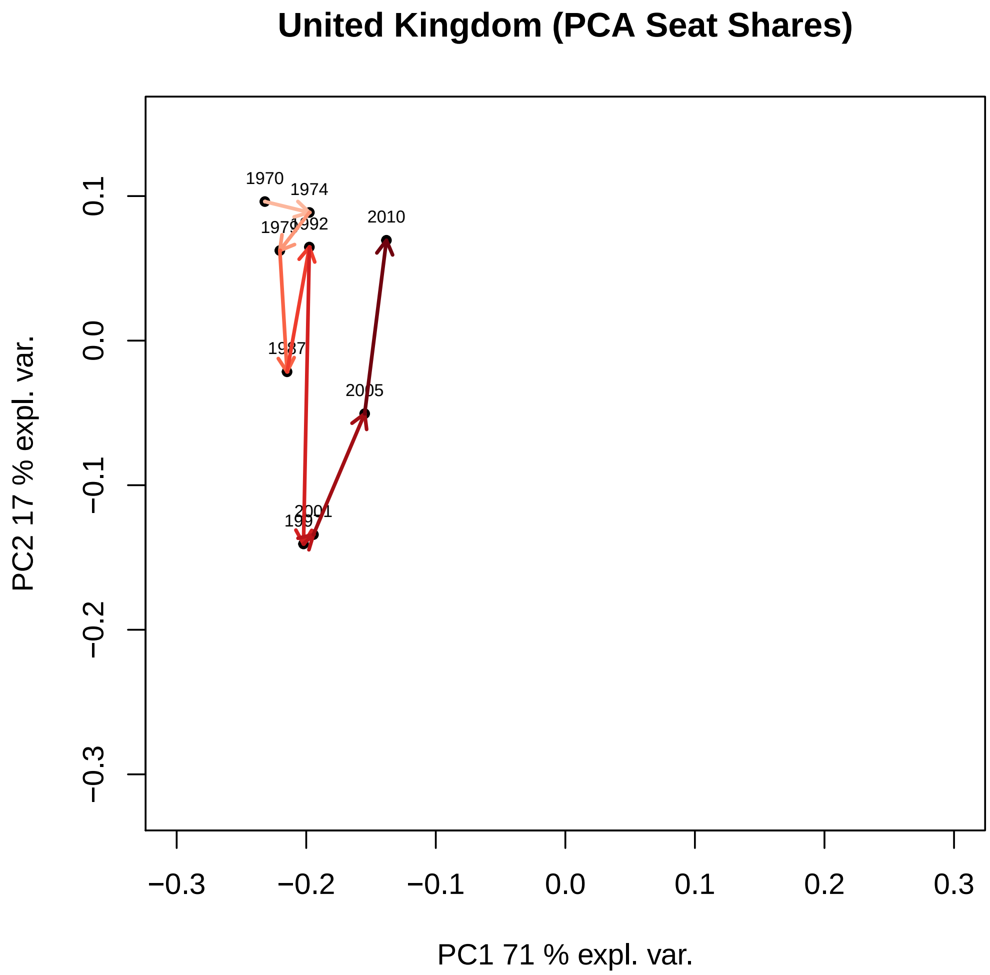
Notes: The plot shows the Spanish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.15: Spain on the PCA Dimensions



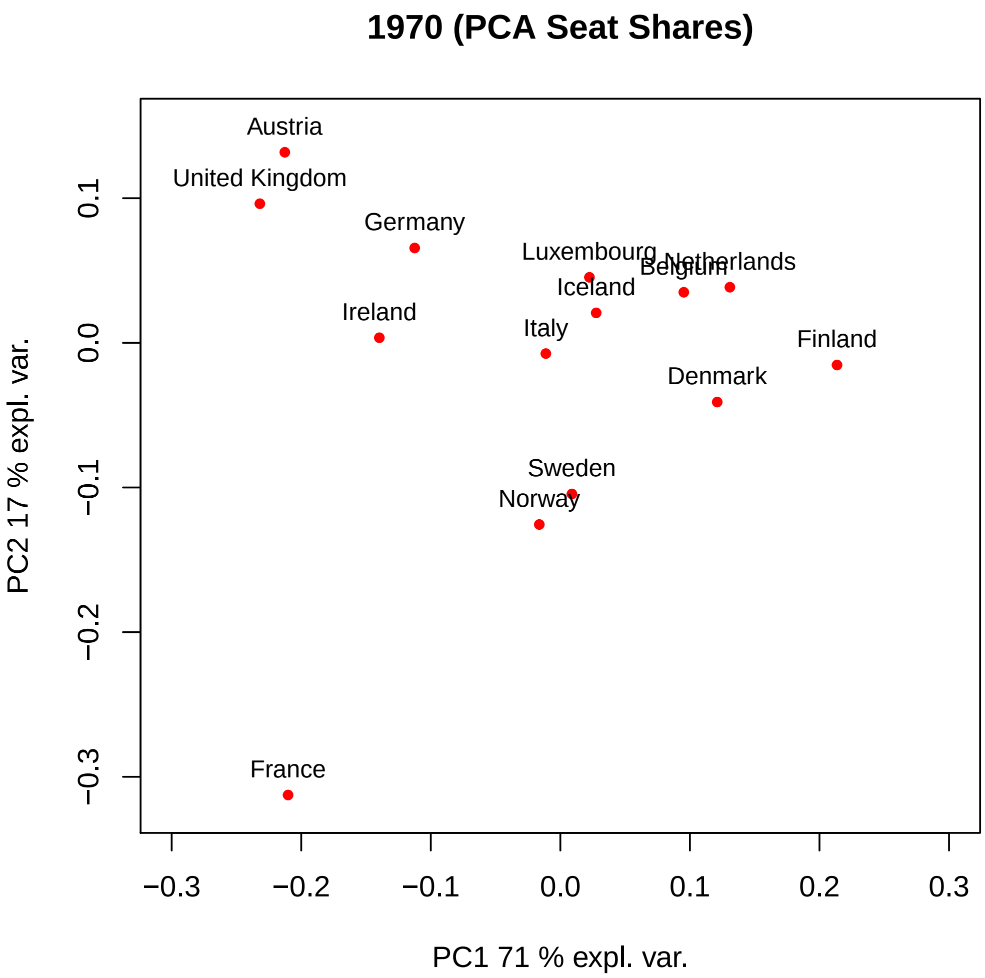
Notes: The plot shows the Swedish party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.16: Sweden on the PCA Dimensions



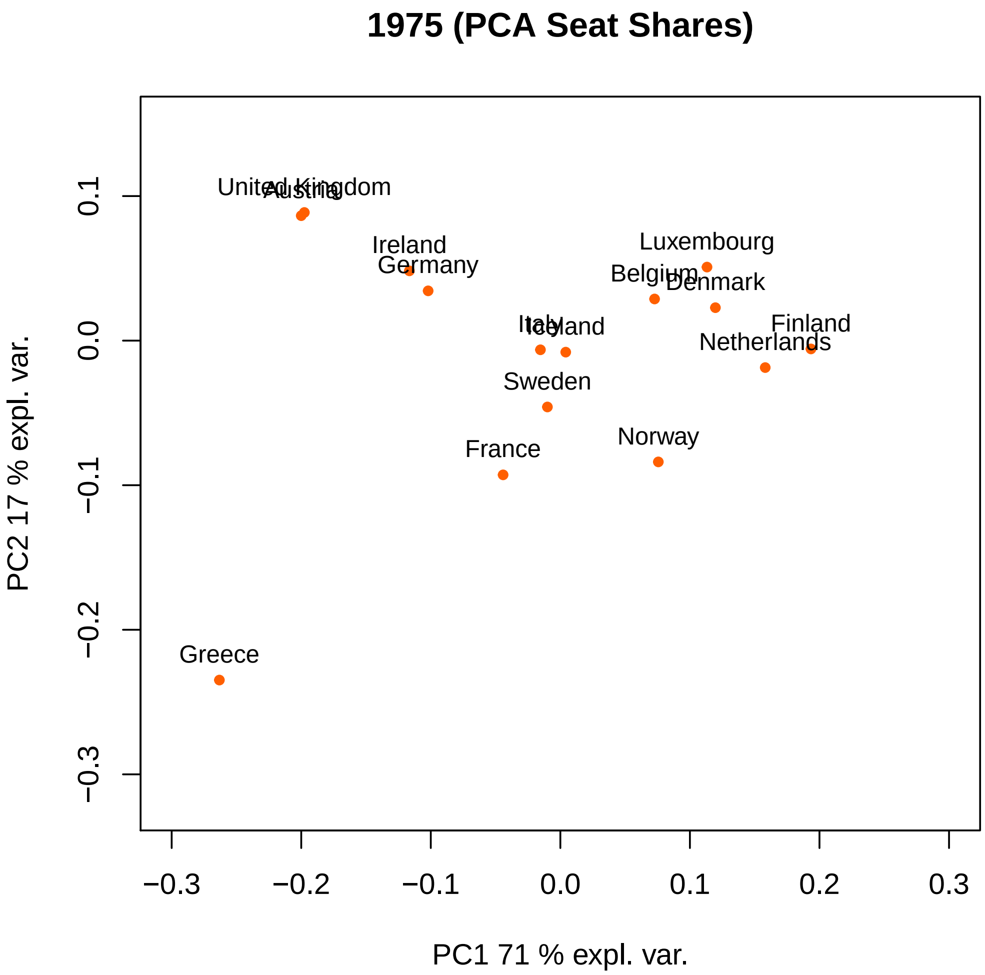
Notes: The plot shows the British party system in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis. The arrows progress from earlier to later years. The shading indicates the direction of the progress; the shading of the arrows becomes darker in later years.

Figure A.17: The United Kingdom on the PCA Dimensions



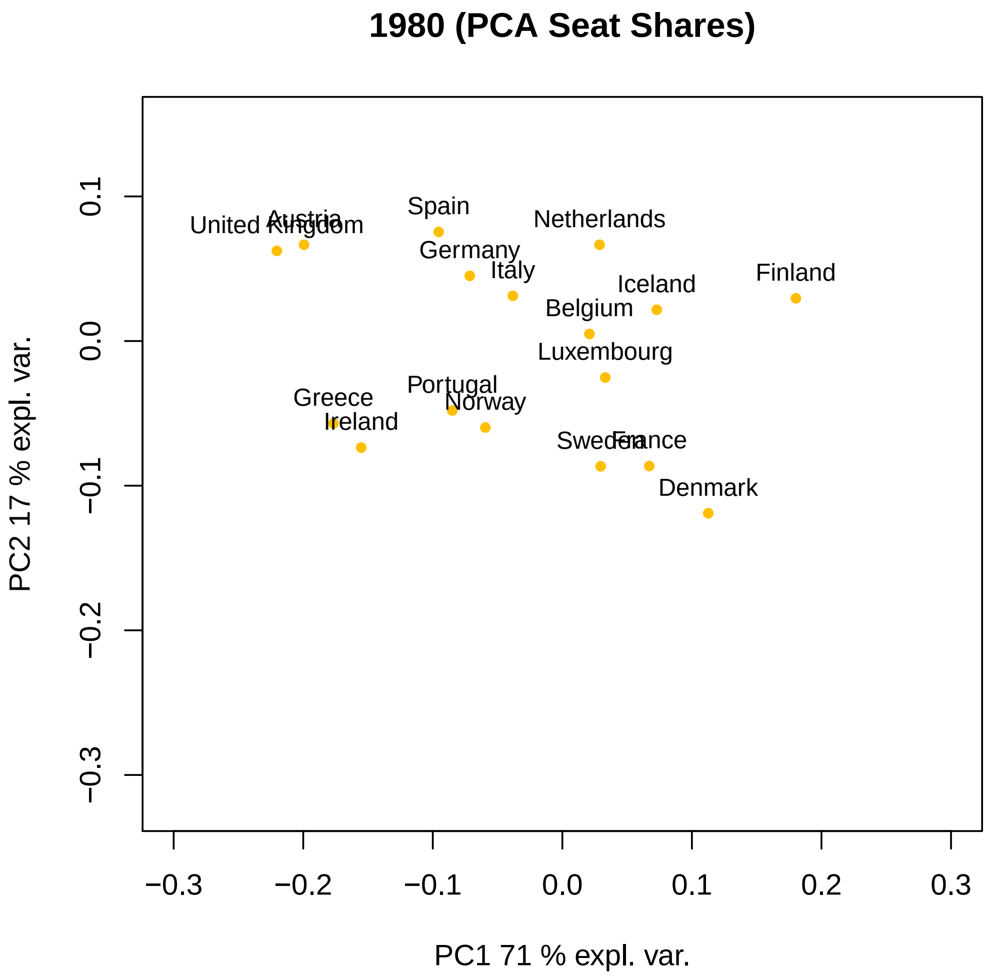
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1970. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.18: The Party Systems of 17 European Countries in 1970 on the PCA Dimensions



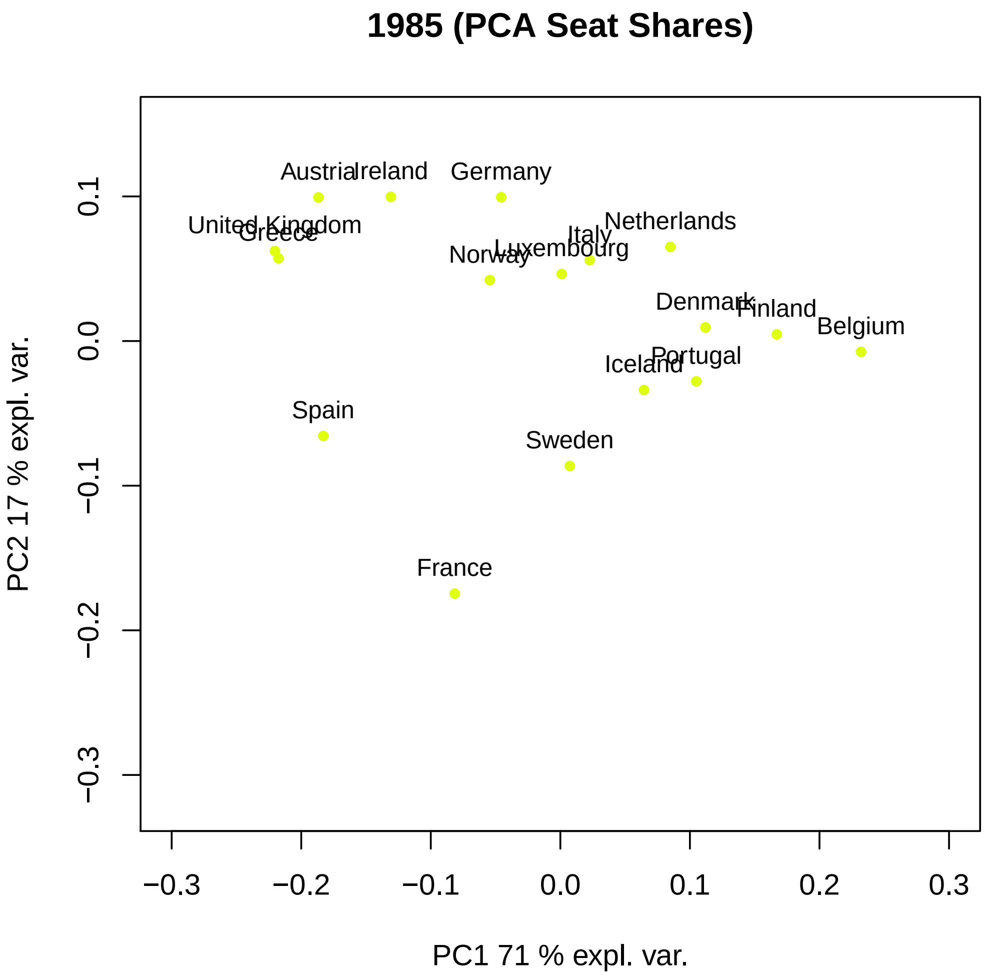
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1975. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.19: The Party Systems of 17 European Countries in 1975 on the PCA Dimension



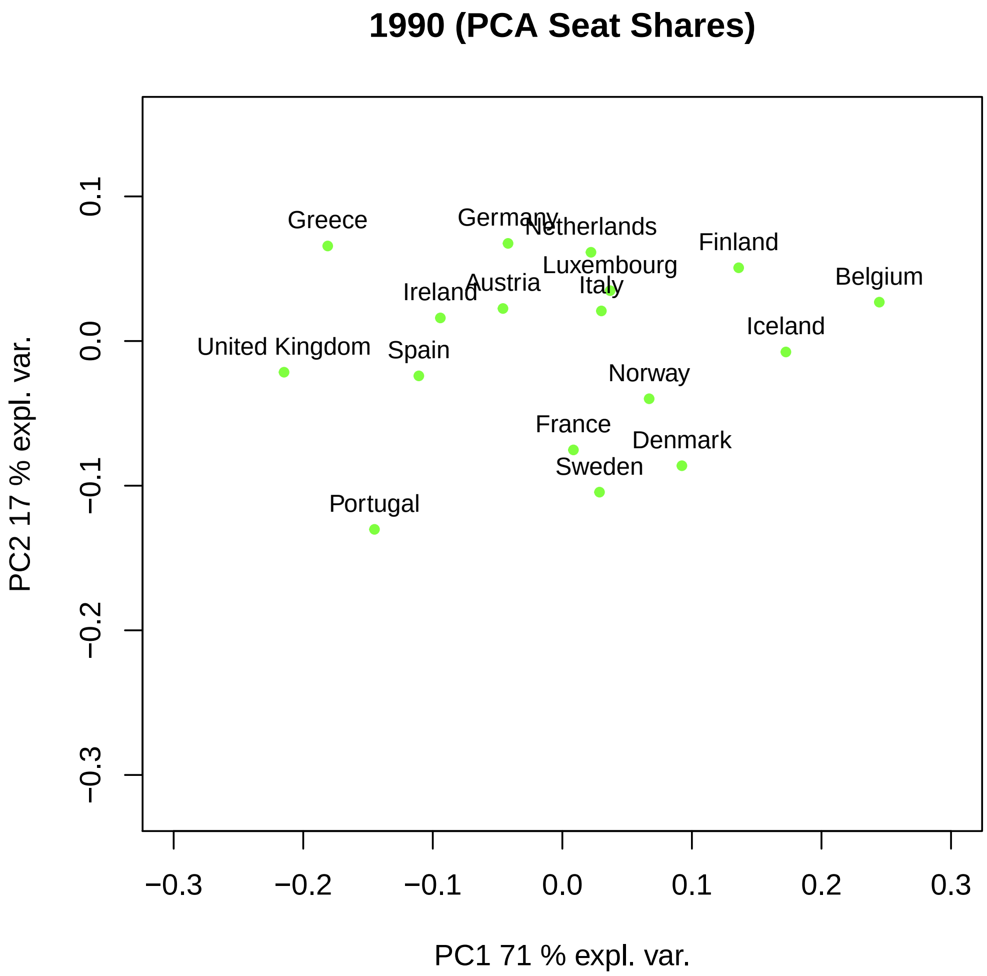
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1980. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.20: The Party Systems of 17 European Countries in 1980 on the PCA Dimensions



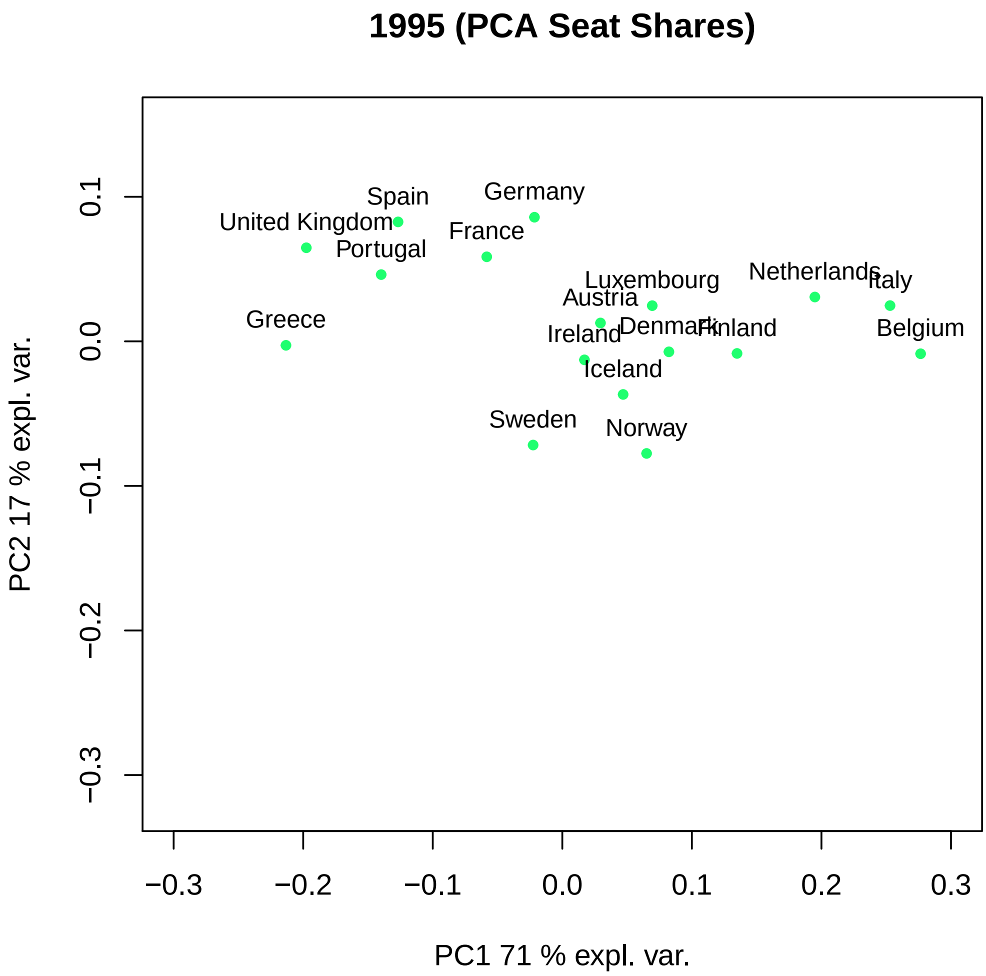
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1985. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.21: The Party Systems of 17 European Countries in 1985 on the PCA Dimensions



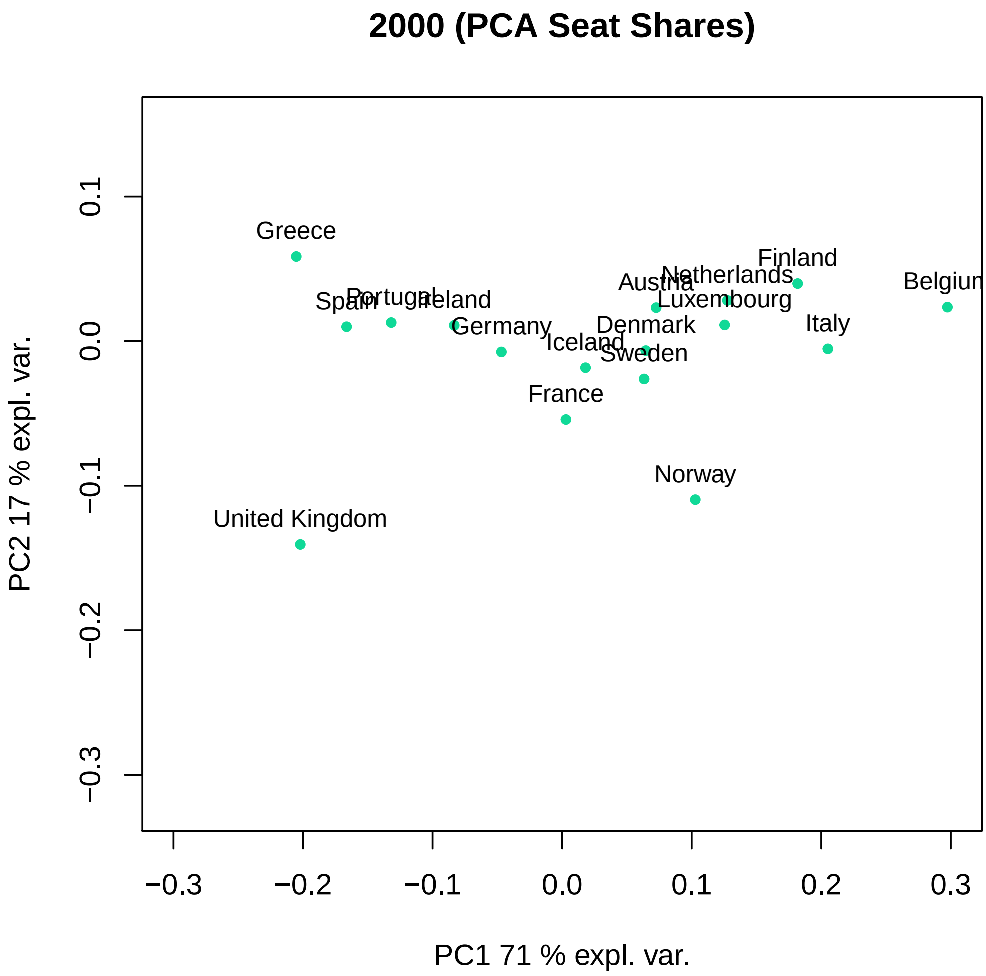
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1990. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.22: The Party Systems of 17 European Countries in 1990 on the PCA Dimensions



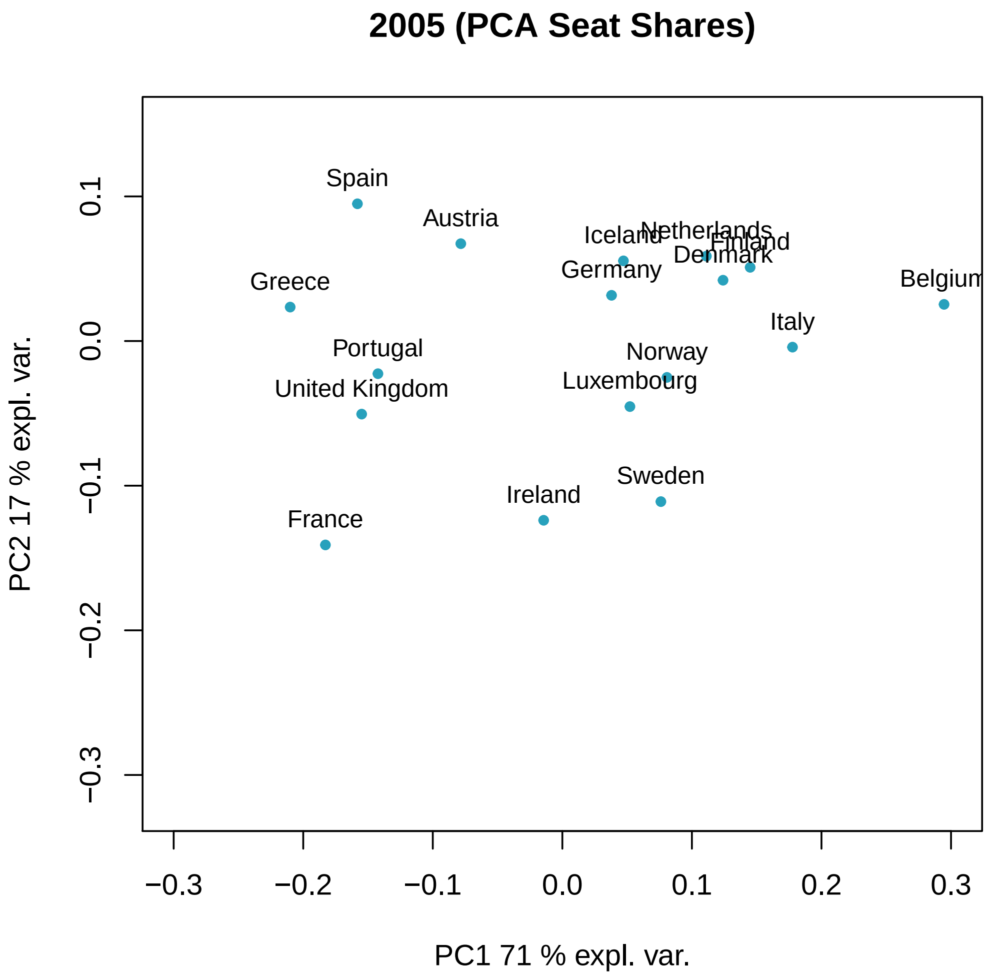
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 1995. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.23: The Party Systems of 17 European Countries in 1995 on the PCA Dimensions



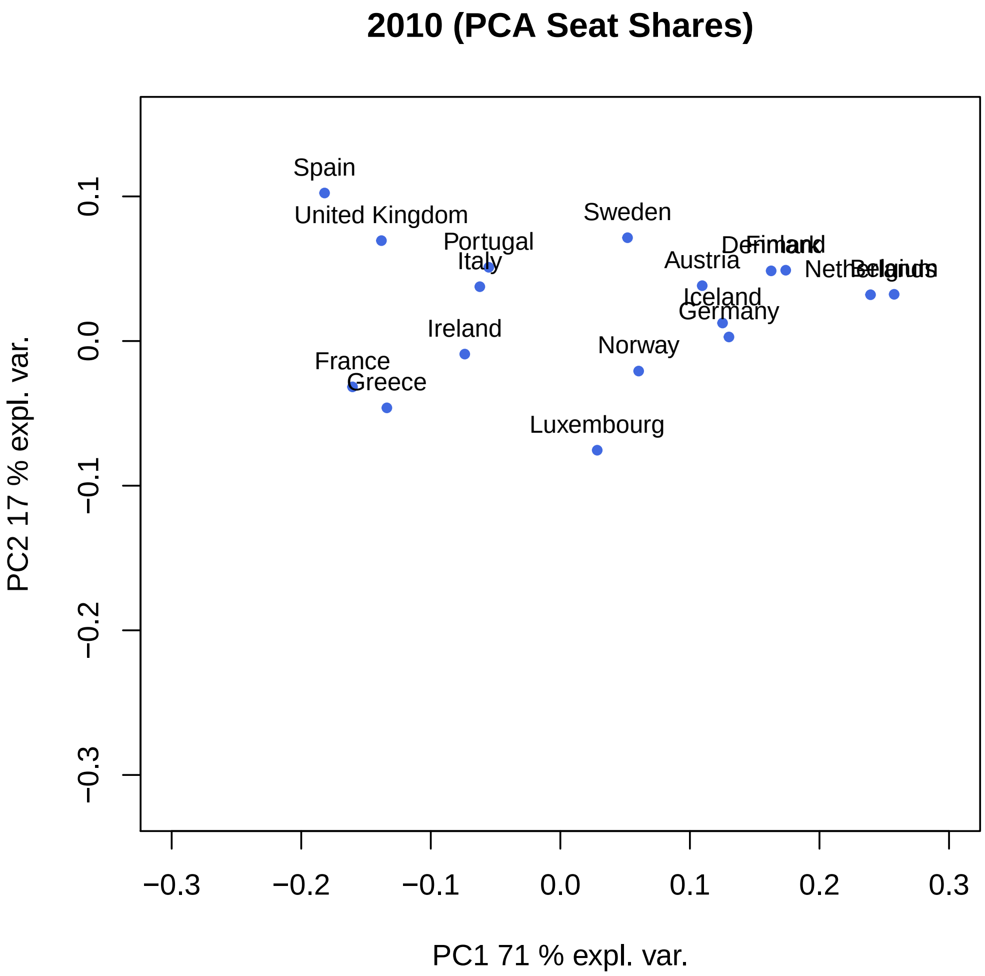
Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 2000. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.24: The Party Systems of 17 European Countries in 2000 on the PCA Dimensions



Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 2005. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.25: The Party Systems of 17 European Countries in 2005 on the PCA Dimensions



Notes: The plot shows the position of 17 countries in the two dimensional plane defined by the first, and the second dimensions of the Principal Component Analysis in 2010. The countries are: Austria, Belgium, Denmark, Finland, France Germany, Greece, Iceland, Italy, Ireland, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom.

Figure A.26: The Party Systems of 17 European Countries in 2010 on the PCA Dimensions

B. Issues with the Compositional Dataset

As we have seen, the result of the PCA depends on whether we have the full dataset or just part of the data. The cause of this problem is the structure of the data. My data is a compositional dataset, the party seat shares add up to one $\sum s_i = 1$. The size of each individual data point depends on the size of the others within a case (Aitchison, 1983). The potential issue with this type of data is that the correlations between the variables might have a negative bias (Jolliffe, 2002).[1] Also Aitchison (1983) notes that one of the issues with the compositional dataset is that the dataset does not have *subcompositional coherence*. This means that if we have only a subset of the data, the PCA on the covariance of this subset will lead to a different result from an analysis on the entire dataset. It has been widely debated in the literature how to run a PCA on a compositional data set. One solution would be to leave one party out of all the party systems and calculate the PCA on the remaining data. However, in my dataset the party systems vary widely in size. This means that leaving out one party from all party systems could change the analysis considerably (as the smallest parties in some countries are relatively big compared to other countries). Below, I apply some of the techniques that previous authors suggested to analyze compositional data. First, I log-transform the variables, second, I perform a PCA on non-centered variables.

B.1. Principal Component Factor Analysis

I first run a factor analysis to analyze how the variables (party seat shares) relate to each other. Factor analysis is another method to reduce the dimensionality of a multi dimensional dataset. However, the assumptions are different from the principal component analysis. When conducting a factor analysis we assume that there are underlying variables that drive the variation in the observed variables of the dataset. The latent, underlying variables are called factors.(Rencher and Christensen, 2012) Similarly to the principal component analysis factor analysis reduces the original dataset into a smaller number of dimensions or factors. These factors are linear combinations of the latent variables (f1,f2...).

Mathematically we can express this as

$$y1 - \mu 1 = \lambda 11 f1 + \lambda 12 f2 + \ldots + \lambda 1 m f m, + \epsilon 1$$

$$y2 - \mu 2 = \lambda 21 f1 + \lambda 12 f2 + \ldots + \lambda 2 m f m, + \epsilon 2$$

$$\vdots$$

$$yp - \mu p = \lambda p1 f1 + \lambda p2 f2 + \ldots + \lambda p m f m, + \epsilon p$$

[1] If we have a D part composition $[x_1, ...x_D]$ where $\sum_{i=1}^{D} = 1$ the covariance is going to be $cov(x_1 x_1 + ... + x_D) = 0$ thus $cov(x_1, x_2) + ... + cov(x_1, x_D) = -var(x_i)$. According to Aitchison (1999) this means that there will be at least one negative element per row in the covariance matrix.

Where μ is the mean vector and ϵ is a random error term.ϵ_i are independent of each other $E(\varepsilon) = 0$, $var(\epsilon_i) = \psi_i$, and $cov(\epsilon_i, \epsilon_j) = 0$.. $cov(, fj) = 0$

With the assumptions made above, the variance of y_i can be expressed as:

$$var(y_i) = \lambda_{2i1} + \lambda_{2i2} + \vdots + \lambda_{im} + \psi_i$$

We can partition the variance of observation y_i due to the common factors, which is called the communality and another called the specific variance. The factors are grouped into a new term denoting the communality, h_{2i}, with the error term ψi representing the specific variance:

$$var(y_i) = (\lambda_{2i1} + \lambda_{2i2} + \vdots + \lambda_{im}) + \psi_i = h_{2i} + \psi_i$$

in matrix notation this is

$$y - \mu - +\epsilon$$

whereΛ is the matrix of λs.
With the factor analysis we model the covariances among the variables y-s. We express these variances in a simplified way with the factor loadings and the specific variances ψs.

$$\Sigma = cov(y) = cov(\Lambda f + \epsilon)$$

Since Λ and $f + \epsilon$ are not correlated the covarianvematrix can be broken down to:

$$\Sigma = \Lambda\Lambda' + \Psi$$

I conduct a factor analysis based on the principal component analysis to find the communalities and the specific variance remaining in the variables. This means that I use the principal components as a basis of the factor analysis. This method uses the observed covariance matrix in place of the Σ. Thus we propose that

$$S \cong \hat{\Lambda}\hat{\Lambda}' + \hat{\Psi}$$

The principal component factor analysis proposes that we can decompose $S \cong \hat{\Lambda}\hat{\Lambda}'$ by decomposing the sample covariance matrix into eigenvalues and eigen vectors. This is exactly the same approach as the principal component analysis.

We can estimate $\hat{\Lambda} = C_1 D_1^{1/2}$ Where D_1 has the m largest eigenvalues at its diagonal and C_1 contains the corresponding eigen values. Thus $\hat{\Lambda}$ is proporional to the eigenvectors S. Thus the factors that we find with this method are going to e proportional to the principal factors. However, using the results now we can estimate the common variation of the variables and the specific variation of the variables. The sums of squares of the rows and columns of this $\hat{\Lambda}$ are going to be the communalities and the eigen values.

In my estimation I set the number of factors to two to mirror the principal component solution. In *Table B.1* I present the results of this analysis. This table shows that the biggest parties are the most related to each other. The biggest party has the biggest common variance, which can explain why we can understand a lot about party systems by knowing the sizes of the biggest parties in the system. The analysis also shows that the size of party 3 has the biggest independent variance within the party system. Overall this analysis highlights the compositional structure of the data.

Table B.1: Results of the Principal Component Factor Analysis

	load1	load2	communalities	spec_var
1	0.1021	0.0368	0.0118	0.0000
2	0.0593	-0.0534	0.0064	0.0001
3	-0.0417	0.0047	0.0018	0.0015
4	-0.0356	0.0071	0.0013	0.0005
5	-0.0271	0.0047	0.0008	0.0004
6	-0.0205	0.0011	0.0004	0.0004
7	-0.0138	0.0003	0.0002	0.0002
8	-0.0081	-0.0009	0.0001	0.0001
9	-0.0057	-0.0006	0.0000	0.0001
10	-0.0043	-0.0001	0.0000	0.0000
11	-0.0021	0.0000	0.0000	0.0000
12	-0.0011	0.0001	0.0000	0.0000
13	-0.0006	0.0001	0.0000	0.0000
14	-0.0003	0.0000	0.0000	0.0000
15	-0.0001	0.0000	0.0000	0.0000
16	-0.0001	0.0000	0.0000	0.0000
17	-0.0001	0.0000	0.0000	0.0000
18	-0.0000	0.0000	0.0000	0.0000
19	-0.0000	0.0000	0.0000	0.0000
20	-0.0000	0.0000	0.0000	0.0000
Variance Accounted for	0.8105	0.1895	1	
Proportion of total variance	0.7062	0.1651	0.8712	
Cumulative Proportion	0.7062	0.8712	0.8712	

B.2. Issues with the Standardization

PCA can be performed on standardized or non-standardized variables. Standardization means that we transform each variable to have unit variance, by dividing the mean centralized variables with their standard deviation (which equals to performing the PCA on the correlation matrix). If we do not standardize the variables, variables that are bigger in size can overpower the analysis, as size may become an important feature that the PCA analysis recovers. Thus, PCA on non-standardized variables can only be performed if the variables are measured on the same scale (Jolliffe, 2002). In this part of the appendix I show that in the case of the party-system size dataset, standardization is not the optimal solution. First, I argue that the variables are measured on the same scale and the size differences between the variables (parties) are inherently important features of the dataset. Second, I will compare the PCA solutions on the standardized and the non-standardized variables to show that the analysis on non-standardized variables recovers dimensions that are more relevant for the analysis of the party systems than the ones recovered through the analysis of standardized variables.

First, the party system size data consists of party seat shares which are measured on the same scale, as the percentage of the total votes. In addition, *party sizes* in the party-size dataset are important features as we can see from both typologies and party system indices. In the party system dataset the variance of the two first variables are big, thus they influence the solution the most out of the variables. With the standardization we would give the variables (party sizes) equal weights, and would increase the influence of small parties. However, an entry or the exit of a small party does not change the party system drastically, and does not make two party systems much more alike/distant. Standardization is not the best solution if we believe theoretically, that the sizes of the variables are important features of the data (Vitt et al., 1997).

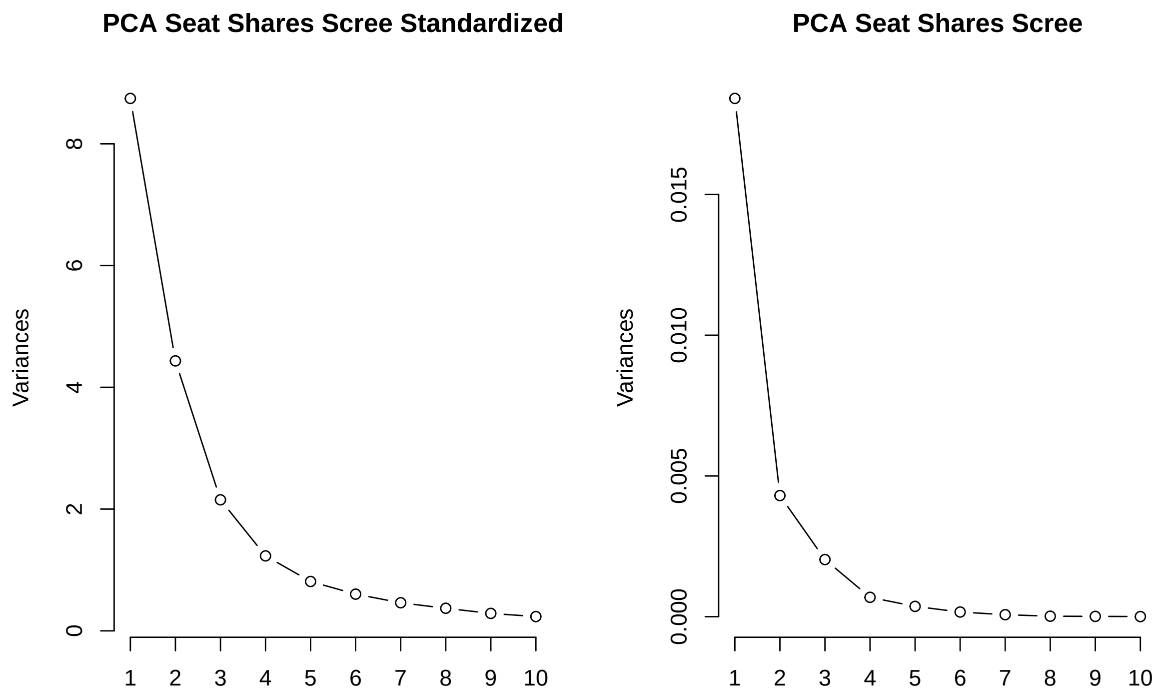
Second, I calculate the PCA with the standardized variables and compare the results to the PCA solution with the non-standardized variables. The results show that the solution that the PCA on the correlation matrix recovers fits the data less well than the solution that the PCA recovers based on the covariance matrix. Tables (*Table B.2*) and (*Table B.3*) show what percent of the total variance the eigen values from each of the two solutions explain. The tables show that if I calculate with the standardized variables, the first eigen value explains only 43.73% of the total variance as opposed to 70.62 % in the non-standardized calculation. The second eigen value explains 22% of the total variance in case of the calculation with the standardized variables as opposed to 16.51% in case of the calculation with the non-standardized variables. Altogether the first two eigen values explain 65.9% of the total variance in case we use the correlation matrix, in contrast to 87.12% in case we use the covariance matrix. Overall 18 eigen values explain 100% of the total variance in case of the standardized solution while only 12 in case of the non-standardized solution. The same information is conveyed in the scree plots of the two solutions (*Figures B.2a* and *B.2b).* The first eigen value explains a much bigger share of the total variance in case the variables are not standardized.

	Eigen Vales	Variance	Cumvariance
1	8.7451841772	43.73	43.73
2	4.4340310061	22.17	65.90
3	2.1520284956	10.76	76.66
4	1.2312671582	6.16	82.81
5	0.8102872814	4.05	86.86
6	0.6026061637	3.01	89.88
7	0.4610328126	2.31	92.18
8	0.3702286313	1.85	94.03
9	0.2861109172	1.43	95.46
10	0.2345027420	1.17	96.64
11	0.1601849027	0.80	97.44
12	0.1347176338	0.67	98.11
13	0.1262302928	0.63	98.74
14	0.0901707336	0.45	99.19
15	0.0727437963	0.36	99.56
16	0.0431652372	0.22	99.77
17	0.0356517568	0.18	99.95
18	0.0098562617	0.05	100.00
19	0.0000000000	0.00	100.00
20	0.0000000000	0.00	100.00

Table B.2: PCA Scaled Seat Shares Eigen

	Eigen Values	Variance	Cumvariance
1	0.0194012464	71.04	71.04
2	0.0045310786	16.59	87.63
3	0.0020397785	7.47	95.10
4	0.0006762824	2.48	97.57
5	0.0003635705	1.33	98.91
6	0.0001867591	0.68	99.59
7	0.0000764051	0.28	99.87
8	0.0000170232	0.06	99.93
9	0.0000113822	0.04	99.97
10	0.0000036437	0.01	99.99
11	0.0000023248	0.01	100.00
12	0.0000009147	0.00	100.00
13	0.0000003156	0.00	100.00
14	0.0000000699	0.00	100.00
15	0.0000000216	0.00	100.00
16	0.0000000152	0.00	100.00
17	0.0000000039	0.00	100.00
18	0.0000000007	0.00	100.00
19	0.0000000000	0.00	100.00
20	0.0000000000	0.00	100.00

Table B.3: PCA Unscaled Seat Shares Eigen



(a) Scree Plot of the PCA on Standardized Variables (Seat Shares)

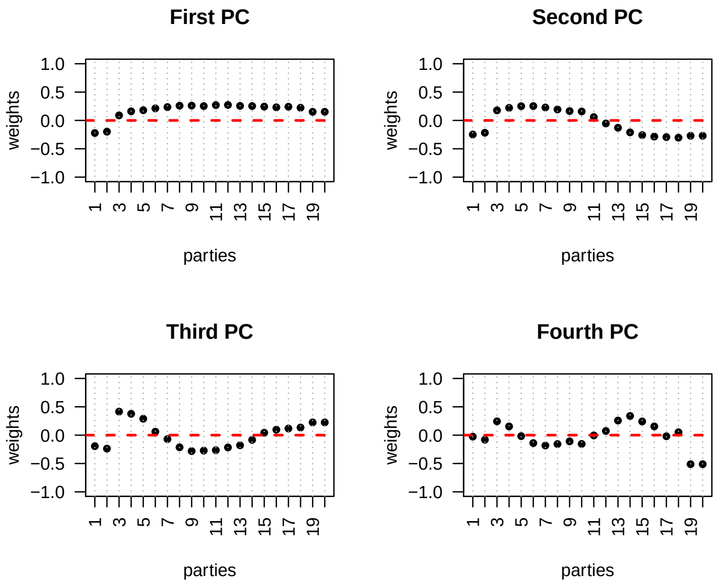
(b) Scree Plot of the PCA on Non-standardized Variables (Seat Shares)

Figure B.1: Screeplot, PCA on Standardized Variables

Next, I examine the dimensions that the PCA on the scaled variables recovers in further detail. In *Figure B.3* I plot the loading plots from the PCA on the correlation matrix. The loadings show a much less clear picture about which parties are important for each dimension than in the case when I perform the PCA on the covariance matrix. This makes sense as all 20 parties have equal variances at this point. The loadings plot *Figure B.3* shows that maybe the first two parties are important for all dimensions. In *Dimension 1* the rest of the parties have an opposite sign but weigh into the dimension. In the *Dimension 2* the first and the last parties have opposite loadings. In *Dimension 3* Parties 3-6 have an opposite loadings from the first two parties. And in *Dimension 4* Parties 3, and 15 have loadings to one direction while 19-20 to the other direction. Overall, the pictures are not clear.

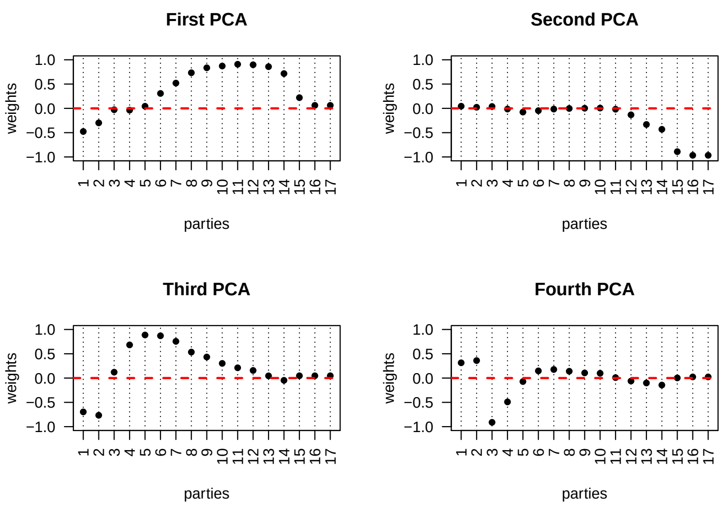
As the loadings are not clear I also rotate the dimensions. As in factor analysis, in principal component analysis we can also rotate the loadings to make their interpretation easier (Jolliffe, 2002). I calculate the varimax rotated loadings and I present the results in *Figure*??. The rotated loadings, show a clearer picture. Now we can see that smaller parties have a big influence in separating party systems in this solution. *Dimension 1* for instance separates the party systems in which there are parties 7-14, *Dimension 2* separates party systems that have parties 14-17. In reality, however, we know that the entry and exit of a small party from a party system will not fundamentally change the character of the party system. Thus, this solution, although mathematically correct, finds a sub-optimal solution. It shows that variables that would have no significant meaning if we had not standardized them have a role in differentiating party systems.

Finally, there is one more reason why it could be problematic to standardize the variables. If we standardize the variables and the variance of small parties are equalized with the variance of big parties, the entry and exit of the small parties can radically change the PCA solution itself. Thus, if we fail to collect data on any small party that can change the solution for all the party systems. I depict how the number of parties that I use in the analysis affects the PCA solution in *Figure B.5*. This figure shows that the PCA solution on the scaled variables varies as we include more or less parties in the analysis. I discuss this point further when I discuss the NLCA calculation and sensitivity (Online Apendix C.2).



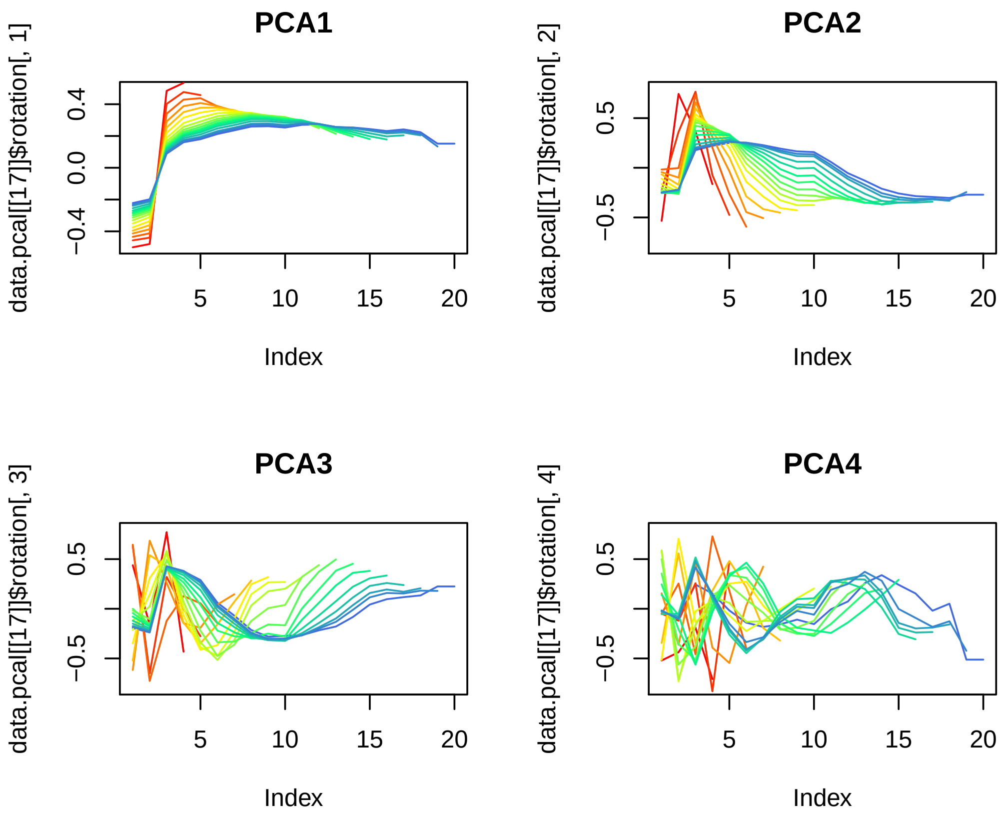
Notes: The plot shows the loadings gained from the PCA analysis on standardized variables.

Figure B.3: Loadings, PCA analysis standardized variables



Notes: The plot shows the loadings gained from the PCA analysis on standardized variables. Varimax rotation.

Figure B.4: Loadings, PCA analysis standardized variables-Varimax



Notes: The plot shows how the PCA changed when we have more or less party data.

Figure B.5: Sensitivity of the Scaled Variables to the Number of Parties

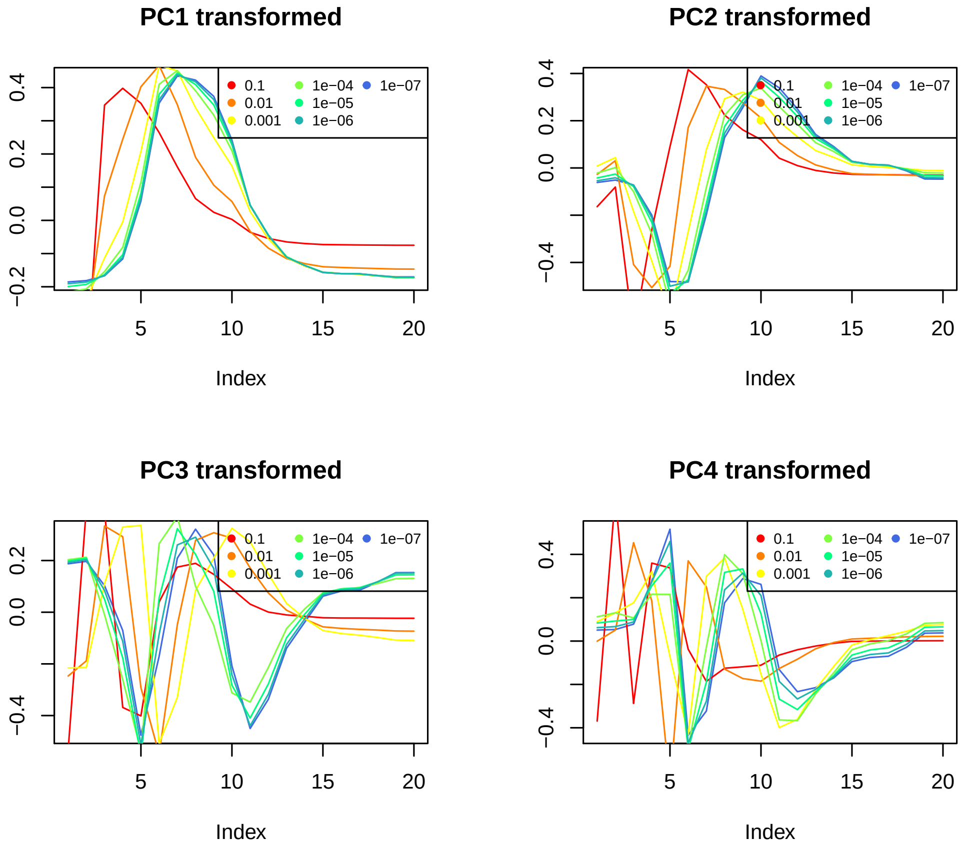
B.3. PCA on Log-ratio Transformed Variables

One recommendation about how to perform any calculation on a compositional dataset comes from Aitchison (1986), who suggests the log-ratio transformation of the data. He argues that this transformation makes the observations uncorrelated, and solves the issue of subcompiositional coherence (Aitchison, 1983).[2] Aitchison (1986) is aware of the problem that some datasets may have zeros in them that we cannot transform with the log-ratio transformation. He suggests adding a small number to the zeros so that the transformation can be done. In this analysis, first I add 1^{-5} to each zero in the dataset, and then transform the variables with centered log-ratio (clr) transformation. Aitchison (1986, 156) suggests that if we add a small number to the zeros, next we should do a sensitivity analysis to check how much the this manipulation changes the results of the PCA. In *Figure B.6* I present the results of this sensitivity analysis. We can see in this plot that if we add a number smaller that 10^{-4} to the zeros we will arrive to a stable solution.

Figure B.7 shows that Dimension 1 of the PCA on the log-ratio transformed variables separates the small party systems (and extremely big party systems) from the bigger ones. *Party 5* to *Party 10* have a big influence on this dimension. Dimension 2 separates the moderately big party systems from the very big party systems: *Party 5* and *Party 10* have opposite loadings in this dimension.

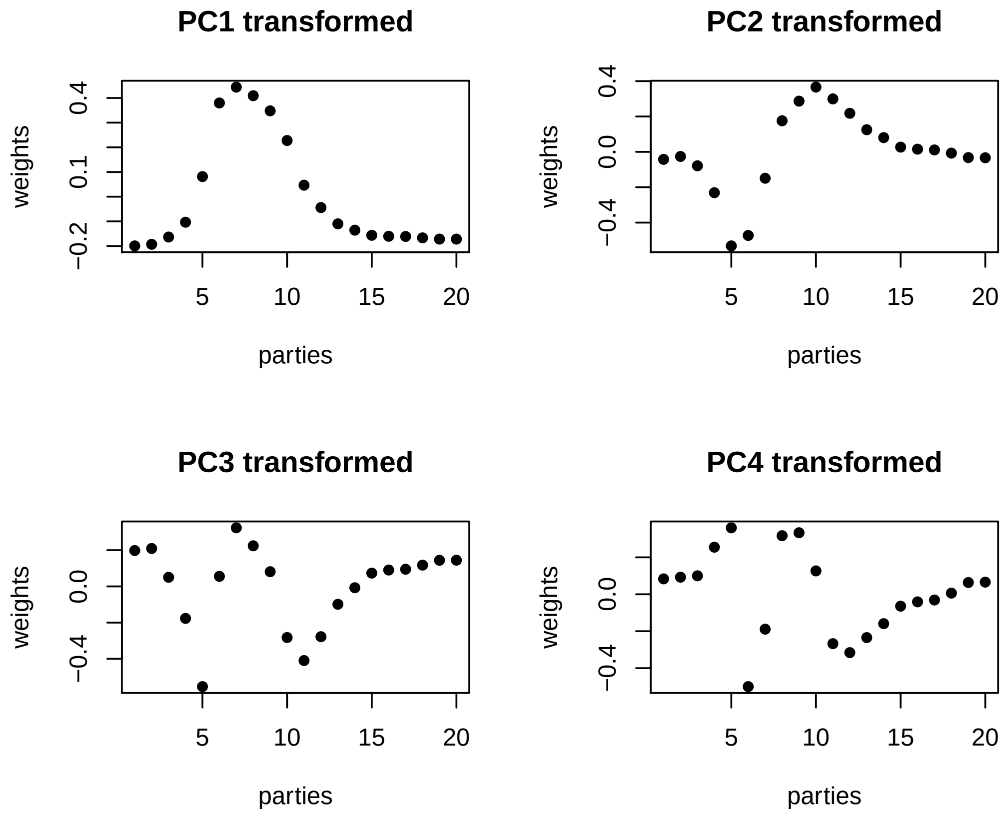
Figure B.8 shows the biplot of this PCA. An advantage of this method is that the countries are separated in quite clear groups. In *Figure B.8*, we can see that the PCA on the transformed variable sorts the countries in groups based on the number of parties in the legislature. Thus, while we can see that the PCA on transformed variables recovered an important feature of the party system (the number of parties) the result is not very informative. In line with this conclusion, Baxter argues that if there are a lot of zeros and small values in the dataset, performing the PCA on the original dataset is potentially more informative than any of the other approaches as the absolute variation in the variables may be an important feature of the data (Baxter and Freestone, 2006).

[2] Aitchison suggests the transformation of the data in such a way that the new data is going to be: $v = log[x/g(x)]$ where $g(x) = (\prod_{i=1}^{p} x_i)^{\frac{1}{p}}$. This means that we divide each variable with its geometric mean and do a logarithmic transformation the following way: $v_j = logx_j - \frac{1}{p}\sum_{i=1}^{p} logx_i, j = 1, 2, ...p$ (Jolliffe, 2002).



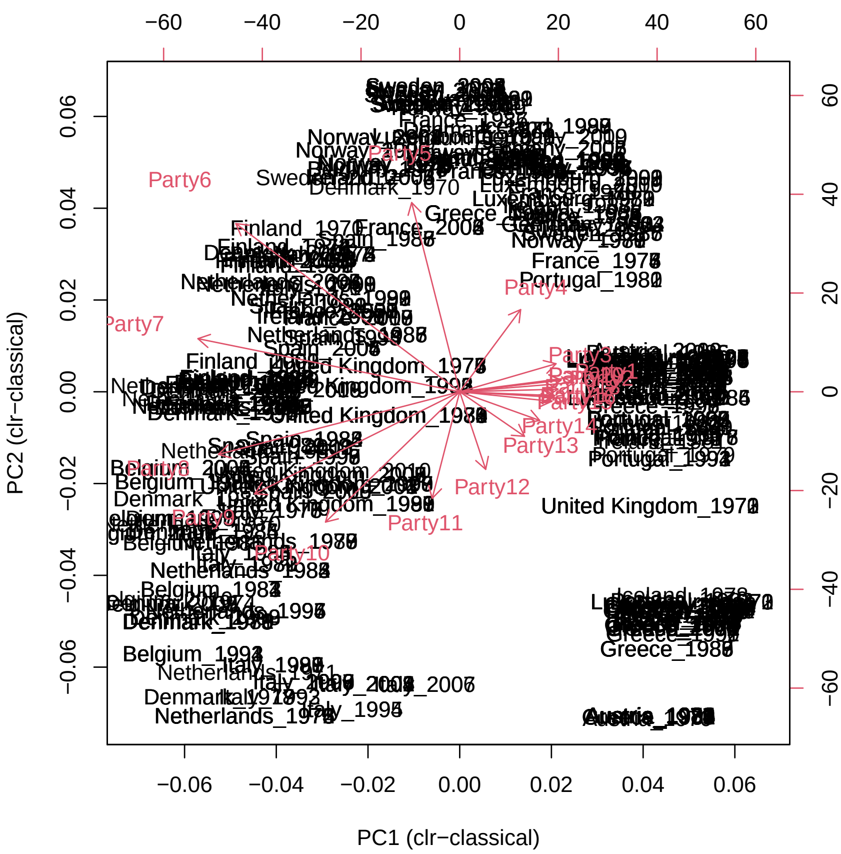
Notes: The plot shows how the PCA on the log-ratio transformed analysis changes when we add different small numbers to the 0 seat shares of the parties.

Figure B.6: Sensitivity of the Log-ratio Transformed Variables to the Size of an Added Value to Zeros



Notes: The parties defining the principal components when the variables are log-ratio transformed.

Figure B.7: Loadings, PCA on Log- ratio Transformed Variables

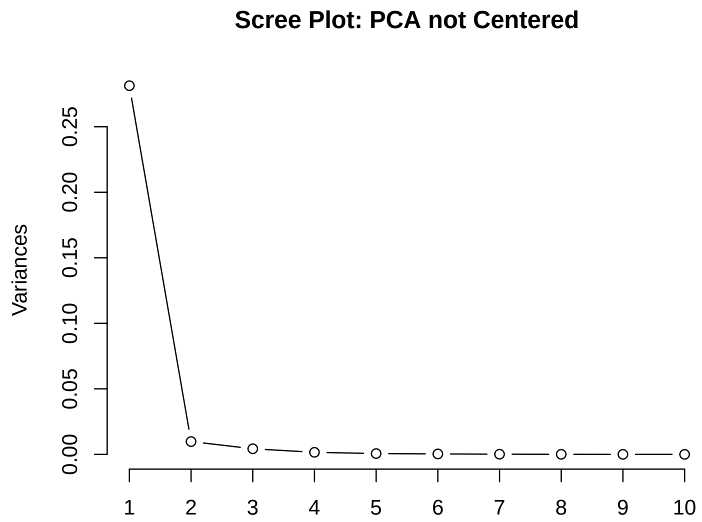


Notes: Biplot of the PCA when the variables are log-ratio transformed.

Figure B.8: Biplot, PCA on Log- ratio Transformed Variables

B.4. PCA on Non-centered Variables

Another approach to reduce the dimensionality of a compositional dataset is to conduct a PCA on the non-centered variables (ter Braak, 1983). The reason follows from the geometrical properties of the compositional dataset. As Aitchison (1983) discusses we can understand each observation in a compositional dataset, as a point on an n-dimensional simplex. This means that each country-year could be represented as a point or vector on a 20 dimensional space, where the 20 coordinates are the seat shares of the 20 parties in the dataset. In case of a compositional dataset thus it is informative to find the space going through the origin of the data as this defines the simplex. This projection can show us the locations of the points on the simplex.[3]



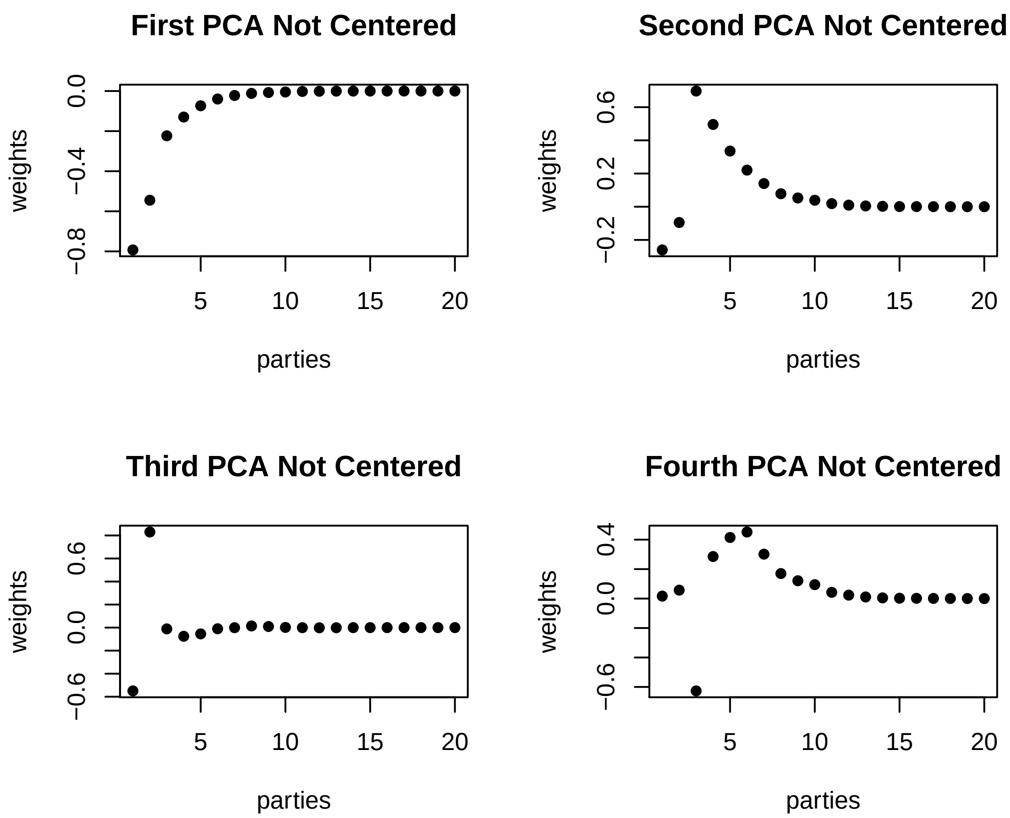
Notes: Screeplot of the PCA on the not centered variables

Figure B.9: Screeplot, PCA on Non-centered Variables

A non-centered PCA does exactly this, it projects the data to the best fitting plane through the true origin and not the center of the data. The data and the direction are projected to this plane (ter Braak, 1983). Thus while we get a different projection of the data, this can be useful if we want to find within group variance as opposed to simply between group variance (ter Braak, 1983). The result is an ordination plot. On this plot

[3]If we denote the seat shares of Party k as s_{ki} in year i so that $\sum_{k=1}^{20} s_{ki} = 1$, each of the individual country -year can be represented as $s_i = (s_{1i}, ...s_{20i})$. The space that these points are on is determined by the basis vectors that are length 1 orthogonal in each of the 20 directions.

the countries that have unstable party systems will be far from the origin, and countries that have stable party systems will be close to the origin. In addition, country-years that have similar party systems will be grouped together (ter Braak, 1983).

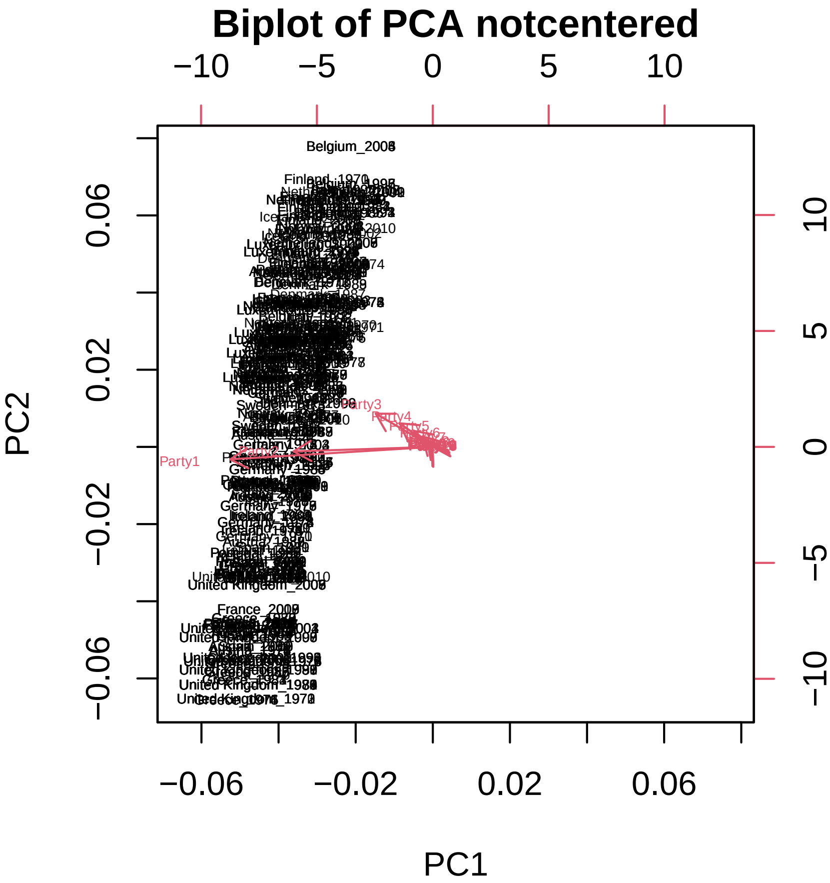


Notes: The parties defining the principal components when the variables are not centered.

Figure B.10: Loadings, PCA on Non-centered Variables

The drawback of this technique is that if the observations are a long way away from the origin, the first Principal Component that the analysis finds is the center of the data. This is what we can observe here too, if we conduct this analysis. As *Figure B.10* shows the first two dimensions absorb the influence of variation in the size of the biggest two parties. The two later principal components (here PC3 and PC4) are exactly the same as PC2 and PC3 were in the original PCA.

The biplot (*Figure B.11*) of this analysis shows that the countries line up mostly based on what extent is their party system concentrated. Indeed, countries that were identified to have smaller party systems (Britain, Germany, France, United Kingdom) are on one end of the dimension, while party systems that are generally considered fragmented



Notes: Biplot of the PCA on the Non-centered Variables.

Figure B.11: Biplot, PCA on Non-centered Variables

(Belgium Finland, the Netherlands) are on the other end. The order is similar to what we have seen on Dimension 1 in the normal PCA. Only three distinct groups arise: in one of these groups we find Greece, Britain and France in certain years, in the second, we can see all the multi-party countries (in the middle of the plot) and in the third group Belgium in the 2000s is its own category.

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C. Introducing Non-linearity to the PCA

The PCA has limitations. During the PCA, the data is linearly projected to the new dimensions. The old data is decomposed as a linear combination of lower lever dimensions (eigenvectors) and weights (eigenvalues). This projection finds the correct solution if the data is close to Gaussian distributed; however, if the data is non-linear, we may not find the most important dimensions of the data (Bishop 1995).

In the following sections, after analysing whether a rotation could improve the interpretation of the dimensions, I am going to present two ways that non-linearity can be introduced into the PCA, and I am going to analyze the dataset through these methods. The first method, the kernel PCA (kPCA), non-linearly transforms the dimensions on which we are projecting the data, while the Non-linear Principal Component Analysis (NLPCA) finds the optimal, potentially non-linear, quantifications (transformations) of the data, at the same time as it projects the new data on linear lower level dimensions.

C.1. Kernel Principal Component Analysis (kPCA)

The Kernel Principal Component Analysis (kPCA) offers one solution to how to find the appropriate reduced dimensional space if the data is non-linear. With this method, we first map the data to a higher dimensional non-linear feature space. After this, in this non-linear subspace, we do a traditional PCA calculation (Scholkopf 1997) Thus, the result will be non-linear on the original data space. Scholkopf et al.(2002) find that kernel PCA provides a better classification rate than does the linear PCA, and more components can be extracted with this method than with the linear PCA.

As I discussed above, the minimum of the sum of squared errors in the PCA estimation can be found when the covariance matrix is diagonalized. The kernel PCA proceeds as the regular PCA. However, the covariance matrix of the data is transformed by the kernel function. The covariance matrix of the non-transformed data is the following $\Sigma = \frac{1}{N}\sum_{i=1}^{N} x_i x_i^T$ (where $x_k, k = 1....N$, $x_k \in R^N, \sum_k^N x_k = 0$).

In this instance, we transform the data to a feature space F by a function ϕ, which will result in: $R^N \to F, x \to X$. Hence the data will be the following: $\phi(x_1)....\phi(x_N)$ and the covariance matrix of the data will look like this: $\bar{\Sigma} = \frac{1}{N}\sum_{i=1}^{N}\phi(x_i)\phi(x_i)^T$. After this transformation, the solution is similar to the regular PCA. To minimize the loss function we find the eigenvectors satisfying $\lambda_i u_i = \bar{\Sigma} u_i$. u_k are the directions of the space. Let's define $u_k = \sum_{j=1}^{N}\alpha_j^k \phi(x_j)$. The inner product space is $K = k(x,y) = \langle\phi(x), \phi(y)\rangle = \phi(x)^T\phi(y)$ by definition. We can use the kernel trick here, since for the estimation of the data matrix the PCA uses the inner products of component scores (eigenvalues) and component loadings (eigenvectors), consequently, we do not need the explicit function to calculate these. The kernel trick means that we do not explicitly use the high order function, but, instead, we directly evaluate kernel k. We use the inner product $\langle\phi(x), \phi(y)\rangle$ between the images of two data points x, y in the "feature space" (ϕ space).

While the features $\phi(x_1)....\phi(x_N)$ are not unique, their dot product is unique.

As we do not use the explicit function, however, we cannot compute the principal components themselves, only the kernel projected data which is computationally given by: $\langle u_k, \phi(x_j)\rangle = \langle \sum_{j=1}^{N} \alpha_j^k \phi(x_j), \phi(x_j) =\rangle \sum_{j=1}^{N} \alpha_j^k \langle \phi(x_i)\phi(x_j)\rangle = \sum_{j=1}^{N} \alpha_j^k k(x_i, x_j) = K\vec{\alpha}$ (Scholkopf 2002).

Table C.1: Eigenvalues and Explained Variance, kPCA

	Eigenvalues	Explained Variance	Cumulative Variance
Comp.1	0.0036424550	72.27	72.27
Comp.2	0.0008534100	16.93	89.21
Comp.3	0.0004065058	8.07	97.27
Comp.4	0.0001375362	2.73	100.00
Sum	0.0050399071		

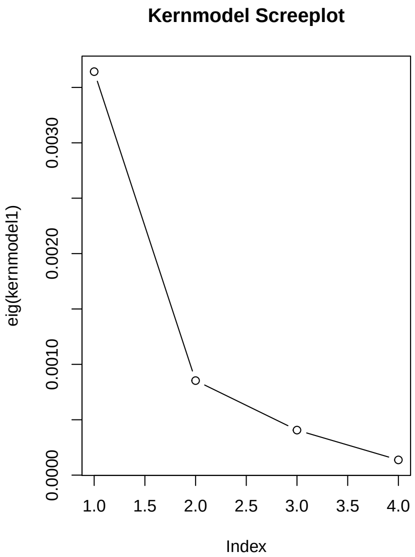
For my analysis I use the Gaussian Radial Basis as my kernel function. This kernel, $k(x, x') = exp(-\sigma||x - x'||^2)$ is a general purpose, smooth kernel that we can use if we do have deeper knowledge about the structure of the data.[4]. Below, I present the results of the kPCA.*Figure C.1* shows the eigenvalues that the kPCA recovers in the high dimensional space in descending order. The first eigenvalue explains most of the variation in the data. *Table C.1* shows that in this case the first eigenvalue explains 72.72% of the variation. Overall, the four eigenvalues explain 100% of the variation.

Figure C.2 shows the observations in the two- dimensional plane determined by the dimensions kPC1 and kPC2. This plot is similar to the PCA plots discussed previously (the PC2 has the loadings in the opposite direction from the PCA calculation, but since any PCA is non-directional method, this does not have any impact on the analysis). Even though we cannot extract the loadings from this estimation process directly, we can see that the first two principal components are very similar to the first two principal components I obtained from the normal principal component analysis. To demonstrate this connection I created show the covariances between the transformed datasets based on the first four PCAs of the linear and the kernel PCA. *(Table C.2)* shows that the respective principal-components are related. Since the results of the PCA are easier to interpret than the results of the kPCA. Because the two sets of results are reasonably similar, I will use the PCA dimensions later in this paper.

C.2. Non-Linear Principal Component Analysis (NLPCA)

I also conducted a Non-Linear Component Analysis (NLPCA) on the data. In this section, I present the results of this analysis, and I compare them to the results of the PCA and the kPCA. The NLPCA is a special case of multiple correspondence analysis or homogeneity

[4]the Gaussian kernel is "universal." It is positive definite and they are invariant under the Euclidean group. These are desirable properties if we want to estimate bounded continuous functions (Hofmann 2008)



Notes: The plot shows the screeplot Kernel PCA.

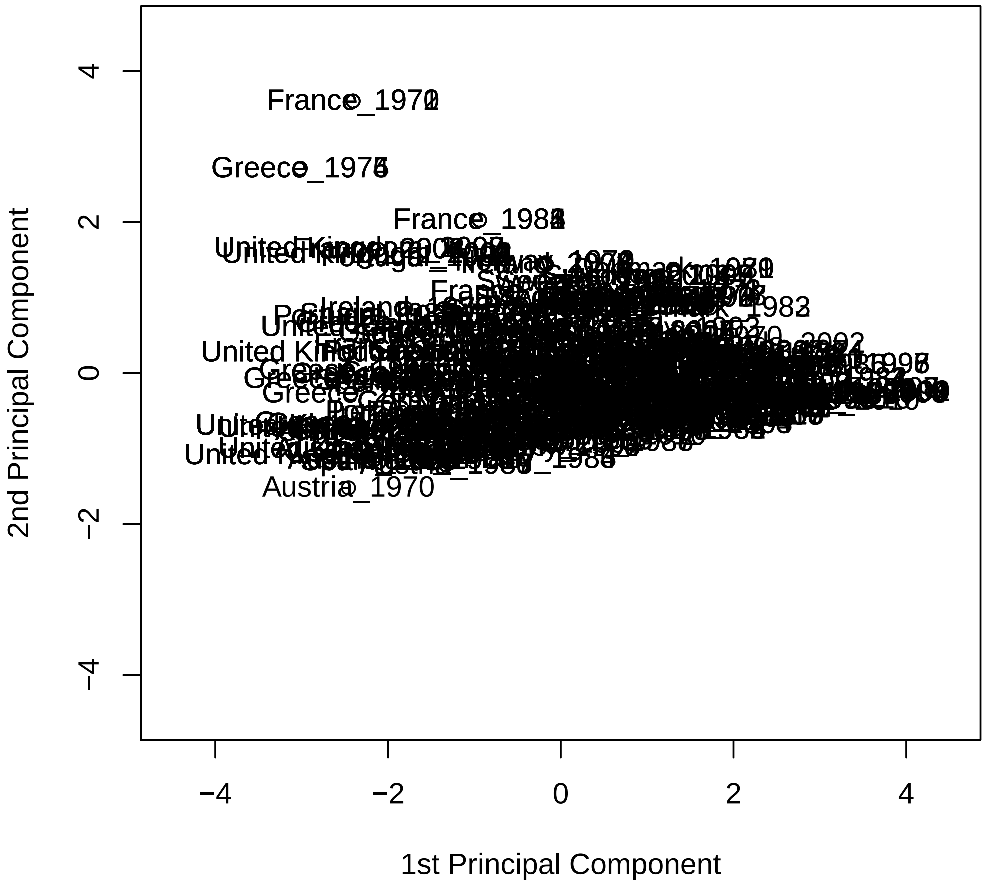
Figure C.1: Scree Plot, kPCA

Table C.2: kPCA and PCA Covariances

	PC1	PC2	PC3	PC4
kPC1	0.21	-0.00	-0.00	-0.00
kPC2	-0.00	-0.05	-0.00	0.00
kPC3	-0.00	0.00	-0.02	-0.00
kPC4	-0.00	-0.00	0.00	-0.01

analysis. Homogenity analysis maximizes the correlation between variables at the same time as it does optimal scaling of the variables (optimal quantification of the variables). One generalization of this method is a non-metric principal components analysis, for which we can use not only categorical but also ordinal and ratio variables (de Leeuw 1998).

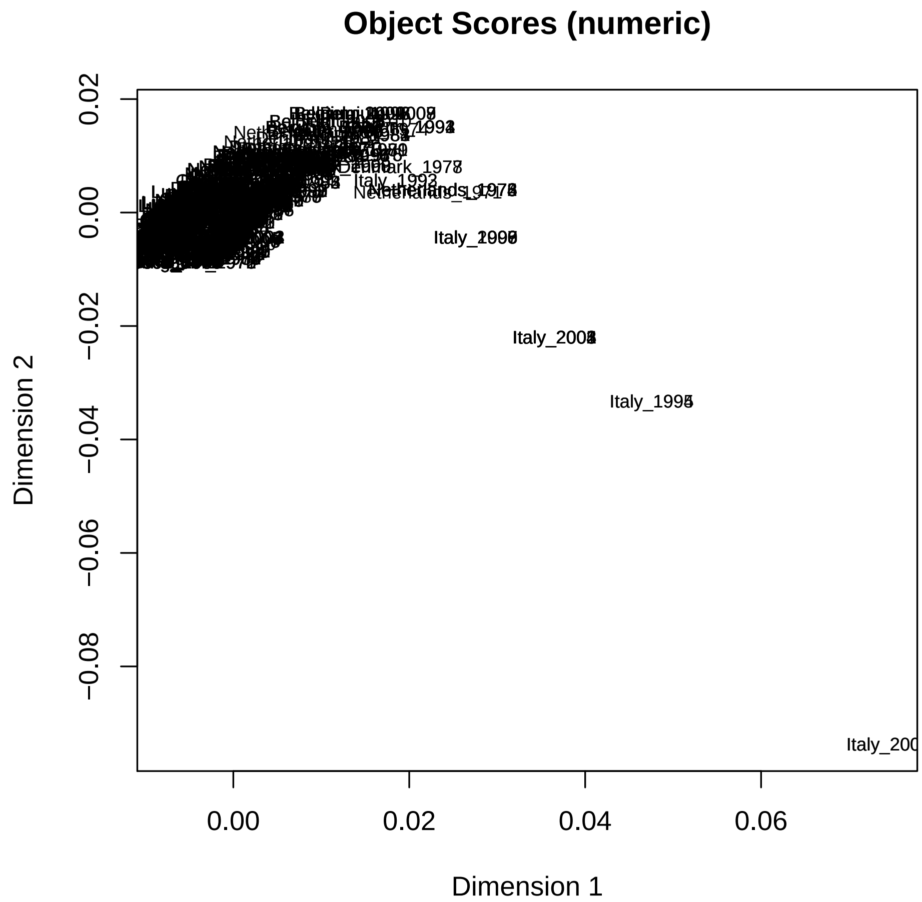
Thus, contrary to the kPCA, the non-linearity of the NLPCA does not come from the transformation of the space on which we project the data, but from the potentially non-linear optimization of the data matrix. During the process, the data matrix is optimized to ensure that the variable variances are explained to the greatest degree possible. The traditional PCA minimizes the loss function over the eigenvectors and eigenvalues. The NLPCA also minimizes the loss over the admissible transformations of the data columns (de Leeuw 2005) The NLPCA can be used if the data is non numerical or if it is rank ordered since this method handles the non-quantifiable distances between variables and can also clarify the results if there is non-linear relationship between the variables(de Leuuw 2005) In this analysis I use the party seat shares as numerical data, since in the



Notes: The plot shows the result of the Kernel PCA

Figure C.2: Biplot, kPCA

dataset the parties are ordered from largest to the smallest. Even though I specify the data as numerical, the NLPCA method considers these variables as categorical. Thus, each observed numerical value becomes a category (Linting 2007).

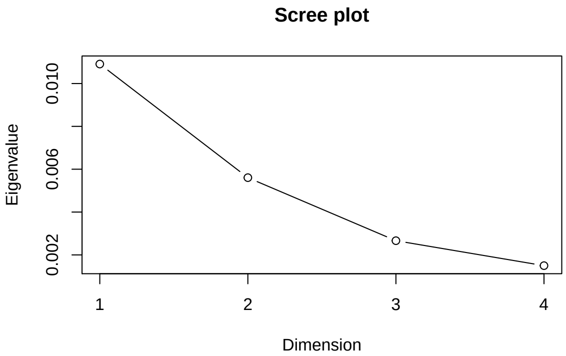


Notes: The plot shows the location of the 17 countries on the two- dimensional plane determined by the first two dimensions that the NLPCA found (with all the parties in the dataset).

Figure C.3: NLPCA Objectplot, All Parties

As I discussed above, I did not standardize the data frame in the PCA and kPCA calculations. As I also discussed above the two solutions from the PCA and the kPCA were similar to each other. However, the NLPCA solution is quite different from these two solutions (*Figure C.3*). This is because the NLPCA method essentially standardizes the variables when it creates optimal quantifications. By dividing the mean centered variables with their standard deviation we can standardize the variables to have unit variance (which equals to performing the PCA on the correlation matrix). As we have seen above, in the party system dataset the variances of the two first variables are big, and thus they influence the solution the most. Through standardization, we give all the variables equal weight. This diminishes the influence of the variances of the biggest two parties and could lead to a solution which shows the structure determined by the sizes of the other parties .[5]

As the biplot of the NLPCA shows (*Figure C.3*), Italy is separated from the rest of the countries on the first dimension. Italy is a unique case because the country had the most parties in the legislature out of all countries (20 in 2006 and 2007). Neither the non-standardized kPCA nor the PCA revealed that Italy is a special case previously.[6]



Notes: The plot shows the scree plot of the NLPCA, when all parties are in the data matrix.

Figure C.4: NLPCA, Scree Plot, All Parties

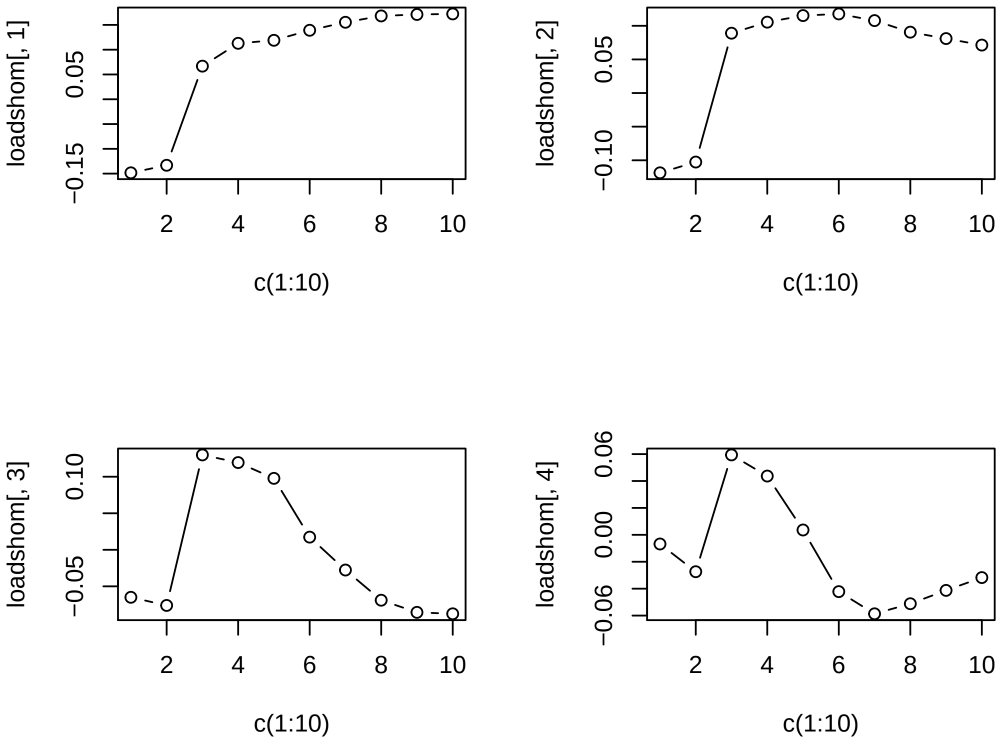
The screeplot (*Figure C.4*) shows that the first eigenvalue that the NLPCA extracts, explains less variation in the data, compared to the the first eigenvalue that the PCA and the kPCA methods have found.[7] This is because most of the variation in the data has been generated by the variation in the sizes of the first two parties.

Because I reduced the variance of the variables the first and second principal components that the NLPCA finds are influenced less by the sizes of the two biggest parties. Because all parties get equal weights in determining the dimensions, NLPC1

[5]Thus the variation of the bigger parties (Party 1 and Party 2) becomes smaller and the variation of the smaller parties (Parties 8-20) becomes bigger. Thus after the standardization Dimensions 1(NLPC1) and 2 (NLPC2) are influenced more by smaller parties than the first and second dimensions I recovered with PCA and kPCA.

[6]If we remove Italy from the dataset, the first dimension separates Belgium from the rest of the countries (as we have seen, the kPCA and PCA solutions also put Belgium at the far end of the dimension that separated countries with two big parties from the rest of the countries).

[7]The scree plot of the NLPCA *Figure C.4* is less steep than the ones we have seen before: *Figure* **??**, *Figure* **??**.



Notes: The plot shows the weight of parties in determining the principal components based on the NLPCA.

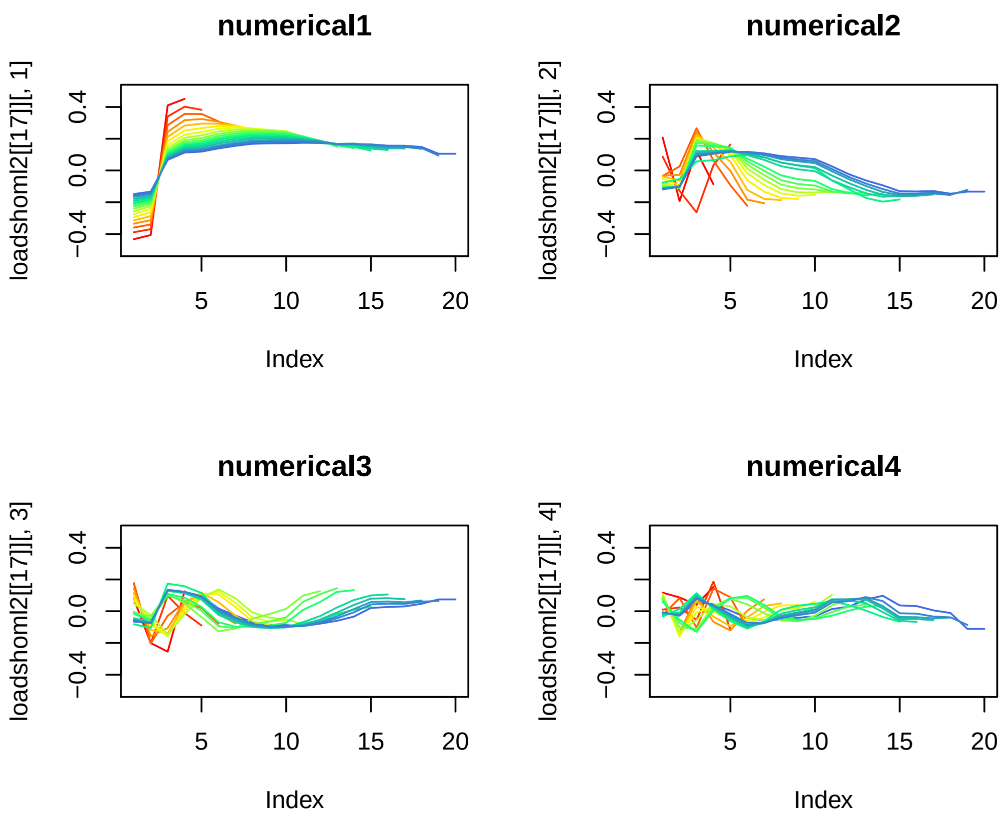
Figure C.5: NLPCA, Loadings, All Parties

and NLPC2 separate the countries based on the actual number of parties. At the same time, NLPC3 separates moderate party systems (party systems up to 5 parties) from the very large party systems (*Figure C.5*).[8] In order to be able to compare the results of the NLPCA to the kPCA and to the PCA results, in the following section, I reduce the number of parties to ten. This way I can avoid that the countries with fragmented party systems would define the Dimension 1 of the NPLCA.

Before reducing the number of the variables however, I examine the impact of this change on the NLPCA and the PCA results. *Figure C.6* shows how the loadings

[8] *Figure C.5* shows that even though the sizes of the first two parties are in the opposite direction from the rest of the parties, the sizes of the smaller parties weigh almost the same as the sizes of the bigger parties. As Italy has many small parties that have a high weight in this analysis, the country gets separated from the other countries. In NLPC1 apart from Party 1 and Party 2, Parties 7-16 have the highest loadings. NLPC2 is determined by the sizes of Party 1 and 2 and also it is influenced by Parties 3-11. NLPC3 is influenced by Party 1 and Party 2 and Parties 3-5, while Dimension 4 has high loadings from the smaller parties, Parties 15-20.

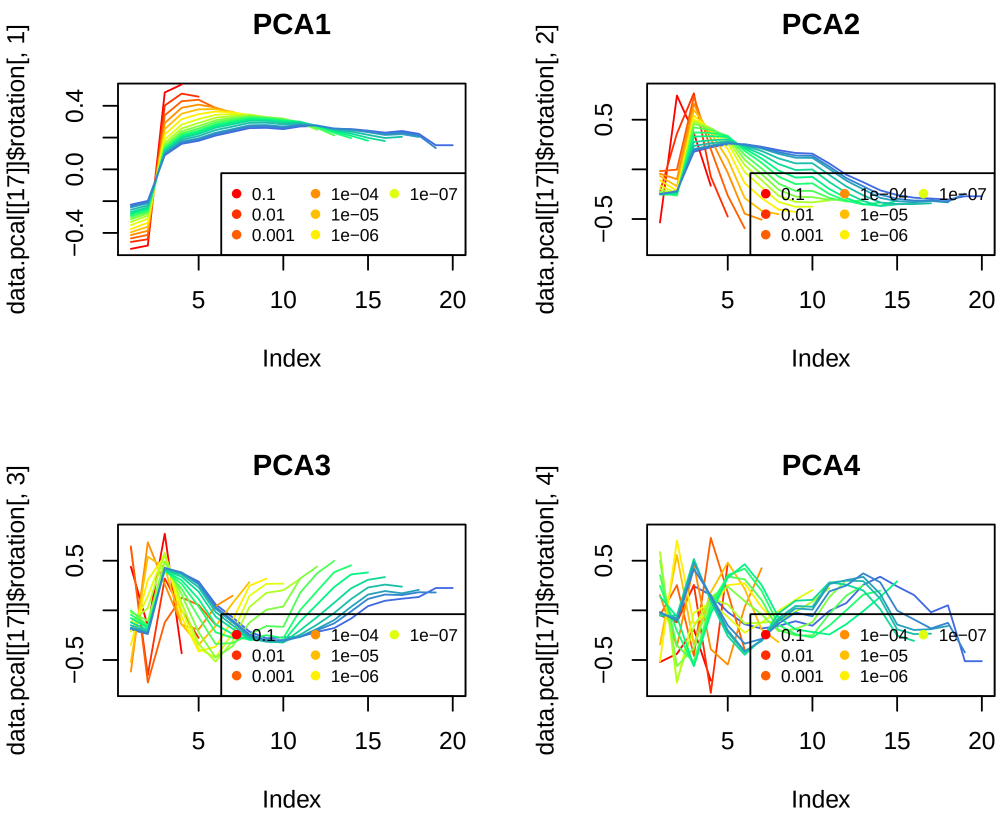
change if we change the number or parties in the NLPCA. The colors show the number of parties in the analysis. The number of parties start at 4 and go up to 20. *Figure C.6* shows that the NLPCA does not find exactly the same solution when we increase the number of parties. When we increase the number of parties the first parties get less weight. However, the solutions are similar in their underlying structure. We can contrast the NLPCA solution *(Figure C.6)* with the scaled *(Figure C.7)* and the unscaled *(Figure C.8)* PCA solutions.



Notes: The plot shows how the NLPCA changes when we run the analysis on fewer and fewer parties. The color of the lines indicates the number of parties in the analysis.

Figure C.6: The Sensitivity of the NLPCA Results to the Change in the Number of Parties

Figure C.8 shows that even when we limit the number of parties radically, the unscaled PCA finds the same first two dimensions, and while the sign of the loadings might change, the PC3 and PC4 remain very similar as well. This is because the small parties get less weight in this analysis than the bigger parties, and the parties after the fourth party tend to be small. This is not the case when we scale the variables *Figure C.7*.

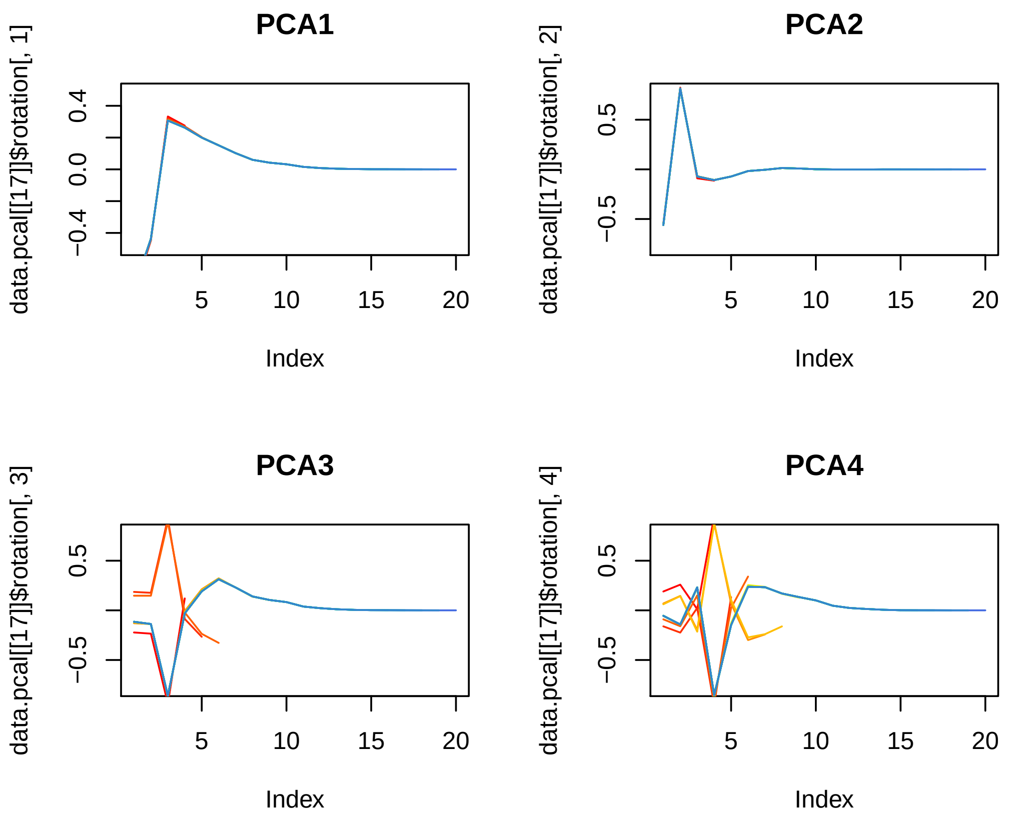


Notes: The plot shows how the scaled PCA changes when we run the analysis on fewer and fewer parties. The color of the lines indicates the number of parties in the analysis.

Figure C.7: The Sensitivity of the Scaled PCA Results to the Change in the Number of Parties

A comparison between the scaled PCA *Figure C.7* and the NLPCA *Figure C.6* reveals that the NLPCA solution is not the same as the scaled PCA solution. While eventually the dimensions that the two methods find seem to be similar (apart from the fact that Dimension 4 is still unclear) the loadings change more when we change the number of parties in case of the scaled PCA. Overall, it seems that small changes in the party system can influence the dimensions that the scaled PCA recovers more than it can influence the dimensions that the NLPCA recovers. In contrast, the unscaled PCA solution remains pretty steady when we include the smaller parties *Figure C.8* This may indicate that we have to consider a trade-off: the NLPCA may be more suitable if we want to explore party system changes when small disturbances happen within a single country, while the PCA may be more suitable for cross-country, cross-era comparison.

Next, I analyze the results of the NLPCA results that I get when I limit the



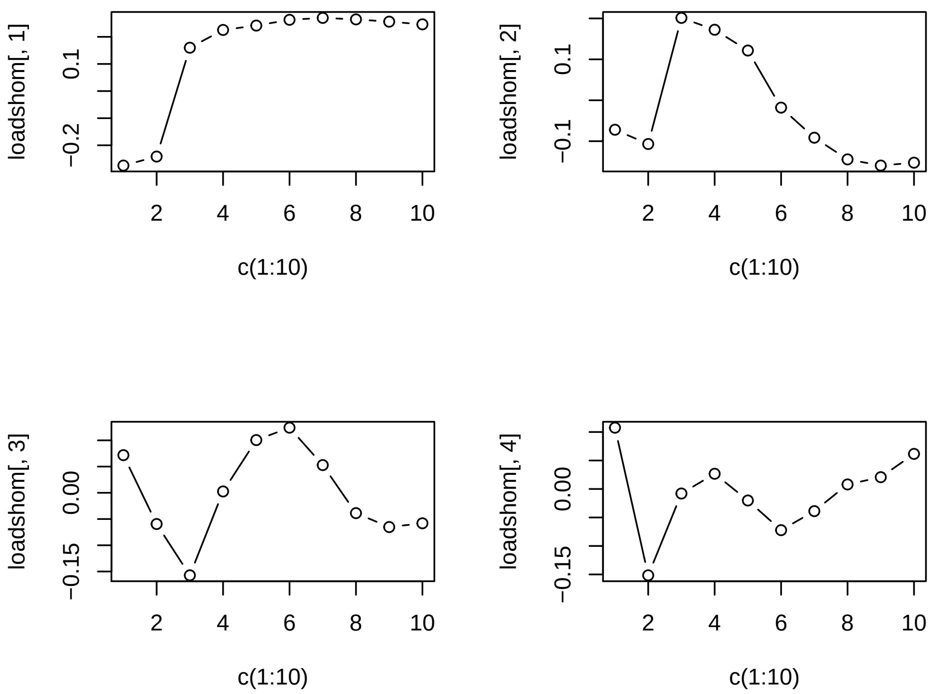
Notes: The plot shows how the unscaled PCA changes when we run the analysis on fewer and fewer parties. The color of the lines indicates the number of parties in the analysis.

Figure C.8: The Sensitivity of the Unscaled PCA Results to the Change in the Number of Parties

parties to the 10 biggest parties in the legislature. As the following plots show, the dimensions that the NLPCA recovers from the limited data, are similar to the ones that the methods finds with the full dataset- although there are some differences.

Figure C.10 shows that when there are fewer parties, the first eigenvalue that the NLPCA finds explains more variation of the data compared to the rest of the eigenvalues than when all the parties are included. Again, this happens because if there are only the biggest 10 parties included, the first two parties get more weight than if all parties are included. *Figure C.11* shows the two-dimensional plane that the NLPCA (with 10 parties) finds, and the object scores of the countries on these dimensions (de Leeuw, 2005).

This means that the PCA loss function: $E_M = \frac{1}{2}\sum_{n=1}^{N}\sum_{i=M+1}^{d}(z_i^n - b_i)^2$ is not only minimized with respect to b_i but also with respect to z_i (or x_i, since $z_i = u_i^T x$). Thus,



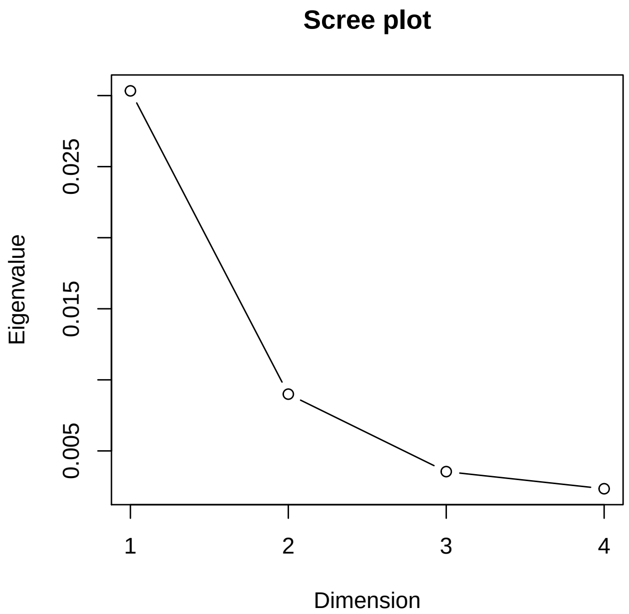
Notes: The plot shows the weight of parties in determining the principal components based on the NLPCA on the 10 biggest parties in each country.

Figure C.9: Loadings, NLPCA, Ten Parties

the solution will be: $\Sigma u_i = \lambda_i u_i(X)$.[9]

Thus, this could lead to the trivial solution that all transformations will be set to zero. To avoid this, de Leeuw argues that we have to make another restriction: we can redefine the cone to only contain centered vectors, so the cone $K_j \cap S$ is going to be a convex cone of centered vectors. Because of this the optimization problem finds admissible transformations of the variables where the sum of the $n - p$ smallest eigenvectors of the correlation matrix is minimized (or the sum of the p largest eigenvectors is maximized). Thus the final form of the NLPCA is $\max_{x_j \in K_j \cap S} \sum_{s=1}^{p} \lambda_S(R(X))$, where the real valued

[9]Furthermore, de Leeuw (2005)(de Leeuw, 2005) discusses that not all transformations are admissible: the first restriction is that the transformed variables must be in a convex cone K. Convex cones are defined by $x \in K$ implies $\alpha x \in K$ for all real $\alpha \leq 0$ and $x \in K$ and $y \in K$ implies $x + y \in K$. However, since α is in the cone, this means that its positive linear function: $\alpha x + \beta$ with $\alpha \leq 0$ must also be in a convex cone.



Notes: The plot shows the scree plot of the NLPCA on the 10 biggest parties in each country.

Figure C.10: Scree Plot, NLPCA, 10 Parties

function ϕ is defined as the sum of the p largest eigenvalues of the correlation matrix $R(X)$ (de Leeuw, 2005).

This means that the PCA is performed while the variables are also optimized. As an algorithm, the method alternates between the two processes in an iterative way, until the loss function is minimized, and the algorithm converges. At this point, neither the variable quantifications nor the PCA solution change (Linting et al., 2007).

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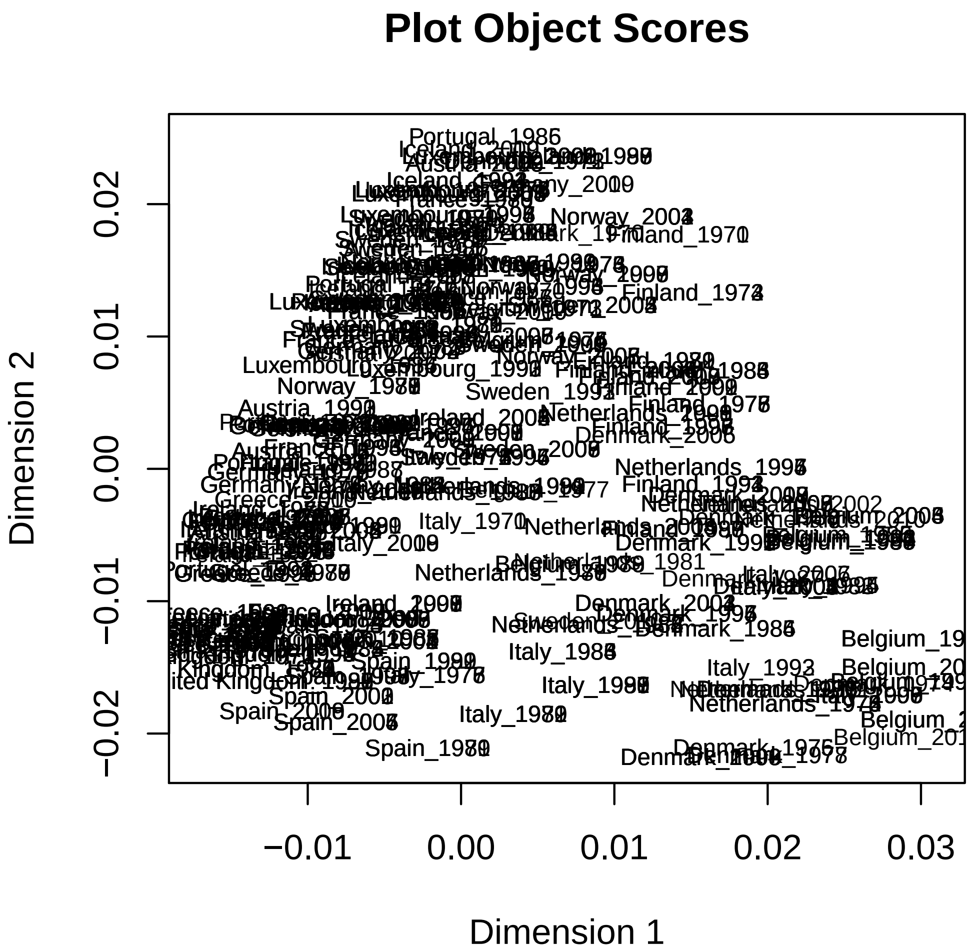


Notes: The plot shows the location of the countries based on the NLPCA on the 10 biggest parties in each country.

Figure C.11: Objectplot, NLPCA, 10 Parties

D. Typologies and Summary Measures

D.1. Typologies

In any parliamentary system, a majority is needed to pass legislation. Normally, this legislative majority chooses the prime minister and the government. The rest of the parties are considered to be in the opposition. The most canonical difference in party systems across countries is between two-party and multi-party systems. In a two-party system, the winning party always holds the legislative majority by itself. In contrast, if there are many parties, no party may hold a majority by itself. If none of the parties wins a majority, some parties have to form a governing coalition.

Duverger (1954) argues that plurality electoral systems (in which only one candidate can win in a given district) lead to a two-party competition, at least on the district level. This is because the voters do not to waste their vote on third party candidates, thus small parties fall out from the competition. By contrast, proportional representation electoral system (PR) leads to a multi-party party system. Under PR, several candidates can win seats within a given electoral district. The parties get seats based on their vote shares in the election (thus a party that got 15% of the votes receives roughly 15% of the seats in the legislature). Under this system, small parties can gain legislative representation. Duverger considers the two-party system the ideal type, while he thinks that multi-party systems are unstable and inchoate, as the coalition governments are less stable than single party governments. In practice, however, there are very few countries with ideal two-party systems (countries that have close to two-party systems, at least in the 1970s include Britain, the United States, Canada, New Zealand, Austria and Australia (Sartori, 1976)).

The rest of the countries are multi-party countries. Within the countries with non-majoritarian electoral systems, there is a wide variety of different sized and structured party systems. One reason for this again is the electoral system: In some of the PR countries, electoral districts are relatively small, - there are electoral districts in which only a few seats get allocated. Even though *within* the electoral districts seats are allocated proportionally, the smallest parties cannot gain seats (for instance if there are only 5 seats available a party with 15% of the votes may not gain seats). However, the variation is not limited to electoral causes. Even in countries with the same electoral system, different party systems have developed, and keep evolving. To impose order in the chaos (to group *similar* countries together), political scientists classify the multi-party countries into more refined categories (Blondel, 1968; Rokkan, 1970; Sartori, 1976).

Blondel (1968) is the first to recognize that not only the number of parties, but also their relative sizes, are important to compare party systems, as small parties are less important than big ones.[10] Most of the typologies following Blondel (1968) sort the countries based on the number of the parties and based on how the parties compete.

[10] He sorts the party systems into two-party systems, two and a "half" party systems, multi-party systems with a dominant party, and multi-party systems without a dominant party.

Depending on their approach, some authors argue that the competition style is a direct outcome of the party system size and structure, while others argue that the competition between the parties is an independent feature, a separate dimension. Rokkan (1970) classifies the countries based on whether the parties in the party system are roughly the same sizes (compared to each other) or whether there is one or more dominant parties facing small parties.[11] In a related paper, Laver and Benoit (2015) create a party system classification based on the government, and coalition potential of the different parties.[12]

Other authors consider competition a separate feature. Dahl (1966)sorts the countries into different categories based on whether the parties only compete or at the same time cooperate with each other (which happens in party systems in which parties regularly have to build coalitions). He argues that the competition style is directly influenced by the party system.[13] On the other hand, Sartori (1976) argues that party fragmentation and the ideological distances between the parties are two separate characteristics and these two dimensions determine the type of political competition in a country.[14] Finally, Mair classifies party systems based on whether a country has open or closed party competition, whether new parties can enter the race. Thus the party system defines the type of the country (Mair, 2002).[15]

D.2. Summary of Typologies

In a later part of this paper, I examine the most important features that separate party systems, and I compare these to the typologies I discussed above. *Table D.1* summarizes the typologies created by previous literature. The table lists the countries that the authors bring up as examples for the categories. Most of the typologies were created in the 1960s and 1970s and as a result the universe of the cases that the authors discuss is primarily

[11] Rokkan's categories are named after the sizes of the parties in these groups. For example the British-German "1 vs. 1+1" system describes a two and a half party system – a dominant party facing one dominant and one small party (Rokkan, 1970).

[12] Laver and Benoit (2015) establish categories based on how the ranked parties (biggest, second biggest etc.) could form winning coalitions (reach the 50% seat share threshold). Thus the authors classify countries based on their party seat share constellations. The authors do not explicitly show the countries that belong to each category, as they argue that the multi-party countries shift in and out of these categories quite frequently, based on small changes in the electoral results.

[13] Dahl (1966)claims that the opposition is competitive in two-party systems –in which only two parties compete– while it is cooperative-competitive in multi-party systems –in which small opposition parties have a chance to join the government coalition without changing the entire government.

[14] Sartori draws a distinction between countries in which two ideologically close party groups compete (limited or moderate pluralisms), and between countries in which the opposition is fragmented, and ideologically diverse (extreme pluralisms). In his classification, the cut off between moderate and polarized pluralism is around five or six parties (Sartori, 1976, 328).

[15] Closed party systems are those where the alternation in government is fully predictable and new parties have no chance of gaining power. In contrast, it is unclear how the next government is going to look in an open system. Mair argues that open competition emerges in transitional (inchoate) party systems, or is a sign of party system failure which is reminiscent of how Duverger characterized multi-party systems (Mair, 2002).

Table D.1: Party System Classifications

Author	Criteria	Typology	Countries
Duverger (1954)	Numbers of Parties	1.Two-party Systems 2. Multi-party Systems	Sartori: 1. England, United States, New Zealand, Australia, Canada, Austria 2. All else
Dahl (1966)	Competitiveness of the Opposition	1. Strictly competitive 2. Co-operative-competitive 2a. two-party 2b. multi-party 3. Coalescent-competitive 3a.Two-party 3b. Multi-party 4. Strictly coalescent	1. Britain 2a. United States 2b.France, Italy 3a Austria, Wartime Britain 3b. (no example) 4. Colombia
Blondel (1968)	Numbers of parties Relative size of parties	1.Two-party systems 2. Two-and-a-half-party systems 3. Multi-party systems with one dominant party 4. Multi-party systems without dominant party	1.United States, New Zealand, Australia, England, Austria 2. Germany, Canada, Belgium, Ireland 3. Denmark, Norway, Sweden, Iceland, Italy 4. Netherlands Switzerland, France, Finland
Rokkan (1970)	Numbers of parties Proximity to the majority Evenness of the competition	1.The British-German “1 vs. 1+1” system 2. The Scandinavian “1 vs. 3-4” system 3. Even multi-party systems “1 vs. 1 vs. 1+ 2-3” 3a. scandinavian “split working class” systems 3b. segmented pluralism	1 Austria, Ireland, some periods Belgium 2. Sweden, Denmark, Norway 3a. Finland, Iceland 3b. Netherlands, Belgium, Luxembourg Switzerland
Sartori (1976)	Party fragmentation (number of parties) Ideological distance	1. Predominant party regimes 2. Two-party systems 3. Moderate pluralism 4. Polarized pluralism	1. Norway(or 3), Sweden (or 3), Japan, Uruguay, India Turkey 2. Canada, Australia, Austria, England, New Zealand, United States 3. Switzerland, Netherlands, Israel, Denmark, Iceland, Luxembourg, Belgium, Ireland, France (after 1958), Germany 4. Finland, Chile, France(before 1958), Italy
Mair (2002)	Type of Competition (alternation of the government, new parties: in the system, in the government)	1. Open Party System 2. Closed Party System	1. Denmark, the Netherlands, post authoritarian systems 2. United Kingdom, New Zealand (till mid 1990s), Japan(1955-93, Switzerland, Ireland (1948-89)
Laver and Benoit (2015)	Type of Competition (potential winning coalitions)	1. Single Winning Party 2. No Single Winning party 2a. Strongly dominant party ($S_2 + S_3 < W$) 2b.Top- three ($S_2 + S_3 \leq W$) 2c. Top-two 2d.Open	countries change categories

Note: The table is modified from Table 1 in Mair (2002). The countries in the different categories are the authors' own except for Duverger, where I take my information from (Sartori, 1976).

limited to European democracies. Often the authors are cautious about discussing the political institutions in non-democracies or newly democratized countries. Greece, Spain and Portugal are also missing for the same reason (Greece becomes a democracy in 1974, Spain in 1978 and Portugal in 1976).

As the authors are writing in the same decade (apart from Mair (2002) and Laver and Benoit (2015)) the typologies are comparable to each other. Most of the authors sort two-party systems in their own separate category.[16]

There is less consensus about countries with more parties. As the number of parties within a country increases, the consensus on the ideal category for the country decreases. Countries that have party systems close to a two-party system (Germany, Ireland), get their own separate category in most of the classifications. However, it is unclear whether the Scandinavian countries are their own category or not.[17] The most problematic countries to categorize into the typologies are Finland and France.[18]

Overall, it seems, that finding the proper categorization of multi-party countries is more difficult than dividing two-party and multi-party countries. However the typologies, in fact, are not too different from each other. Rokkan's (1970) idea to categorize parties based on how parties face each other within party system is made more precise 40 years later by Laver and Benoit (2015). Sartori's (1976) distinction between moderate and polarized pluralisms creates a very similar categorization to Mair (2002). All typologies suggest, that apart from the size and relative power of the parties, we should consider the competition within the party system to separate countries into groups.

D.3. Indices

In contrast to the authors who sort countries into party system categories, other authors summarize party systems with a single, continuous variable. Later, I will calculate some of these measures to compare them with the results of the PCA.

[16] There is considerable agreement that New Zealand, the United States and Australia are within this category, and Austria (at this time) as well. Sartori (1976) argues that the consensus is that most of the anglo-saxon countries are close to the two-party system ideal (Britain, the United States, Canada, New Zealand, Australia). However, in Canada there is a clear third party, and in Australia a single party competes with a two-party coalition. Often Austria also listed as a two-party system, although it does not adhere to the "two-party competition" ideal. In Austria in the 1960s and 1970s the two biggest parries, SPÖ and ÖVP formed a coalition to keep the radical right FPÖ out of the government

[17] While Sweden and Norway are usually in the same category, the appropriate category for Denmark and especially Iceland is less clear.

[18] France has several parties but these parties form coalitions, so depending on the author the country is categorized as either a quasi two-party system; or a party system with several, equally strong parties. Finland on the other hand gets categorized with the Netherlands (and France) by Blondel (1968) as the country has many small parties, Rokkan (1970) puts the country into the same category as Iceland (Scandinavian split working class country), while Sartori (1976) sorts the country to a category in which countries with a dominant parties are (along with Italy).

D.4. Maximum Entropy

Kesselman (1966) develops an entropy-based hyperfractionalization index to characterize the shapes of party systems (Taagepera and Shugart, 1989, 5). The entropy measure evaluates the probability of the i-th bin in a histogram. It counts the number of ways how we could rearrange the parties while still arriving at the same histogram(**?**).[19] Kesselman defines his index as I=$exp[-\sum_i^k p_i \log_e p_i]$[20] where k is the number of candidates or lists, p_i is the proportion of vote for i-th list and $\sum_i p_i = 1$ (Kesselman, 1966).

Thus the hyperfractionalization indices uniquely characterize each party distribution. However, entropy-based indices are sensitive to the smallest changes in the distribution. This can make the measure unreliable, as similar party systems may end up with very different numbers (Laakso and Taagepera, 1979).

D.5. Concentration

Next, in order to give more weight to bigger parties in the system and minimize the weight of smaller parties (to make the measures more reliable), political scientists adapt an economic measure. The basis of this family of measures is the Herfindahl-Hirschman concentration index, which is the sum of squares of the market share of each company in a given market HH= $\sum s^2$. (Where s is the market share of each company). The range of this index is 0 to 1 where a 1 means that the market is dominated by one company and 0 means that all companies are equal.[21] Rae and Taylor argue that this measure shows the probability that two randomly selected voters would vote for the same party (Molinar, 1991).

Laakso and Taagepera (1979) argue that an intuitive transformation is 1/HH ($\frac{1}{\sum s^2}$), which shows how many equal sized parties would be equivalent to the current party system. They call this measure the Effective Number of Parties (ENP). Currently the ENP is probably the most widely used measure of party-system concentration. However the measure has been criticized both because it insufficiently weights big parties, and because it does not show small changes in the party system. The generic formula for this

[19] For the i-th bin there are $N_i!$ such ways how we could arrange the objects and arrive at the same histogram. Where N is the number of objects and N_i is the number of objects in each bin. Altogether the multiplicity of the objects can be given by $W = \frac{N!}{\prod_i N_i!}$ the entropy is the negative logarithm of the multiplicity. $S = -\frac{1}{N}\{lnN! - \sum_i lnN_i!\}$ if we assume that $N \to \infty$ and use Stirling's approximation we find that $S = -\sum_i p_i ln p_i$, the entropy. Consequently, a very high peaked histogram has a very low entropy (a histogram with one bin would have an entropy of 0) while a uniformly distributed one has a high entropy.(**?**).

[20] According to Wildgen (1971) this measure measures "the voters' tendencies to diverge or converge relative to parties or candidates."

[21] Rae and Taylor (1970) calculate a fractionalization index by exchanging the companies' market shares to seat shares in the formula, and changing the formula to 1-HH or 1-$\sum s_i^2$ where the $(s_1, ..., s_n)$ are the legislative seat shares of the parties. This measure is in fact the Effective Number of Legislative Parties. Depending on the issue at hand, this measure can be calculated as 1-$\sum v_i^2$ where $(v_1, ..., v_n)$ is the vote share of all the parties that ran in the elections.

family of indices is: $N_a = [\sum_1^x v_i^a]^{1/(1-a)}$ (Dunleavy and Boucek, 2003). Where we raise the decimal vote shares to a power (a) add these numbers together and raise the resulting summed number to 1 divided by (1-a). We can see that the ENP is a special case of this formula where a=2 (Dunleavy and Boucek, 2003).

D.6. Party Power and the Number of Parties

The first criticism of the ENP is that it overestimates the number of relevant parties. The critics argue that we should only consider parties to be relevant if they have a real probability of joining a governing coalition (Kline, 2009) or at least of influencing the behavior of parties that do have coalition potential (Sartori, 1976). Thus new measures are created to put more weight on the bigger parties if they have a higher coalitional potential or "power".[22]

The Shapley-Shubik power index shows how many times a party would be pivotal in coalitions (Shapley and Shubik, 1954).[23] The Banzhaf index measures how many times a coalition would shift from winning to losing if a particular actor were to change their vote (Banzhaf, 1965).[24] Caulier and Dumont (2005), Grofman (2006), and Kline (2009) all suggest using the sum of squared power shares instead of the seat shares of the parties in the formula of the ENP in order to address the potential over-valuing of small parties. Mathematically this measure is: $LTB = 1/\sum_{i=1}^{n} B_i$ where B_i is the Banzhaf score of i-the party.[25]

Several other measures have been created to increase the weight of bigger parties.[26] Dunleavy and Boucek suggest that because all of these measures are correlated

[22]In practice all power indices use the same data as the party number indices: the seat shares (or the vote shares) of the parties. The only difference is that, based on some combinatorial rules, the parties may receive bigger or smaller weight than their original seat (vote) shares.

[23]This measure starts from the premise that all possible coalitions are ordered as the parties join them in particular order. After listing all coalitions, in each coalition the pivotal player is identified. The pivotal player is the player that can make the coalition's total vote share pass the threshold that is needed to win the particular vote. The index is calculated for each actor (party) and it shows how many times a player would be pivotal out of all possible permutations of party coalitions.

[24]Mathematically the Shapley index for a simple game of n players for party i is the following: $\Phi_i = \frac{1}{n!}\sum_{\{i-swings-in-S\}}(s-1)!(n-s)!$, $(s = |S|)$. Where the sum is taken over all such coalitions S that i is in S, S is winning but $S-(i)$ is losing. With similar notation the Banzhaf index is the following: $\beta_i = \frac{1}{2^{n-1}}\sum_{\{i-swings-in-S\}} 1$ (Straffln Jr., 1988). In practice, bigger parties could get a higher Banzhaf power value than Shapley-Shubik value. This is because in an oversized coalition, a big party may be the only one whose leaving could swing the coalition from winning to losing so it would be the only party that is relevant for the calculation of the Banzhaf index. But the big party still may not be a majority party and may therefore need coalition partners, so it would not be the only party relevant for the calculation of the Shapley-Shubik value.

[25]This measure in practice ends up having sharper step-downs in the number of parties than the ENP when certain thresholds (of coalitional potential) are hit. Especially around these thresholds, the measure diverges from ENP. Kline argues that we should use this measure when we are interested in outcomes related to coalitional potential such as government duration (Kline, 2009, 21)

[26]For instance Molinar (1991) argues that we should always count the winning party as one, and then he

with the size of the biggest party, we might as well use the latter to measure the size of the party system (2003). They suggest using $\frac{1}{V_1}$ where V_1 is the vote share of the biggest party. This is also suggested by Taagepera (1999).

In practice, studies find that there are sharper step-downs in the number of parties in the measures modified by the power of parties (Kline, 2009). In fact, this modification amplifies that problem that ENP has, that some very different party configurations end up with the same index numbers.

D.7. Full distribution

The second major criticism about the ENP measure is coming from the other direction. Some authors argue that by weighing big parties more than small parties, a lot of different party configurations end up with the same ENP value, thus the index may mask important differences among the party systems.[27] Thus in recent years, some political scientists have created measures, to describe the full distribution of parties in order to measure the nuanced changes in the party system. These efforts create predicted vote shares of each party by using the log-ratio transformed party vote shares. (Katz and King, 1999; Rozenas, 2011).[28]

D.8. Indices and PC Dimensions

In this part of the appendix I compare the PCA results to the party system size indices that the previous scholarship identified. Based on the literature review, below I compare the PCA results to several measures. From indices that are more sensitive to the sizes of small

suggests that we should calculate the ENP with all the other parties and add the two values together. Mathematically this index is the following $M = 1 + \left(\frac{1}{\sum_1^x v_i^2} * \frac{\sum_1^x v_i^2 - V_i^2}{\sum_1^x v_i^2} \right)$, where v are all the parties. and V_i the voteshare of the opposition parties. This index is criticized by Dunleavy and Boucek (2003) as it behaves erratically under certain circumstances.

[27] However, the original goal of Laakso and Taagepera (1979) was to create exactly such a measure. They believed that the party systems that they characterized with the same value were indeed similar. "The effective number of parties is the number of hypothetical equal-size parties that would have the same total effect on fractionalization of the system as have the actual parties of unequal size" (Laakso and Taagepera, 1979). The goal of the authors with the index was to create a measure that will not change significantly when there is an additional small party in the party system

[28] Katz and King (1999) use district level electoral data from England to calculate the changes of party vote shares within the system. With the full distribution, they predict the expected vote share for each party in the districts and can calculate whether the politicians have incumbency advantage. Rozenas (2011) uses the relative sizes of the parties similarly. The parties are not defined by their names but by their electoral results (biggest, second biggest etc.). Both of these papers use the mathematical transformation that is suggested by Aitchison (1986) for compositional data. For party J let the voteshares in the districts i ($i = 1, ..., n$) be $V_i = (V_{i1}, ...V_{iJ-1})$. In addition let Y_i be the vector of J-1 log-ratios. $Y_{ij} = ln(\frac{V_{ij}}{V_{iJ}})$. Then we transform the voteshares as $V_{ij} = \frac{exp(Y_{ij})}{1+\sum_{j=1}^{J-1} exp(Y_{ij})}$ where Y_i is the vector of $J-1$ to get the observed voteshares Katz and King (1999).

parties – *Fractionalization Index* (Rae, 1967), *Entropy* (Kesselman, 1966) to indices that are less sensitive to the sizes of small parties – *ENP* (Laakso and Taagepera, 1979), *Shapley ENP* (ENP in which I replace the parties' seat shares with their Shapley-Shubik indices) (Grofman and Kline, 2011), and the size of the *Biggest Party* in the legislature (Dunleavy and Boucek, 2003; Taagepera, 1999). In addition, I include the *Number of Parties in the Government*, as this measure became popular for finding political outcomes(Bawn and Rosenbluth, 2006). I show in three different ways how the PC dimensions relate to the different indices. First, I use a correlation table to show the correlations between the PC dimensions and the various indices. Second, I plot the relationship in two dimension and indices using logistic regressions. Finally, I use the method of *Sum of Ranking Differences* SRD to represent the relationship between the indices and the dimensions.

First, I represent the correlations between these measures and the first two dimensions of the principal component analysis in the first six columns and bottom two rows of *Table D.2.* This correlation matrix shows the Pearson correlation coefficients between the variables and their significance levels. All of these measures are highly correlated (0.7 to .98) with *Dimension 1* (PC1) and with each other. The correlation table shows that the *Effective Number of Parties*, the *Fractionalizaton Index* and the *Hirschman–Herfindahl Index* are all highly correlated with *Dimension 1.*

I also plot the indices on PC *Dimension 1* and *Dimension 2* to show how the indices relate to these dimensions. To construct the plot I regress each index on the PC1 and PC2 dimensions. Since the dimensions are not correlated I can use the regression coefficients directly to construct the lines that show how the measures relate to the dimensions. Note, that I am using the absolute value of the coefficients in all cases since I do not care about the directionality of the relationship between the indices and the dimensions.*Figure D.1* also shows that most of the indices that the current scholarship uses are closer to *Dimension 1. Figure D.1* This offers a way in which we can decide between measures with multi-criteria decision-making process, as the curve can serve as the basis of a pareto-optimization. The more we would like to take into account competition between the parties with our measure and less the number of parties, the closer index we should choose to *Dimension 2.* Alternatively, if our theory calls for the size of the party system we can choose one of the variables close to *Dimension 1.*

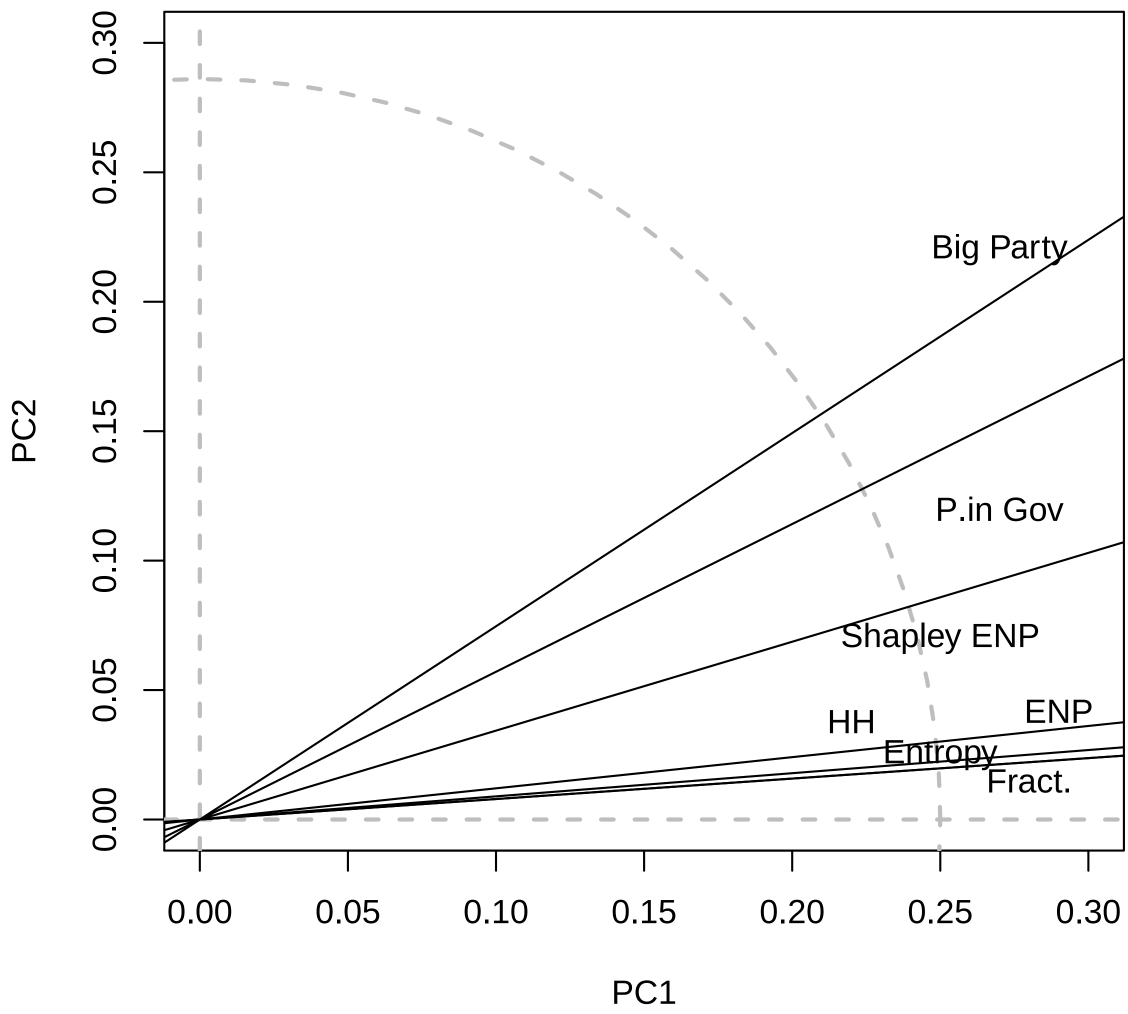
Finally, I calculate the *Sum of Ranking Differences. Sum of Ranking Differences* is a methodology developed to evaluate the success of models, measures, methods (Bajusz, Rácz, and Héberger, 2015). The method ranks the variables across the cases and compares to the reference method, measure or model. In practice the cases and the variables are organized in a matrix, the objects (or cases) are in the rows and the variables of interest are in the columns (Kalivas, Héberger, and Andries, 2015). After this, we have to calculate how the objects (rows) are ranked by the solutions (columns). In addition, we have to calculate how the objects (rows) are ranked by the ideal measure, method or model (which are the variables, in the columns). In due course, we deduct the rankings of the ideal solution from the rankings of each of the models, methods that we are testing. The sum of these differences is the *Sum of Ranking Differences.* In this particular case my

Table D.2: Correlation table, Traditional Measures of Party System Size and Opposition Structure and Principal Components

	ENP	Fraction.	HH	P.in Gov	Big Party	Entropy	Shap. ENP
Fraction.	0.91						
	(0.02)						
HH	0.95	0.96					
	(0.01)	(0.01)					
P.in.Gov	0.75	0.70	0.73				
	(0.03)	(0.03)	(0.03)				
Big.Party	-0.90	-0.94	-0.89	-0.71			
	(0.02)	(0.01)	(0.02)	(0.03)			
Entropy	0.95	0.96	1.00	0.73	-0.89		
	(0.01)	(0.01)	(0.00)	(0.03)	(0.02)		
Shap.ENP	0.91	0.87	0.87	0.76	-0.91	0.87	
	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	
PC1	0.93	0.98	0.94	0.70	-0.94	0.94	0.87
	(0.01)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.02)
PC2	0.04	0.06	-0.04	0.12	-0.34	-0.04	0.24
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

Notes: Computed correlation using Pearson-method with pairwise deletion. Standard Errors are in parentheses. Fraction: Fractionalization Index,HH:Herfindahl–Hirschman Index, Entropy: Entropies, ENP: Effective Number of Parties, P.in Gov: Number of parties in the Government, Shap.ENP: Effective Number of Parties(Shapley), Big Party: Size of the biggest party over the size of the legislature, P.s in Gov: Parties in Government, *ENOP*: Effective Number of Opposition Parties,

cases are the country-years. To make the solution easier I eliminate repeated rows in the matrix and I discard the idea to have a ranking for each county-year. My final matrix has 200 rows. My columns are the various indices. Then, for each index I rank the cases in ascending order. If two or more values are equal all cases are given the middle value. As the PCA is non-directional, I repeat the ranking in descending order as well. After this, I examine how the PC dimensions rank the cases. I repeat this procedure to as many of the PC dimensions as I chose to use. Finally, I deduct the ranking of the PC dimensions from all of the rankings of the variables and I sum the results. Out of the two sums I get for each index (ascending, descending) I choose the lower number. The closer this final value to zero the better the index approximates the reference category (Héberger and Kollár-Hunek, 2011; Bajusz, Rácz, and Héberger, 2015; Kalivas, Héberger, and Andries, 2015). After this I normalize the SRD numbers. According to Héberger and Kollár-Hunek (2011) we can normalize the SRD values such as $SRD_{nor} = 100/SRD_{max}$



Notes: The plot shows how party system size measures relate to each other and the two first dimensions of the PCA. PC1: Dimension 1, Sizes of the two biggest parties. PC2: Dimension 2, Competition between the biggest and the second biggest parties. Fract: Fractionalization Index, Entropy:Entropies, ENP: Effective Number of Parties, P.in Gov: Number of parties in the Government, Shap.ENP: Effective Number of Parties(Shapley),Big Party: Size of the biggest party over the size of the legislature, P.s in Gov: Parties in Government, HH:Herfindahl–Hirschman Index

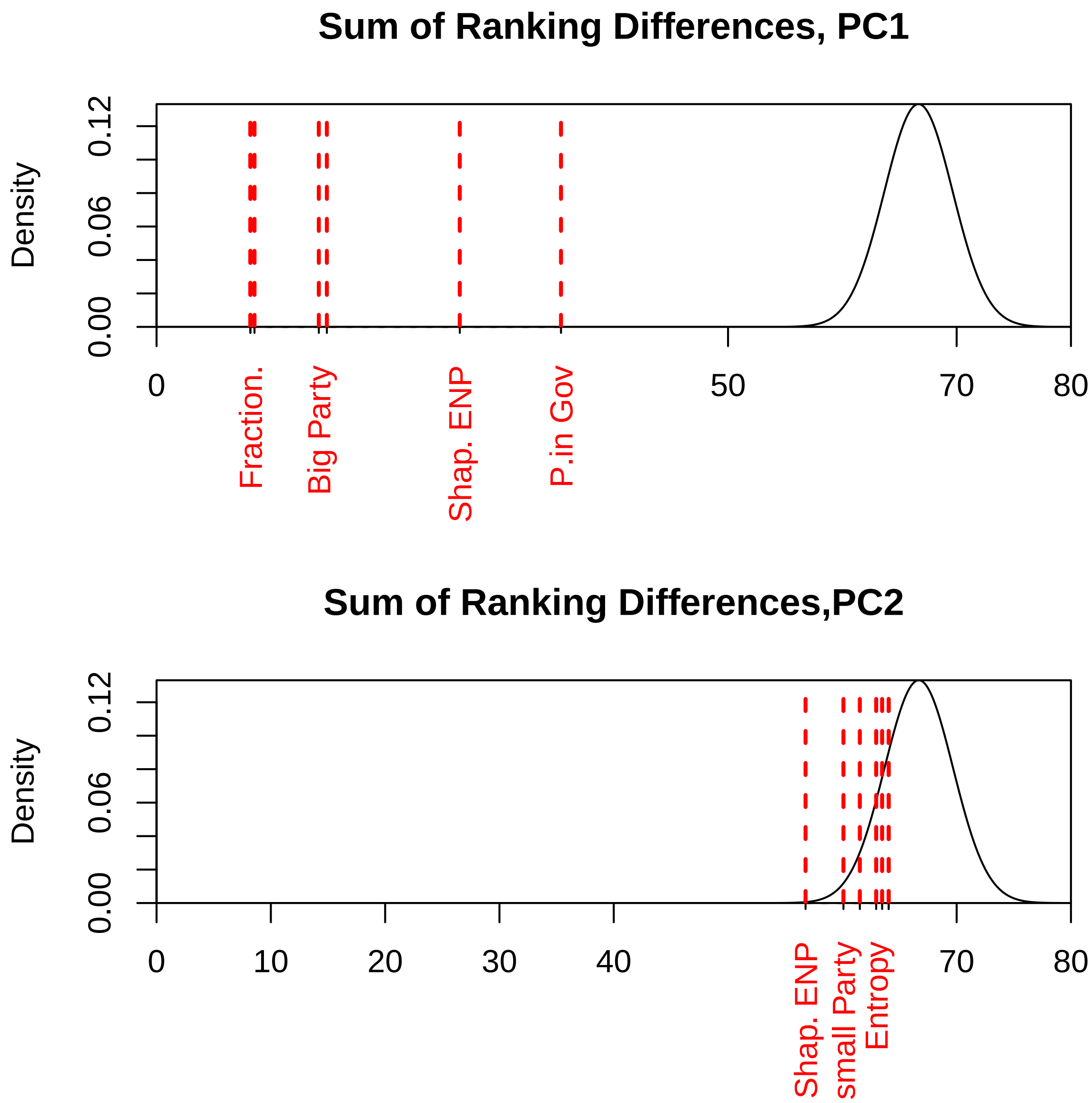
Figure D.1: Measures on the PC Dimensions where

$$SRD_{max} = \begin{cases} 2\sum_{j=1}^{k}(2j-1) = 2k^2 \text{ if } n_r = 2k \\ 4\sum_{j=1}^{k} j = 2k(k+1) \text{ if } n_r = 2k+1 \end{cases} \tag{1}$$

After the normalization I calculate the theoretical distribution that would arise from a large number of measures that randomly ranks the cases. According to Héberger and Kollár-Hunek (2011) above 13 cases we can approximate the theoretical SRD distribution as a normal curve. Since, I have 200 rows I can do this calculation. I estimate the mean and standard deviation of this curve by bootstrapping random samples from the original dataset. In Figure D.2 I plot the indices and the normal distribution that results from this algorithm. As the indices lay outside of the normal curve we can be sure that they do not represent random rankings of the cases in case of *Dimension 1*. Moreover, the *Fractionalization Index* approximates the most the ranking of the PC1 dimension (closely followed by the *ENP* and *HH* indices). Finally, we can see that compared to the ranking of the PC2 the measures do not hold up well as all the indices could be the results of random rankings.

D.9. Measuring the Party System: Summary

Overall, there is a trade-off between how comprehensively we would like to describe the party system on the one hand, and how much we would like to identify the bigger more relevant parties. The former approach yields a measure that weights smaller parties more, while the latter yields a measure that weights larger parties more. All the measures were created to reduce the dimensionality of the party system data matrix by extracting the most important information in the dataset. The debate between scholars has been over which information to keep and which information to discard. Currently, in most empirical studies that evaluate whether certain factors influence government policies, the author picks one or more controls for the party system size (which is usually the *ENP*) without sufficient attention to what the indices actually measure. This may be one of the reasons why previous studies on the influence of the size of the party system did not lead to substantive results.



Notes: The plot shows how party system size measures relate to each other and the two first dimensions of the PCA. PC1: Dimension 1, Sizes of the two biggest parties. PC2: Dimension 2, Competition between the biggest and the second biggest parties. Fract: Fractionalization Index, Entropy:Entropies, ENP: Effective Number of Parties, P.in Gov: Number of parties in the Government, Shap.ENP: Effective Number of Parties(Shapley),Big Party: Size of the biggest party over the size of the legislature, P.s in Gov: Parties in Government

Figure D.2: Sum of Ranking Differences

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E. PCA and Typologies, typologies of Rokkan (1970) and Sartori (1976)

In this appendix, I show how the first two PCA dimensions relate to two earlier typologies: the typology of Rokkan from 1970 (Rokkan, 1970) and the typology of Sartori from 1976 (Sartori, 1976). I show in the following pages the possible demarcation lines that separate the groups that these authors identified. This reveals that the categories that Rokkan (1970) and Sartori (1976) establish are not very different from one another even though the authors call the categories differently.

Using the plots superimposed with the demarcation lines, I will show how we can categorize countries that the authors left out from the typology(es). Second, I will show how we can potentially evaluate these typologies. I argue, that we can better understand why the authors in some cases hesitate about the classification of certain countries. Finally, I will superimpose the demarcation lines onto the two-dimensional plane that the PCA recovered, and show how we can categorize all the countries in all the years in our sample, based on these typologies. I argue that the PCA can help researchers to sort these countries as different "types" and compare how political elites in countries with similar party systems behave. This method can be used with other party system typologies as well beyond the two discussed in this part. The PCA in itself shows which party systems are alike on the main separating dimensions, thus it is probably more important to understand where certain clusters of countries emerge than the exact name (typology) someone uses to identify these clusters.

In *Figures E.1a and E.2a* I plot the position of the countries in the first two PCA dimensions in the years that the typologies were written, 1970 in case of (Rokkan, 1970) and in 1976 in case of (Sartori, 1976). I indicate the different categories that the authors identified with different symbols. Rokkan (1970) classifies the countries based on whether the parties in the party system are roughly the same sizes (compared to each other) or whether there is one or more dominant parties facing small parties into three categories: 1. the British-German system (in which one party faces another or another big party and a small one), 2. the Scandinavian system (in which a bigger party faces 3-4 smaller parties), 3. Even multi party systems (in which there are 2-5 even sized parties). He splits the final group into two different categories Scandinavian split working-class systems (in which the workers do not back the same parties) and segmented pluralisms, in which different identity groups create roughly equal sized parties (Rokkan, 1970). Sartori (1976) also classifies the countries into four types, based on the number of parties and the polarization of the competition in the party system. His categories are: 1. Predominant party regimes, 2. Two-party regimes, 3. Moderate pluralisms, and Polarized pluralisms. In *Figures E.1 and E.2* we can also see that the authors did not categorize all the countries in the dataset into one of the types.

In *Figures E.1b and E.2b* I draw the demarcation lines between the groups with the help of logit models. In case of the estimation of Sartori's classification system I leave Italy and France out of the estimation process.Sartori in his text hesitates about the classification of France, and the PCA shows why. The country does not seem to fall neatly into one of the groups. In the logistics regression I regress positive and negative cases (dummies) of the different categories on the PC1 and PC2 dimensions. The only exception is the dividing line between Rokkan's two groups in category 3, in which case the negative and the positive cases (which is the outcome variable) only belong to the two sub categories. I use the coefficients of the logistics regressions to draw the lines.

Figures E.1b and E.2b, visually show that the typologies of Rokkan (1970) and Sartori (1976) are structurally close to one another. The two authors identify very similar groups. On one hand these authors separate countries with small party systems and big party systems, and on the other hand countries with little or more competition within the party systems. The PCA also separates party systems from each other in these two dimensions. The figures show that countries that have a dominant party are in the lower half of the plot. At the same time, two-party systems are on the left, while multi-party systems are on the right. The PCA separates the categories that Rokkan (1970) and Sartori (1976) identify well, with one exception. The countries that Sartori groups as "Polarized pluralisms" separate less well from the other countries than other types.

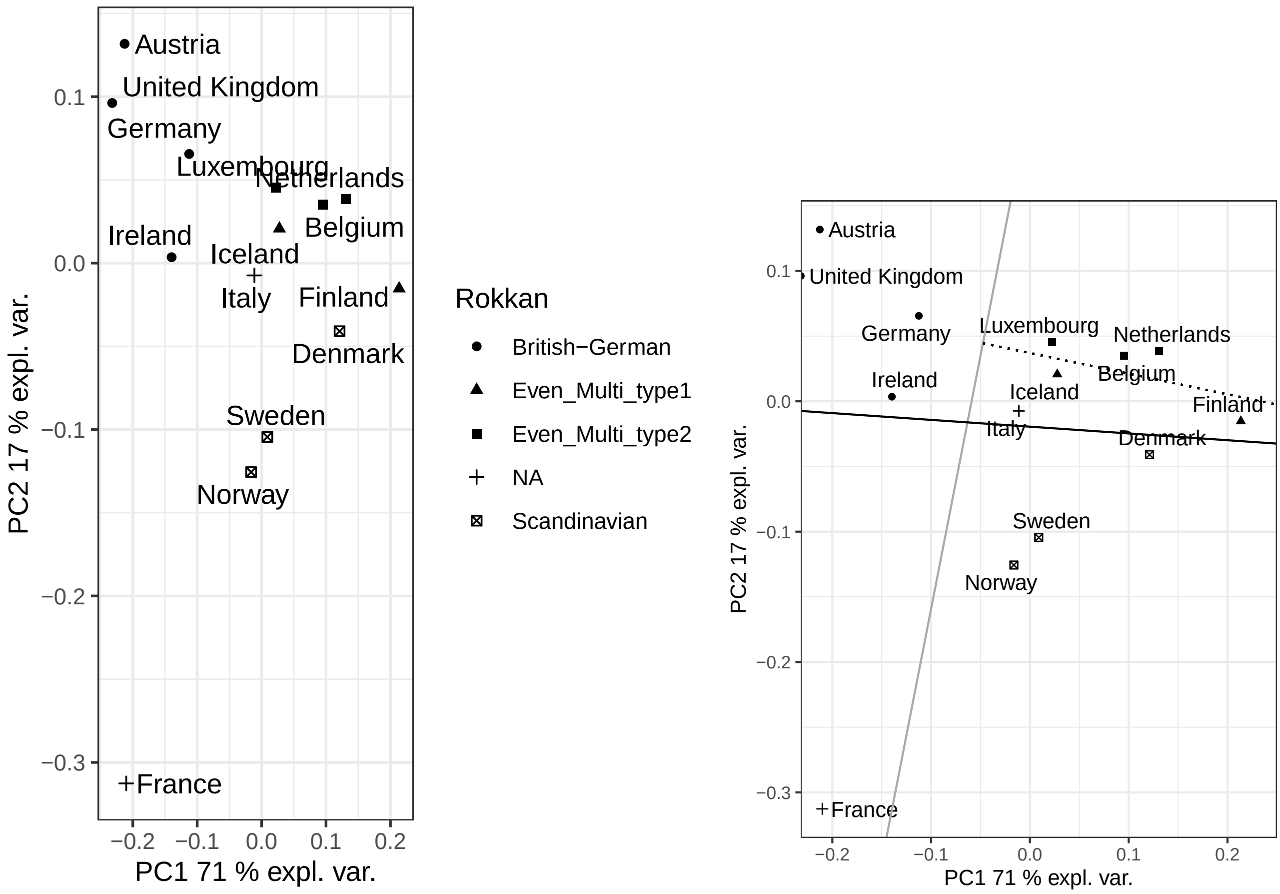
First, as mentioned above we can examine the typologies themselves with the help of the PCA. *Figure E.1b* shows that Rokkan does not create a category that logically should exist. He does not sort into a separate category the countries that have few parties but one of them is dominant. In the plot these countries should be placed into the bottom left quadrant. Technically, it could be the case that no such party existed in 1970, but the plot shows that there is a country that Rokkan does not classify –France– that could fall into this category. Indeed, in 1968 in the French legislative elections the UDR won 354 of the 458 legislative seats of the French National Assembly while the FGDS ended up a distant second with 57 seats.

Figure E.2b shows that Sartori also has some difficulty in sorting countries into the predominant party system category. Unfortunately, I do not have data on non-European countries which makes it somewhat difficult to see where would for instance Japan would fall on the plot. In my sample I have, however, Norway and Sweden and Sartori expresses that he is uncertain whether these countries should be classified as predominant party regimes or moderate pluralisms. We can see on the plot why he is hesitant. While the plot shows that the competition is uneven between the two biggest parties at the time (these countries are lower on *Dimension 2* than the rest of the countries), Sweden and Norway fall into the middle of *Dimension 1* which means that they have relatively large number of parties, a characteristic of pluralisms. In addition, France (that Sartori) also hesitates about, may also fall into this category. As we can see in *Figures E.7a and E.7b* I will discuss in Online Appendix G, the fact, that France is a very difficult country to categorize is not surprising. France's legislative party system changed a lot from one election to another between 1970 to 2013 probably because of the electoral system of the country.

With the help of the PCA we can also classify countries that are in the dataset but were not classified by the authors. Rokkan did not classify Italy. With the help of the plot we can infer that Rokkan probably would have classified Italy as a Scandinavian "split working class" system along with Finland and Iceland (*Category 3a*). On the other hand, Sartori did not classify Greece, which had its first democratic elections after the military junta in 1974. *Figure E.9b* shows that in the first two elections- 1974 and 1977 there were very few parties and one of these dominated. Indeed, in 1974 the Greek New Democracy party wins 220 seats over the 60 seats of the Centre-Union, new Forces Party. Thus we can argue that Greece would fall into the "predominant party system" category.

Finally, *Figures E.4 to E.19* show how the 17 countries moved in and out of the categories that Rokkan (1970) and Sartori (1976) established between 1970 and 2013 (or their democratic periods). In addition, *Figures E.20 to E.24* show the party systems in different groups from 1975 to 2010. It seems that the classification changes are very similar in most cases. A country changes category both according to Rokkan (1970) and Sartori (1976)'s classifications when it becomes a multi- party country from a two-party country. Both classifications reveal when a party system becomes dominated by one big party. The only notable difference is how countries change categories within multi-party system category, whether they move from/ to working-class systems (in which the workers do not back the same parties) and segmented pluralisms in Sartori's classification system or from/to moderate and polarized pluralisms in Rokkan's classification

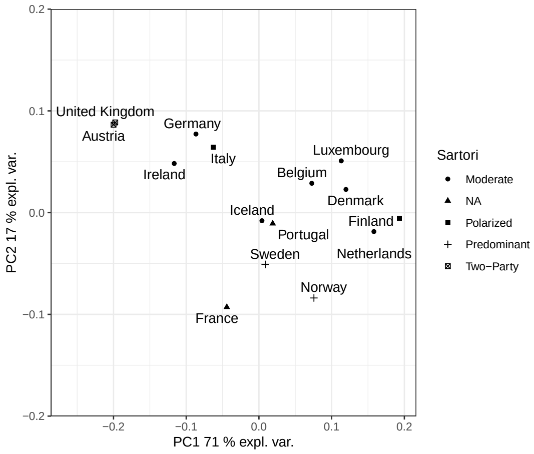
In this appendix, I have shown how the PCA can help us evaluate and compare different typologies of the past. In addition, it can help us sort countries in different years into different types according to these typologies, thus it can help researchers to use these typologies in their future works.



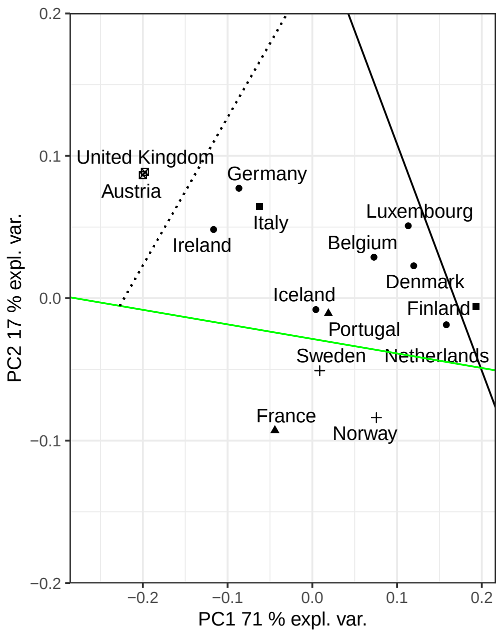
(a) (b)

Rokkan (1970)	**Basis of categories:** Numbers of parties Proximity to the majority Evenness of the competition
1.The British-German “1 vs. 1+1” system 2. The Scandinavian “1 vs. 3-4 system 3.Even multi-party systems “1 vs. 1 vs. 1+ 2-3”	1 Austria, Ireland, some periods Belgium 2. Sweden, Denmark, Norway 3a. Finland, Iceland 3b. Netherlands, Belgium, Luxembourg Switzerland

Figure E.1: Comparison of Rokkan (1970)’s typology to PCA results



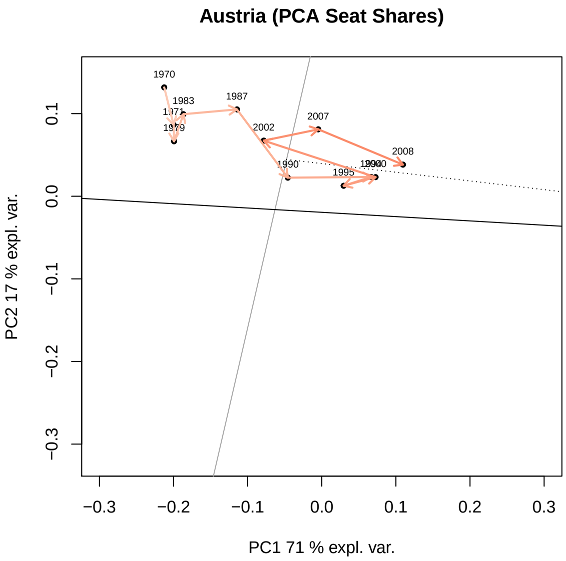
(a)



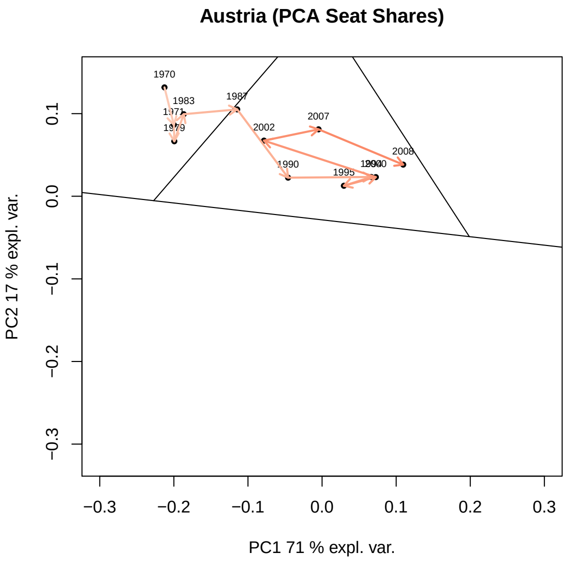
(b)

Sartori (1976)	**Basis of categories:** Party fragmentation (number of parties) Ideological distance
1. Predominant party regimes 2. Two-party systems 3. Moderate pluralism 4. Polarized pluralism	1. Norway(or 3), Sweden (or 3), Japan, Uruguay, India Turkey 2. Canada, Australia, Austria, England, New Zealand, United States 3. Switzerland, Netherlands, Israel, Denmark, Iceland, Luxembourg, Belgium, Ireland, France (after 1958), Germany 4. Finland, Chile, France(before 1958), Italy

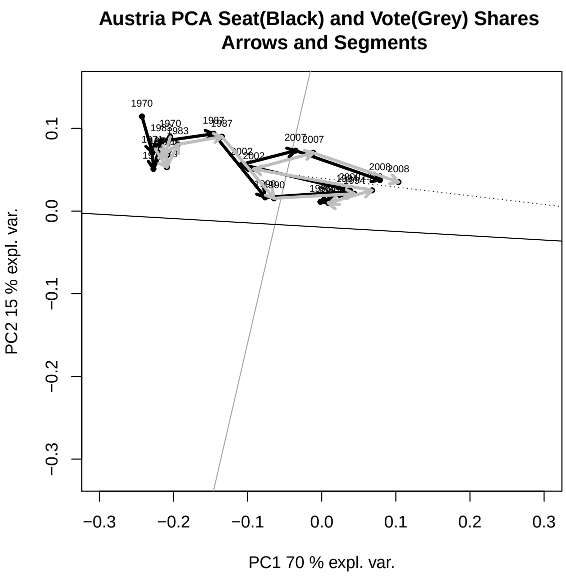
Figure E.2: Comparison of Sartori (1976)'s typology to PCA results



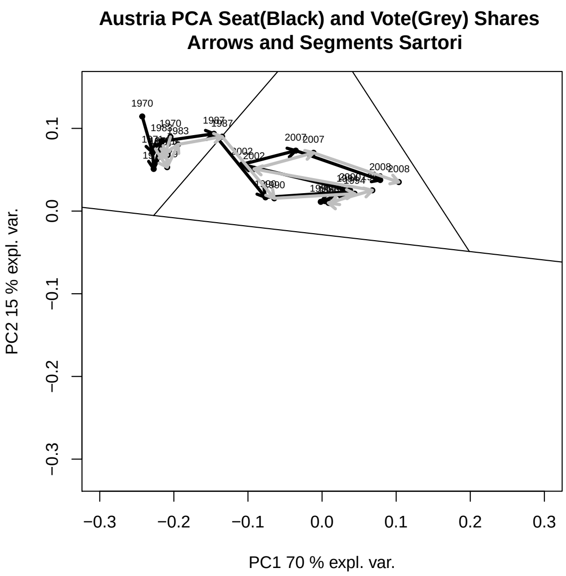
(a) Austria on the PCA Dimensions/Rokkan Classification



(b) Austria on the PCA Dimensions/Sartori Classification

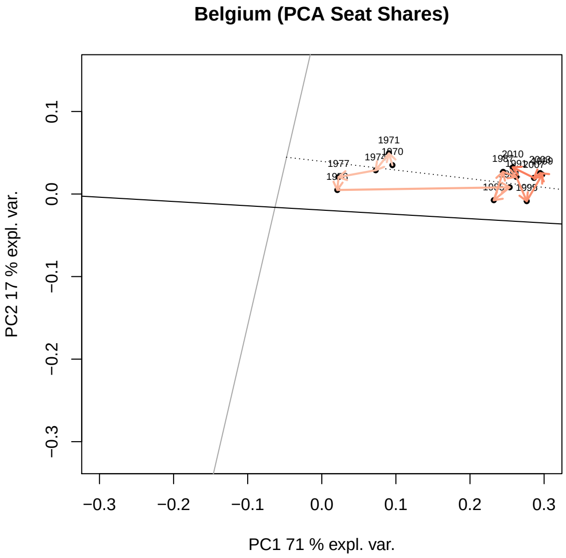


(c) Austria on the PCA Dimensions/Rokkan Classification

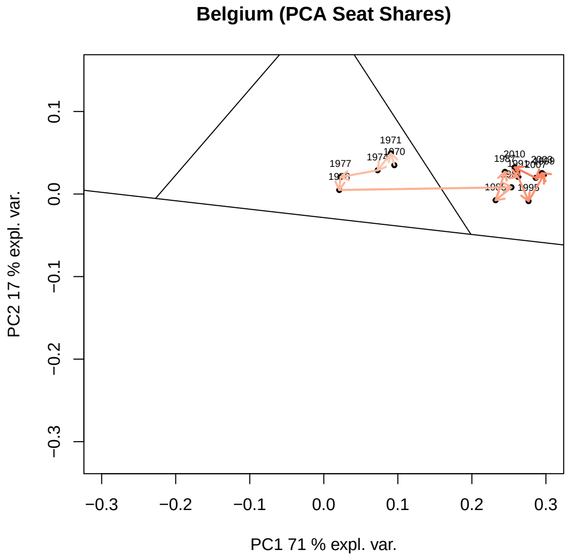


(d) Austria on the PCA Dimensions/Sartori Classification

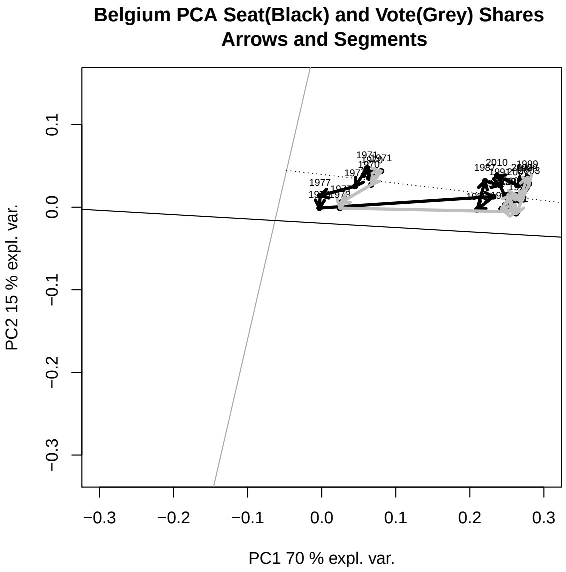
Figure E.3: Notes: The plot shows the Austrian party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



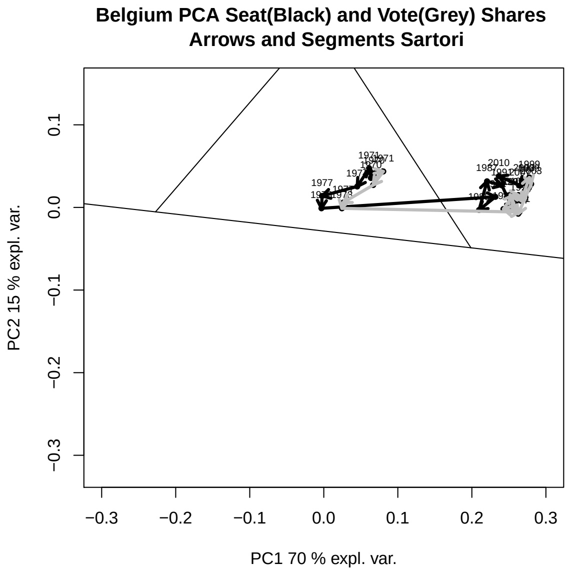
(a) Belgium on the PCA Dimensions/Rokkan Classification



(b) Belgium on the PCA Dimensions/Sartori Classification

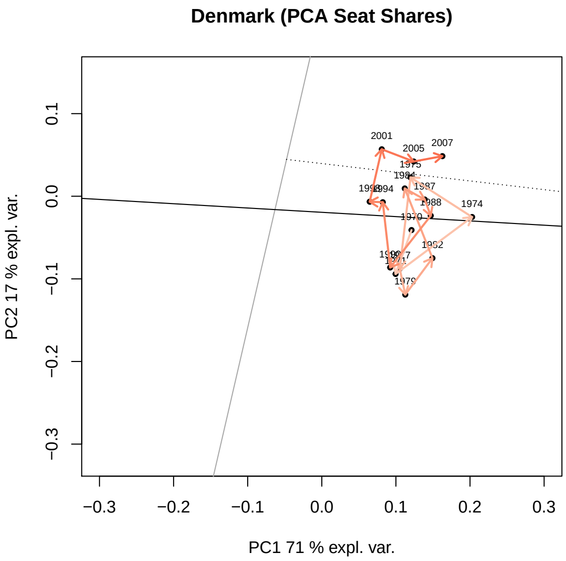


(c) Belgium on the PCA Dimensions/Rokkan Classification

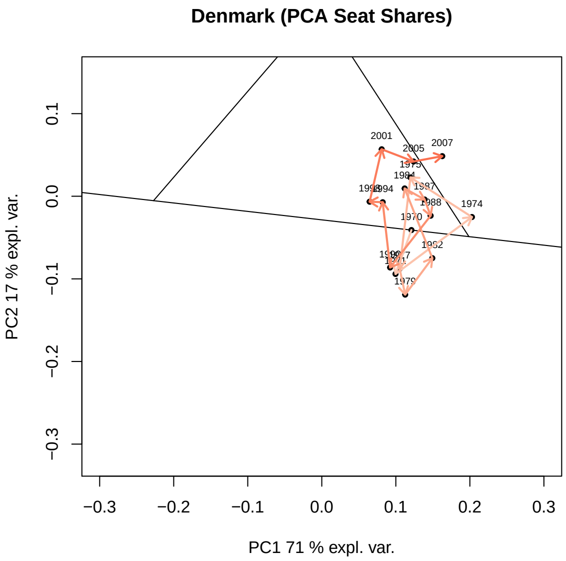


(d) Belgium on the PCA Dimensions/Sartori Classification

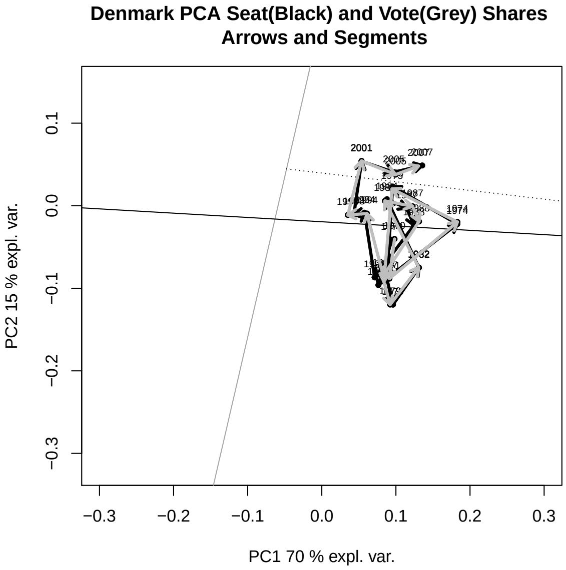
Figure E.4: Notes: The plot shows Belgian party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



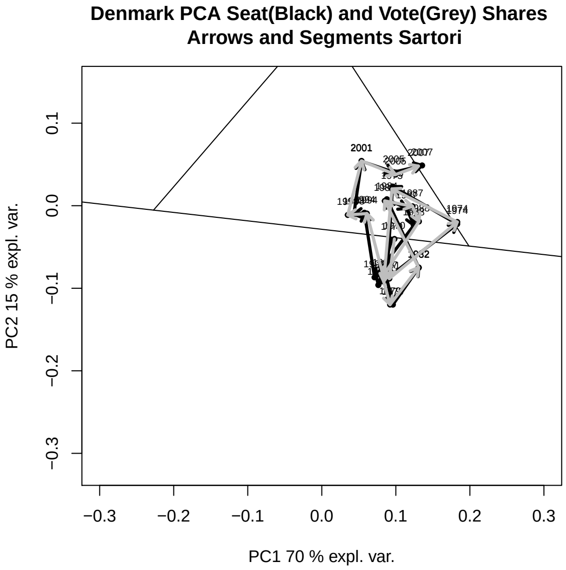
(a) Denmark on the PCA Dimensions/Rokkan Classification



(b) Denmark on the PCA Dimensions/Sartori Classification

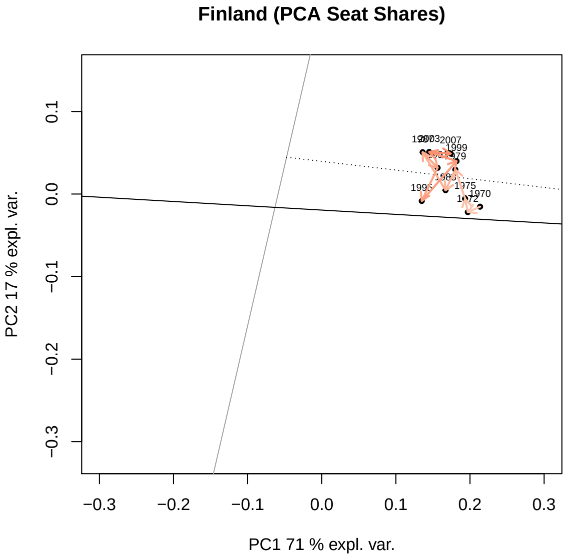


(c) Denmark on the PCA Dimensions/Rokkan Classification

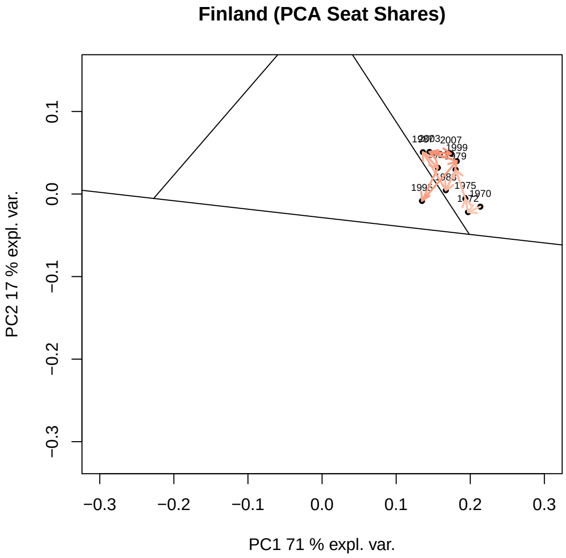


(d) Denmark on the PCA Dimensions/Sartori Classification

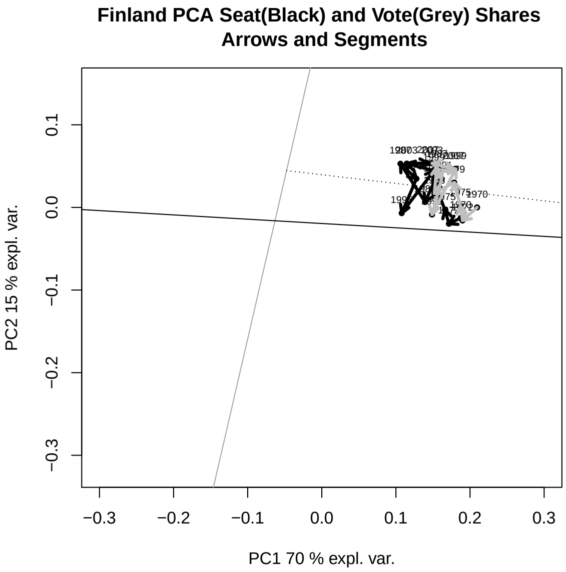
Figure E.5: Notes: The plot shows the Danish party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



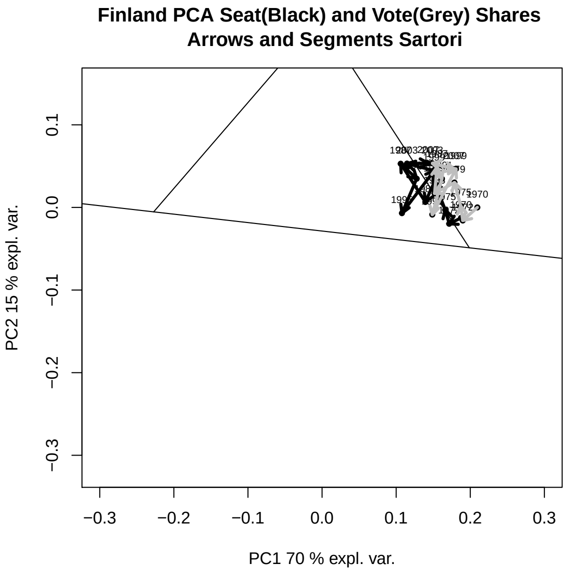
(a) Finland on the PCA Dimensions/Rokkan Classification



(b) Finland on the PCA Dimensions/Sartori Classification

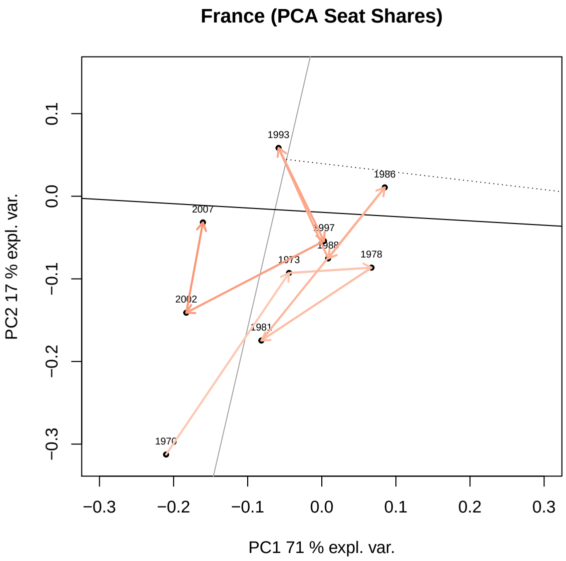


(c) Finland on the PCA Dimensions/Rokkan Classification

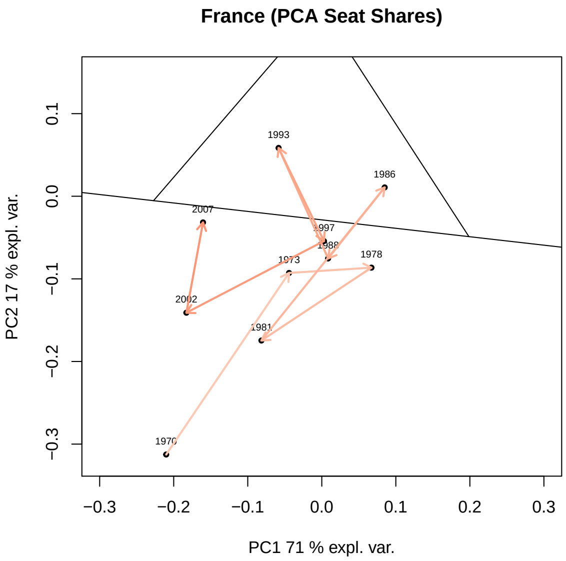


(d) Finland on the PCA Dimensions/Sartori Classification

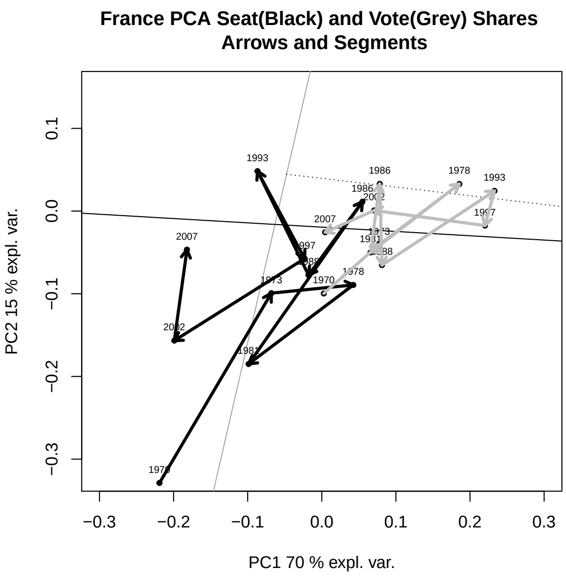
Figure E.6: Notes: The plot shows the Finnish party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



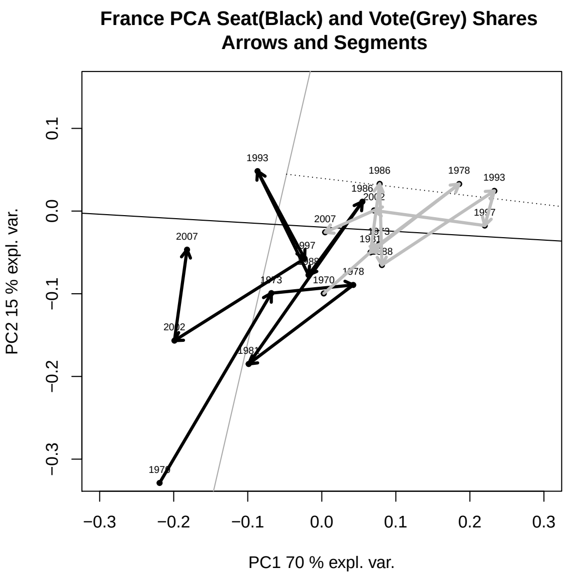
(a) France on the PCA Dimensions/Rokkan Classification



(b) France on the PCA Dimensions/Sartori Classification

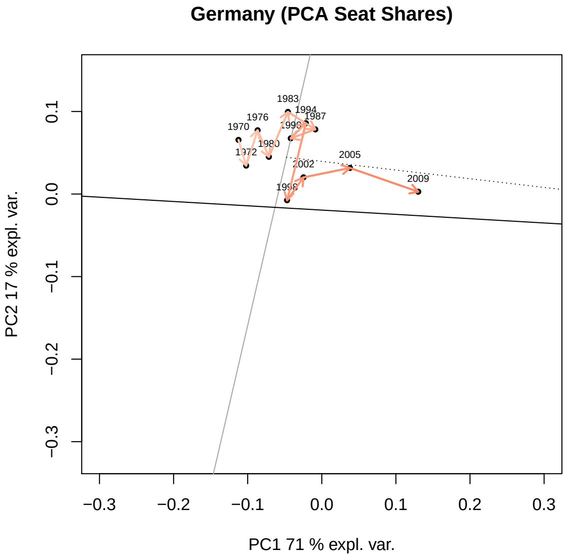


(c) France on the PCA Dimensions/Rokkan Classification

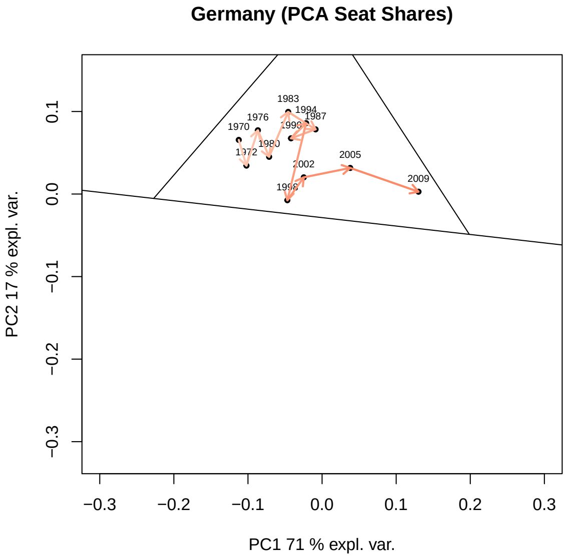


(d) France on the PCA Dimensions/Sartori Classification

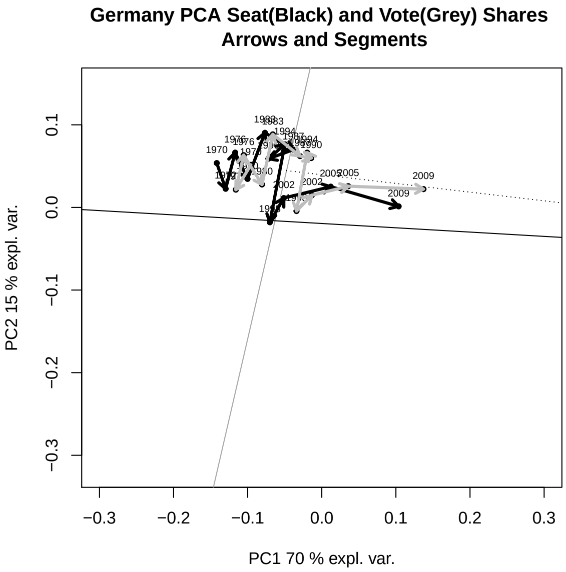
Figure E.7: Notes: The plot shows the French party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



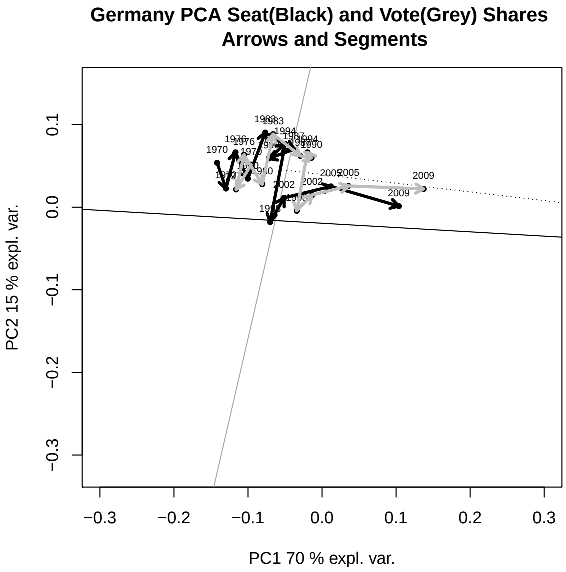
(a) Germany on the PCA Dimensions/Rokkan Classification



(b) Germany on the PCA Dimensions/Sartori Classification

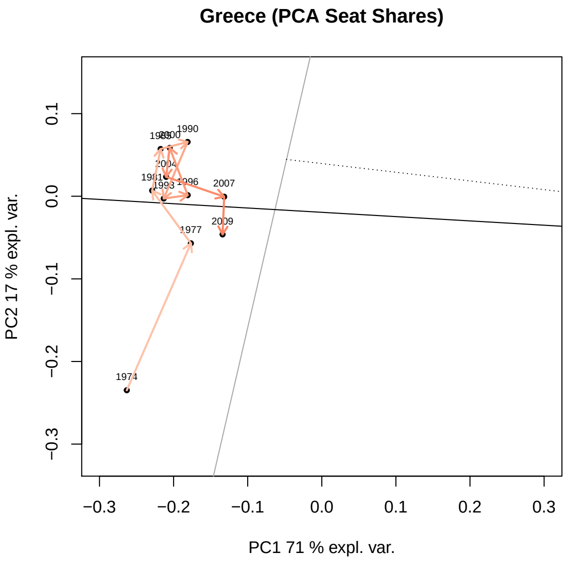


(c) Germany on the PCA Dimensions/Rokkan Classification

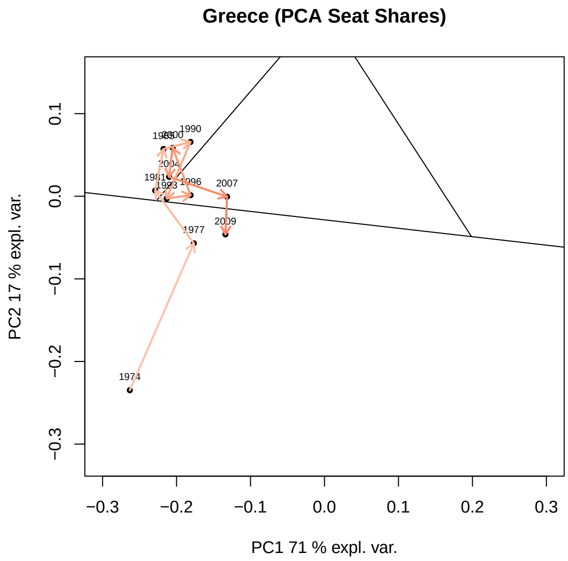


(d) Germany on the PCA Dimensions/Sartori Classification

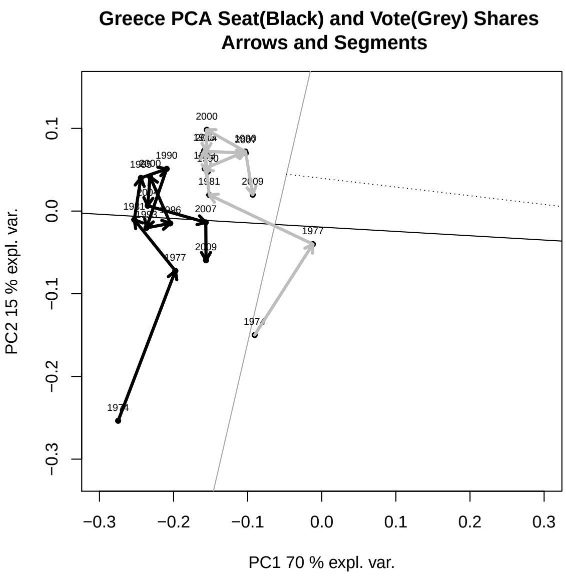
Figure E.8: Notes: The plot shows the German party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



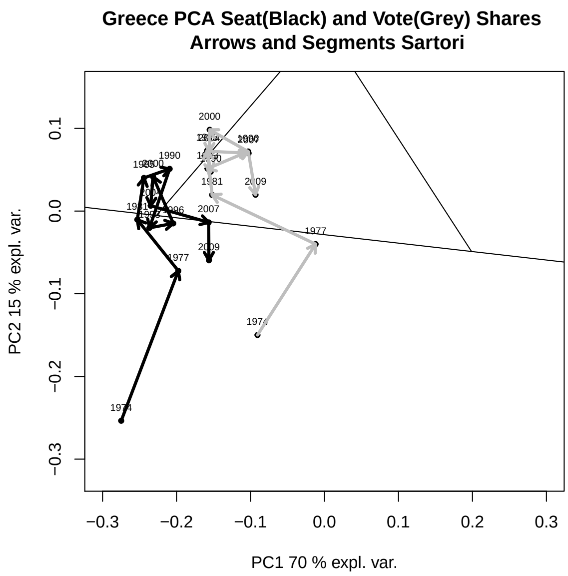
(a) Greece on the PCA Dimensions/Rokkan Classification



(b) Greece on the PCA Dimensions/Sartori Classification

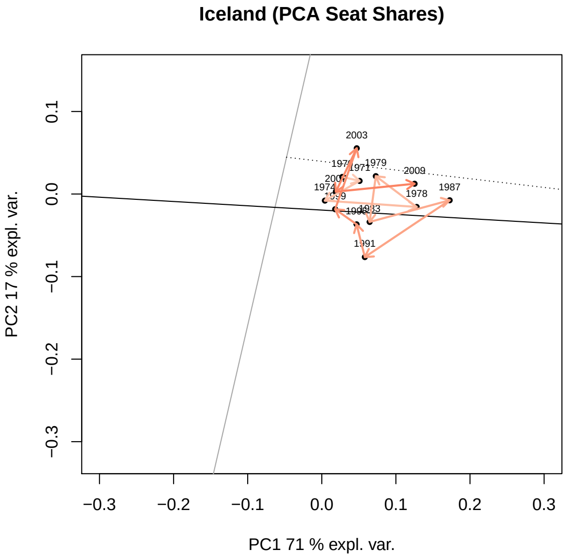


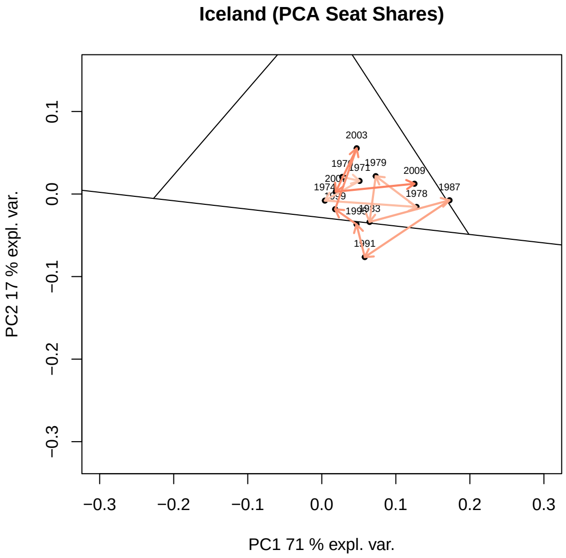
(c) Greece on the PCA Dimensions/Rokkan Classification



(d) Greece on the PCA Dimensions/Sartori Classification

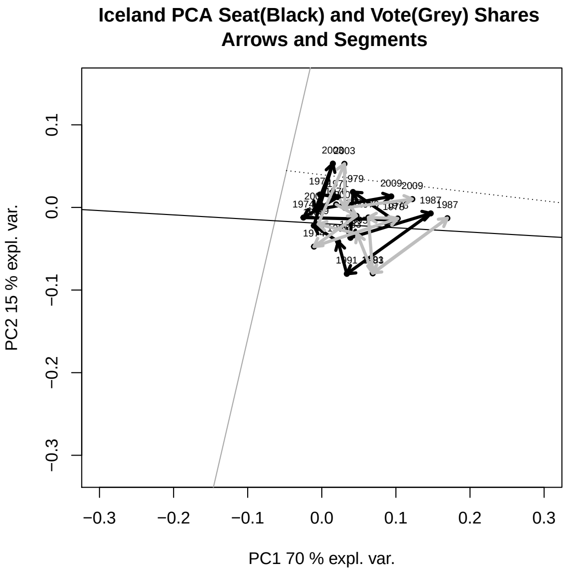
Figure E.9: Notes: The plot shows the Greek party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).

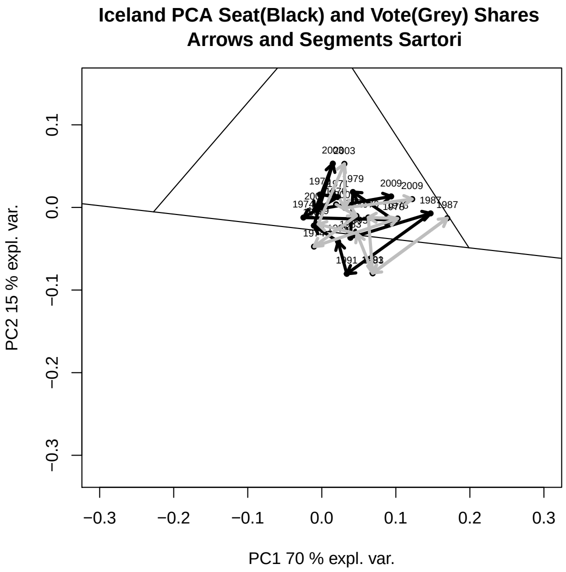




(a) Iceland on the PCA Dimensions/Rokkan Classification

(b) Iceland on the PCA Dimensions/Sartori Classification

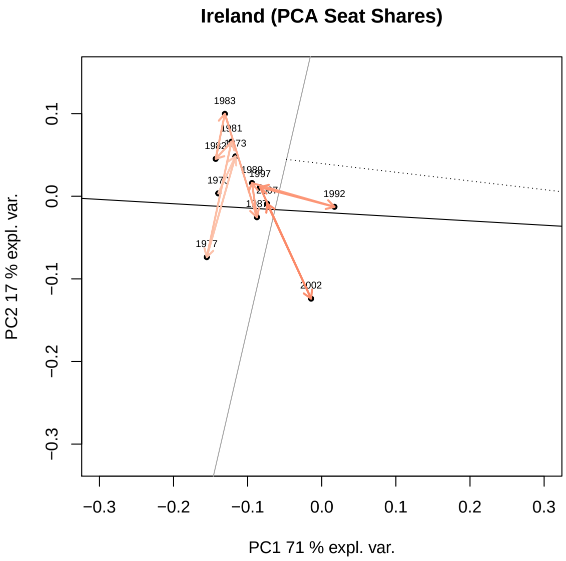




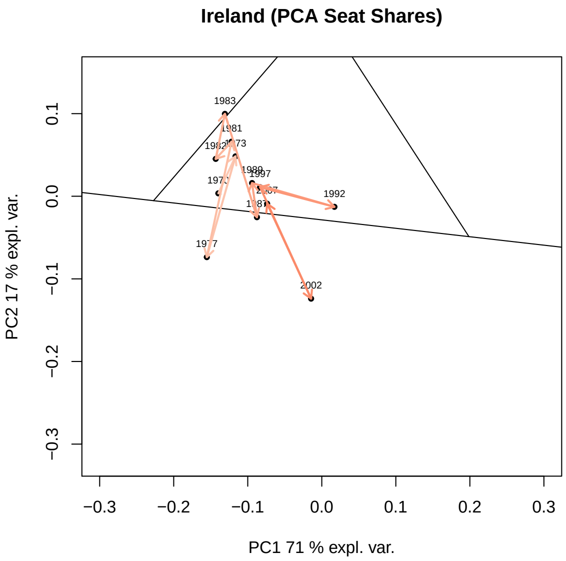
(c) Iceland on the PCA Dimensions/Rokkan Classification

(d) Iceland on the PCA Dimensions/Sartori Classification

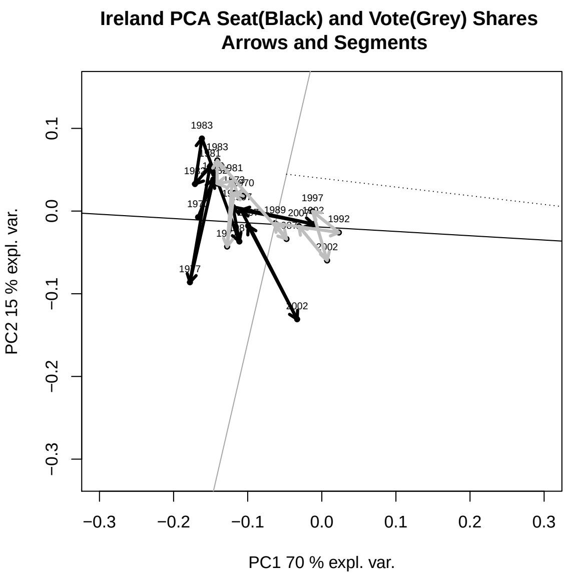
Figure E.10: Notes: The plot shows the Icelandic party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



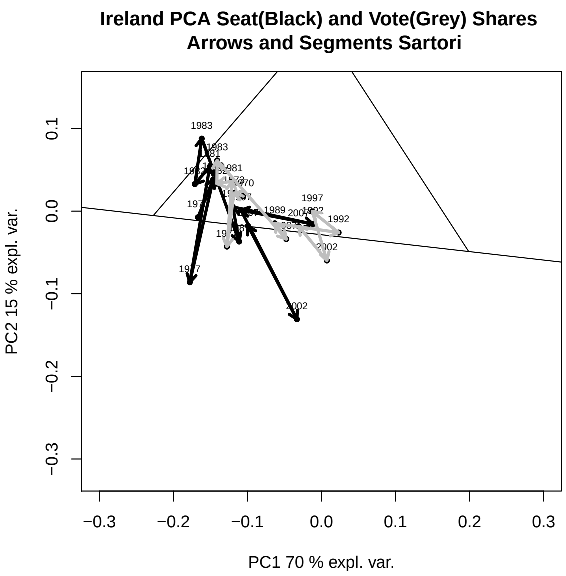
(a) Ireland on the PCA Dimensions/Rokkan Classification



(b) Ireland on the PCA Dimensions/Sartori Classification

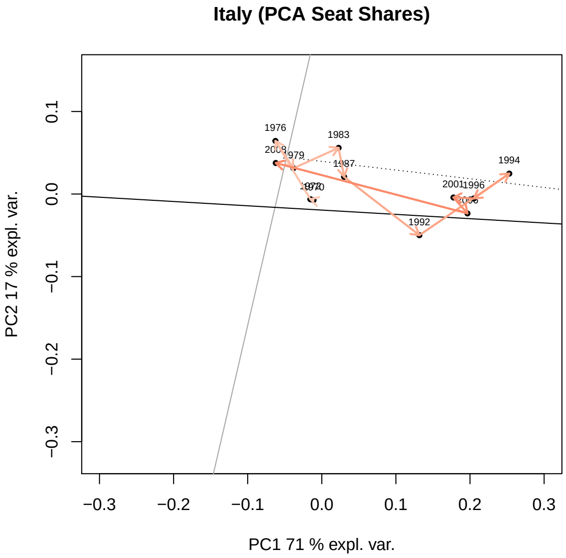


(c) Ireland on the PCA Dimensions/Rokkan Classification

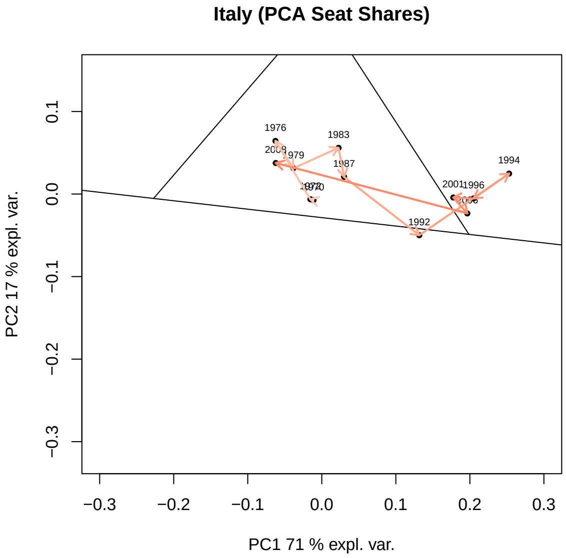


(d) Ireland on the PCA Dimensions/Sartori Classification

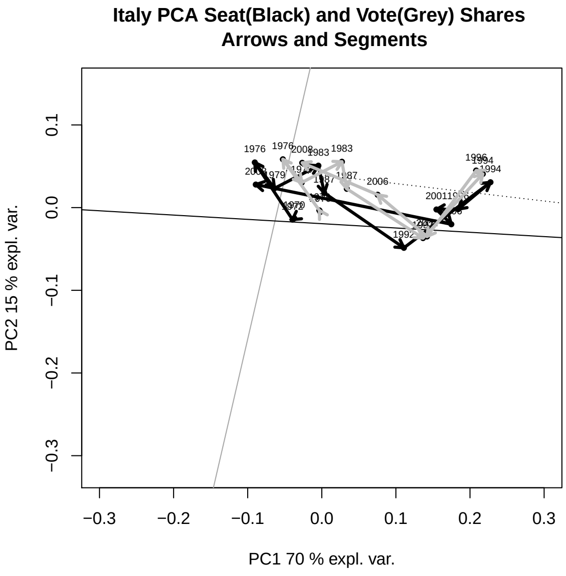
Figure E.11: Notes: The plot shows the Irish party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



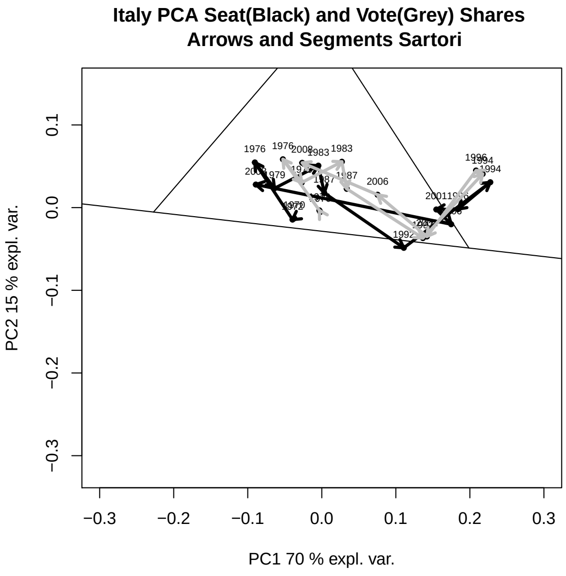
(a) Italy on the PCA Dimensions/Rokkan Classification



(b) Italy on the PCA Dimensions/Sartori Classification

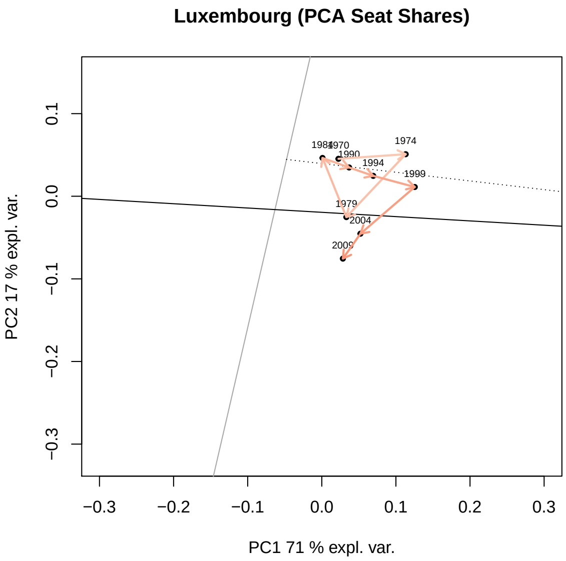


(c) Italy on the PCA Dimensions/Rokkan Classification

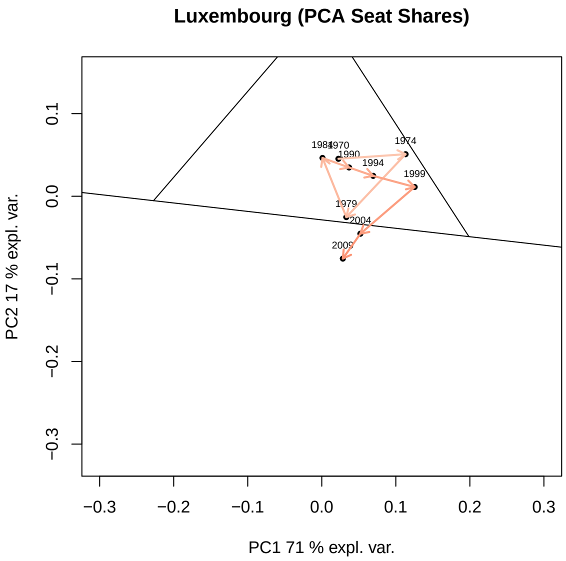


(d) Italy on the PCA Dimensions/Sartori Classification

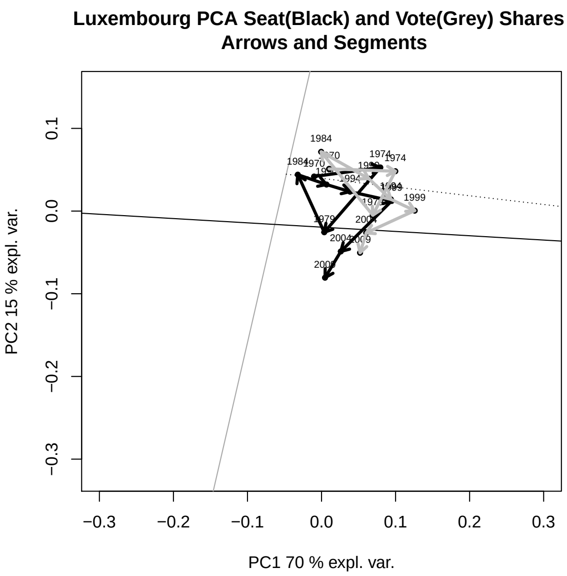
Figure E.12: Notes: The plot shows the Italian party system with Rokkan's and Sartori's classifications.Seat shares(up) Seat and Vote shares (down).



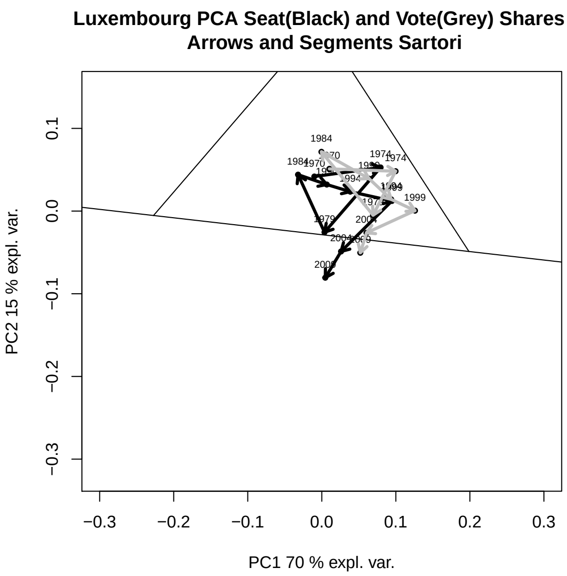
(a) Luxembourg on the PCA Dimensions/Rokkan Classification



(b) Luxembourg on the PCA Dimensions/Sartori Classification

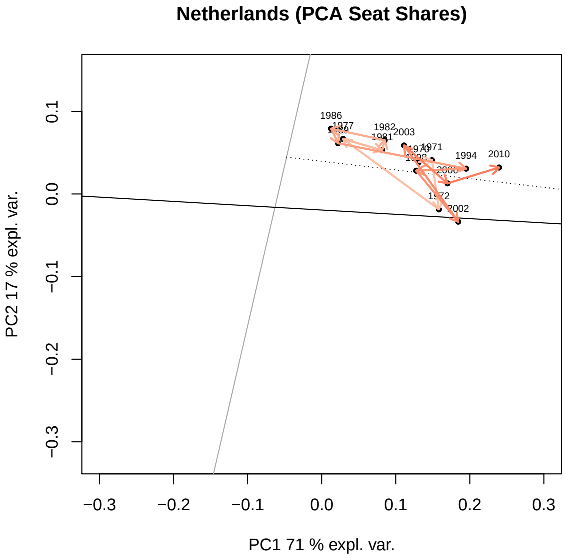


(c) Luxembourg on the PCA Dimensions/Rokkan Classification

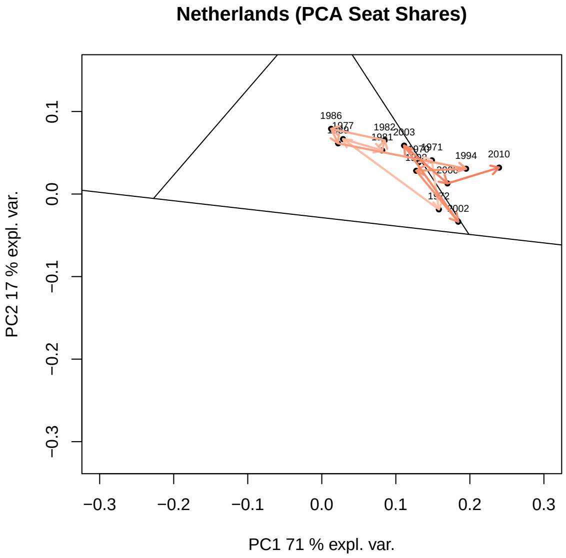


(d) Luxembourg on the PCA Dimensions/Sartori Classification

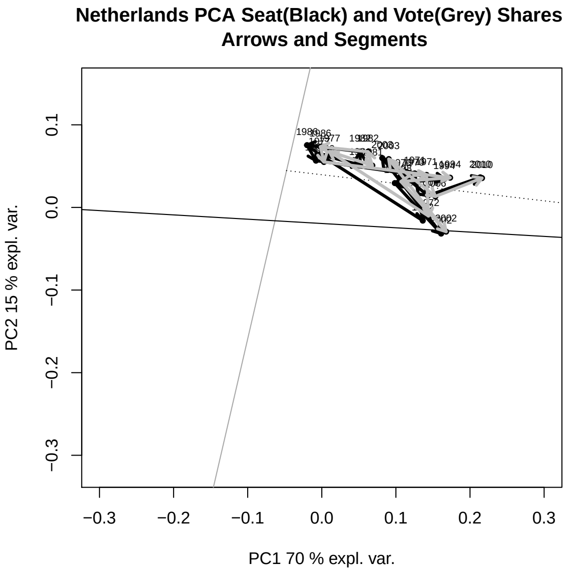
Figure E.13: Notes: The plot shows the Luxembourgian party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



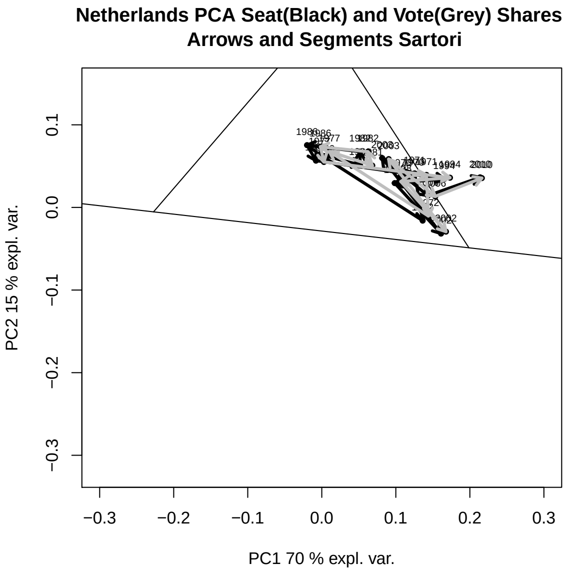
(a) Netherlands on the PCA Dimensions/Rokkan Classification



(b) Netherlands on the PCA Dimensions/Sartori Classification

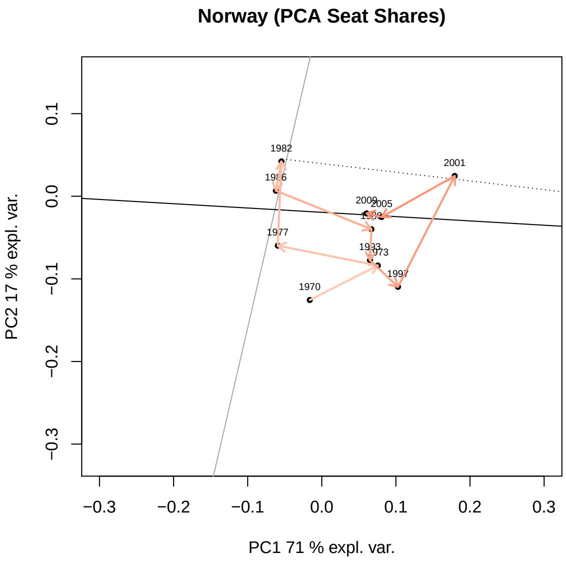


(c) Netherlands on the PCA Dimensions/Rokkan Classification

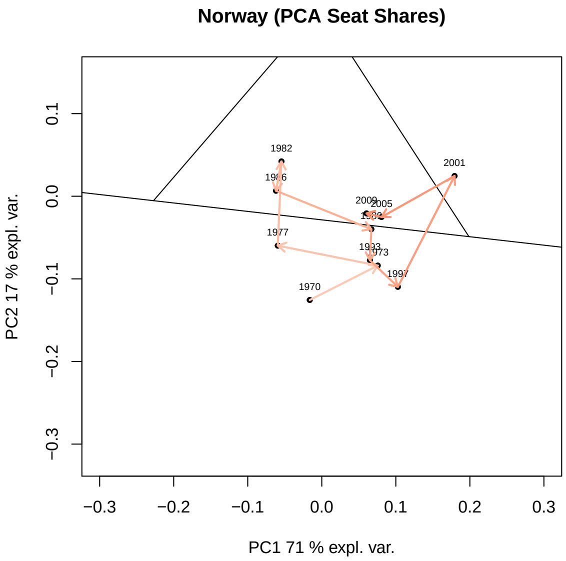


(d) Netherlands on the PCA Dimensions/Sartori Classification

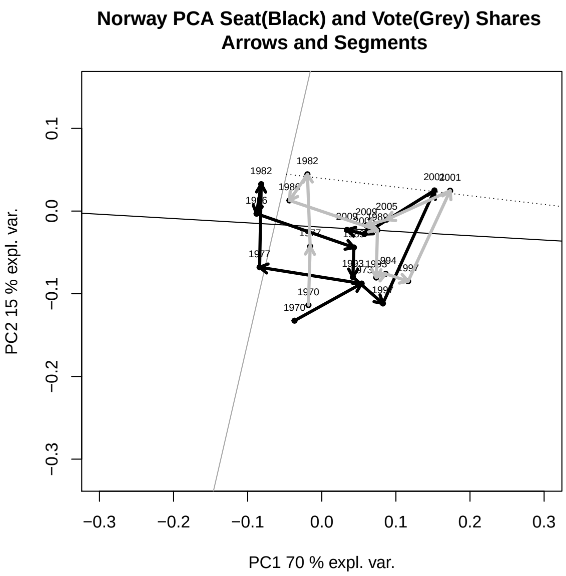
Figure E.14: Notes: The plot shows the Dutch party systems with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



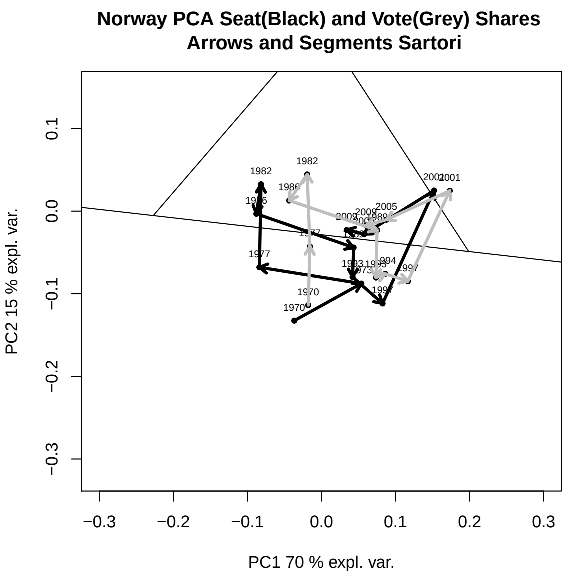
(a) Norway on the PCA Dimensions/Rokkan Classification



(b) Norway on the PCA Dimensions/Sartori Classification

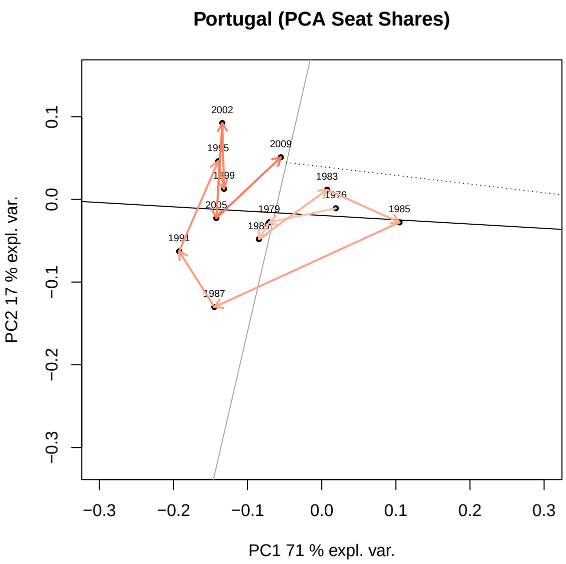


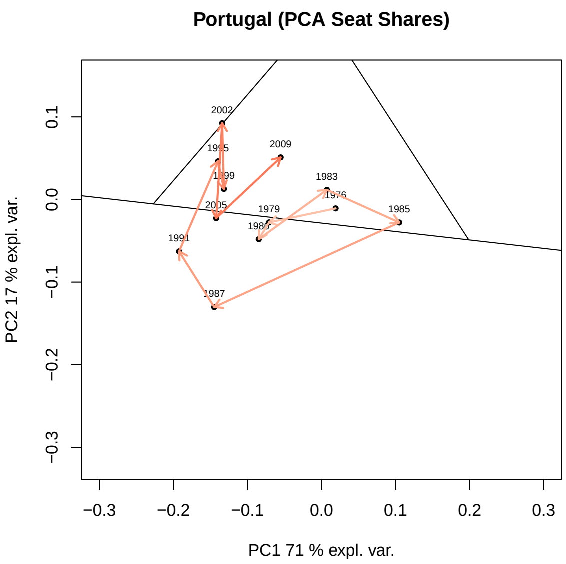
(c) Norway on the PCA Dimensions/Rokkan Classification



(d) Norway on the PCA Dimensions/Sartori Classification

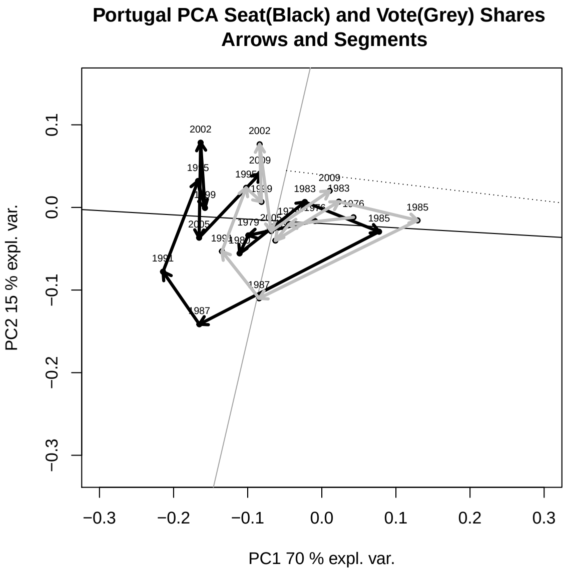
Figure E.15: Notes: The plot shows the Norwegian party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).

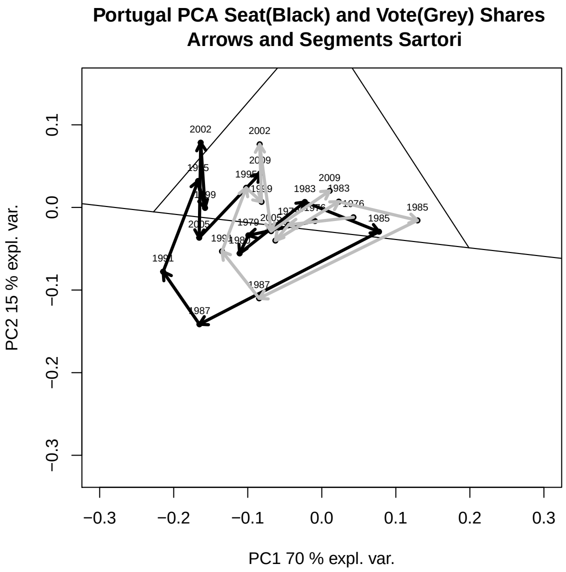




(a) Portugal on the PCA Dimensions/Rokkan Classification

(b) Portugal on the PCA Dimensions/Sartori Classification

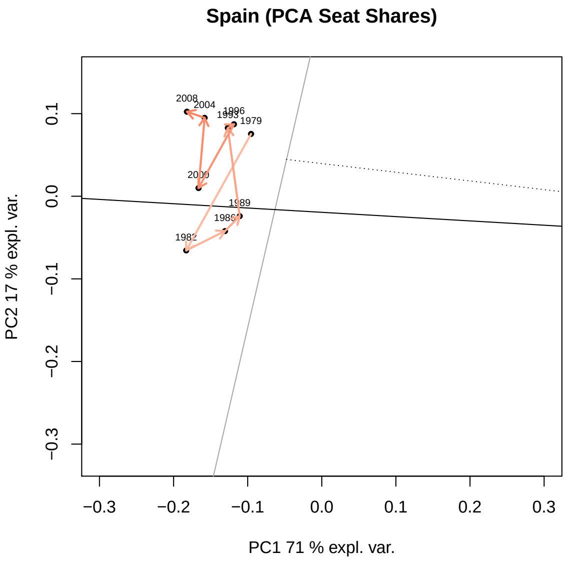




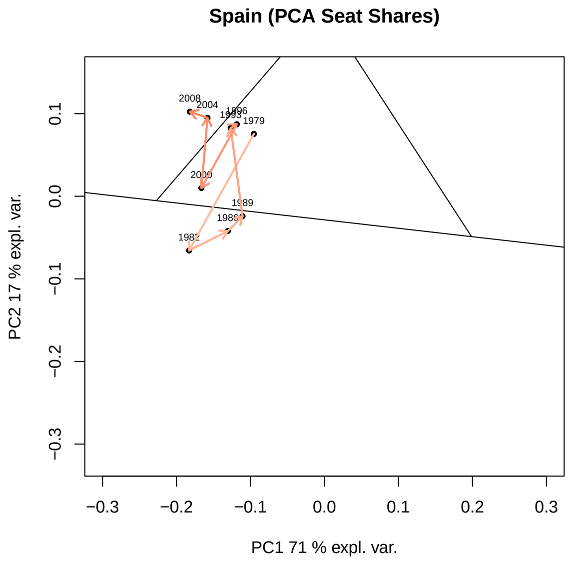
(c) Portugal on the PCA Dimensions/Rokkan Classification

(d) Portugal on the PCA Dimensions/Sartori Classification

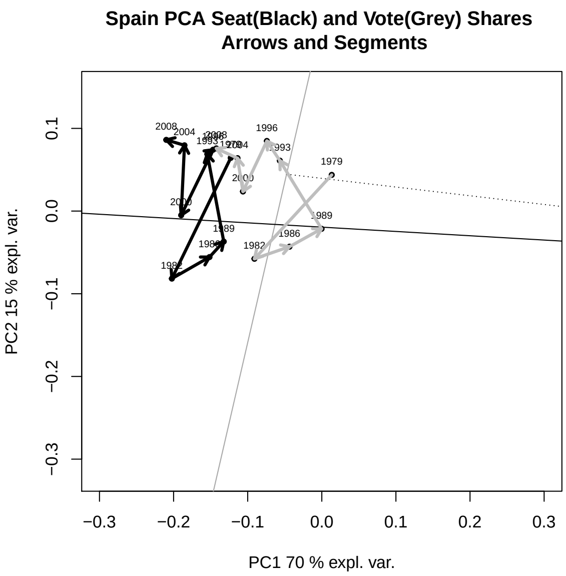
Figure E.16: Notes: The plot shows the Portugal party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



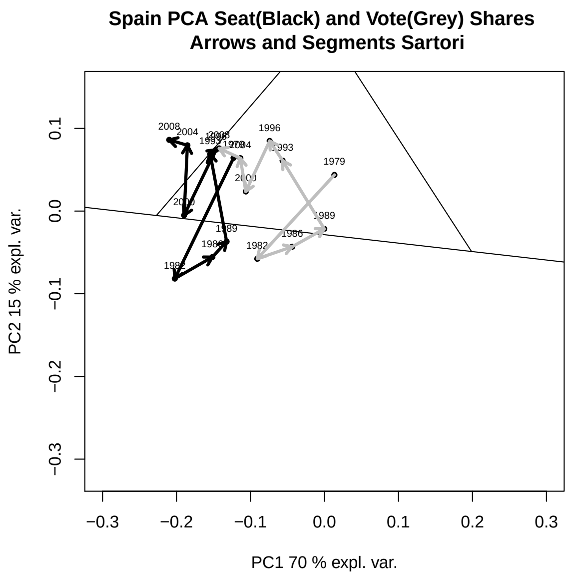
(a) Spain on the PCA Dimensions/Rokkan Classification



(b) Spain on the PCA Dimensions/Sartori Classification

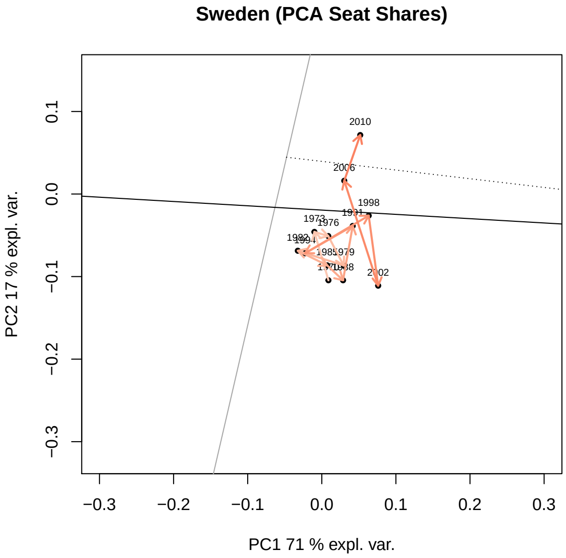


(c) Spain on the PCA Dimensions/Rokkan Classification

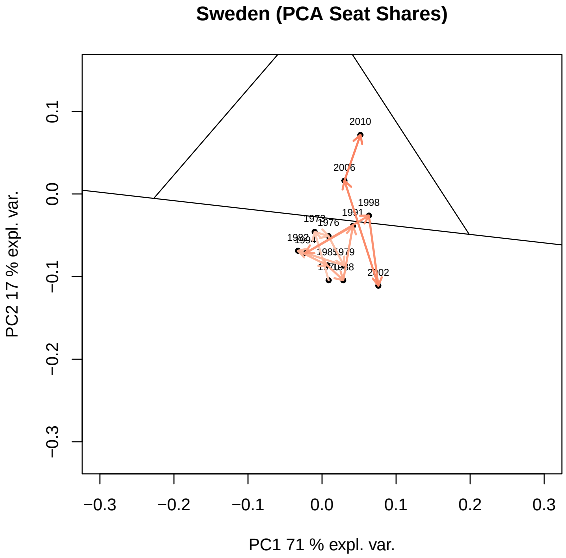


(d) Spain on the PCA Dimensions/Sartori Classification

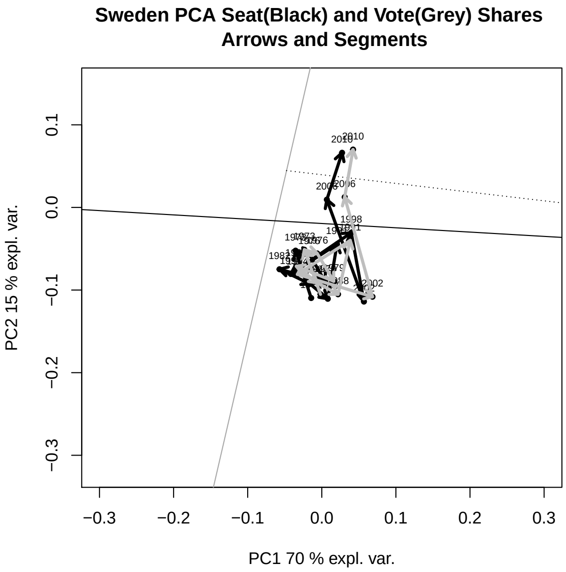
Figure E.17: Notes: The plot shows the Spanish party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



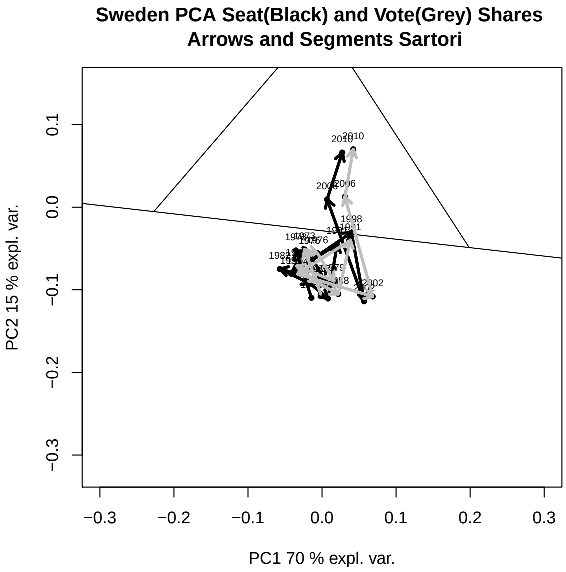
(a) Sweden on the PCA Dimensions/Rokkan Classification



(b) Sweden on the PCA Dimensions/Sartori Classification

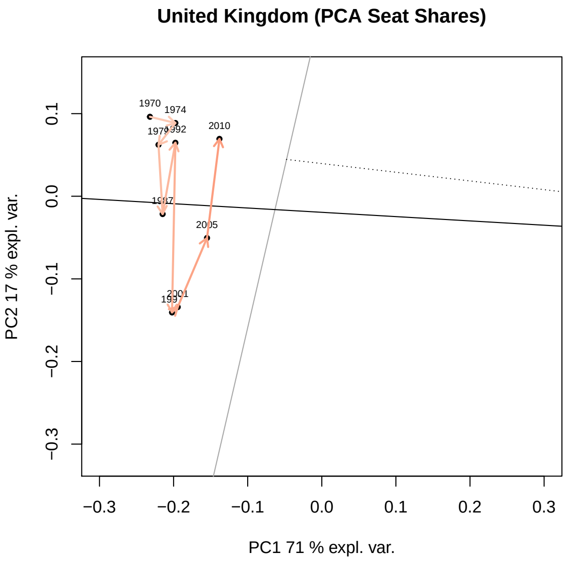


(c) Sweden on the PCA Dimensions/Rokkan Classification

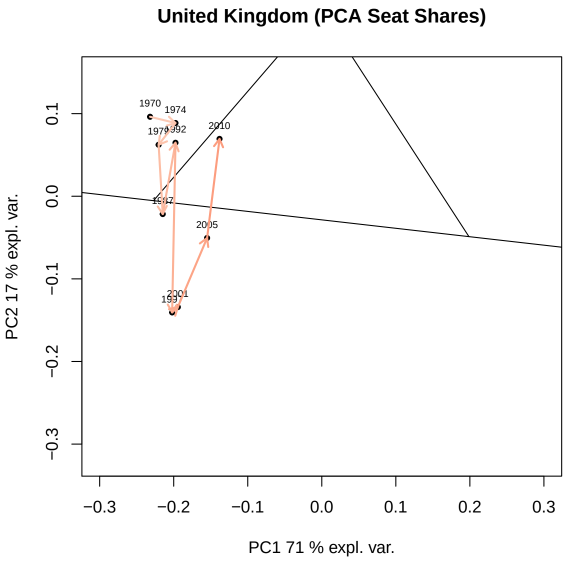


(d) Sweden on the PCA Dimensions/Sartori Classification

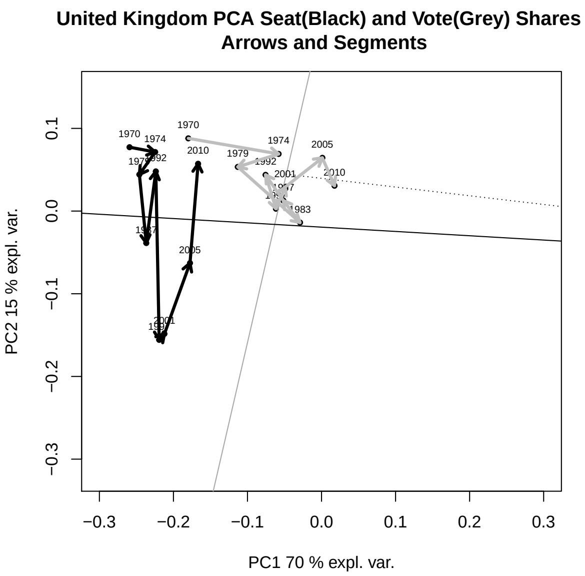
Figure E.18: Notes: The plot shows the Swedish party systems with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



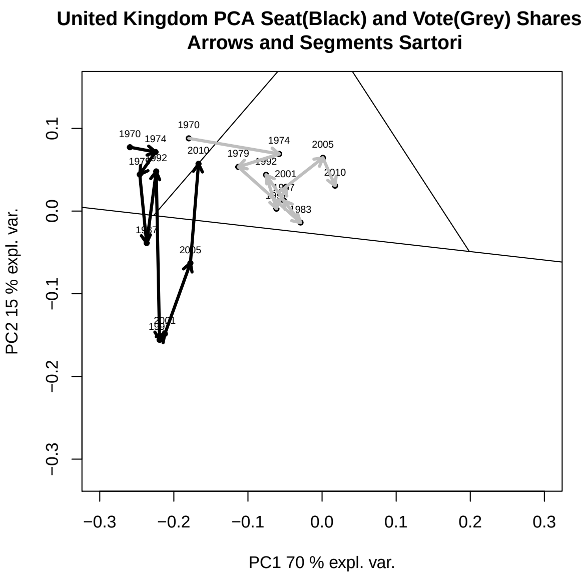
(a) United Kingdom on the PCA Dimensions/Rokkan Classification



(b) United Kingdom on the PCA Dimensions/Sartori Classification

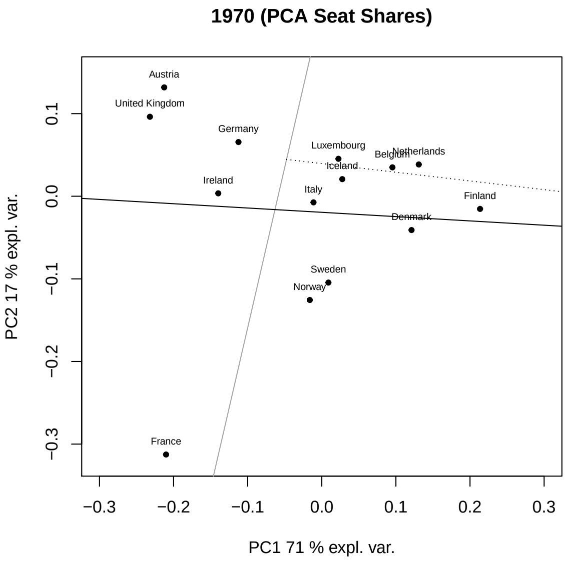


(c) United Kingdom on the PCA Dimensions/Rokkan Classification

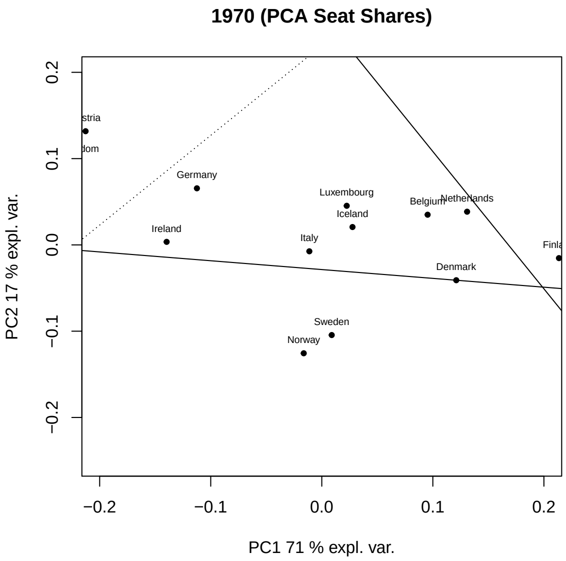


(d) United Kingdom on the PCA Dimensions/Sartori Classification

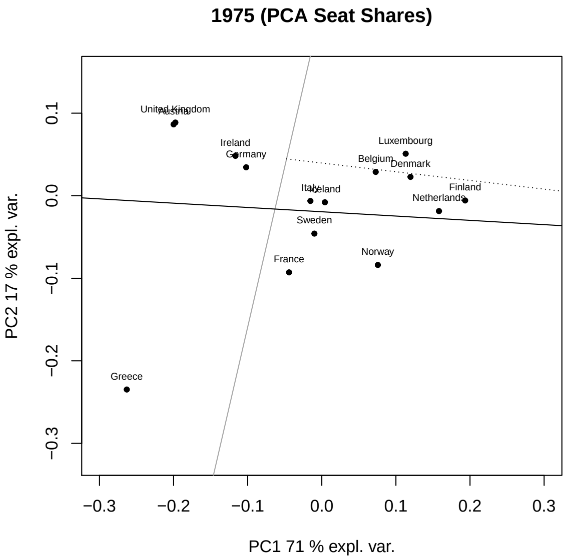
Figure E.19: Notes: The plot shows the UK party system with Rokkan's and Sartori's classifications. Seat shares(up) Seat and Vote shares (down).



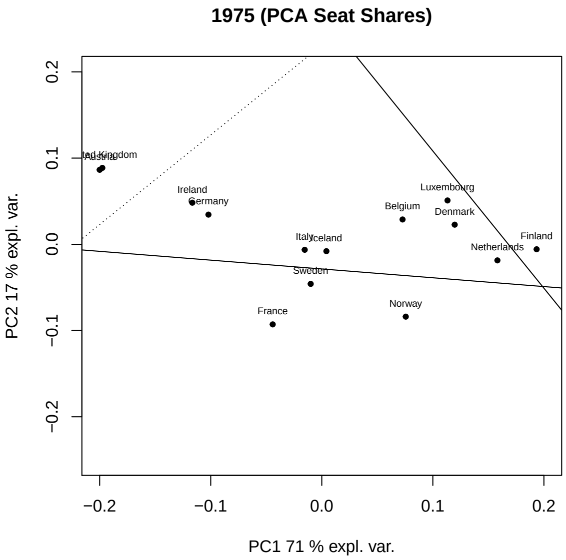
(a) 1970 on the PCA Dimensions/Rokkan Classification



(b) 1970 on the PCA Dimensions/Sartori Classification

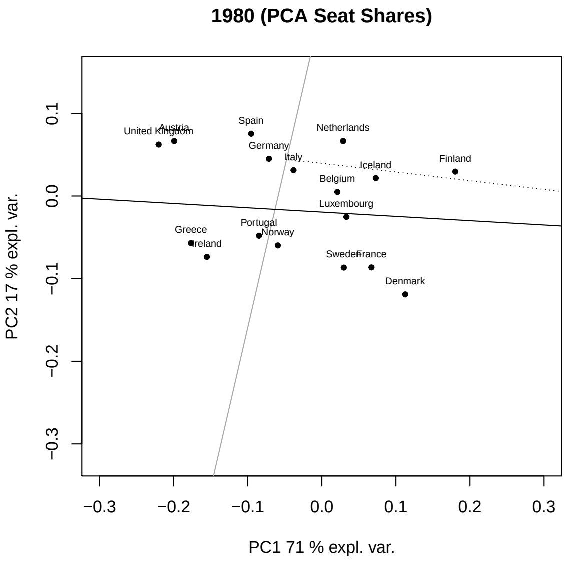


(c) 1975 on the PCA Dimensions/Rokkan Classification

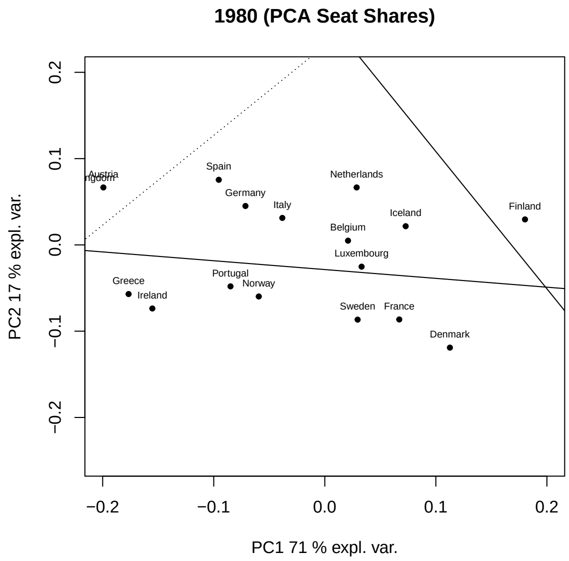


(d) 1975 on the PCA Dimensions/Sartori Classification

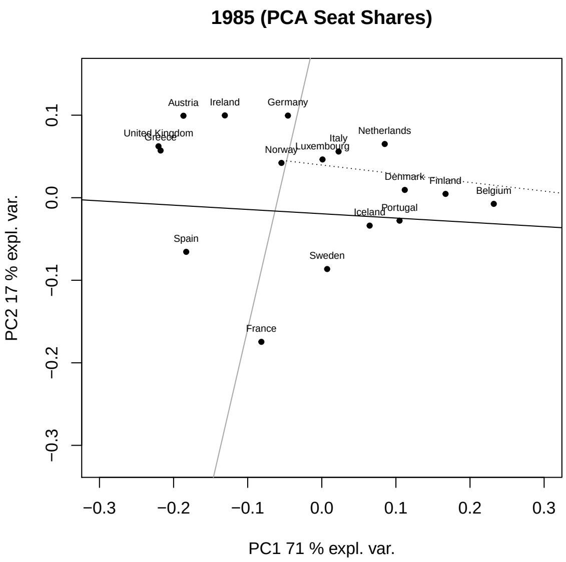
Figure E.20: Notes: The plot shows the party systems in 17 European countries in 1970, 1975 with Rokkan's and Sartori's classifications



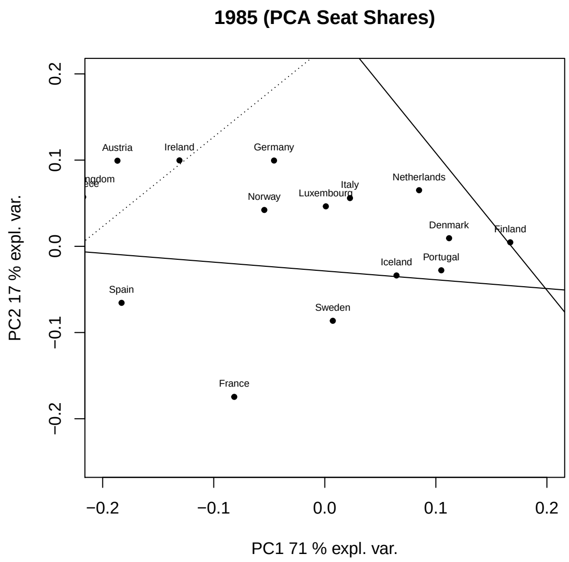
(a) 1980 on the PCA Dimensions/Rokkan Classification



(b) 1980 on the PCA Dimensions/Sartori Classification

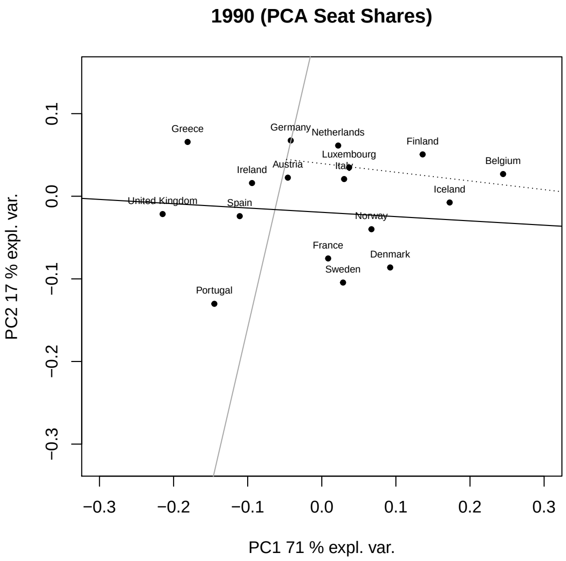


(c) 1985 on the PCA Dimensions/Rokkan Classification

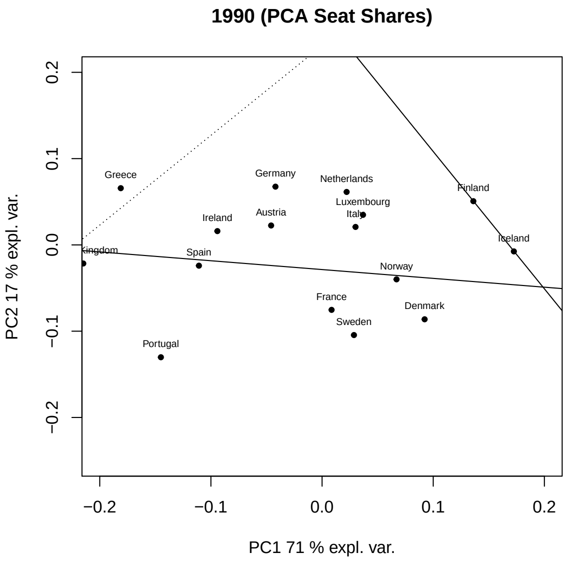


(d) 1985 on the PCA Dimensions/Sartori Classification

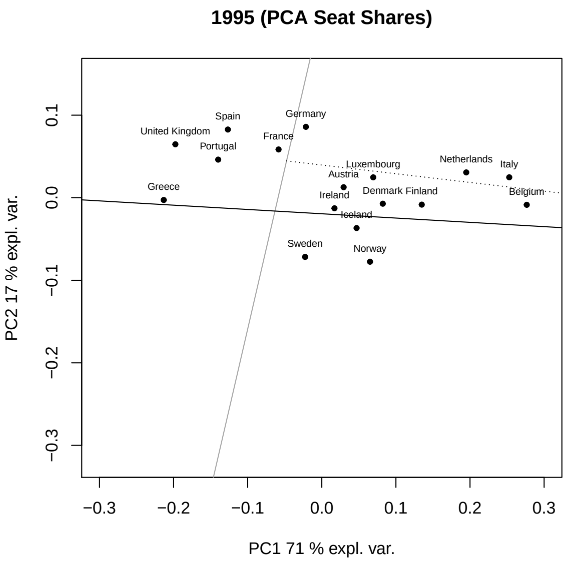
Figure E.21: Notes: The plot shows the party systems in 17 European countries in 1980, 1985 with Rokkan's and Sartori's classifications



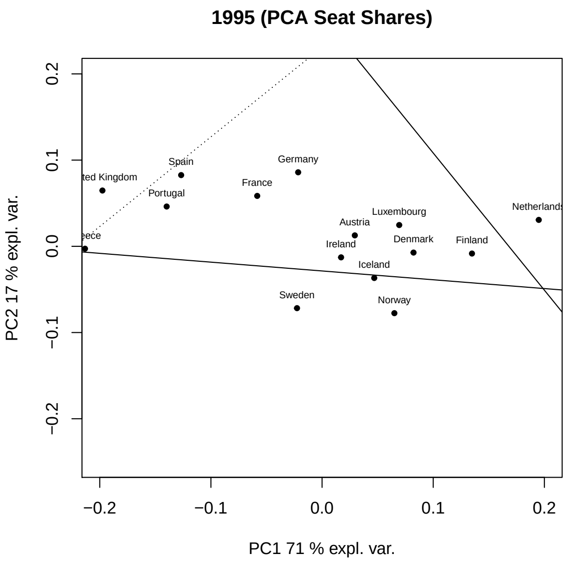
(a) 1990 on the PCA Dimensions/Rokkan Classification



(b) 1990 on the PCA Dimensions/Sartori Classification

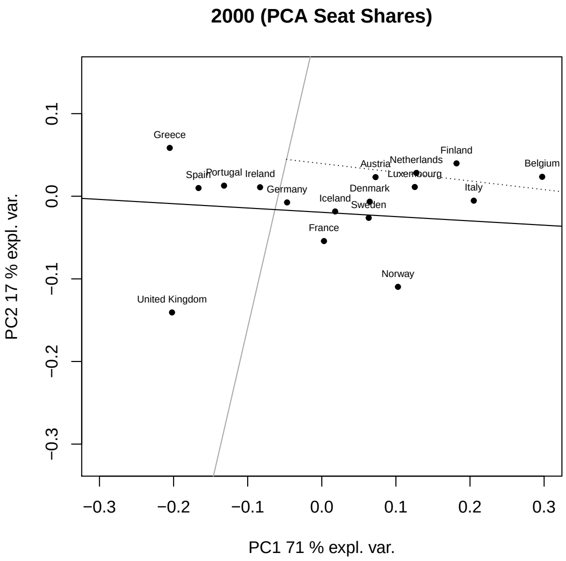


(c) 1995 on the PCA Dimensions/Rokkan Classification

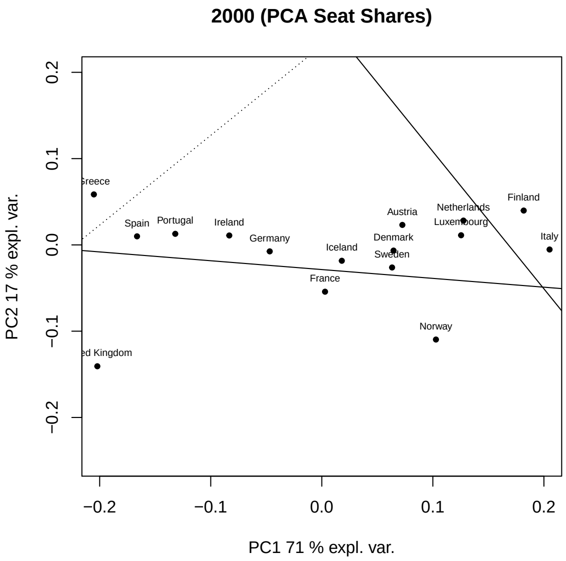


(d) 1995 on the PCA Dimensions/Sartori Classification

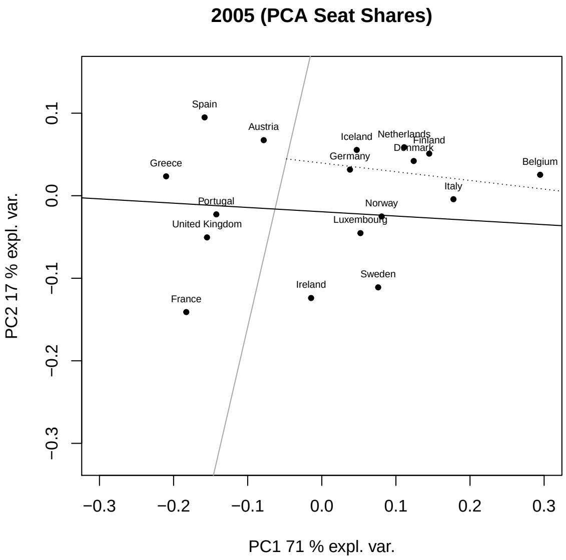
Figure E.22: Notes: The plot shows the party systems in 17 European countries in 1990, 1995 with Rokkan's and Sartori's classifications



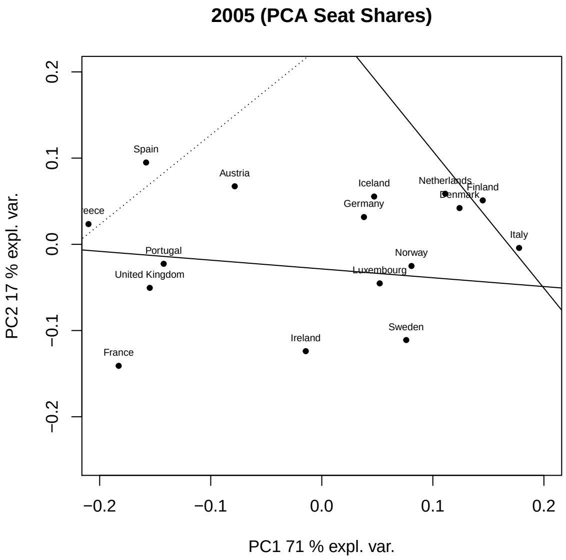
(a) 2000 on the PCA Dimensions/Rokkan Classification



(b) 2000 on the PCA Dimensions/Sartori Classification

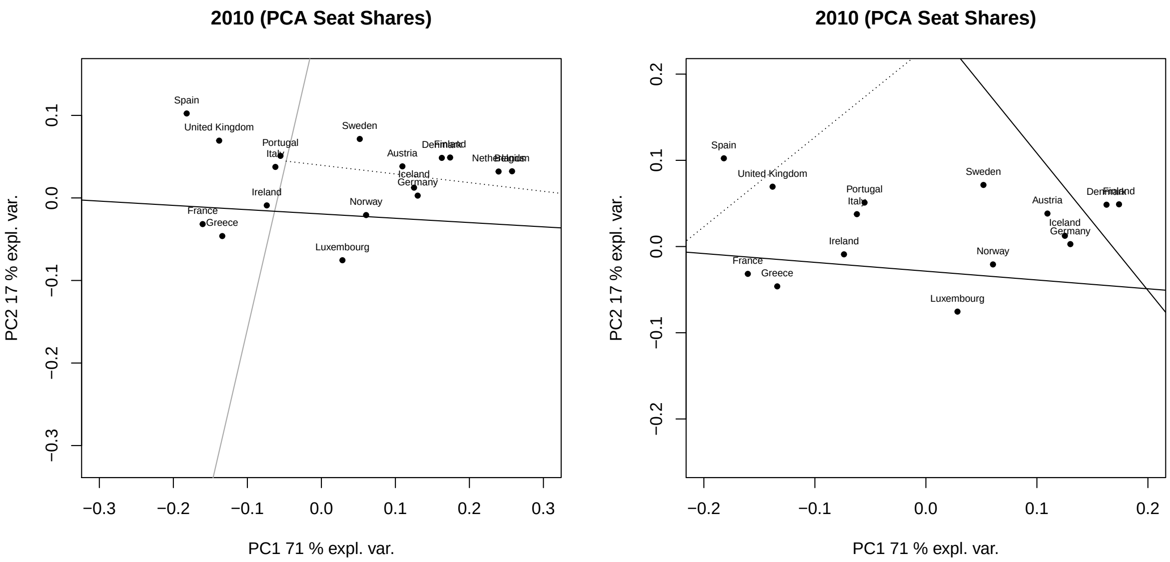


(c) 2005 on the PCA Dimensions/Rokkan Classification



(d) 2005 on the PCA Dimensions/Sartori Classification

Figure E.23: Notes: The plot shows the party systems in 17 European countries in 2000, 2005 with Rokkan's and Sartori's classifications



(a) 2010 on the PCA Dimensions/Rokkan Classification

(b) 2010 on the PCA Dimensions/Sartori Classification

Figure E.24: Notes: The plot shows the party systems in 17 European countries in 2010 with Rokkan's and Sartori's classifications

F. Analysis with Vote Shares

While many parties can get (some) votes in elections, not all of these votes translate to seats in the legislature. How the vote shares translate to seat shares depends on the and permissiveness of the electoral system, the threshold to enter the legislature, and the geographical concentration of voters who prefer the same party. I call the number and sizes of parties that get votes in the elections the electoral party system. A legislative party system, on the other hand, is the number and size of the parties in the legislature.

Among the 17 countries that I have in my dataset there are countries with highly proportional electoral systems: Austria, Belgium, Denmark, Finland, Iceland,Greece before 1989,France in 1986, Italy (before 1994), Luxembourg, the Netherlands, Norway (after 1989), and Sweden. These electoral systems are considered to be permissive. Some countries also use proportional representation PR electoral system but there are few representatives from districts so the allocation of seats is less proportional: Spain, Portugal, Greece after 1989 (the winner party here gets extra 50 seats). Germany and Italy have mixed-member electoral systems in which the voters have a PR list vote and a single member vote. Finally, the UK and France have single-member electoral systems (SMDs) although France has a two -round electoral system. These final electoral systems are nopermissive and the seat shares of the parties can diverge significantly from the vote shares of the parties (Powell and Powell Jr, 2000). Below I will show how these electoral systems influence how the legislative party system changes *vis-a-vis* the electoral party system.

In the main body of the paper I use the seat shares of the parties in order that I can compare my results with seat share based indices and typologies in the second half of the paper. I this appendix, I repeat the analysis with the vote share of the parties and compare the results of the two analyses.

F.1. Data

The data that I am using consists of the party vote shares in the legislature of 17 European countries from 1970 to 2013.[29] In each row (country-year) of the matrix, I rank parties based on their sizes. Thus, the first variable is the seat shares of the biggest parties, the second variable is the seat shares of the second biggest parties etc. Thus, the dataset does not contain the identity of any individual party, but it allows me to compare the party systems across countries. If all the parties have been accounted for in a given country-year, the next entry in the row is a 0. The number of parties ranges from 4 in Austria, Greece and Ireland (various times) to 17 in Italy in 1994 and 1995.[30] The matrix that I

[29] The countries in the dataset are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, and the UK. Data available: http://www.parties-and-elections.eu/countries.html. Countries that democratized later than 1970 appear in the dataset after the first democratic elections

[30] There are periods in which there are more legislative parties than electoral parties. This is possible because in some cases the parties can create pre-electoral coalitions and run in the elections together but separate in the legislature. Another possibility is that parties split in the legislature

create has 675 rows and 20 columns.[31]

F.2. Principal Component Analysis- Vote Shares

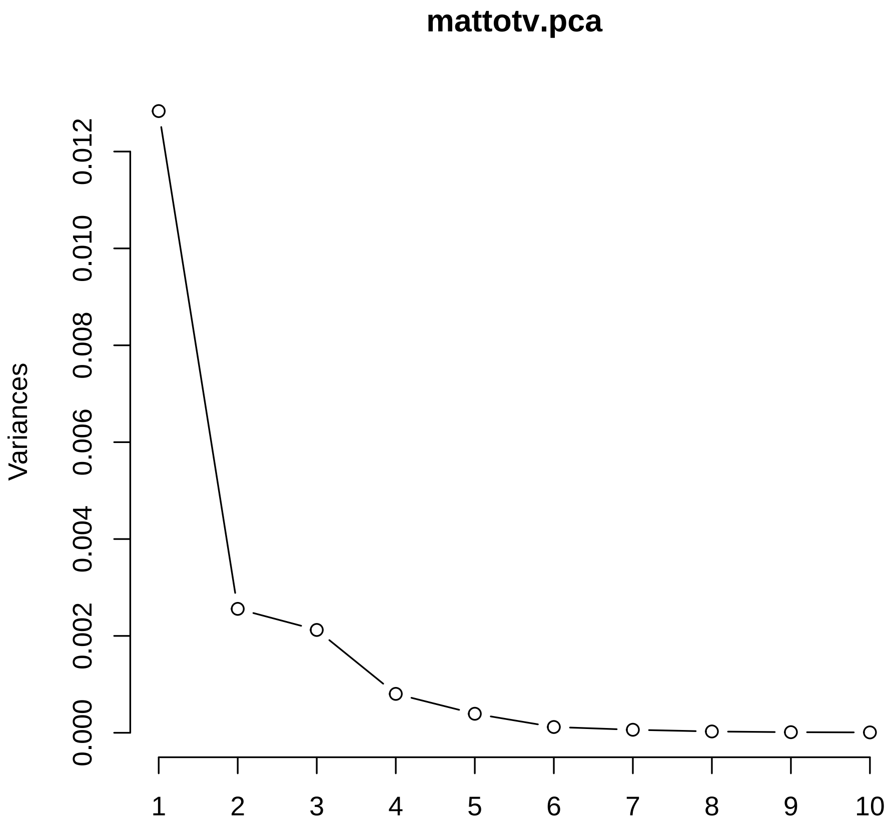
Next, I project the 675-dimensional party vote share data matrix to a 20 dimensional space (the number of variables). I analyze the matrix by row. I first mean center the data. Then, I find the eigenvalues and the eigenvectors of the covariance matrix. I do not scale the variables as previously.

Table F.1: Explained Variance by the Eigen Values (Party Vote Shares)

	eig	variance	cum. variance
1	0.0128352989	67.74	67.74
2	0.0025581430	13.50	81.24
3	0.0021231845	11.21	92.45
4	0.0008033289	4.24	96.69
5	0.0003925861	2.07	98.76
6	0.0001194118	0.63	99.39
7	0.0000623363	0.33	99.72
8	0.0000271358	0.14	99.86
9	0.0000124992	0.07	99.93
10	0.0000080809	0.04	99.97
11	0.0000027318	0.01	99.99
12	0.0000022119	0.01	100.00
13	0.0000002441	0.00	100.00
14	0.0000001366	0.00	100.00
15	0.0000000146	0.00	100.00
16	0.0000000000	0.00	100.00
17	0.0000000000	0.00	100.00
18	0.0000000000	0.00	100.00
19	0.0000000000	0.00	100.00
20	0.0000000000	0.00	100.00

Next, I plot the screeplot, *Figure F.1* which shows in descending order how much of the total variance the eigenvalues explain. *Figure F.1* shows that the first four eigenvalues account for most of the variation in the dataset. *Table F.1* shows that the first four eigenvalues explain 96.69% of the variation in the data and the first two of the eigenvalues explain 81.24% variation in the data. *Figure F.2* shows the first four eigenvectors, or principal components that the analysis has recovered. We can see which parties get a weight in separating the most dissimilar party systems over country-years.

[31] I create a matrix with the same dimensionality as in case of the seat shares so I can easily compare the results of the analysis.

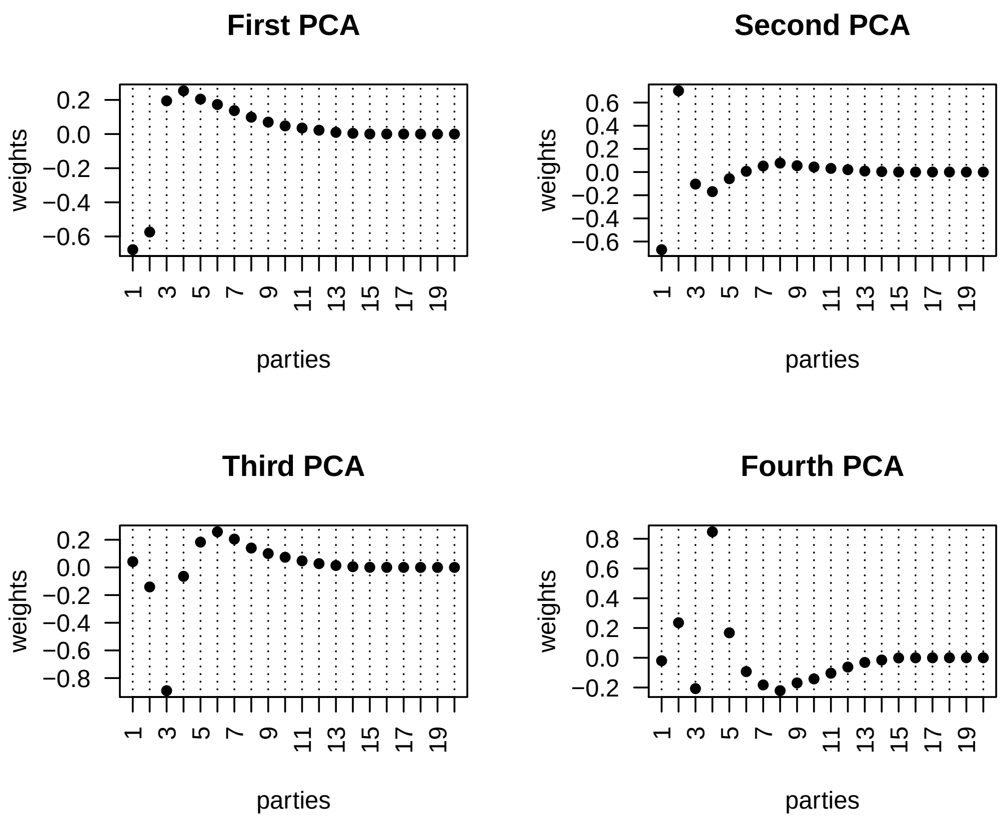


Notes: The plot shows the Scree Plot. On the x-axis are the Eigenvalues, on the y-axis the unexplained variance.

Figure F.1: Unexplained Variance- Scree Plot PCA, Vote Shares

On the X-axes of the plots, we can see the number of parties. On the Y-axes of the plots we can see the weights that each party has in the given principal component. The first three principal components seem to show a clear picture of what makes party systems most unalike. As *Figure F.2* shows, the first dimension (PC1) contrasts countries where the size of the two biggest parties is big relative to the other parties, with countries where the size of the two biggest parties is small relative to the other parties. I call this dimension "Size of the Biggest Two Parties." The second dimension (PC2) contrasts the countries where the size of the two biggest parties are close to each other with countries where the two biggest parties size are far from each other. This contrasts countries with two party competition with countries with one dominant party. We can understand this dimension as the "Competition between the Biggest two Parties." The third dimension

(PC3) is most heavily influenced by the size of the third party, so it contrasts countries with a big third party with countries with a small third party (we can call this dimension "Third Party"), while the fourth dimension (PC4) is somewhat unclear. This dimension seems to be defined by Parties 3-5 and tentatively I call it: "Multipartism."

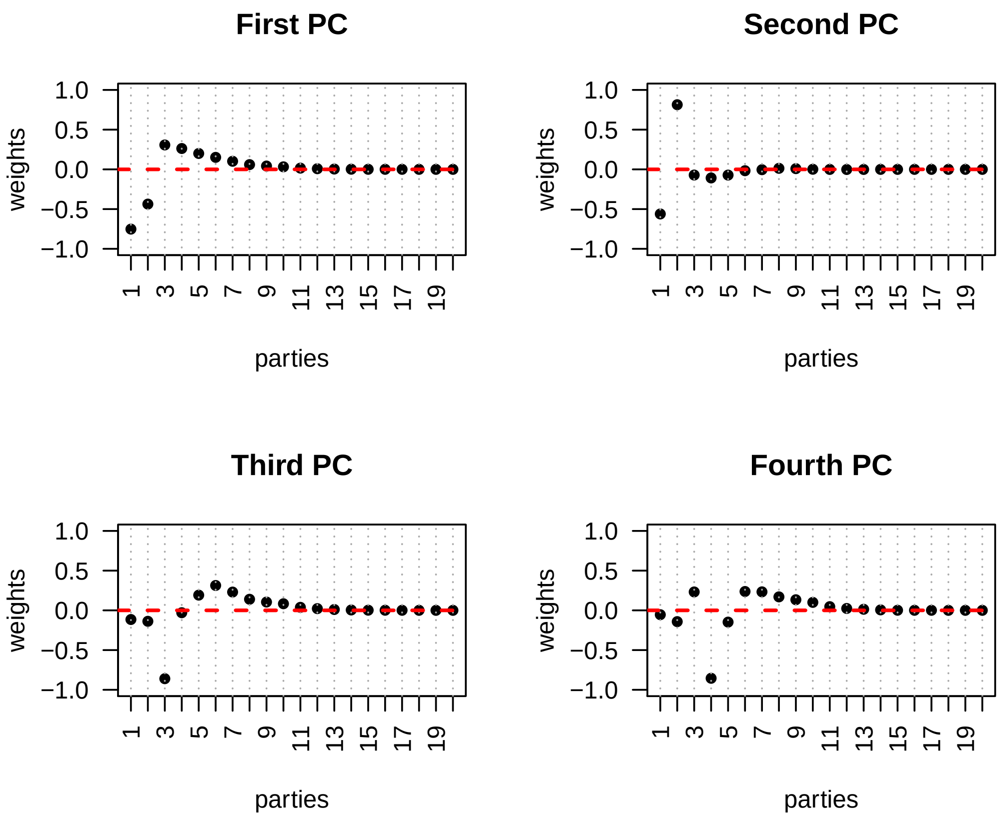


Notes: The plot shows the loadings of the PCA based on party vote shares.

Figure F.2: Loadings, Vote Share

F.3. Comparison of the PCA Results- Vote and Seat Shares

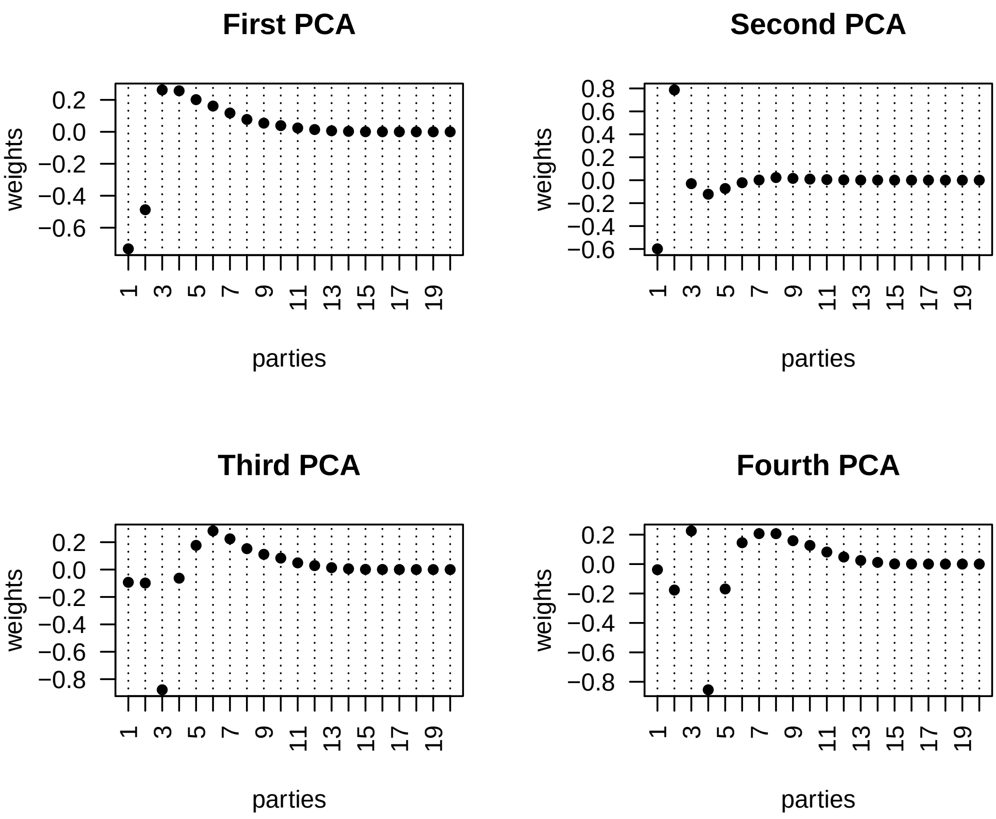
As F.3 shows the first two principal components of the PCA on the vote shares compared to the PCA of the seat shares. The figure shows that these dimensions are very similar to each other. I both cases *Dimension 1* is influenced mostly by the sizes of the biggest two parties, while *Dimension 2* is influenced by the size difference between the first and the second biggest parties. *Table F.1* shows that less variance is explained by the first two dimensions than in case of the seat share PCA (67.74% compared to 71% by the seat share analysis, and 13.50% compared to 17% in case of the seat share analysis). However, the first two dimensions explain over 80% variation in the data which is the rule of thumb threshold to keep PC dimensions that Jolliffe (Jolliffe, 2002) suggests. For comparability I will analyze the similarities and the differences between the seat share and the vote share analysis below. In the future, this analysis can be repeated with three dimensions as well.



Notes: The plot shows the loadings determining the principal components in the PCA by seat shares.

Figure F.3: Loadings, PCA

Because the structure of the data is very similar, next I combine the vote and seat share dataset into one matrix so that the results of the anaysis will be on the same PCA dimensions. The matrix that I create has 1350 rows and 20 columns. Each country-year is represented as two observations, once with its electoral and once with its legislative party system. I run the PCA again. As *FigureF.4* shows, not surprisingly the first two PC dimensions are very similar to the dimensions that the two separate PCAs on the vote shares and on the seat shares have recovered. *TableF.2* shows that the first two PC dimensions account for 84% variation in the data.



Notes: The plot shows the loadings of the PCA based on party vote and seat shares.

Figure F.4: Loadings, Seat and Vote Share, joint analysis

After doing the PCA I map the changes in the legislative and in the electoral party systems onto the two dimesnional plane that the PC1 and PC2 dimensions determine. In addition, I calculate the outer hull that surrounds the changes in time. Later in this appendix I will show how some measures of these hulls can help us in comparing the changes in the legislative and the electoral party systems of the countries.

I present the plots that show the PCA of vote shares, seat shares and the joint PCA at the end of this appendix. *Figures F.5 to F.21* show each of the 17 country. For each country I show the same five plots. In case of Austria *Figure F.5a* shows how the legislative party system changed based on the seat share PCA. *Figure F.5b* shows the convex hull surrounding the changes. *Figure F.5c* shows how the electoral party system changed based on the vote share PCA. *Figure F.5d* shows the convex hull surrounding the changes. Finally *Figure F.5f* shows how these hulls relate to each other based on the joint PCA.

Table F.2: Explained Variance by the Eigen Values (Party Vote and Seat Shares)

	eig	variance	cum. variance
1	0.0161413247	69.58	69.58
2	0.0035189493	15.17	84.75
3	0.0021360093	9.21	93.96
4	0.0007525351	3.24	97.20
5	0.0003825724	1.65	98.85
6	0.0001509375	0.65	99.50
7	0.0000650826	0.28	99.78
8	0.0000259928	0.11	99.89
9	0.0000125100	0.05	99.95
10	0.0000067652	0.03	99.98
11	0.0000030152	0.01	99.99
12	0.0000017763	0.01	100.00
13	0.0000003664	0.00	100.00
14	0.0000001501	0.00	100.00
15	0.0000000203	0.00	100.00
16	0.0000000104	0.00	100.00
17	0.0000000037	0.00	100.00
18	0.0000000014	0.00	100.00
19	0.0000000000	0.00	100.00
20	0.0000000000	0.00	100.00

As we can see from the *Figures* in most countries the electoral and the legislative party system changes were almost identical between 1970 and 2013 (Austria, Belgium, Denmark, Finland, Iceland the Netherlands, Sweden). These are the countries that have the most proportional representation (PR) electoral systems. A few countries however, seem to have different trajectories. These are the countries with PR electoral systems that have low district magnitudes (which means that few representatives get elected from one electoral district). These countries are: Spain, Portugal. In Greece the largest party gets a bonus of 50 seats. We can see that there is some difference between the changes of the electoral and legislative electoral systems, in countries with mixed-member electoral systems (Germany and Italy). In these countries voters vote for a party list and separately they vote in a single-member district (SMD). There is a difference in Ireland where the electoral system is Single Transferable Vote in single member districts. Finally the biggest difference between the electoral and legislative party systems are in countries with single member district electoral systems (SMD): France and the UK. The convex hulls of the electoral and legislative system changes of the UK do not even overlap as *Figure F.21e* shows.

In order to quantify the difference between the party system changes based on vote and seat shares throughout time I use three measures. To make the measures comparable I calculate them from the joint vote-seat share PCA. First, I draw the convex hulls that surround the trajectories of each country's electoral and legislative party systems. Then, I calculate the areas of these hulls. In itself the size of these hulls show how volatile the (electoral or legislative) party systems of the country are. Second, I also calculate the standard deviations of all the points (based on both vote and seat PCA) on both the x and the y axes. In itself, the area of the hull will not allow me to see which of the two directions (one or both) are changing. If we know whether the standard deviation of x or y is bigger we can evaluate whether that party system is more prone to changes in the relative sizes of the two biggest parties compared to the rest of the parties or whether the countries are more prone to changes in the relative sizes of the two biggest parties compared to each other. In fact a single measure can express this quantity of interest the difference between the standard deviation in one direction and in the other direction. $Sd_{diff} = Sd_x - Sd_y$. For this measure a positive value means that the country's party system changed more in the number of parties while a negative number means that the country's party system changed more in its competition structure between the two biggest parties.

The summary measures of each country in alphabetical order are listed in Table F.3. In many cases the seat and vote share hull areas are very close to each other as well as the sign and magnitude of the Sd_{diff} is similar. A bigger hull area means that the party system through 1970-2013 changed more like in France, Norway and Portugal lead, while a smaller one means that the party system of the country from 1970-2013 changed very little like in the Netherlands, Belgium and Finland. As for the Sd_{diff} measure, Belgium, Italy and Austria changed the most on the party system axis compared to the competition dimension, while the United Kingdom, Greece and Spain changed most on the competition dimension compared to the party system size dimension based on the PCA conducted on the seat and vote shares of the parties.

To identify the countries for which this is not so straightforward, I am creating three more tables. First, I calculate the difference between the areas of seat and vote share hulls for each country. Table F.4 shows the countries ranked in the absolute value of the hull differences between vote and seat share hulls $Hull_{diff} = A_{HullS} - A_{HullV}$. This measure can show the effect of the electoral system on the party system or in other words the difference between the electoral party system and the legislative party system. The results confirm that this measure can serve for this purpose, it is clear that the biggest difference between the hulls is in countries where the district magnitude is low. The smaller district magnitude means that fewer legislative seats are distributed within the district (in the extreme, in single member district SMD electoral systems only one), and thus a lot of the votes cast are lost. France, Ireland, Portugal the United Kingdom and Greece are the countries that practically had the biggest difference between the changes in the electoral and in the legislative party systems. I also calculated the area and perimeter ratio of the outer polygon of the countries. This measure F.7 shows how evenly a party moved on the two dimensions. This measure is the largest for the Netherlands and Belgium countries that only moved on the First Dimension while it is the smallest for France, Norway and Denmark in which countries both the competition and the size of the biggest two parties have changed.

Finally, I also calculated the same measures for the years between 1970 and 2010 that I present in Table F.8 and in Figures F.22 and F.23. As a general trend overall it seems that the party systems became more similar to each other over the years specifically on the First Dimension, which means that in Europe the party systems became more similar in the sizes of the two biggest parties. This may be a consequence of the European integration process.

While the change in the area of the hull supported our current knowledge of electoral systems, this analysis can give us new insights into the development and the change in party systems. Namely, by looking at the difference between the standard deviation of how the electoral and the legislative party systems have changed throughout the years, we can get an insight whether the electoral system influences the sheer size of the party system, the competition between the two biggest parties (by overcompensating the biggest party) or both. To calculate these measures I use the difference in standard deviation of the different party systems on the X axis and on the Y axis. Thus my two measures are going to be: $Sd_{diff_X} = Sd_{XV} - Sd_{XS}$ and $Sd_{diff_Y} = Sd_{YV} - Sd_{YS}$. Note that unlike the measure I introduced in the last section these measures do not inform us about whether the standard deviation is bigger on the X axis or on the Y axis (although this is also easily calculable), simply it shows what feature of the party competition changes due to the electoral system. I show the results in Tables: F.5 and F.6. Overall this appendix shows that the difference between electoral and legislative party systems is bigger the more majoritarian electoral system a country has. In mixed member systems, and in PR systems with low district magnitudes, the difference is typically on the dimension of the number of parties, while in majoritarian electoral systems the electoral and legislative party system changes seem to diverge from each other on both the party system size and the competitiveness dimensions.

Table F.3: SD differences (Dim1-Dim2)and Hull Areas, Seat and Vote Shares

	Country	sd_diff_seats	seat_shares_hull	sd_diff_votes	vote_shares_hull
1	Austria	0.08	0.01	0.09	0.01
2	Belgium	0.09	0.00	0.09	0.00
3	Denmark	-0.02	0.01	-0.02	0.01
4	Finland	-0.00	0.00	-0.01	0.00
5	France	-0.01	0.04	0.04	0.01
16	Germany	0.03	0.01	0.04	0.01
17	Greece	-0.05	0.01	-0.02	0.01
6	Iceland	0.02	0.01	0.01	0.01
7	Ireland	-0.01	0.02	0.02	0.01
8	Italy	0.09	0.01	0.06	0.01
9	Luxembourg	-0.01	0.01	0.00	0.01
10	Netherlands	0.04	0.01	0.04	0.01
11	Norway	0.02	0.02	0.02	0.02
12	Portugal	0.03	0.02	0.03	0.01
13	Spain	-0.03	0.01	-0.00	0.01
14	Sweden	-0.02	0.01	-0.02	0.01
15	United Kingdom	-0.06	0.01	0.02	0.01

Table F.4: Hull Differences between Vote and Seat Share PCA

	Country	seat_shares_hull	vote_shares_hull	hull_diff	hull_diff_percent
4	Finland	0.00	0.00	0.00	26.15
2	Belgium	0.00	0.00	0.00	43.68
10	Netherlands	0.01	0.01	0.00	7.39
9	Luxembourg	0.01	0.01	0.00	12.67
14	Sweden	0.01	0.01	0.00	25.12
13	Spain	0.01	0.01	-0.00	-45.43
17	Greece	0.01	0.01	-0.00	-37.38
6	Iceland	0.01	0.01	-0.00	-19.50
16	Germany	0.01	0.01	0.00	18.83
15	United Kingdom	0.01	0.01	0.00	32.74
8	Italy	0.01	0.01	0.00	10.38
1	Austria	0.01	0.01	0.00	8.84
3	Denmark	0.01	0.01	0.00	2.60
7	Ireland	0.02	0.01	0.01	51.14
12	Portugal	0.02	0.01	0.00	21.34
11	Norway	0.02	0.02	0.00	14.66
5	France	0.04	0.01	0.02	61.86

Table F.5: Movement Difference on Party System Size Vote vs. Seat Share

	Country	sd_x_seats	sd_x_votes	sd_diff_xsxv
1	Austria	0.12	0.12	-0.00
2	Belgium	0.11	0.11	0.00
3	Denmark	0.04	0.04	-0.00
4	Finland	0.03	0.02	0.01
5	France	0.10	0.08	0.01
16	Germany	0.07	0.07	-0.00
17	Greece	0.04	0.05	-0.01
6	Iceland	0.05	0.05	-0.00
7	Ireland	0.05	0.06	-0.01
8	Italy	0.12	0.09	0.02
9	Luxembourg	0.04	0.04	0.00
10	Netherlands	0.07	0.07	0.00
11	Norway	0.08	0.07	0.01
12	Portugal	0.08	0.07	0.01
13	Spain	0.03	0.05	-0.02
14	Sweden	0.03	0.03	0.00
15	United Kingdom	0.03	0.06	-0.03

Table F.6: Movement Difference on Competition Vote vs. Seat Share

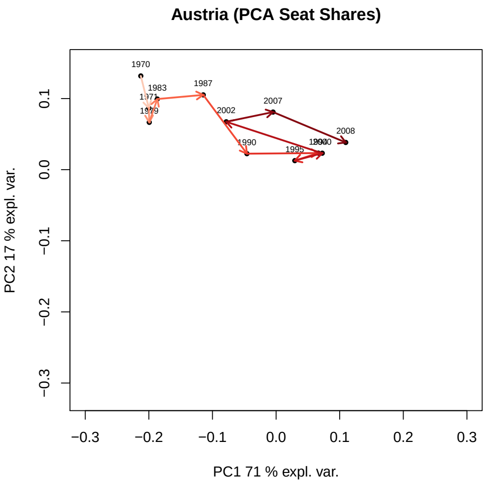
	Country	sd_y_seats	sd_y_votes	sd_diff_ysyv
1	Austria	0.03	0.03	0.00
2	Belgium	0.02	0.02	-0.00
3	Denmark	0.05	0.05	0.00
4	Finland	0.03	0.03	0.00
5	France	0.11	0.04	0.06
16	Germany	0.03	0.03	0.01
17	Greece	0.08	0.07	0.01
6	Iceland	0.03	0.04	-0.00
7	Ireland	0.06	0.04	0.02
8	Italy	0.03	0.03	-0.00
9	Luxembourg	0.05	0.04	0.01
10	Netherlands	0.03	0.03	0.00
11	Norway	0.05	0.05	0.00
12	Portugal	0.06	0.05	0.01
13	Spain	0.07	0.05	0.01
14	Sweden	0.05	0.05	-0.00
15	United Kingdom	0.09	0.03	0.06

Table F.7: Area and perimeter ratio of the outer polygon/countries

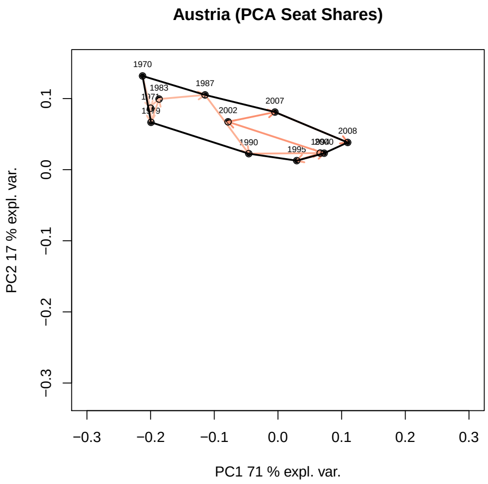
	Country	area_seats	area_votes	peri_seats	peri_votes	a_p_seats	a_p_votes
1	Austria	0.01	0.01	0.72	0.69	57.71	60.60
2	Belgium	0.00	0.00	0.53	0.37	130.50	162.23
3	Denmark	0.01	0.01	0.49	0.49	35.34	36.54
4	Finland	0.00	0.00	0.26	0.22	71.02	80.21
5	France	0.04	0.01	0.91	0.61	24.06	42.05
16	Germany	0.01	0.01	0.45	0.40	47.30	51.99
17	Greece	0.01	0.01	0.40	0.61	44.89	49.66
6	Iceland	0.01	0.01	0.45	0.47	48.33	42.44
7	Ireland	0.02	0.01	0.64	0.47	36.63	55.26
8	Italy	0.01	0.01	0.73	0.62	60.78	57.50
9	Luxembourg	0.01	0.01	0.42	0.38	57.24	59.86
10	Netherlands	0.01	0.01	0.54	0.51	80.17	81.71
11	Norway	0.02	0.02	0.71	0.64	34.62	36.41
12	Portugal	0.02	0.01	0.72	0.61	37.76	40.79
13	Spain	0.01	0.01	0.45	0.48	53.36	39.03
14	Sweden	0.01	0.01	0.48	0.40	57.63	64.78
15	United Kingdom	0.01	0.01	0.56	0.40	57.46	61.00

Table F.8: Area and perimeter ratio of the outer polygon/years

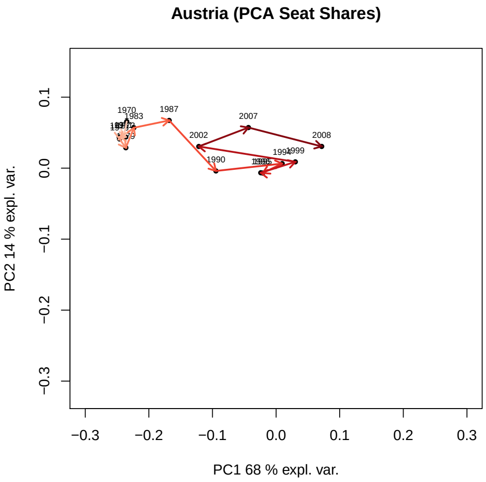
	Year	sd_x_seats	sd_y_seats	sd_x_votes	sd_y_votes	area_seats	area_votes	peri_seats	peri_votes	a_p_seats	a_p_votes
1	1970	0.14	0.11	0.11	0.07	0.04	0.02	1.13	0.98	27.93	39.39
2	1975	0.14	0.08	0.11	0.06	0.03	0.03	1.02	1.00	29.83	29.94
3	1980	0.11	0.06	0.10	0.05	0.04	0.03	1.01	0.96	23.17	28.53
4	1985	0.14	0.07	0.12	0.06	0.05	0.04	1.10	0.96	19.95	26.97
5	1990	0.12	0.06	0.10	0.06	0.05	0.04	1.11	0.98	21.03	27.06
6	1995	0.14	0.05	0.12	0.04	0.04	0.03	1.13	0.97	31.19	35.58
7	2000	0.15	0.05	0.13	0.04	0.03	0.03	1.29	0.97	37.01	34.66
8	2005	0.14	0.07	0.11	0.05	0.06	0.03	1.16	0.96	19.83	30.81
9	2010	0.14	0.05	0.11	0.03	0.03	0.02	1.04	0.87	33.69	44.59



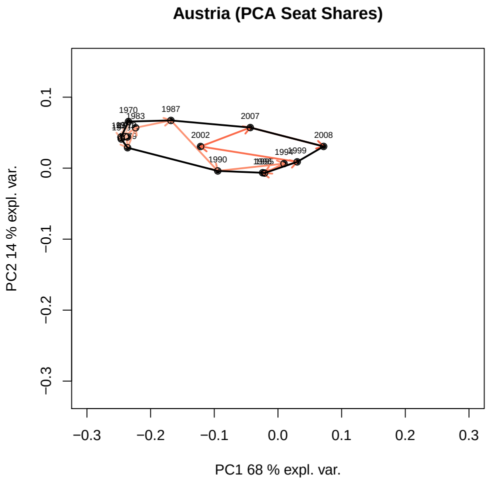
(a) Austria Seat Shares



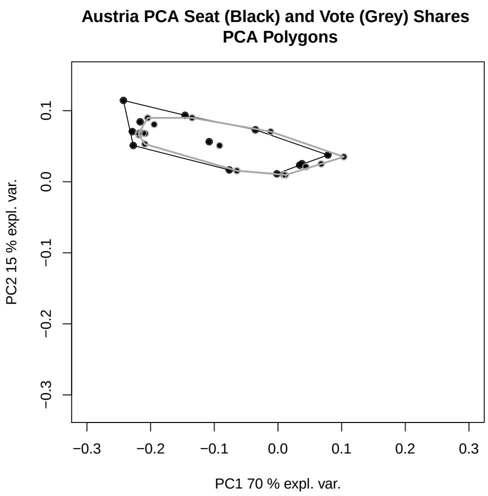
(b) Austria Seat Shares: Polygon



(c) Austria Vote Shares



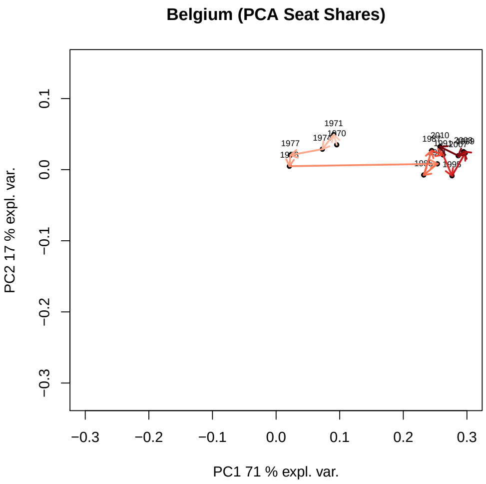
(d) Austria Vote Shares: Polygon



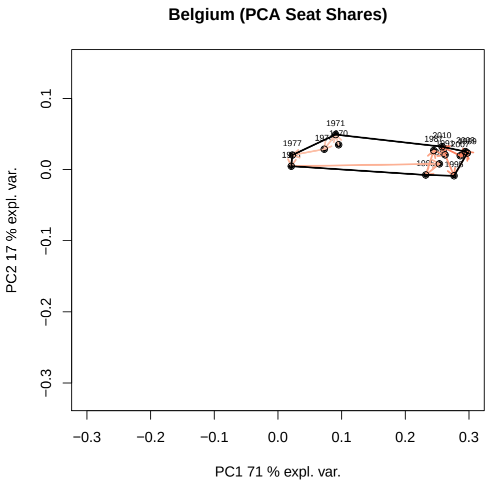
(e)

(f) Austria Seat-Vote Shares Polygons

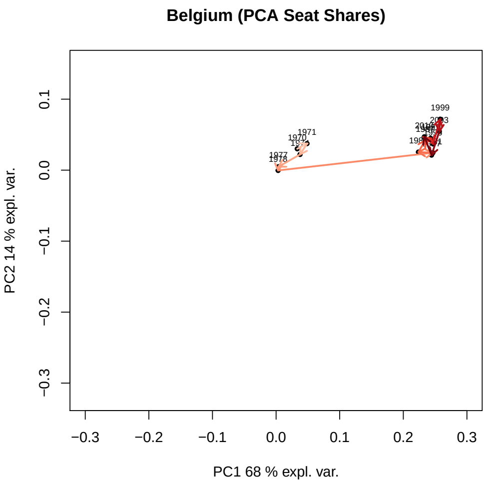
Figure F.5: Austria Changes in the Electoral and Legislative P.Systems



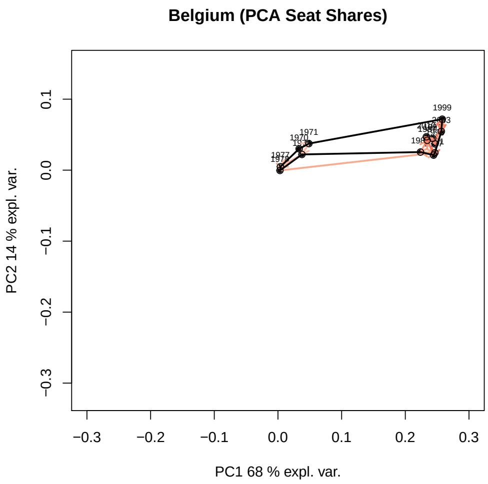
(a) Belgium Seat Shares



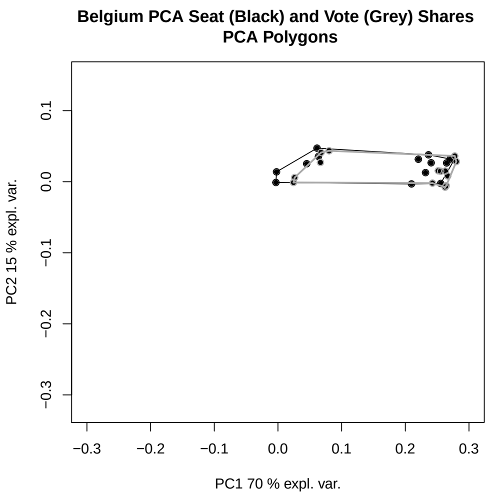
(b) Belgium Seat Shares: Polygon



(c) Belgium Vote Shares



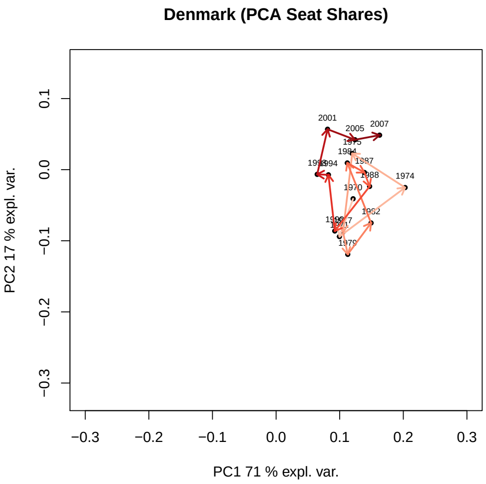
(d) Belgium Vote Shares: Polygon



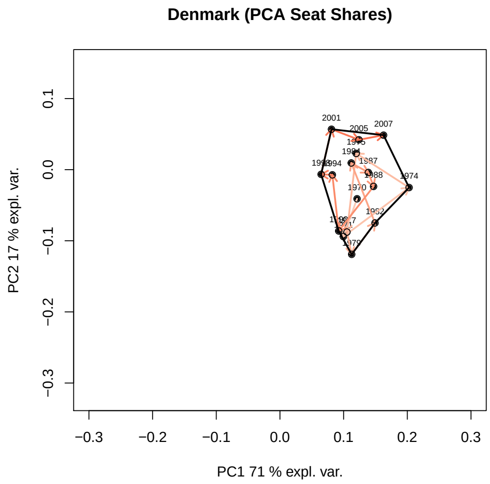
(e)

(f) Belgium Seat-Vote Shares Polygons

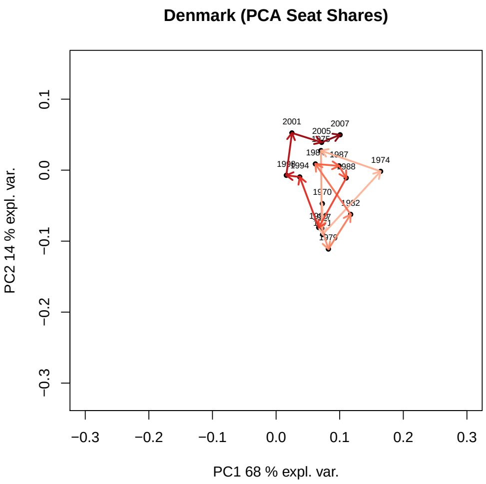
Figure F.6: Belgium Changes in the Electoral and Legislative P.Systems



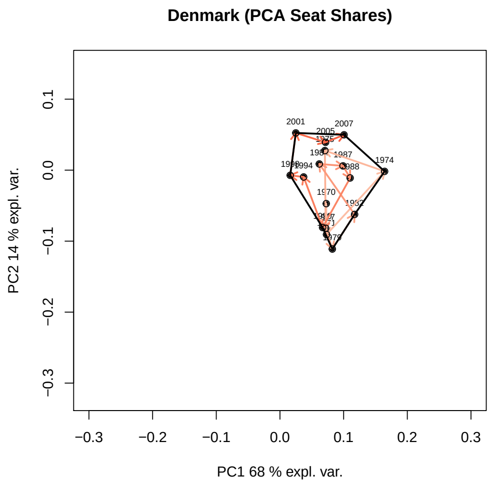
(a) Denmark Seat Shares



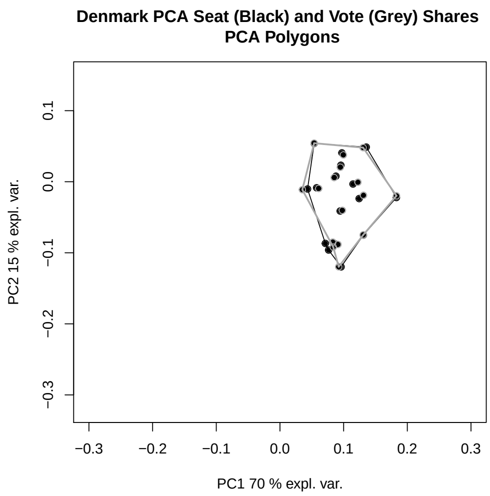
(b) Denmark Seat Shares: Polygon



(c) Denmark Vote Shares



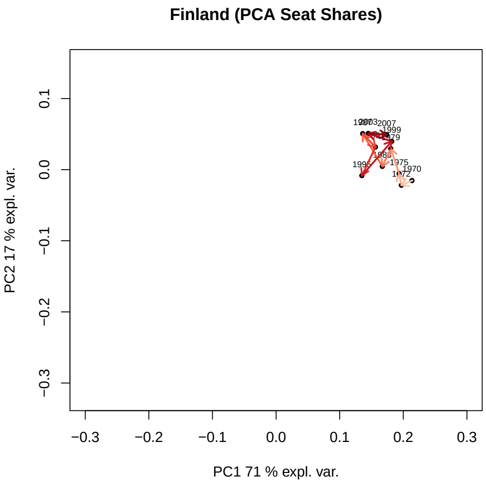
(d) Denmark Vote Shares: Polygon

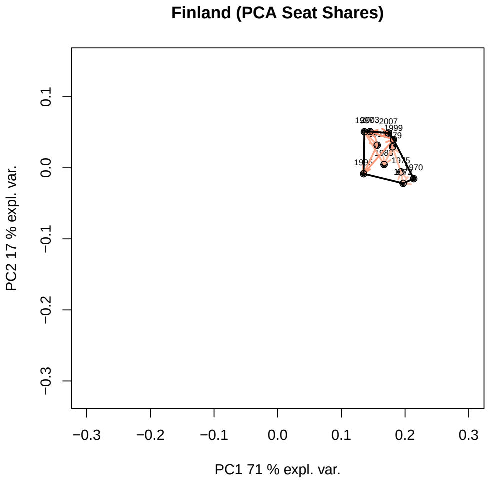


(e)

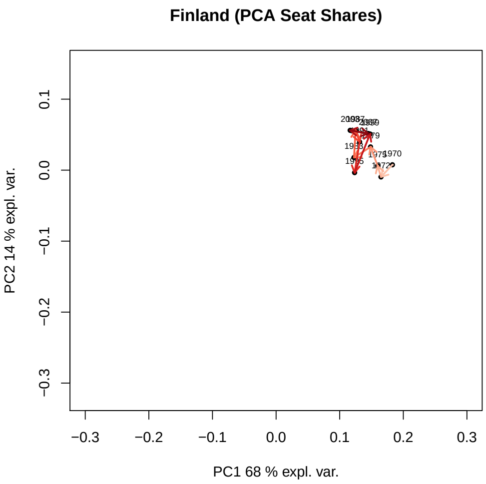
(f) Denmark Seat-Vote Shares Polygons

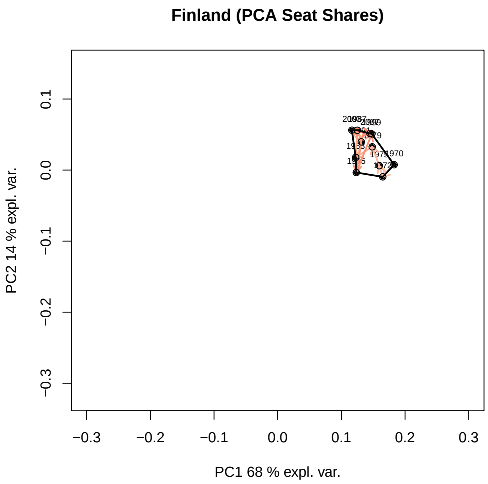
Figure F.7: Denmark Changes in the Electoral and Legislative P.Systems





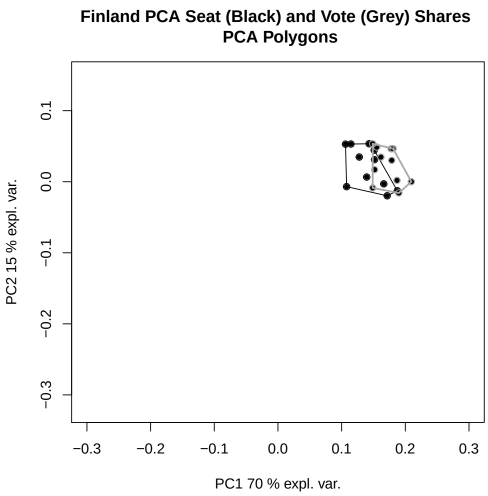
(a) Finland Seat Shares





(b) Finland Vote Shares

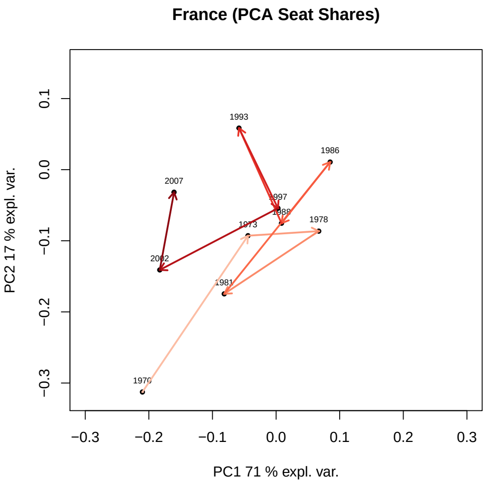
(c) Finland Vote Shares: Polygon



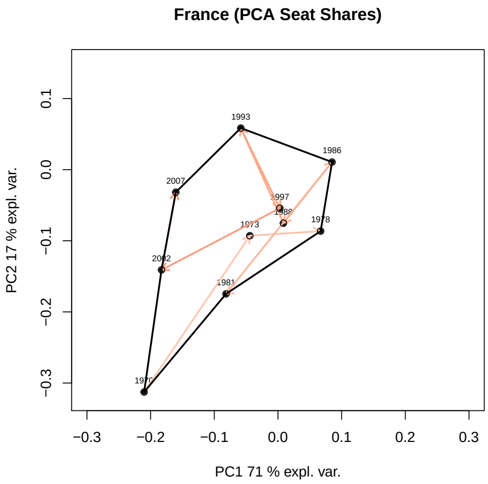
(d)

(e) Finland Seat-Vote Shares Polygons

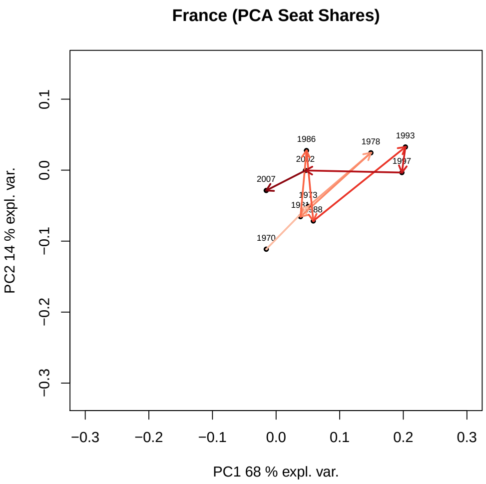
Figure F.8: Finland Changes in the Electoral and Legislative P.Systems



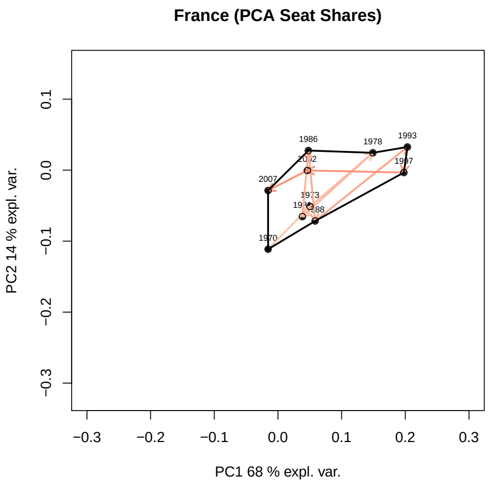
(a) France Seat Shares



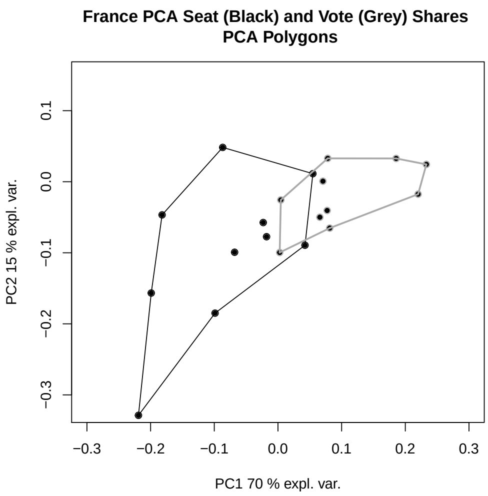
(b) France Seat Shares: Polygon



(c) France Vote Shares



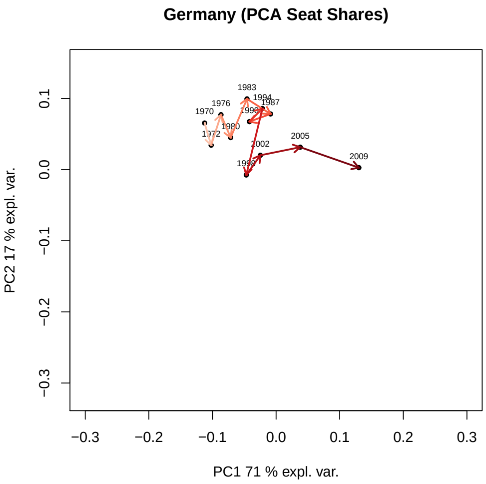
(d) France Vote Shares: Polygon

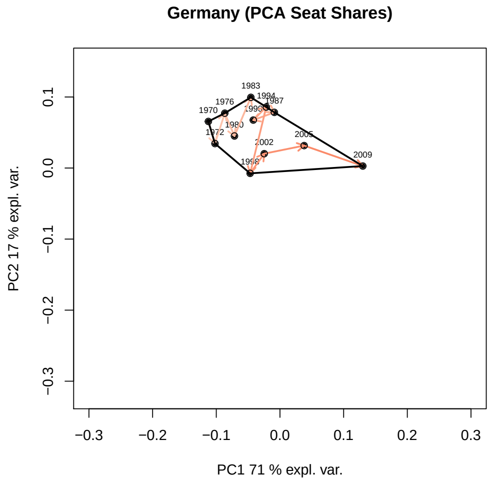


(e)

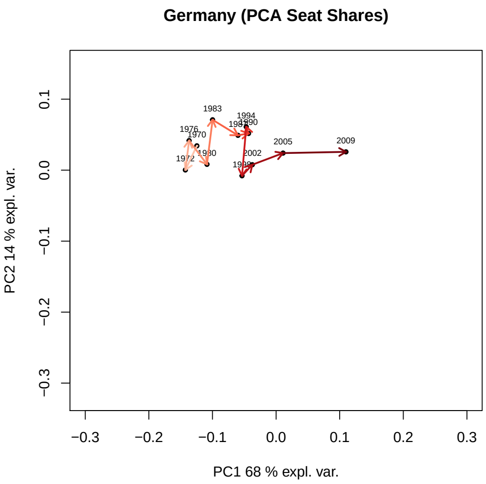
(f) France Seat-Vote Shares Polygons

Figure F.9: France Changes in the Electoral and Legislative P.Systems

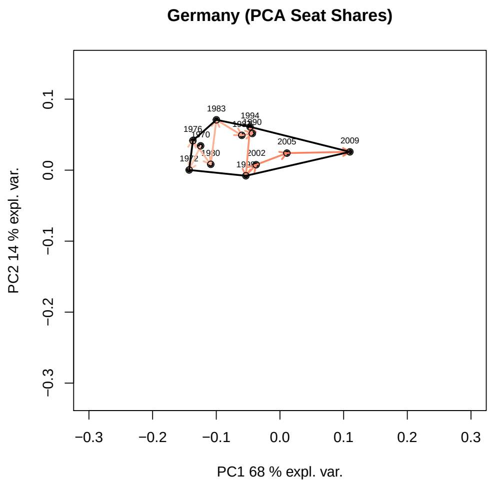




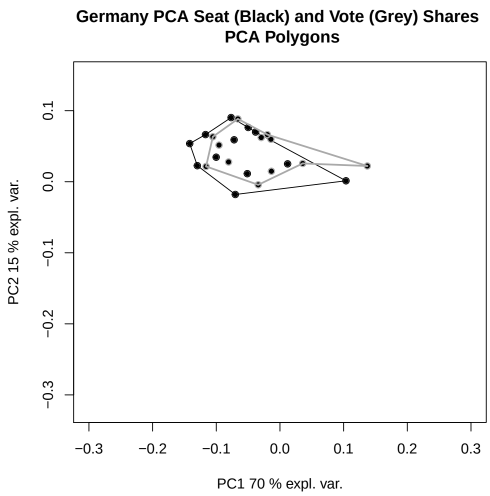
(a) Germany Seat Shares



(b) Germany Vote Shares



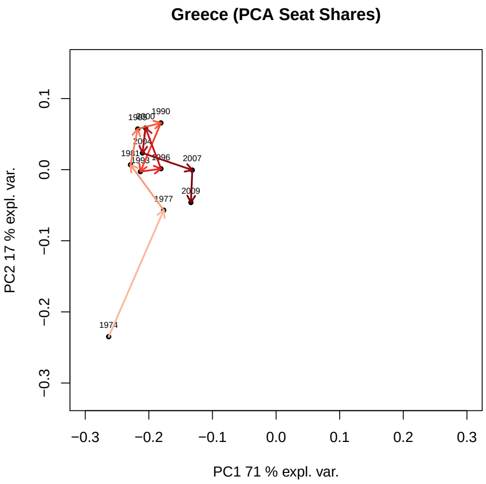
(c) Germany Vote Shares: Polygon



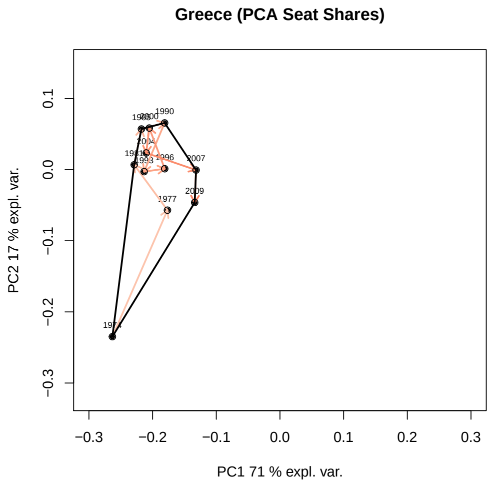
(d)

(e) Germany Seat-Vote Shares Polygons

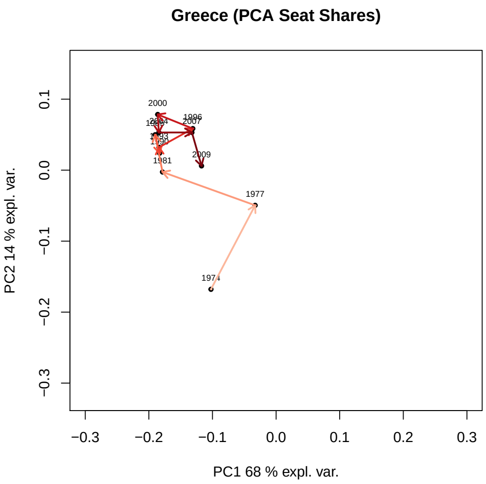
Figure F.10: Germany Changes in the Electoral and Legislative P.Systems



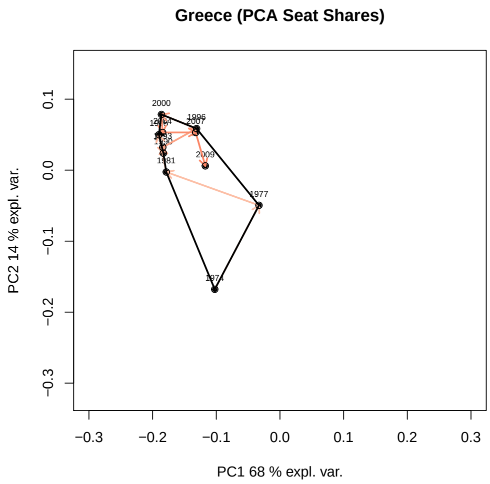
(a) Greece Seat Shares



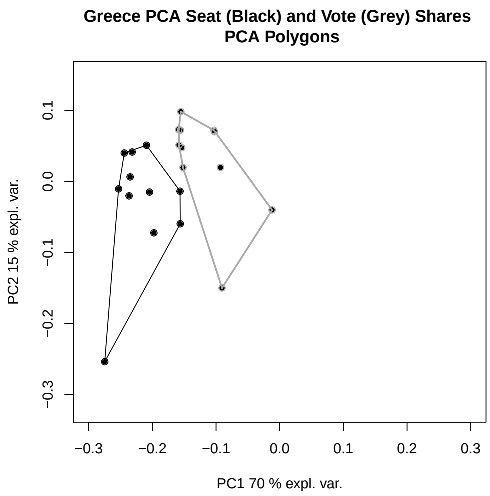
(b) Greece Seat Shares: Polygon



(c) Greece Vote Shares



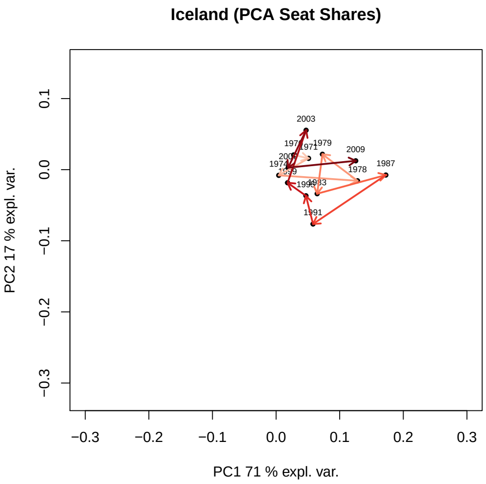
(d) Greece Vote Shares: Polygon



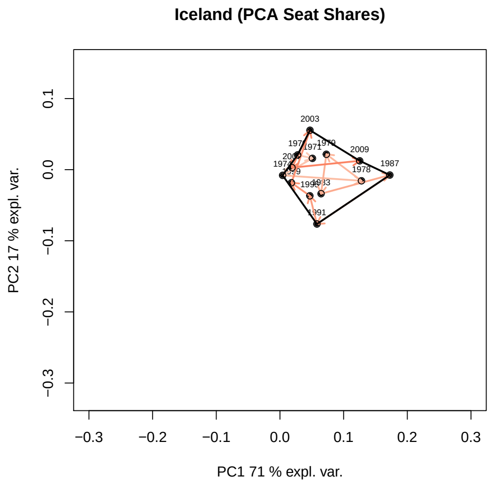
(e)

(f) Greece Seat-Vote Shares Hulls

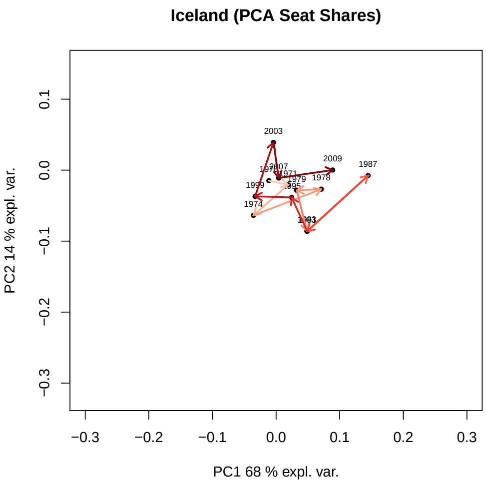
Figure F.11: Greece Changes in the Electoral and Legislative P.Systems



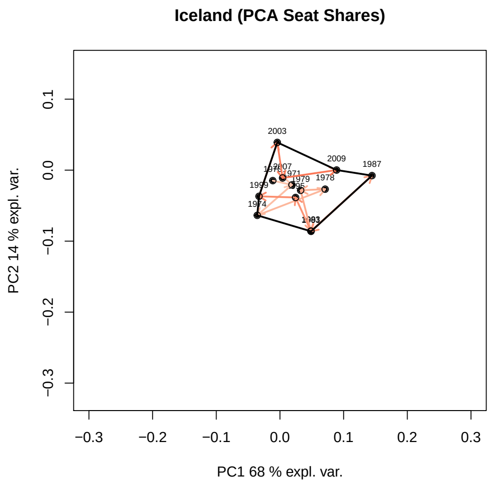
(a) Iceland Seat Shares



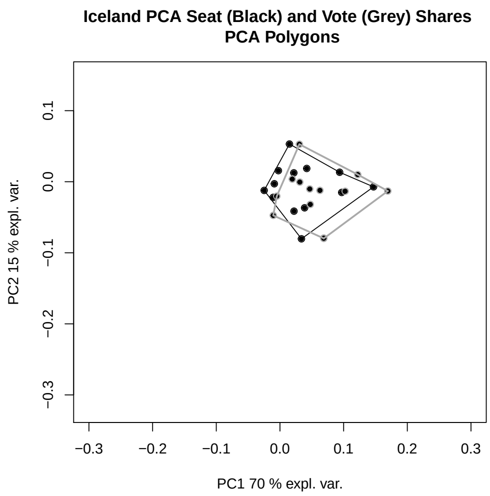
(b) Iceland Seat Shares: Polygon



(c) Iceland Vote Shares



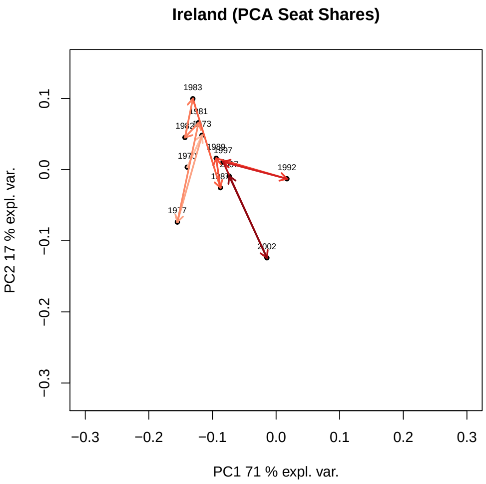
(d) Iceland Vote Shares: Polygon



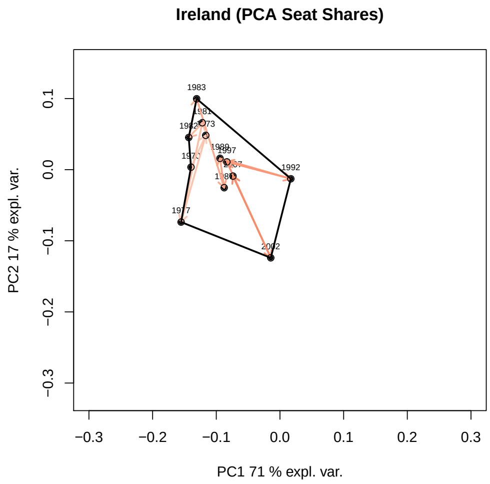
(e)

(f) Iceland Seat-Vote Shares Polygons

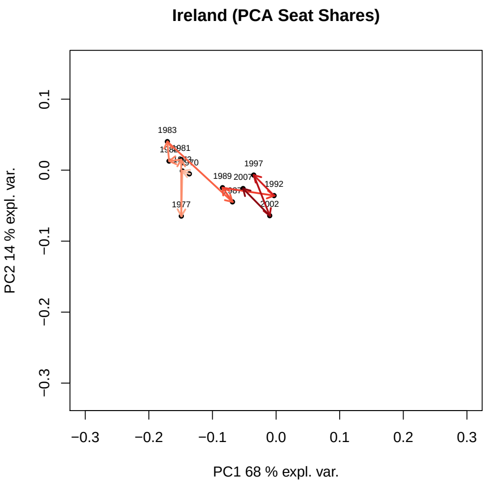
Figure F.12: Iceland Changes in the Electoral and Legislative P.Systems



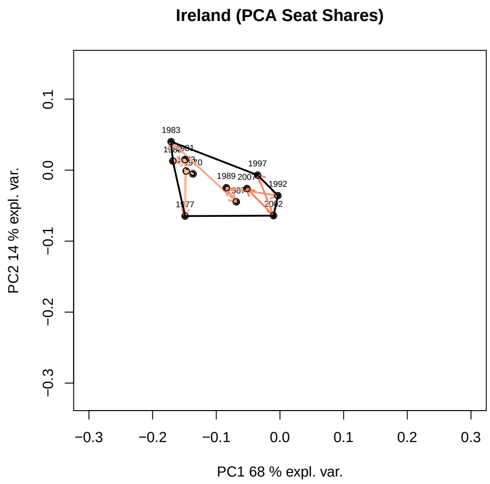
(a) Ireland Seat Shares



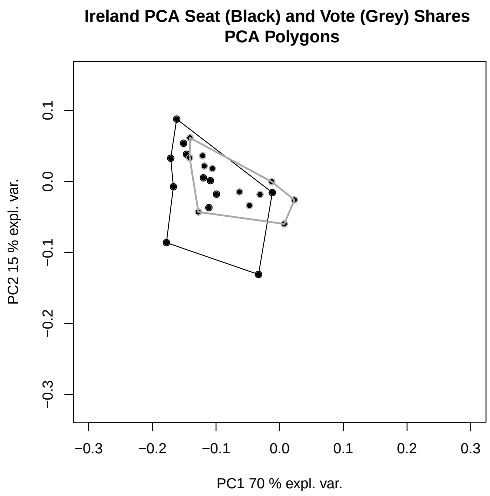
(b) Ireland Seat Shares: Polygon



(c) Ireland Vote Shares



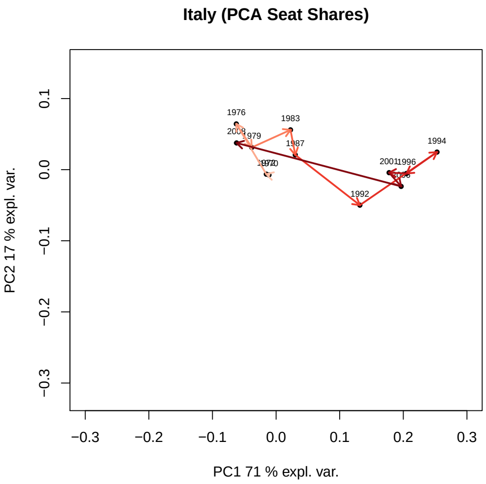
(d) Ireland Vote Shares: Polygon



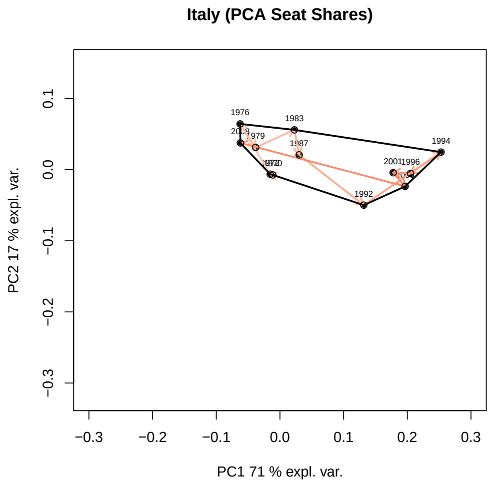
(e)

(f) Ireland Seat-Vote Shares Polygons

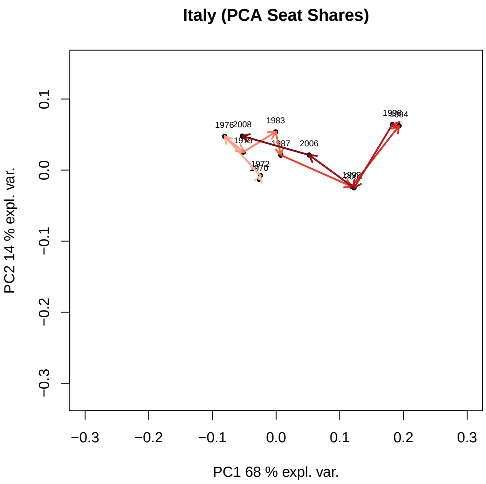
Figure F.13: Ireland Changes in the Electoral and Legislative P.Systems



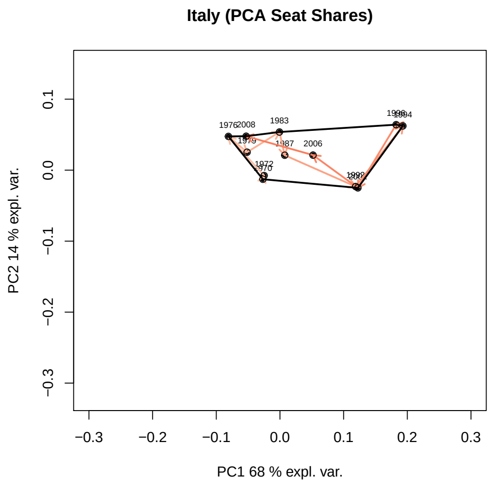
(a) Italy Seat Shares



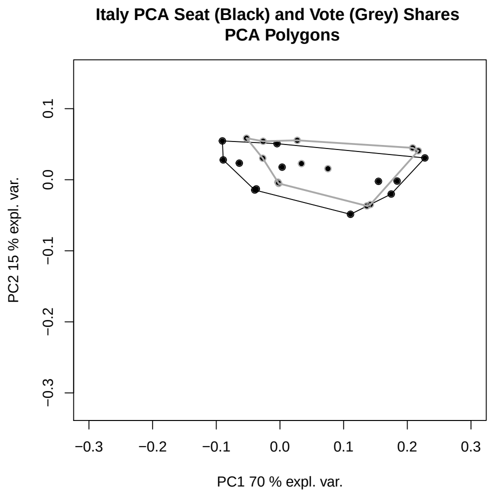
(b) Italy Seat Shares: Polygon



(c) Italy Vote Shares



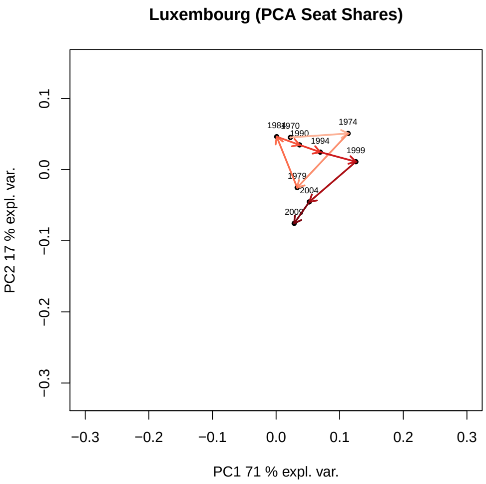
(d) Italy Vote Shares: Polygon



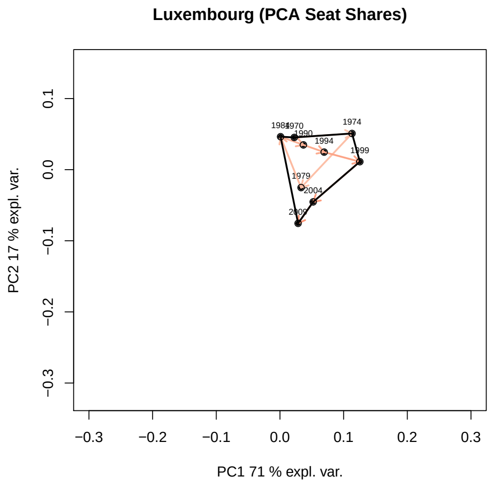
(e)

(f) Italy Seat-Vote Shares Polygons

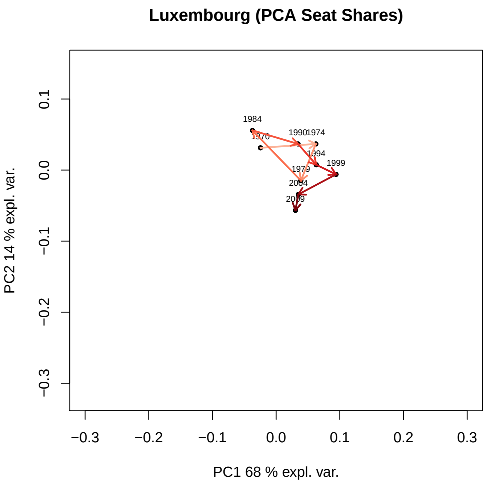
Figure F.14: Italy Changes in the Electoral and Legislative P.Systems



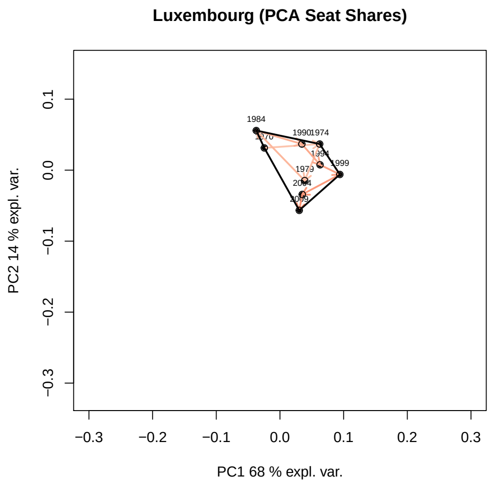
(a) Luxembourg Seat Shares



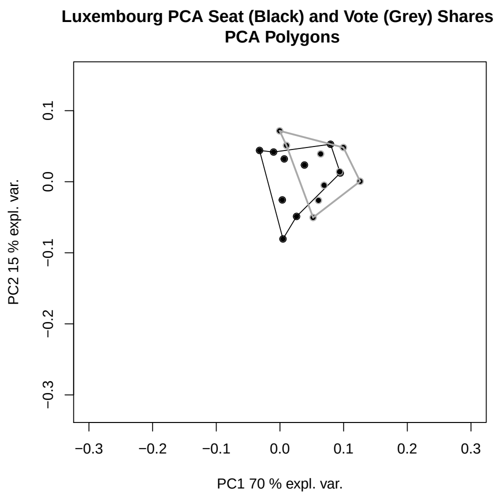
(b) Luxembourg Seat Shares: Polygon



(c) Luxembourg Vote Shares



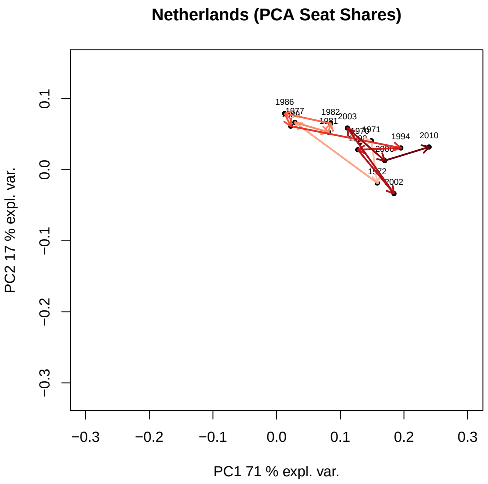
(d) Luxembourg Vote Shares: Polygon



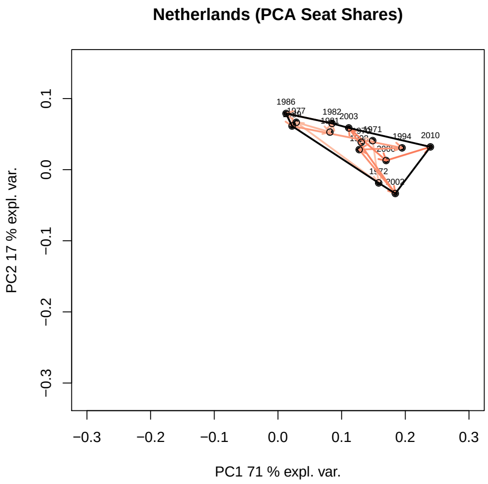
(e)

(f) Luxembourg Seat-Vote Shares Polygons

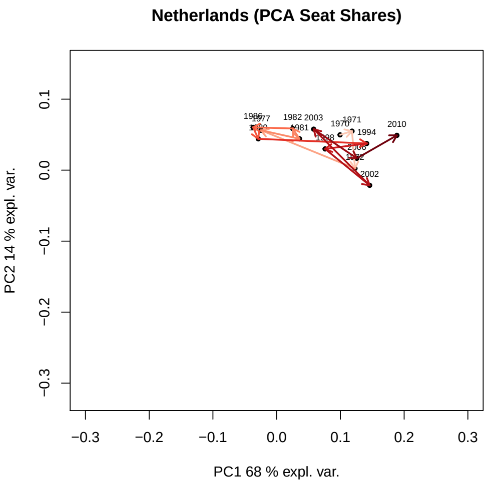
Figure F.15: Luxembourg Changes in the Electoral and Legislative P.Systems



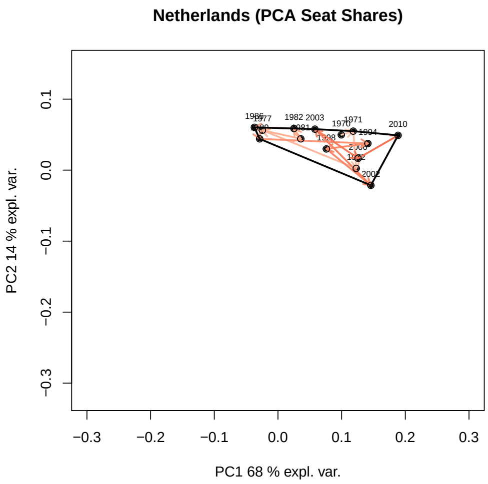
(a) Netherlands Seat Shares



(b) Netherlands Seat Shares: Polygon

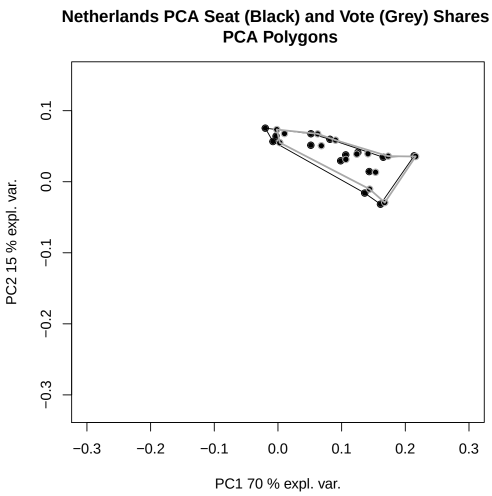


(c) Netherlands Vote Shares



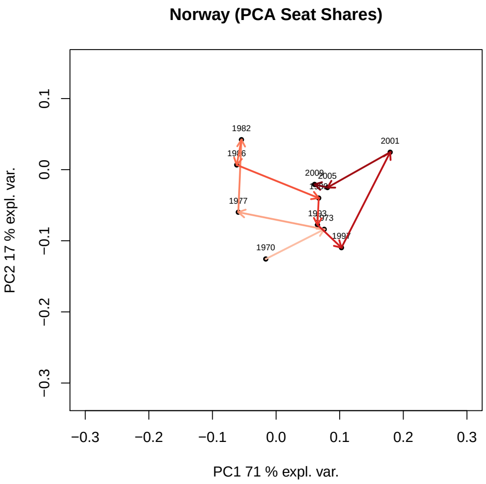
(d) Netherlands Vote Shares: Polygon

(e)

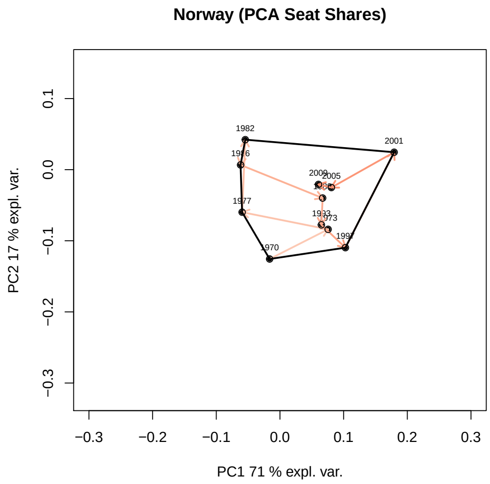


(f) Netherlands Seat-Vote Shares Polygons

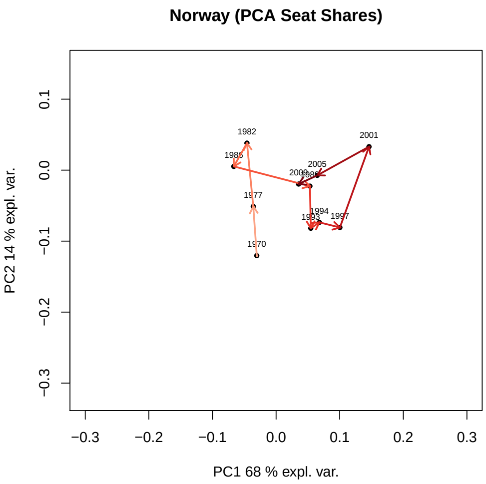
Figure F.16: the Netherlands Changes in the Electoral and Legislative P.Systems



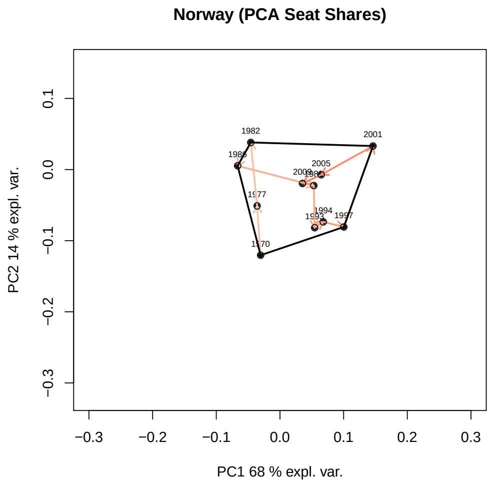
(a) Norway Seat Shares



(b) Norway Seat Shares: Polygon



(c) Norway Vote Shares



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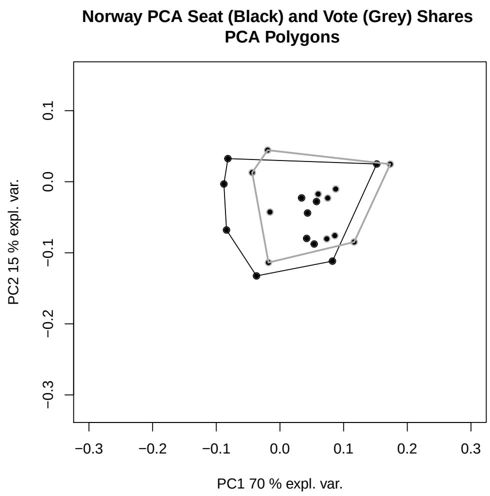
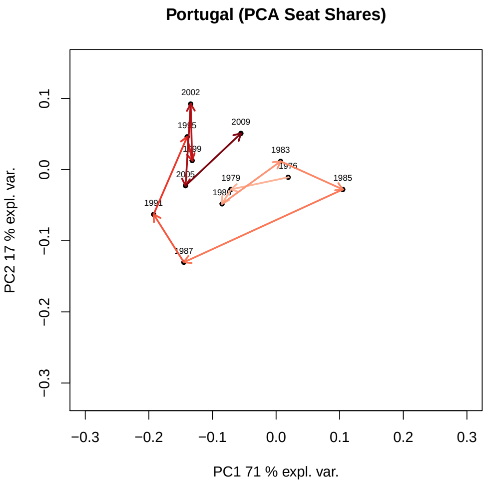
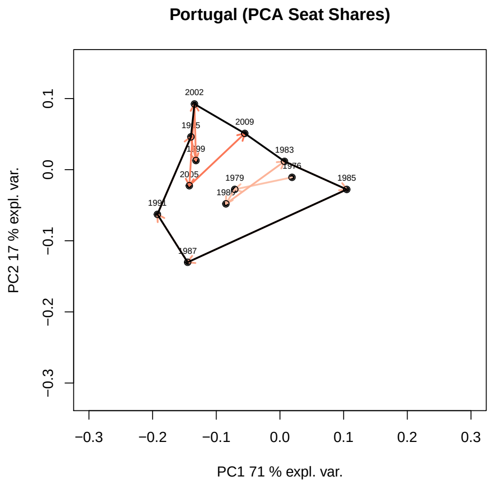


(e) Norway Seat-Vote Shares Polygons

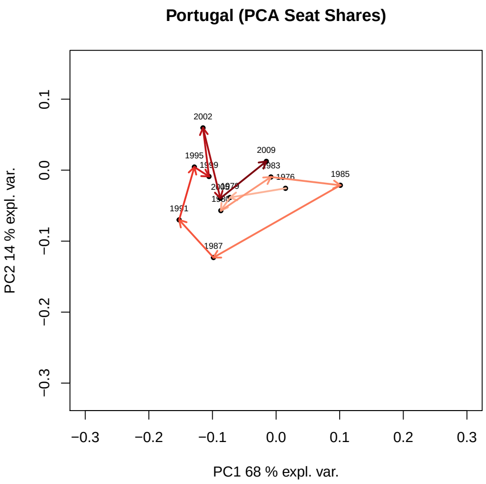
Figure F.17: Norway Changes in the Electoral and Legislative P.Systems



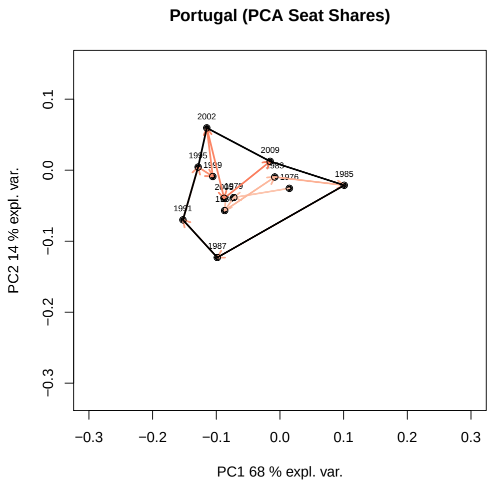
(a) Portugal Seat Shares



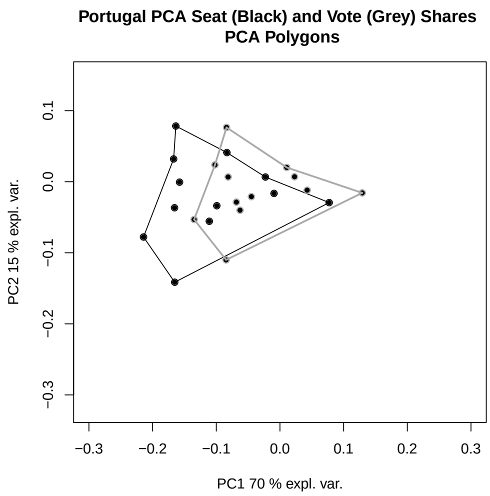
(b) Portugal Seat Shares: Polygon



(c) Portugal Vote Shares



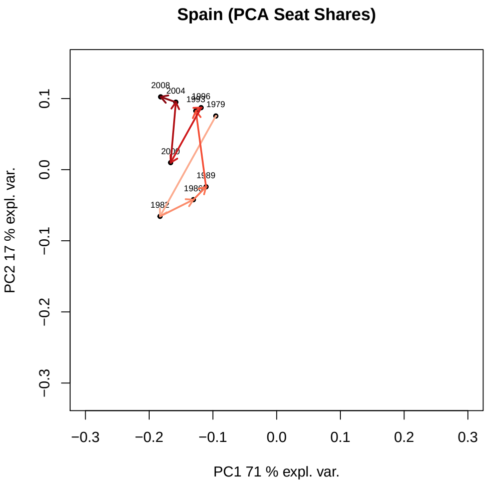
(d) Portugal Vote Shares: Polygon



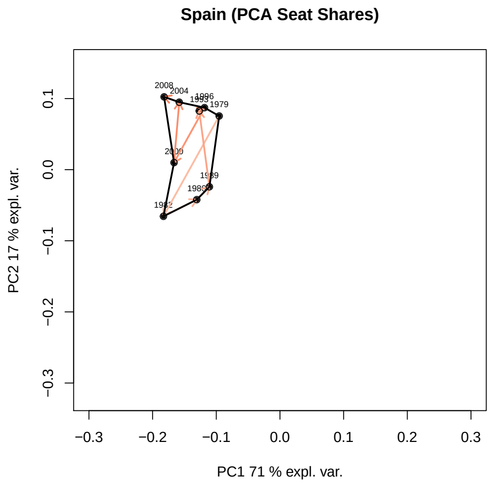
(e)

(f) portugal Seat-Vote Shares Polygons

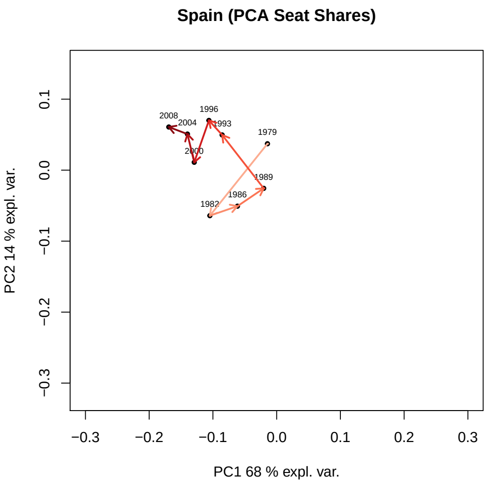
Figure F.18: Portugal Changes in the Electoral and Legislative P.Systems



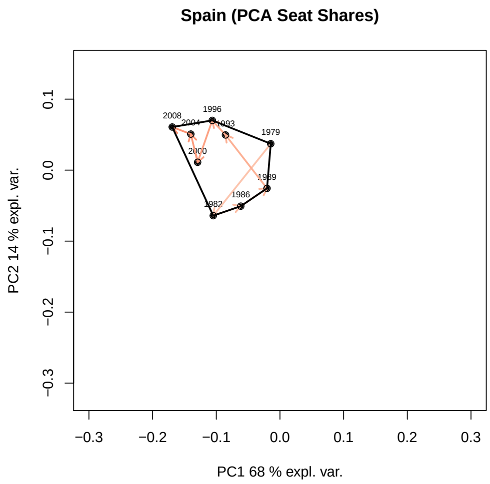
(a) Spain Seat Shares



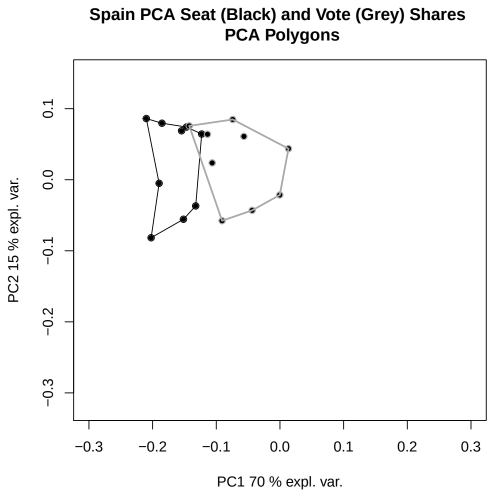
(b) Spain Seat Shares: Polygon



(c) Spain Vote Shares



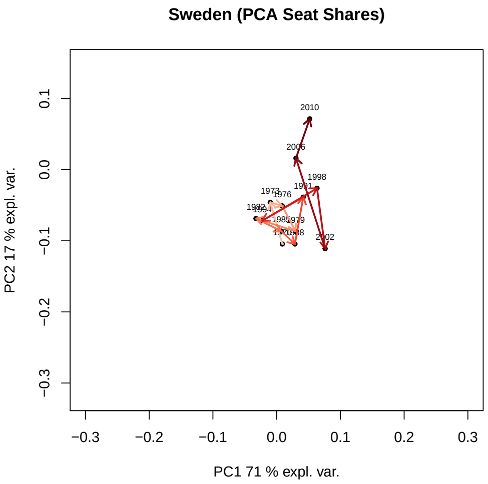
(d) Spain Vote Shares: Polygon



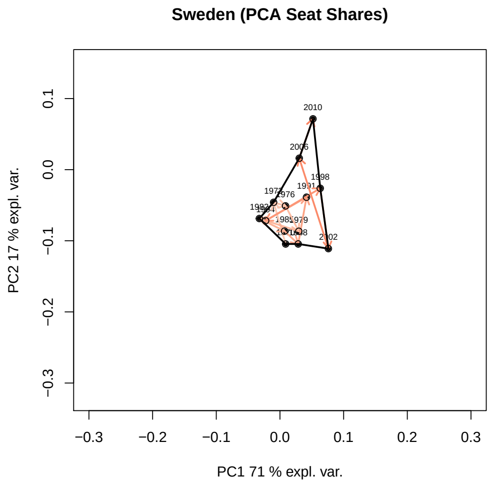
(e)

(f) Spain Seat-Vote Shares Polygons

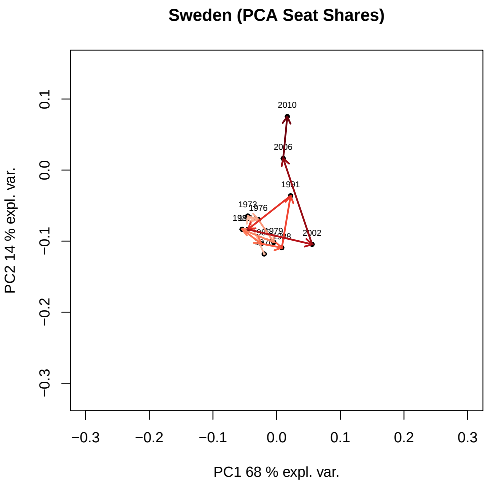
Figure F.19: Spain Changes in the Electoral and Legislative P.Systems



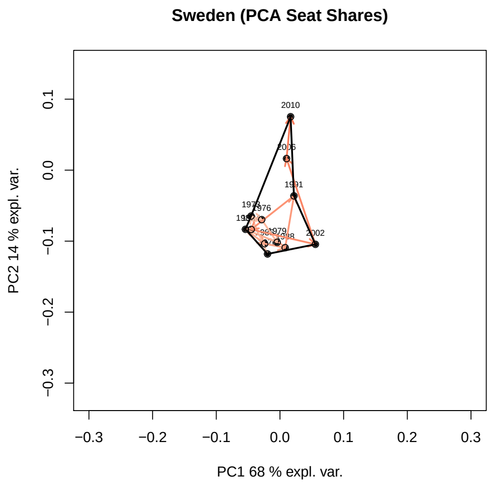
(a) Sweden Seat Shares



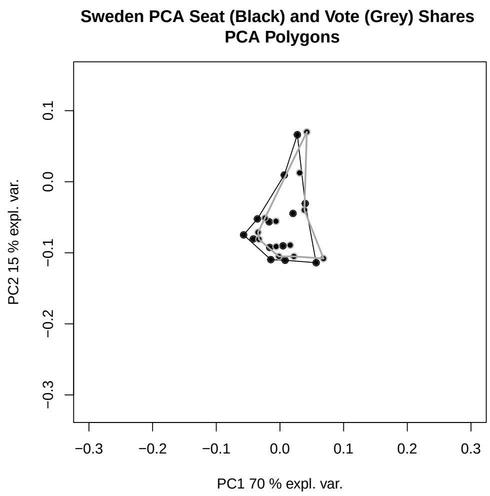
(b) Sweden Seat Shares: Polygon



(c) Sweden Vote Shares



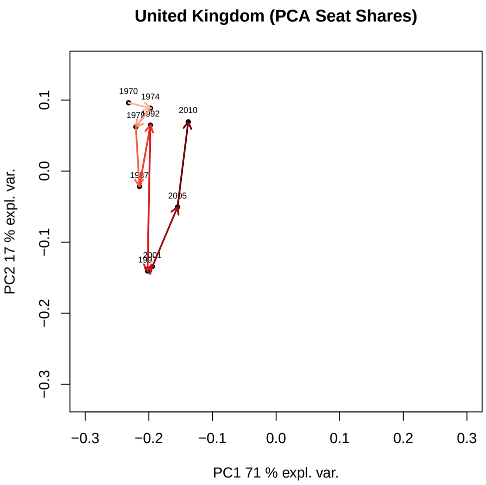
(d) Sweden Vote Shares: Polygon



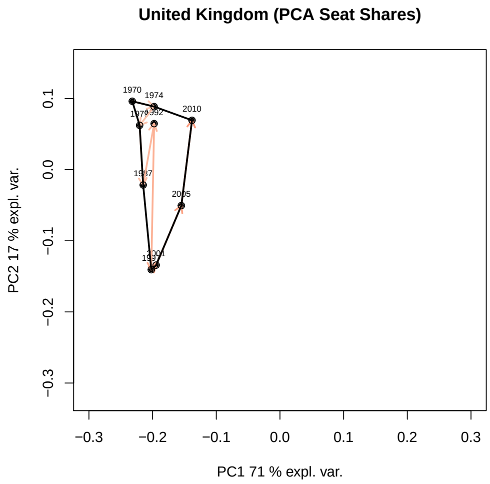
(e)

(f) Sweden Seat-Vote Shares Polygons

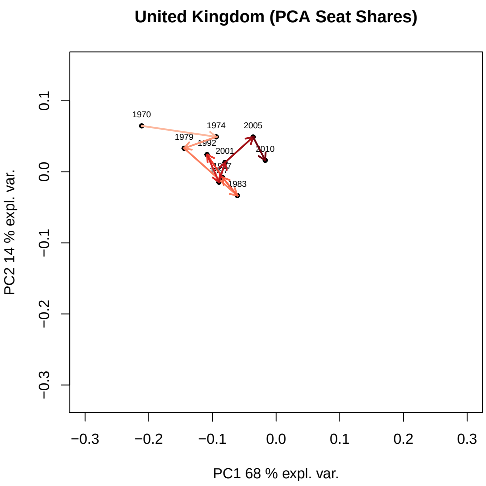
Figure F.20: Sweden Changes in the Electoral and Legislative P.Systems



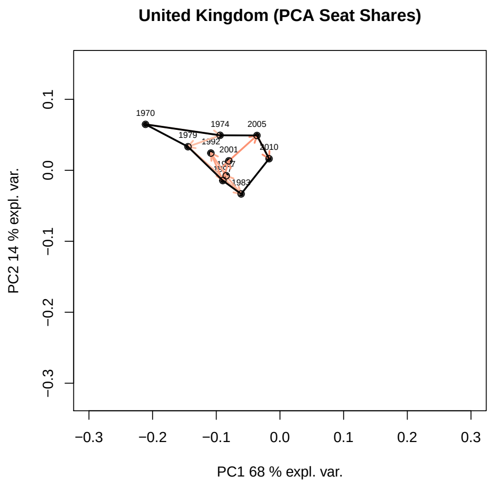
(a) United Kingdom Seat Shares



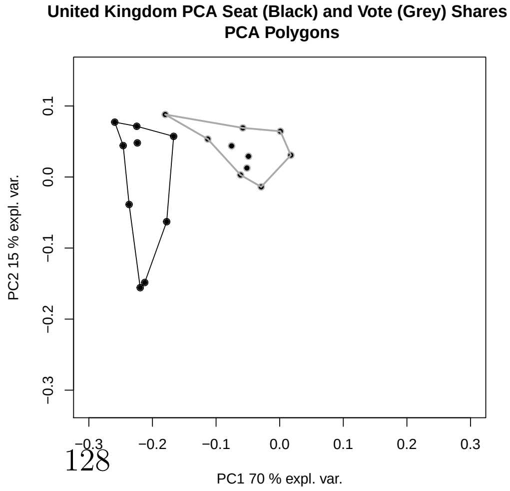
(b) United Kingdom Seat Shares: Polygon



(c) United Kingdom Vote Shares



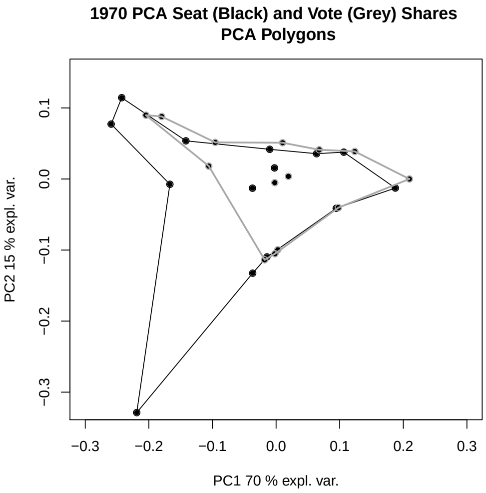
(d) United Kingdom Vote Shares: Polygon



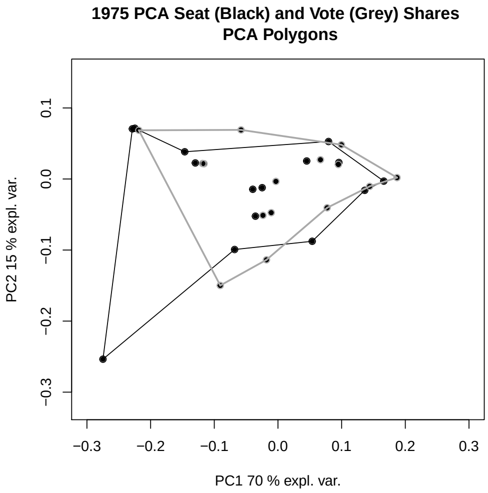
(e)

(f) UK Seat-Vote Shares Polygons

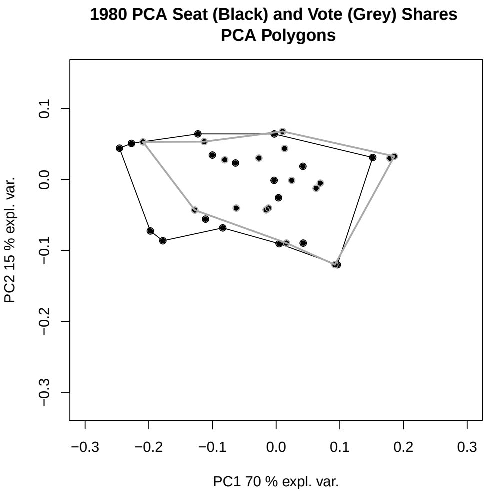
Figure F.21: UK Changes in the Electoral and Legislative P Systems



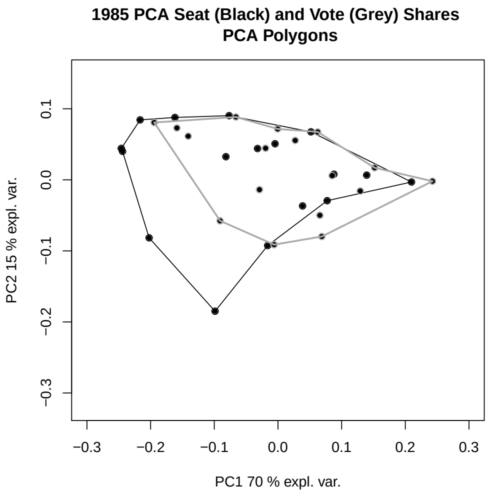
(a) 1970 Seat-Vote Share Polygon



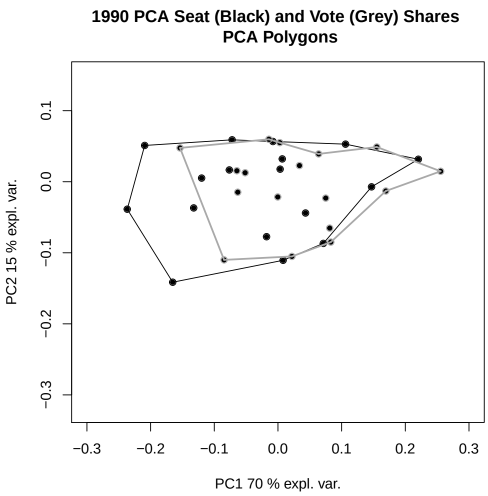
(b) 1975 Seat-Vote Share Polygon



(c) 1980 Seat-Vote Share Polygon



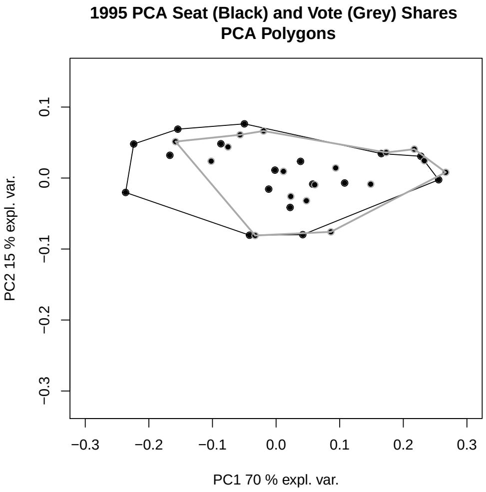
(d) 1985 Seat-Vote Share Polygon



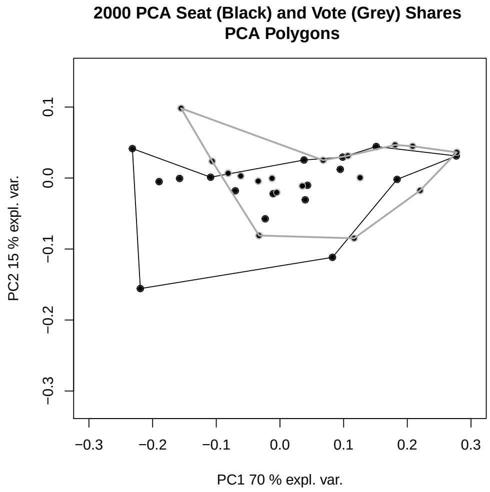
(e)

(f) 1990 Seat-Vote Share Polygon

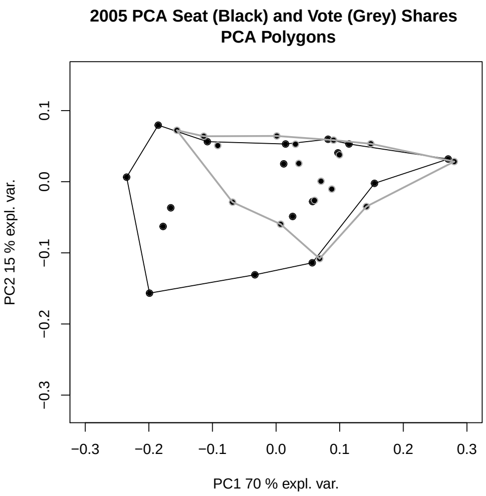
Figure F.22: Years 1970-1990 Changes in the Electoral and Legislative P.Systems



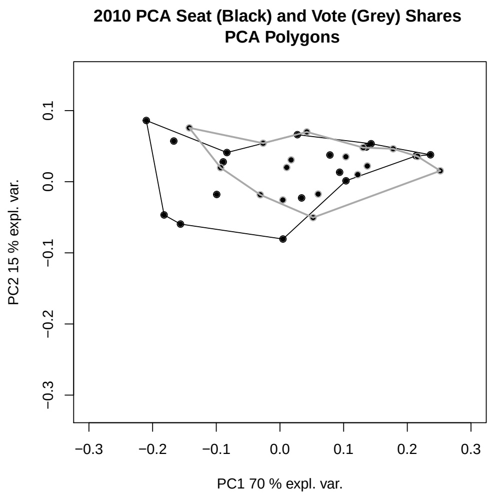
(a) 1995 Seat-Vote Share Polygon



(b) 2000 Seat-Vote Share Polygon



(c) 2005 Seat-Vote Share Polygon



(d) 2010 Seat-Vote Share Polygon

Figure F.23: Years 1995-2010 Changes in the Electoral and Legislative P.Systems

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Powell, G Bingham, and G Bingham Powell Jr. 2000. *Elections as instruments of democracy: Majoritarian and proportional visions*. Yale University Press.

G. The Third Dimension

G.1. Introduction

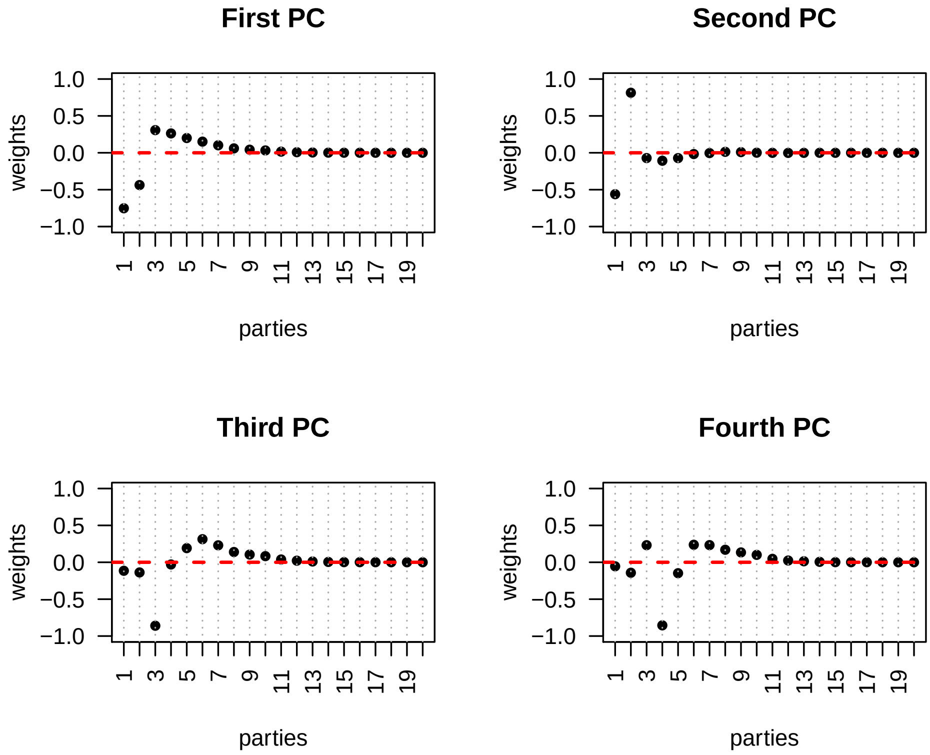
In this appendix, I explore the third PC dimension. In the literature many different selection criteria has been suggested to determine the number of PCA dimensions to retain for analysis. Usually these methods yield to different results and there is no scientific consensus about which one of these methods is the best solution. Thus, after selecting the dimensions we also have to examine whether there is substantive information in the selected dimensions.

Determining the number of significant dimensions can be especially difficult in case that we use the covariance matrix to calculate the PCA (that is we use unstandardized variables in the analysis). This is because, the most popular methods for selecting dimensions, have been adopted from Factor Analysis and suitable only for PCA on the correlation matrix (Jolliffe, 2002). As I discussed earlier, in this paper I use unstandardized variables to analyze the party system size dataset. One measure that can be adopted to evaluate the importance of the eigenvalues that I find through the PCA on the covariance matrix is the Kaiser criterion or Kaiser rule (Kaiser, 1960). According to this rule, if we perform PCA on the correlation matrix we should retain PC dimensions whose variance exceeds one, which means that their eigen values are above one. In the case of the covariance matrix, this rule may be adopted so that we evaluate how many of the eigen values are above the mean of all the eigen values (Jolliffe, 2002). With this criterion I find that three PC dimensions may carry meaningful information in the PCA on party seat shares. I also find this using cross-validation, by using random samples of the data. This third dimension explains 8% variation in the data.

The PC loadings show that *Dimension 3* is most heavily influenced by the size of the third party, and so it contrasts countries that have a big third party with countries that have a small third party (I call this dimension "Third Party") *Figure G.1.* In this appendix I examine first, how the countries moved on this third dimension throughout the years. Second, I examine whether the Rokkan (1970) and Sartori (1976) classifications separate the countries on this third dimension. Finally, I calculate how the indices that measure the party system size relate to this dimension. Overall, I find that while according to the Kaiser criterion this dimension may contain important information for the analysis it has not been incorporated into the party system measures. I argue that this may be so because the the movement of the countries of this dimension is small and also the countries move along this dimension frequently. This means that this dimension may nor help substantively in separating alike and disparate party systems.

G.2. Country-Year movements

Below, I plot how the party systems change on the first and third and on the first and second dimensions throughout the years in the 17 countries Further, I calculate the standard



Notes: The plot shows the loadings, the weight of parties in determining the principal components in the PCA.

Figure G.1: Loadings, PCA

Table G.1: Standard Deviations on Dimensions

Country	seats_sd_x	seats_sd_y	seats_sd_z	votes_sd_x	votes_sd_y	votes_sd_z
Austria	0.12	0.04	0.05	0.12	0.03	0.06
Belgium	0.11	0.02	0.05	0.11	0.02	0.05
Denmark	0.03	0.05	0.03	0.04	0.05	0.03
Finland	0.03	0.03	0.02	0.02	0.02	0.01
France	0.10	0.10	0.05	0.08	0.05	0.05
Germany	0.07	0.03	0.03	0.07	0.03	0.03
Greece	0.04	0.08	0.01	0.05	0.07	0.02
Iceland	0.05	0.03	0.04	0.05	0.03	0.03
Ireland	0.05	0.06	0.03	0.06	0.03	0.03
Italy	0.12	0.03	0.02	0.10	0.03	0.02
Luxembourg	0.04	0.05	0.03	0.04	0.04	0.03
Netherlands	0.07	0.03	0.05	0.07	0.02	0.04
Norway	0.08	0.05	0.02	0.07	0.05	0.02
Portugal	0.09	0.06	0.04	0.07	0.05	0.04
Spain	0.03	0.07	0.01	0.05	0.05	0.02
Sweden	0.03	0.05	0.05	0.03	0.06	0.04
United Kingdom	0.03	0.09	0.02	0.05	0.03	0.04

deviations on *Dimension 3* to be able to assess which countries move the most on this dimension. I present the results in Table G.1. The table shows that the countries that move the most (based on calculations on the PCA on vote shares) are Austria, Belgium, France the Netherlands and Sweden on *Dimension 3*.

We can look at the plots of these movements to see when exactly the movement happened in the countries. I present these plots in *Figures G.7 to G.18*. Austria had two big parties before the elections of 1995, but the competition changed to a three way tie between the biggest parties after 1995. In Belgium the competition became more even between the second and the third parties by 1990, and by the 2000 the leading 5 parties almost evenly split the vote. In France, because of the two-round majority plurality voting system, the balance between the second and the third party can change quite quickly. *Figure G.6* shows this rapid change. In the Netherlands, the changes are also very quick on *Dimension 3*. In this fragmented party system sometimes the second biggest party catches up to the biggest party (2010, 2006, 2003, 1998, 1989, 1986, 1981, 1977, 1971) and sometimes the first three parties or the second and the third parties have evenly balanced seat shares (2002, 1994, 1982, 1972). In Sweden, in the 1970s and 1980s the Socialist party was dominant and the second and the third parties were evenly balanced. This changes sharply in 1998 when the center right becomes important and the third party becomes much smaller than the two biggest parties. Overall, the table and the plots show that few countries change their positions on *Dimension 3* these changes are frequent.

G.3. Other Measures

I also explore whether we can compare the third dimension to any of the previous party system measures or indices. I plot the Rokkan classification and the Sartori classification using *Dimension 3* in Figures G.19 and G.20. However, there seem to be no correlation between the grouping of the countries and their positions on *Dimension 3*.

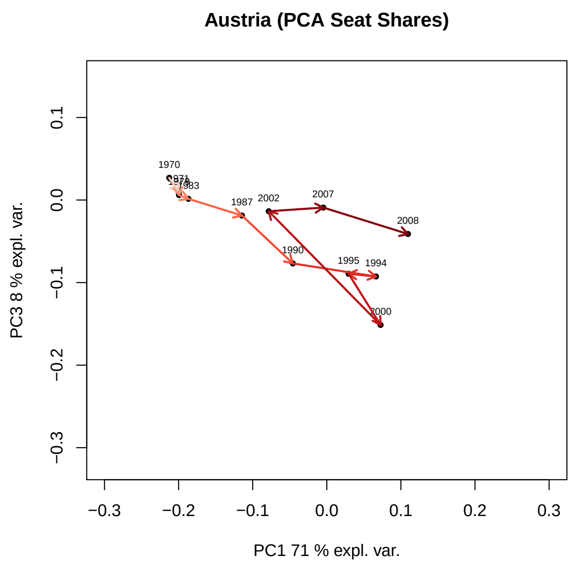
In addition, I calculate a correlation table that compares the various indices to the third dimension. As Table G.2 shows, I only find weak correlations between the indices and *Dimension 3*. Finally, in Figure G.21 I calculate and plot the *Sum of Ranking Differences* of the indices that I discuss in the main body of the paper. *Sum of Ranking Differences* is a methodology developed to evaluate the success of models, measures, methods (Bajusz, Rácz, and Héberger, 2015). The method ranks the variables across the cases and compares to the reference method, measure or model. To make the solution easier I eliminate repeated rows in the matrix and I discard the idea to have a ranking for each county-year. My final matrix has 200 rows. My columns are the various indices. I calculate how the objects (rows) are ranked by the solutions (columns). In addition, I calculate how the objects (rows) are ranked by *Dimension 3*. Next, I deduct the rankings of *Dimension 3* from the rankings of the indices. The sum of these differences is the *Sum of Ranking Differences* of *Dimension 3*. As the PCA is non-directional, I repeat the ranking in descending order as well and I calculate the SRD-s. Out of the two sums I get for each index (ascending, descending) I choose the lower number. The closer this final value is to zero the better the index approximates *Dimension 3* (Héberger and Kollár-Hunek, 2011; Bajusz, Rácz, and Héberger, 2015; Kalivas, Héberger, and Andries, 2015). After this I normalize the SRD numbers. According to Héberger and Kollár-Hunek (2011) we can normalize the SRD values such as $SRD_{nor} = 100/SRD_{max}$ where

$$SRD_{max} = \begin{cases} 2\sum_{j=1}^{k}(2j-1) = 2k^2 \text{ if } n_r = 2k \\ 4\sum_{j=1}^{k} j = 2k(k+1) \text{ if } n_r = 2k+1 \end{cases} \tag{2}$$

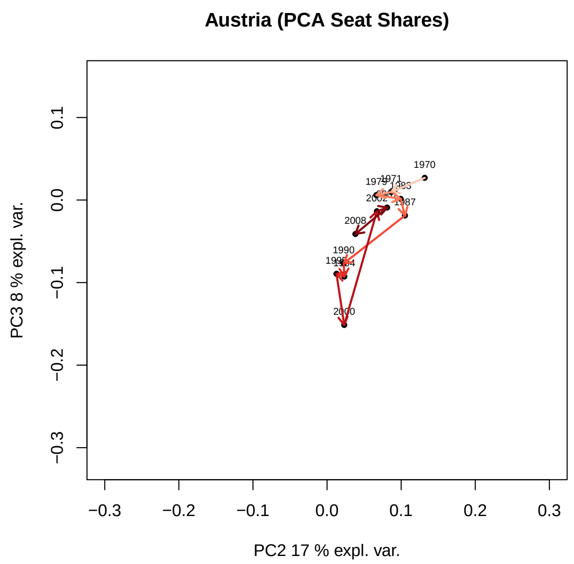
After the normalization I calculate the theoretical distribution that would arise from a large number of measures that randomly ranks the cases. Since, I have 200 rows ,more than 13 required according to Héberger and Kollár-Hunek (2011) I can approximate this curve with a normal distribution. I estimate the mean and standard deviation of this curve by bootstrapping random samples from the original dataset. In Figure G.21 I plot the indices and the normal distribution that results from this algorithm. As all the lines fall within the normal curve, the plot shows that none of the indices are better to measure *Dimension 3* than a random index would be.

Overall, few countries move significantly on *Dimension 3* but those countries move rapidly. Maybe due to this it seems that this dimension has not been used by political scientists to classify or measure party systems. While I do not analyze this issue dimension in the main body of the paper it may be useful to think about the usefulness of this dimension for the classification of party systems in the future.

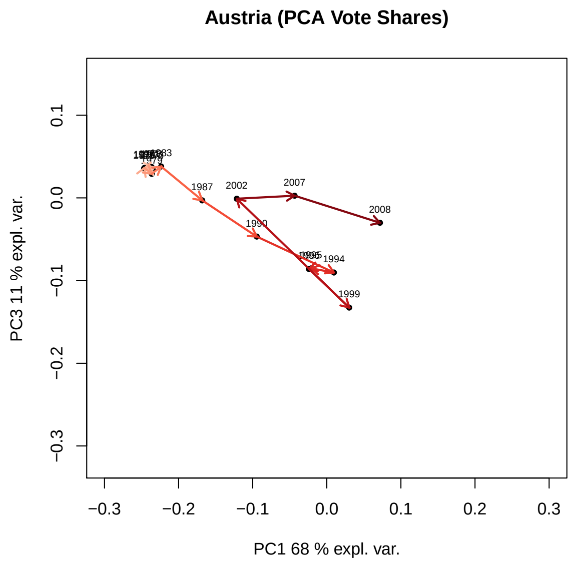
G.4. Country-year Plots on the Third Dimension



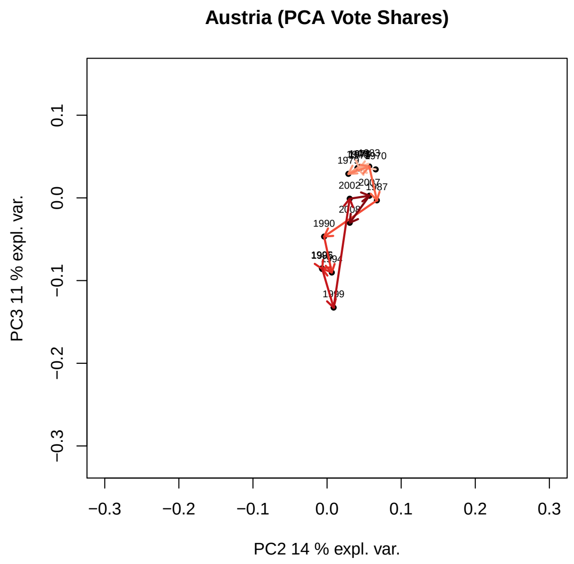
(a) Austria PCA Dimension1-Dimension3-Seats



(b) Austria PCA Dimension2-Dimension3-Seats

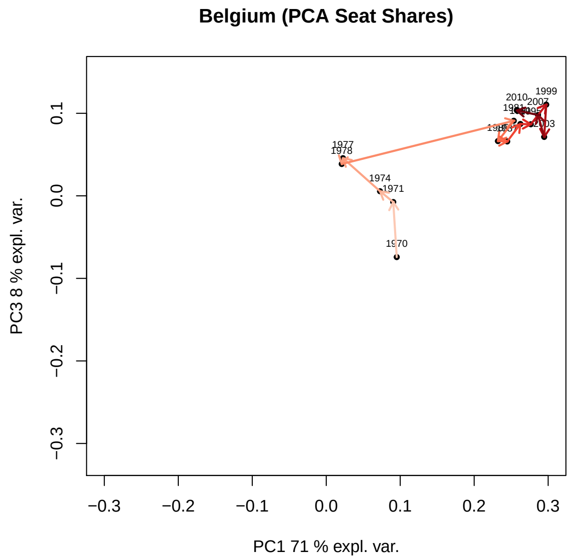


(c) Austria PCA Dimension1-Dimension3-Votes

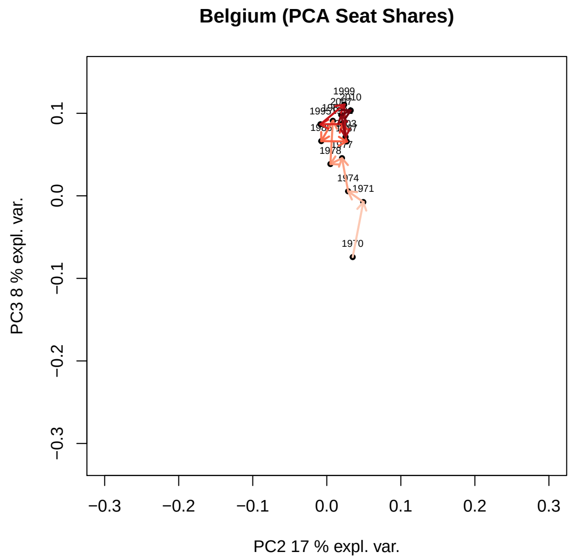


(d) Austria PCA Dimension2-Dimension3-Votes

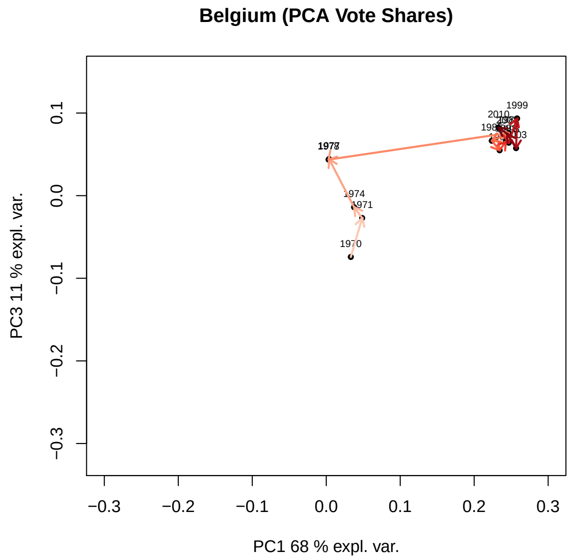
Figure G.2: Notes: The third dimension Austria



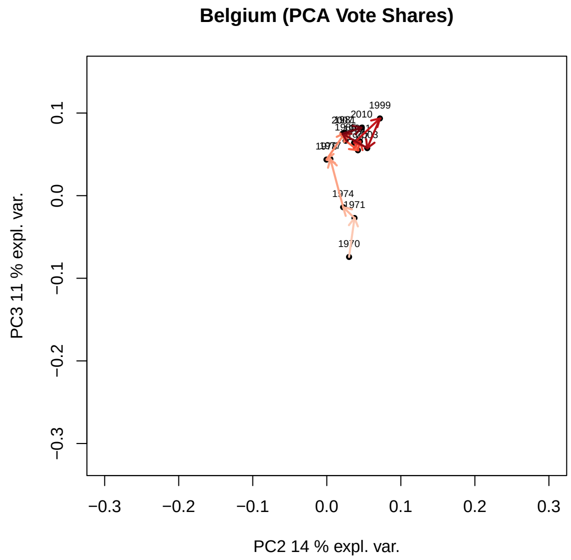
(a) Belgium PCA Dimension1-Dimension3-Seats



(b) Belgium PCA Dimension2-Dimension3-Seats

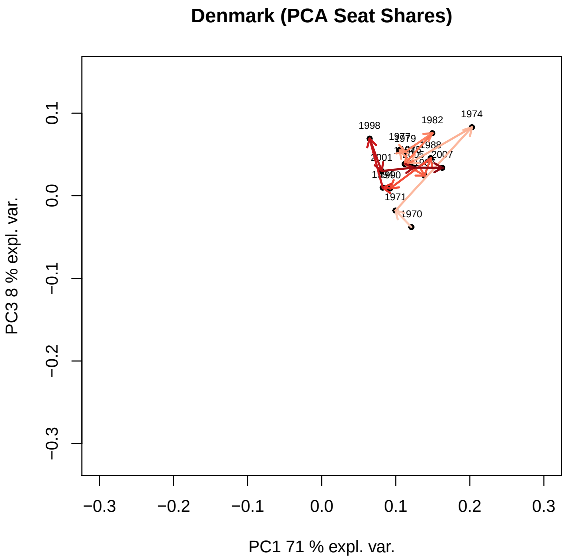


(c) Belgium PCA Dimension1-Dimension3-Votes

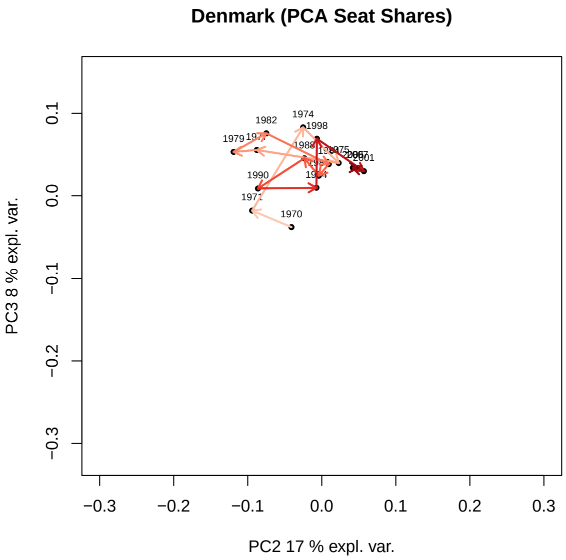


(d) Belgium PCA Dimension2-Dimension3-Votes

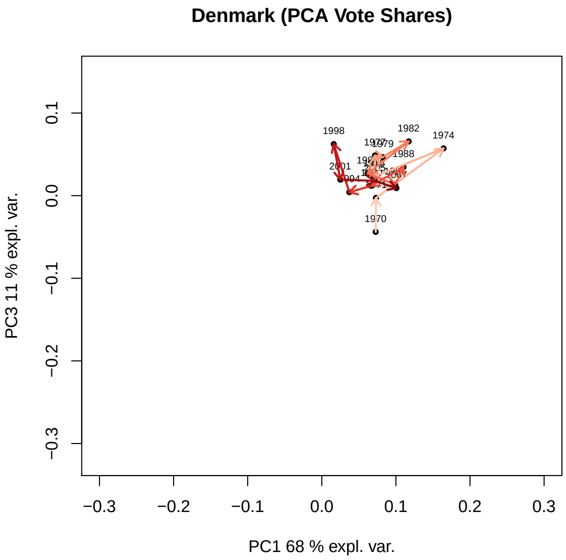
Figure G.3: Notes: The third dimension Belgium



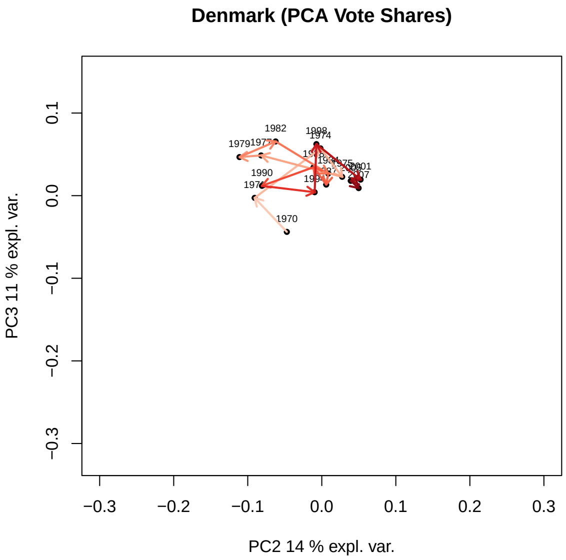
(a) Denmark PCA Dimension1-Dimension3-Seats



(b) Denmark PCA Dimension2-Dimension3-Seats

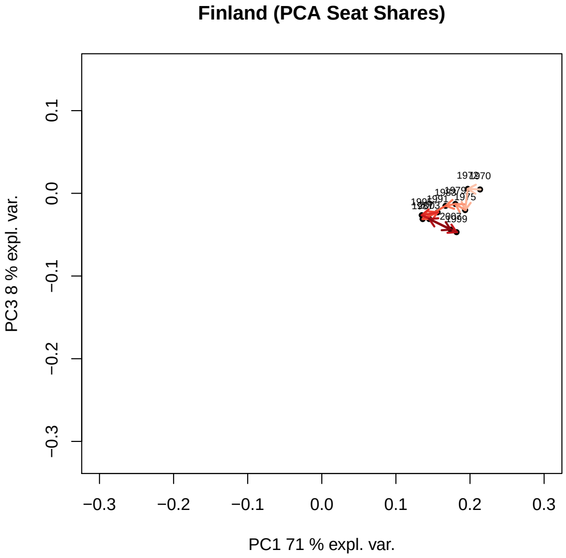


(c) Denmark PCA Dimension1-Dimension3-Votes

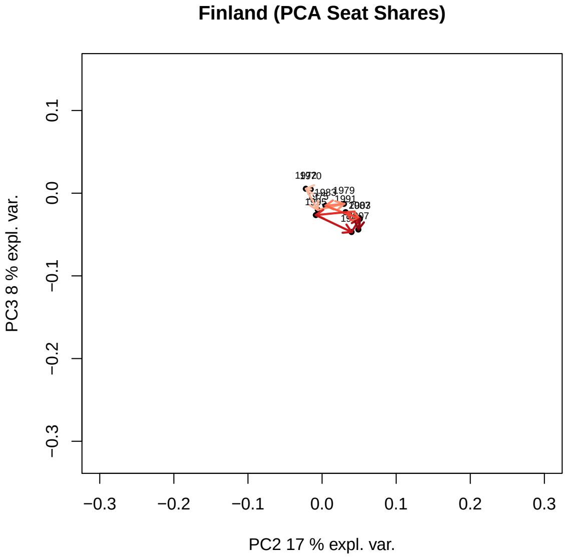


(d) Denmark PCA Dimension2-Dimension3-Votes

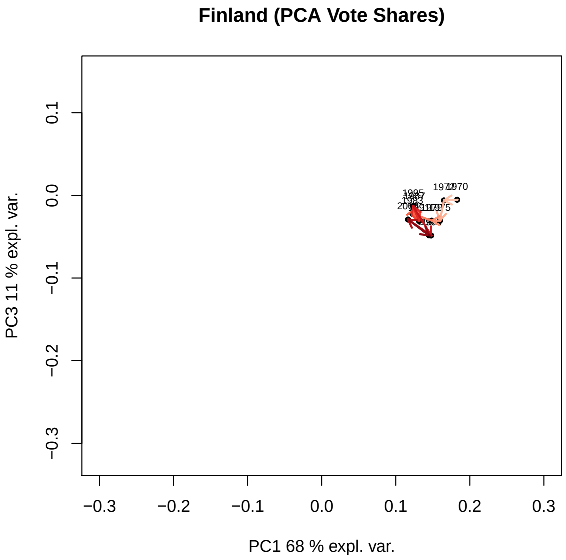
Figure G.4: Notes: The third dimension Denmark



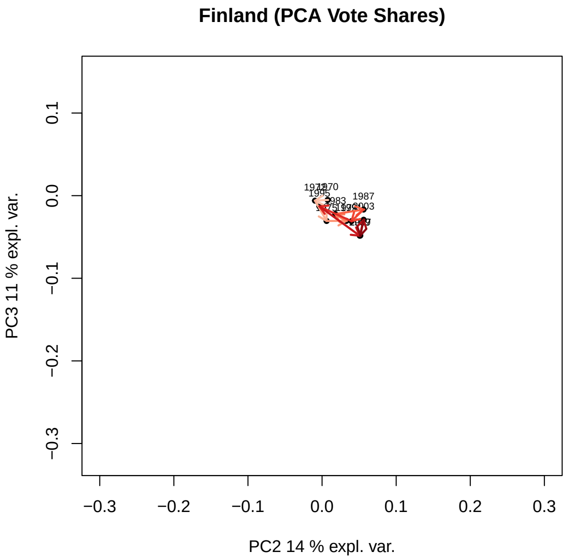
(a) Finland PCA Dimension1-Dimension3-Seats



(b) Finland PCA Dimension2-Dimension3-Seats

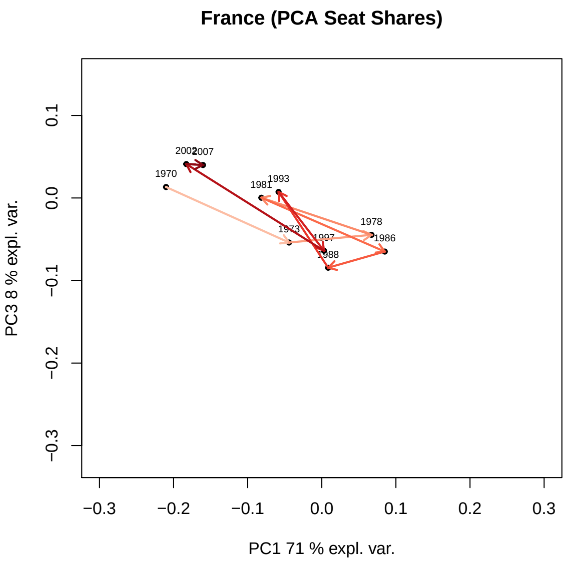


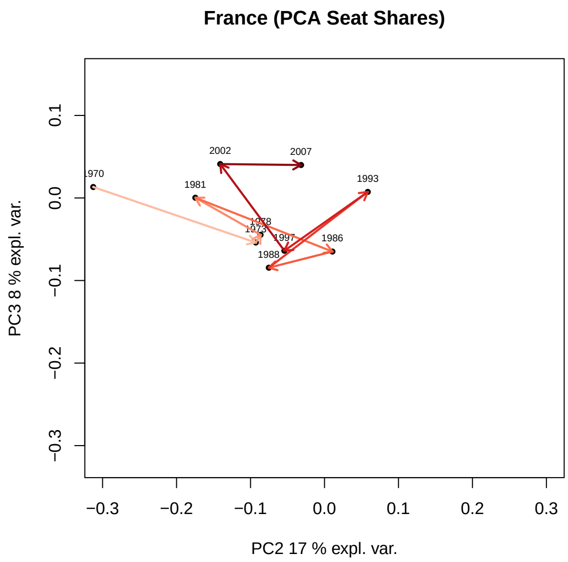
(c) Finland PCA Dimension1-Dimension3-Votes



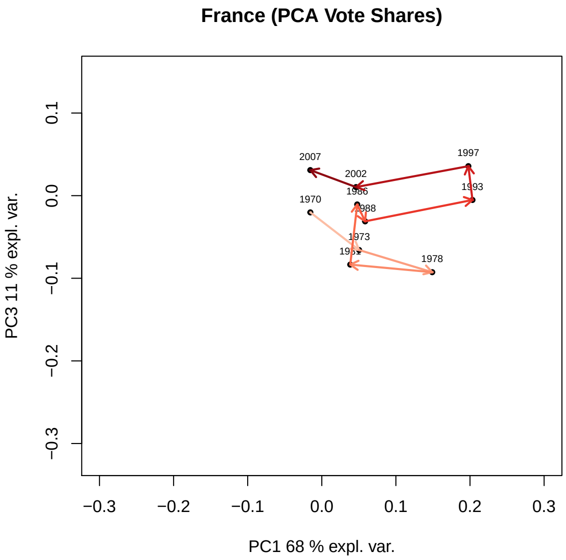
(d) Finland PCA Dimension2-Dimension3-Votes

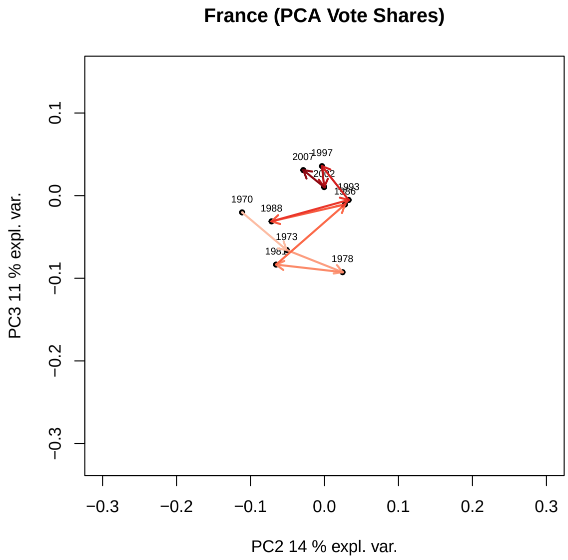
Figure G.5: Notes: The third dimension Finland





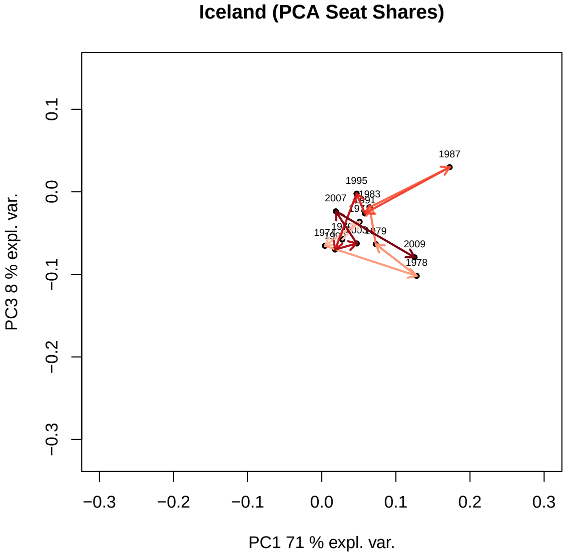
(a) France PCA Dimension1-Dimension3-Seats (b) France PCA Dimension2-Dimension3-Seats



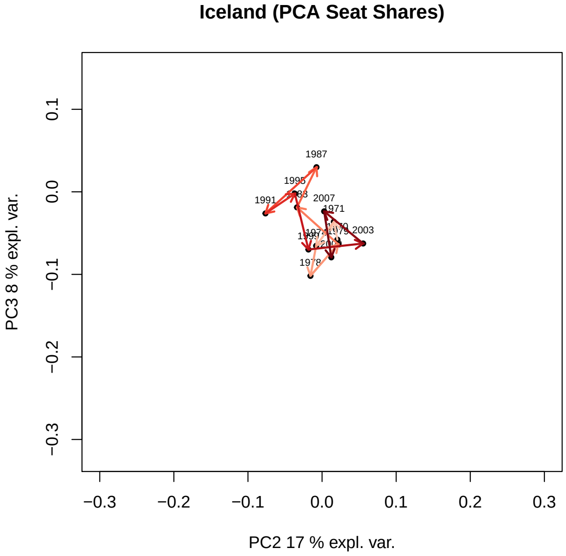


(c) France PCA Dimension1-Dimension3-Votes (d) France PCA Dimension2-Dimension3-Votes

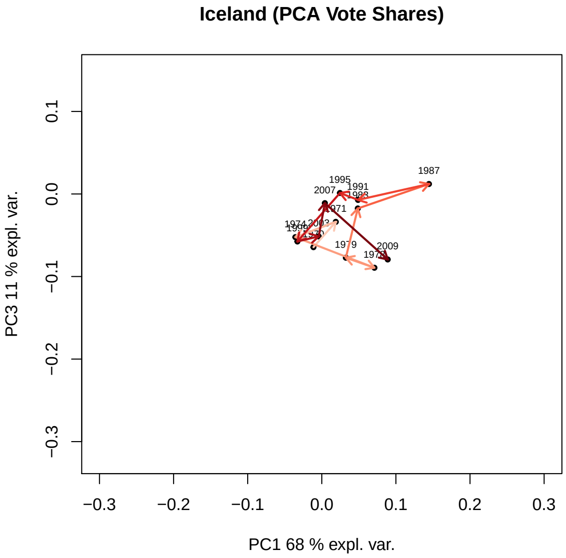
Figure G.6: Notes: The third dimension France



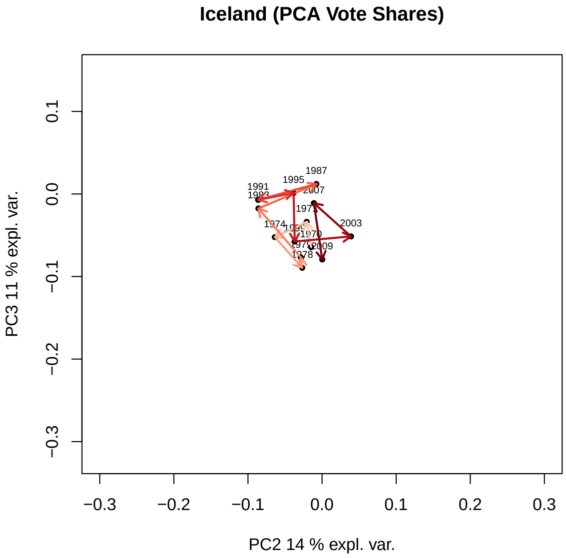
(a) Germany PCA Dimension1-Dimension3-Seats



(b) Germany PCA Dimension2-Dimension3-Seats

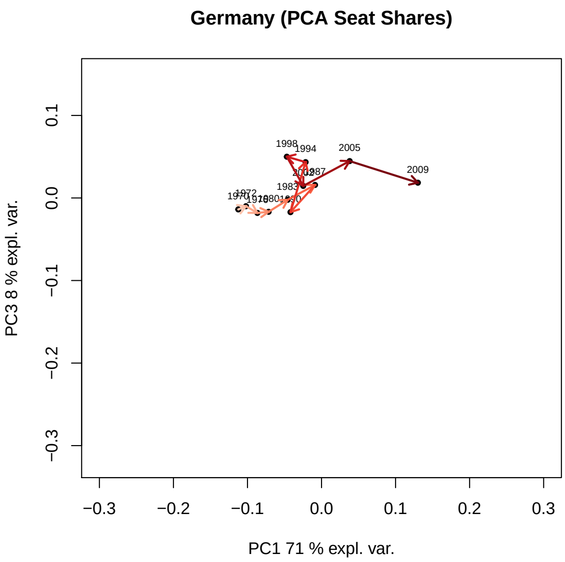


(c) Germany PCA Dimension1-Dimension3-Votes

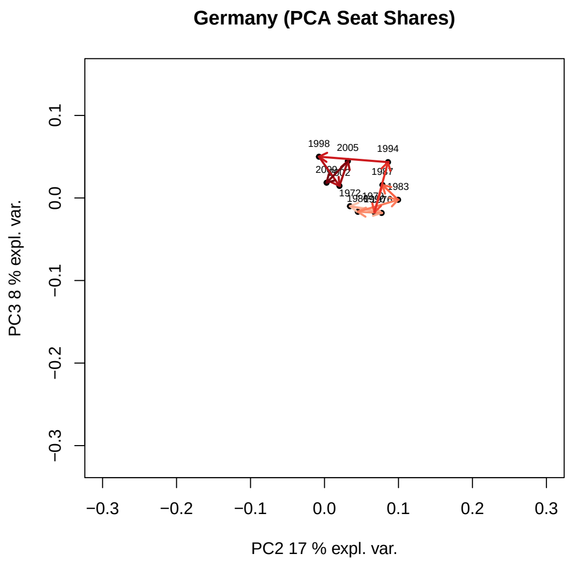


(d) Germany PCA Dimension2-Dimension3-Votes

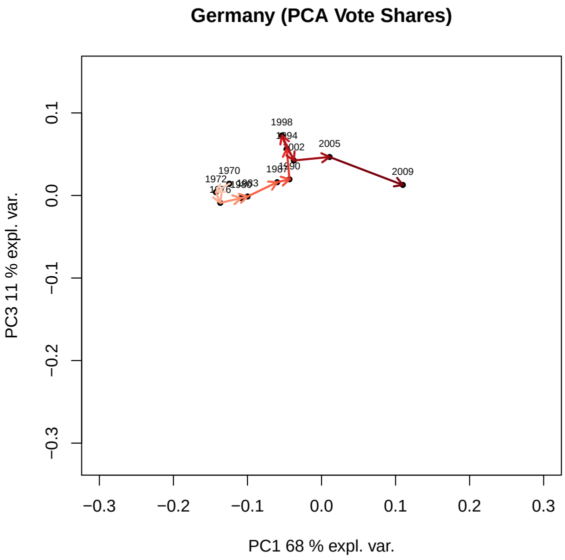
Figure G.7: Notes: The third dimension Germany



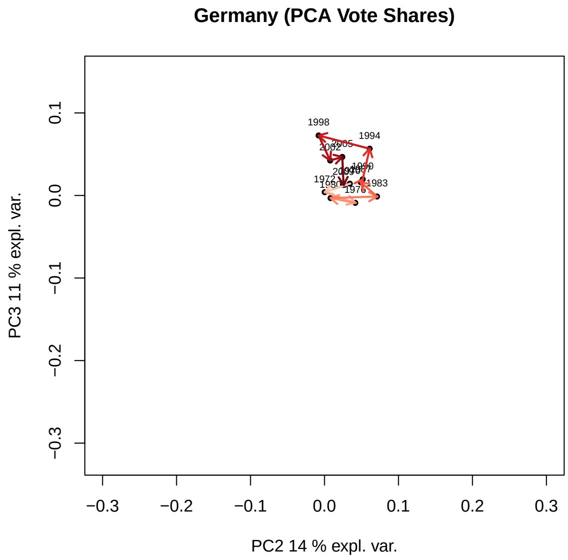
(a) Greece PCA Dimension1-Dimension3-Seats



(b) Greece PCA Dimension2-Dimension3-Seats

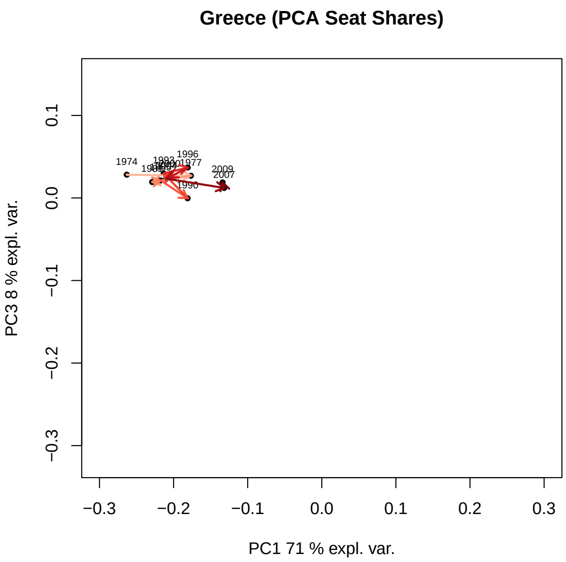


(c) Greece PCA Dimension1-Dimension3-Votes

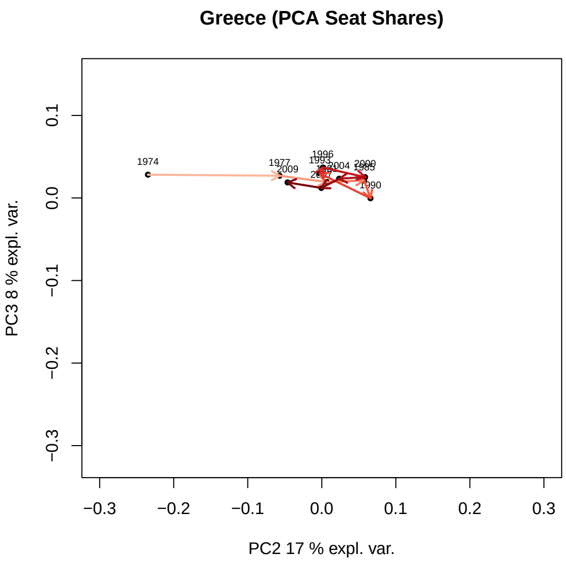


(d) Greece PCA Dimension2-Dimension3-Votes

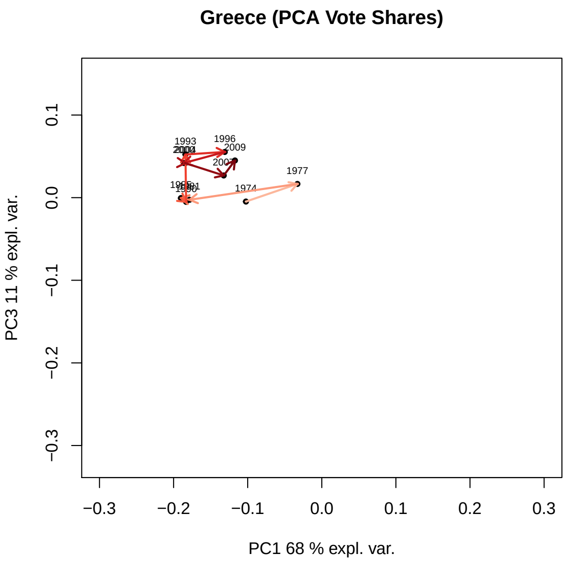
Figure G.8: Notes: The third dimension Greece



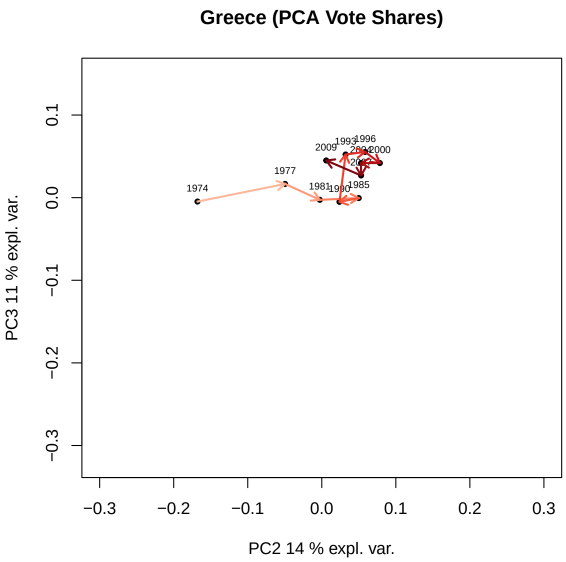
(a) Iceland PCA Dimension1-Dimension3-Seats



(b) Iceland PCA Dimension2-Dimension3-Seats

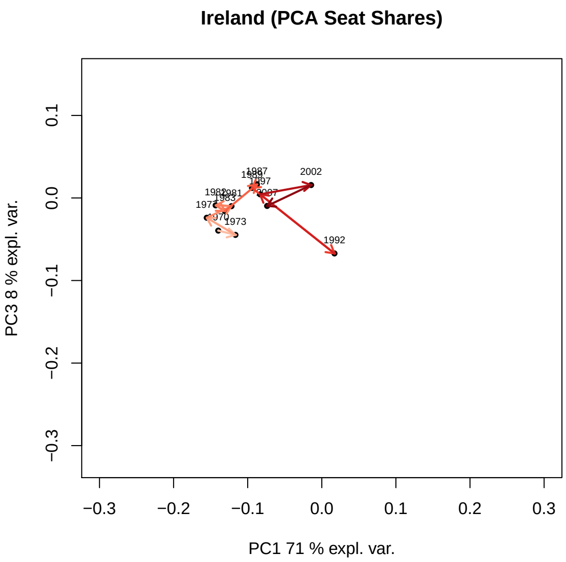


(c) Iceland PCA Dimension1-Dimension3-Votes

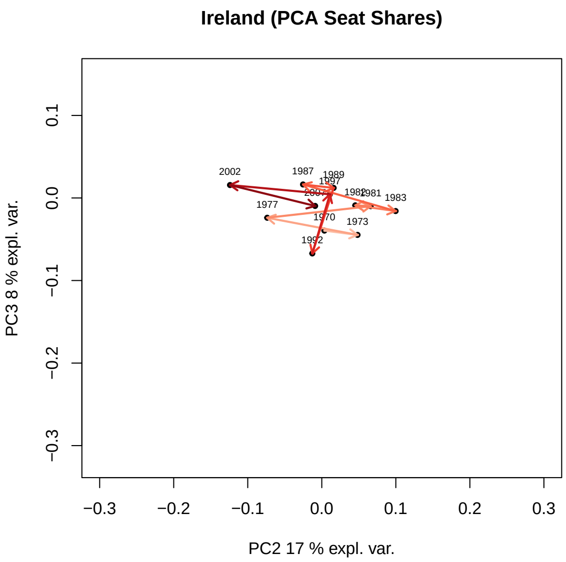


(d) Iceland PCA Dimension2-Dimension3-Votes

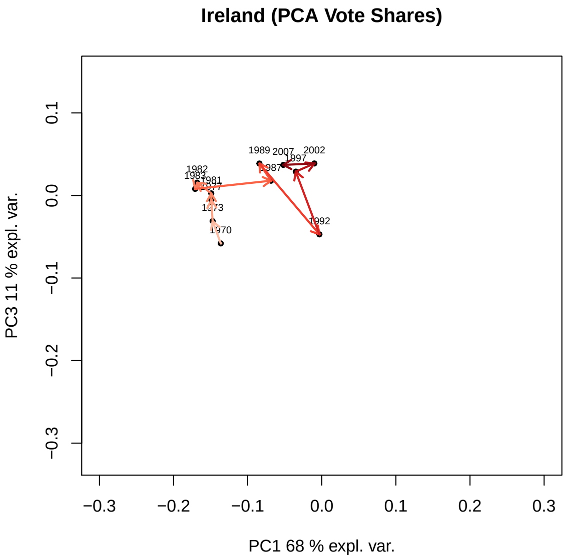
Figure G.9: Notes: The third dimension Iceland



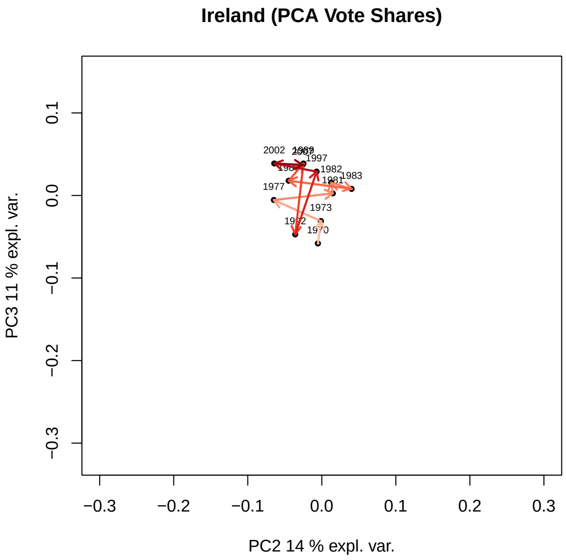
(a) Ireland PCA Dimension1-Dimension3-Seats



(b) Ireland PCA Dimension2-Dimension3-Seats

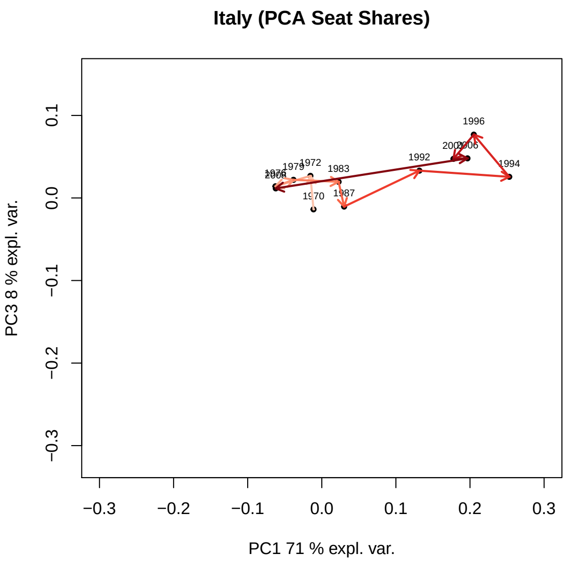


(c) Ireland PCA Dimension1-Dimension3-Votes

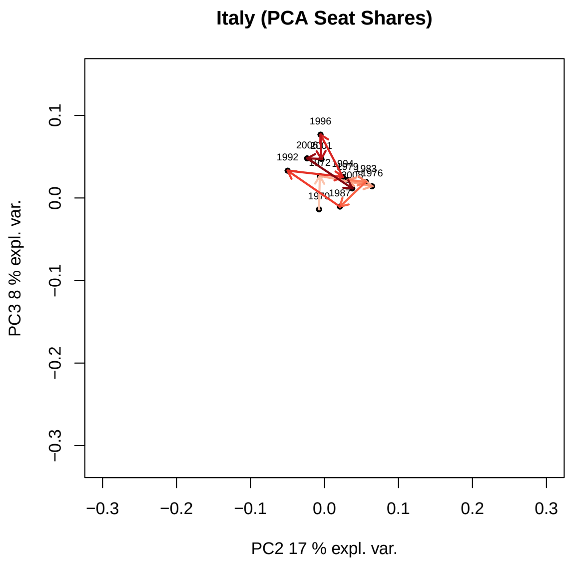


(d) Ireland PCA Dimension2-Dimension3-Votes

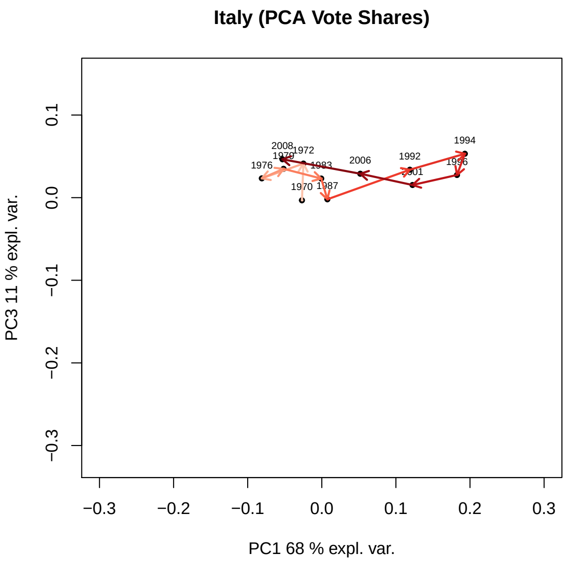
Figure G.10: Notes: The third dimension Ireland



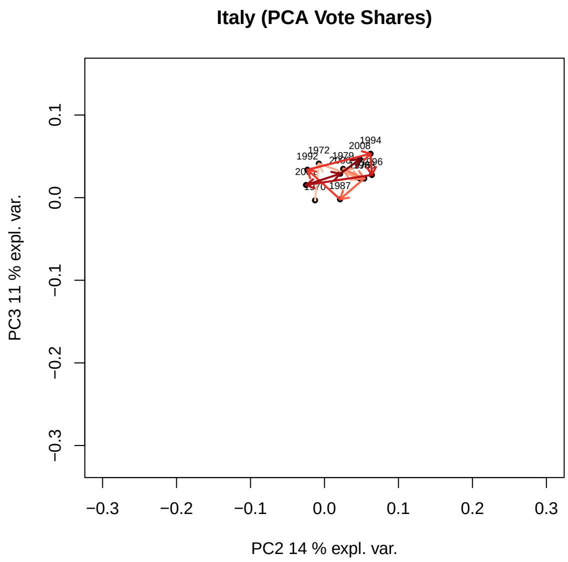
(a) Italy PCA Dimension1-Dimension3-Seats



(b) Italy PCA Dimension2-Dimension3-Seats

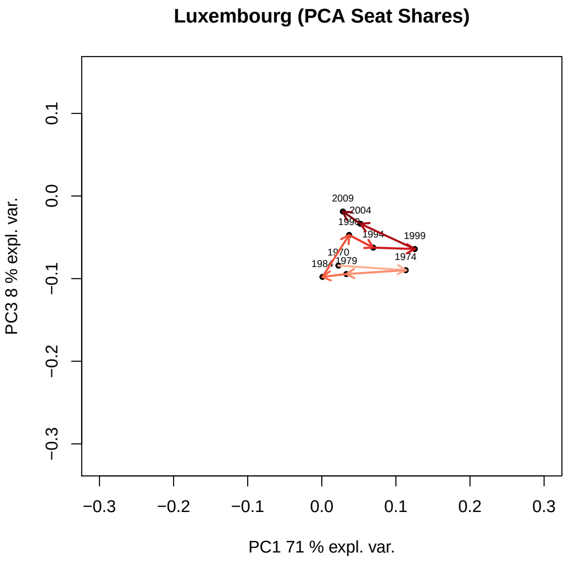


(c) Italy PCA Dimension1-Dimension3-Votes

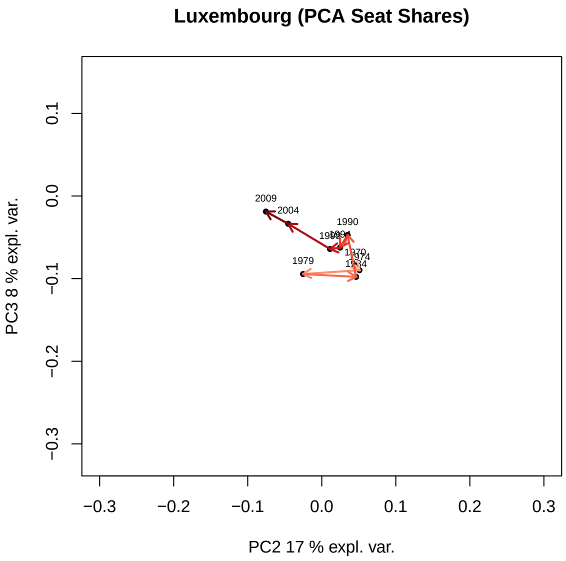


(d) Italy PCA Dimension2-Dimension3-Votes

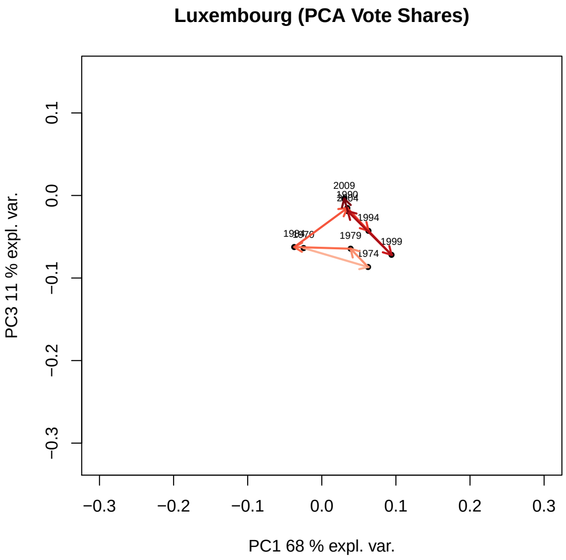
Figure G.11: Notes: The third dimension Italy



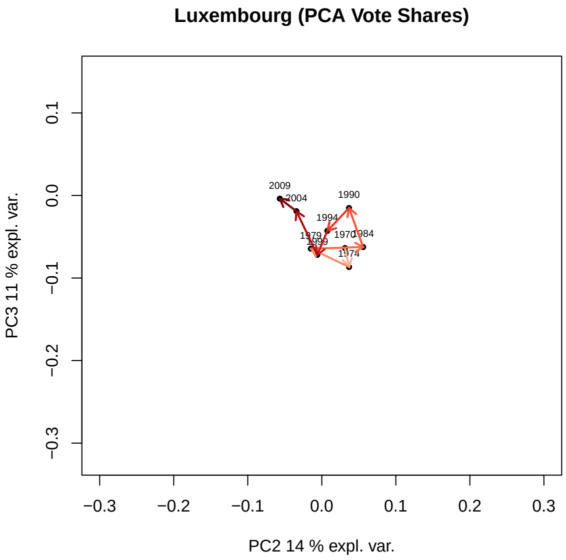
(a) Luxembourg PCA Dimension1-Dimension3-Seats



(b) Luxembourg PCA Dimension2-Dimension3-Seats

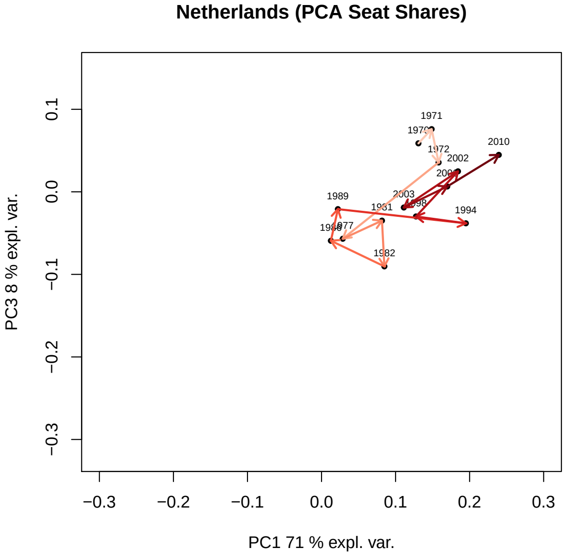


(c) Luxembourg PCA Dimension1-Dimension3-Votes

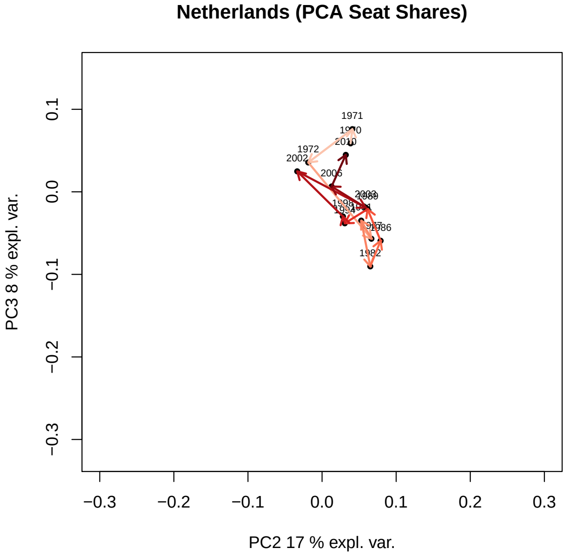


(d) Luxembourg PCA Dimension2-Dimension3-Votes

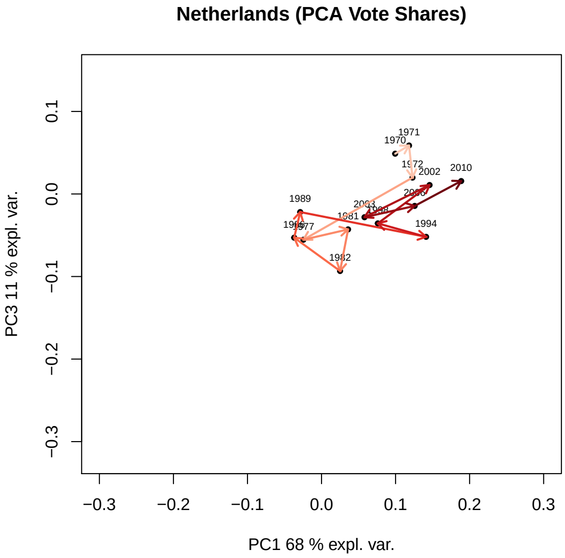
Figure G.12: Notes: The third dimension Luxembourg



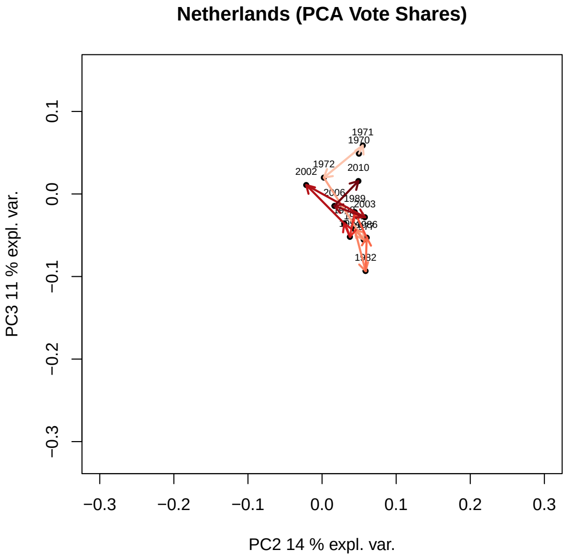
(a) Netherlands PCA Dimension1-Dimension3-Seats



(b) Netherlands PCA Dimension2-Dimension3-Seats

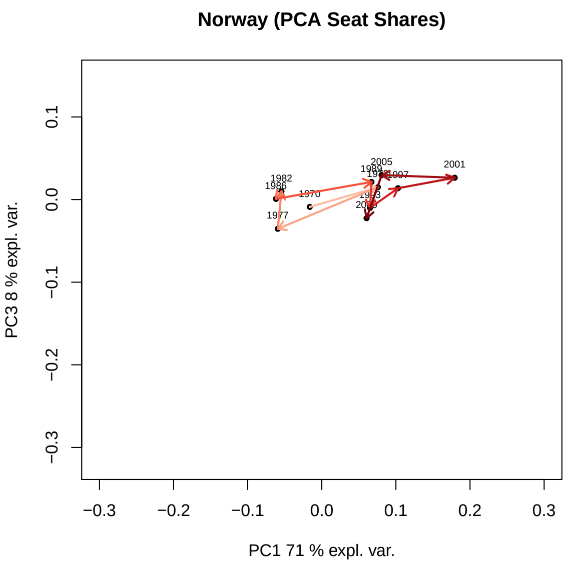


(c) Netherlands PCA Dimension1-Dimension3-Votes

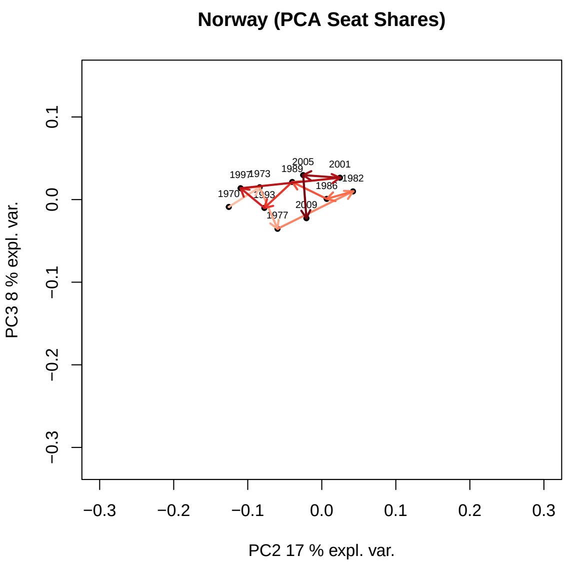


(d) Netherlands PCA Dimension2-Dimension3-Votes

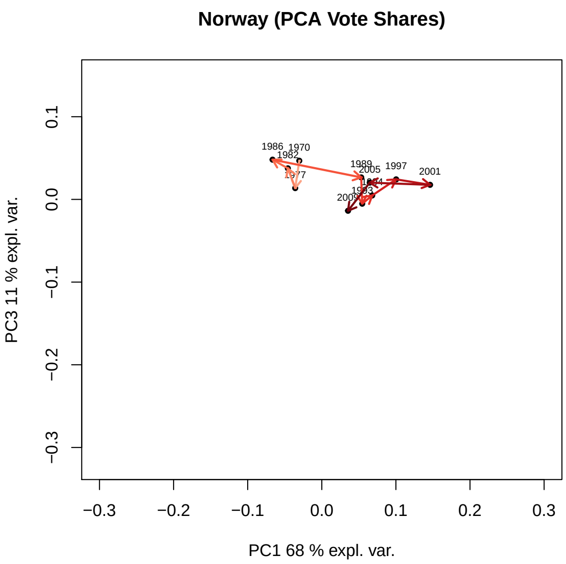
Figure G.13: Notes: The third dimension Netherlands



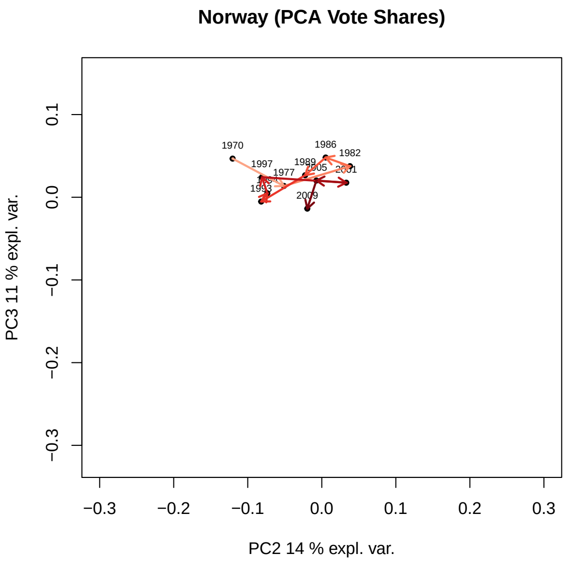
(a) Norway PCA Dimension1-Dimension3-Seats



(b) Norway PCA Dimension2-Dimension3-Seats

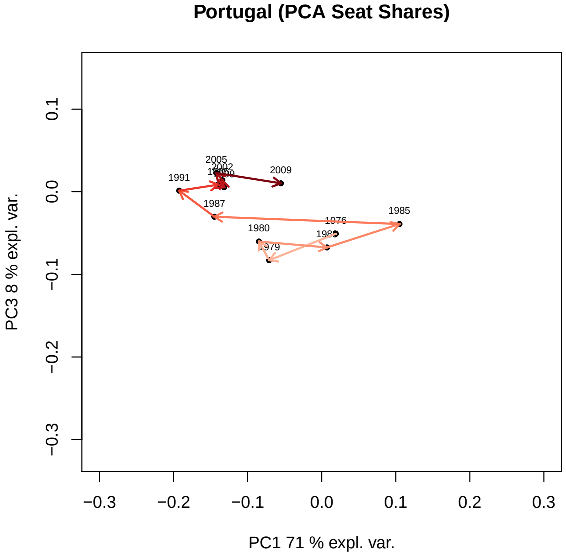


(c) Norway PCA Dimension1-Dimension3-Votes

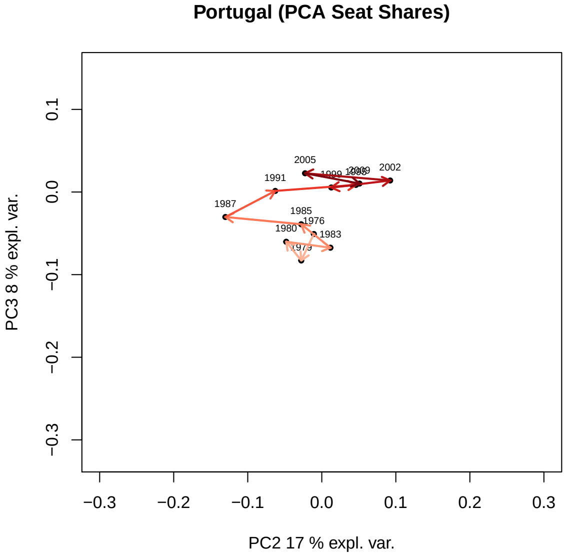


(d) Norway PCA Dimension2-Dimension3-Votes

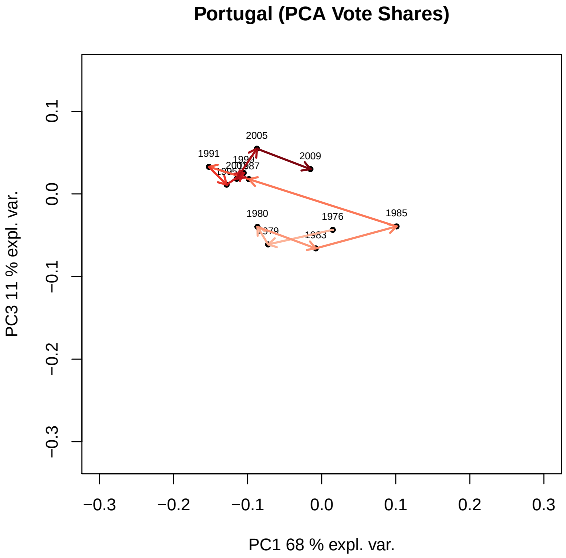
Figure G.14: Notes: The third dimension Norway



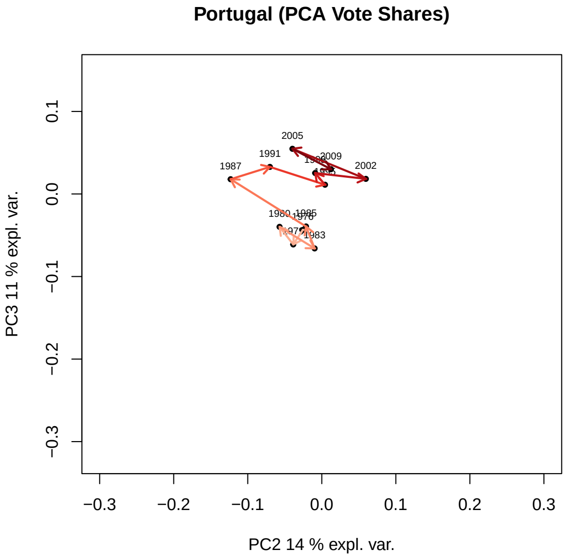
(a) Portugal PCA Dimension1-Dimension3-Seats



(b) Portugal PCA Dimension2-Dimension3-Seats

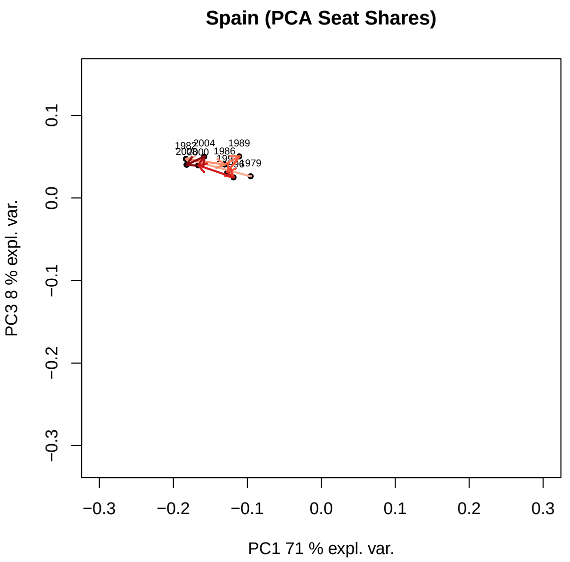


(c) Portugal PCA Dimension1-Dimension3-Votes

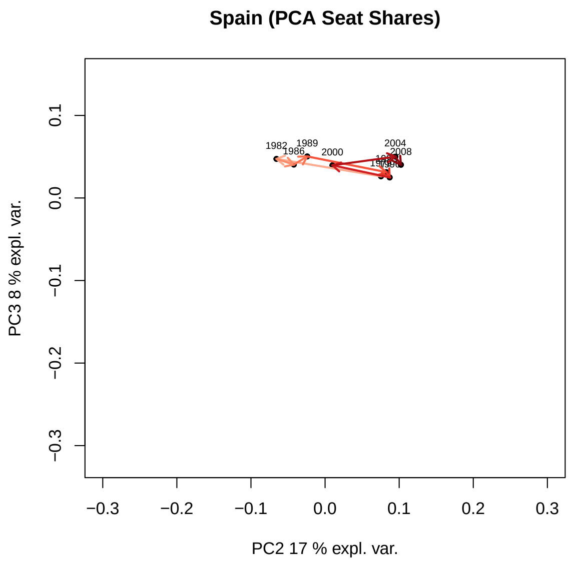


(d) Portugal PCA Dimension2-Dimension3-Votes

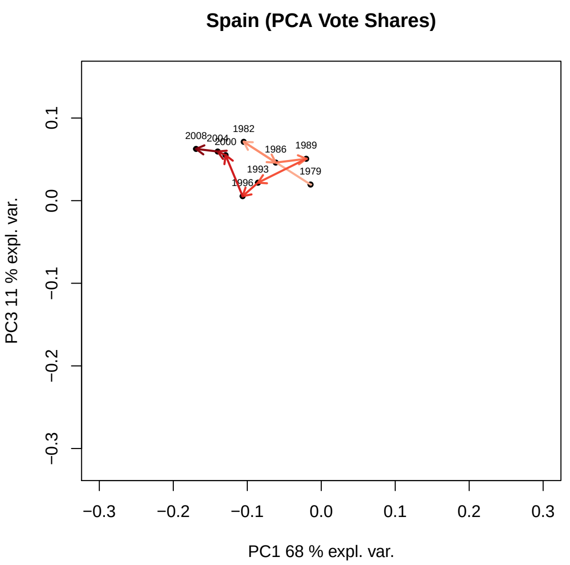
Figure G.15: Notes: The third dimension Portugal



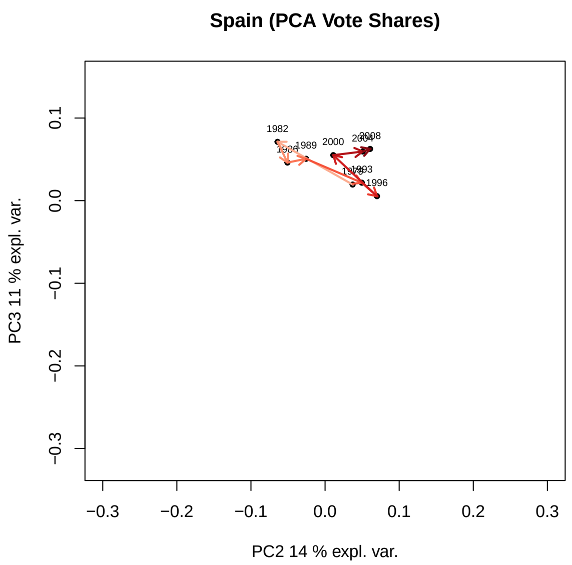
(a) Spain PCA Dimension1-Dimension3-Seats



(b) Spain PCA Dimension2-Dimension3-Seats

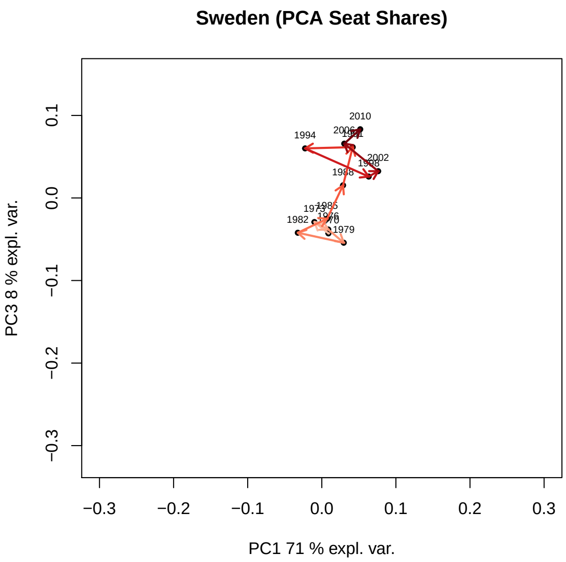


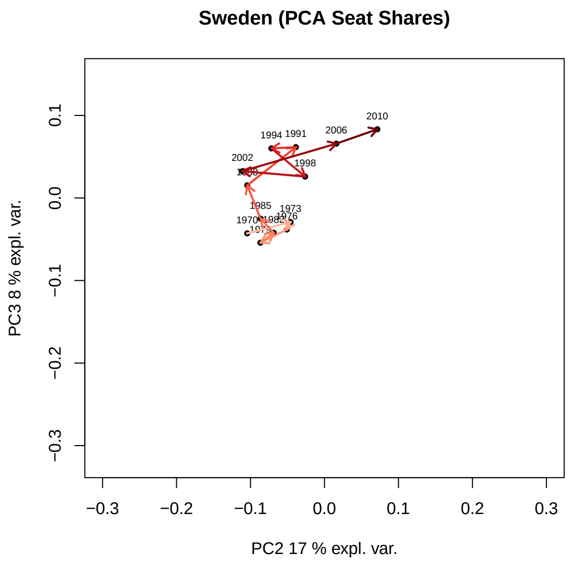
(c) Spain PCA Dimension1-Dimension3-Votes



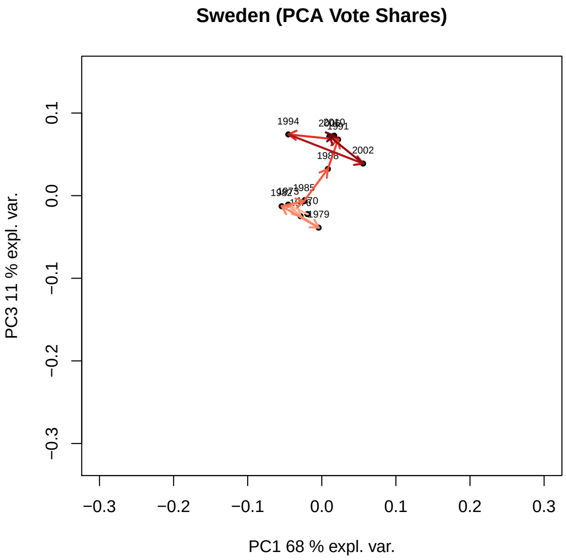
(d) Spain PCA Dimension2-Dimension3-Votes

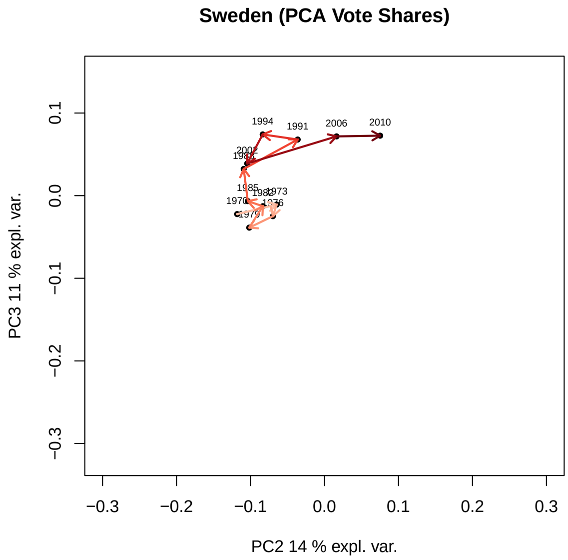
Figure G.16: Notes: The third dimension Spain





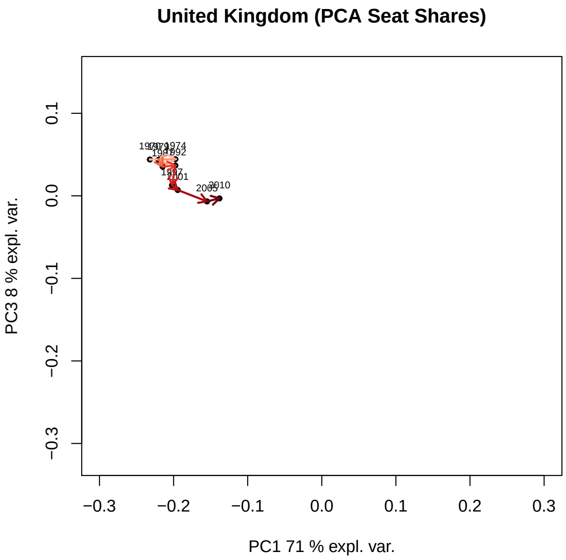
(a) Sweden PCA Dimension1-Dimension3-Seats (b) Sweden PCA Dimension2-Dimension3-Seats



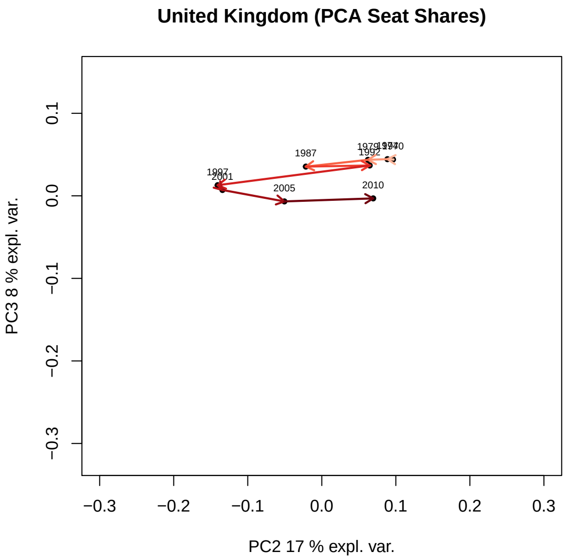


(c) Sweden PCA Dimension1-Dimension3-Votes (d) Sweden PCA Dimension2-Dimension3-Votes

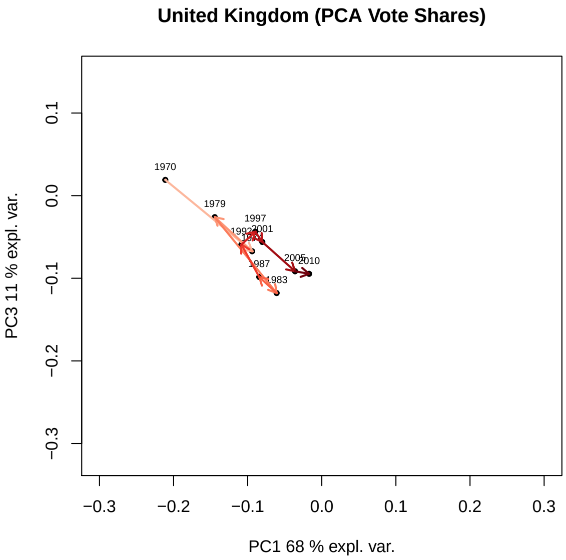
Figure G.17: Notes: The third dimension Sweden



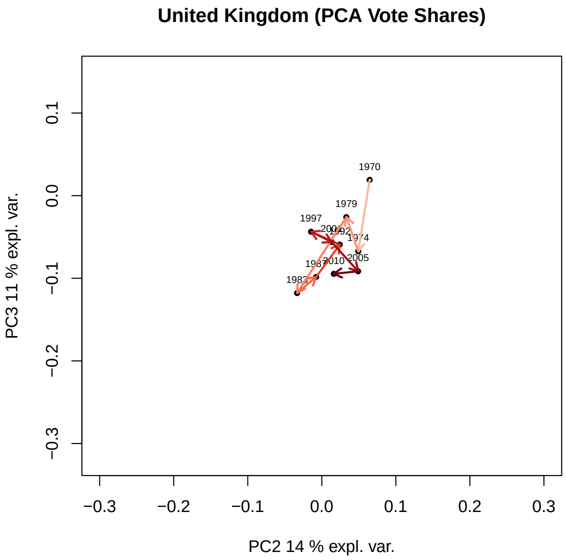
(a) United Kingdom PCA Dimension1-Dimension3-Seats



(b) United Kingdom PCA Dimension2-Dimension3-Seats



(c) United Kingdom PCA Dimension1-Dimension3-Votes

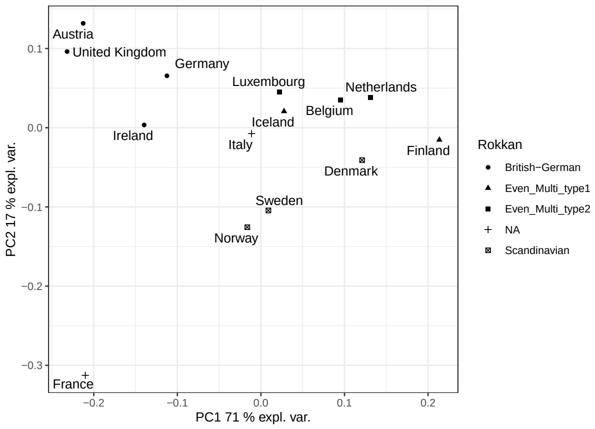


(d) United Kingdom PCA Dimension2-Dimension3-Votes

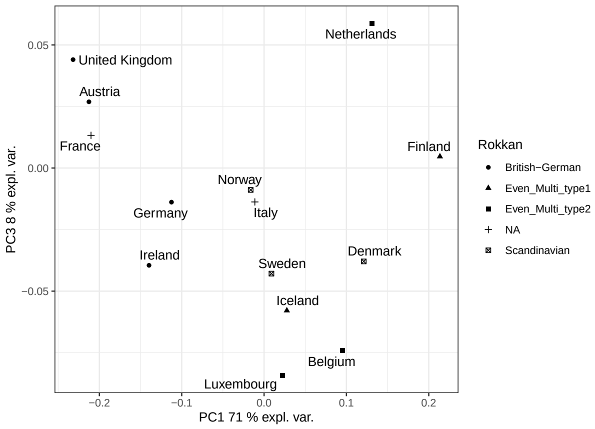
Figure G.18: Notes: The third dimension United Kingdom

G.5. Rokkan and Sartori in Three Dimensions

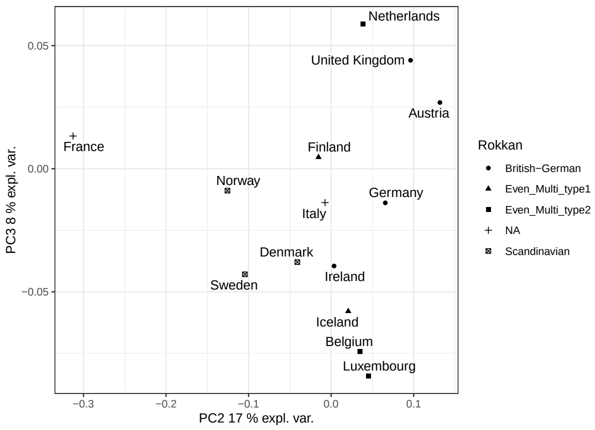
Rokkan



(a) Rokkan PC1-PC2



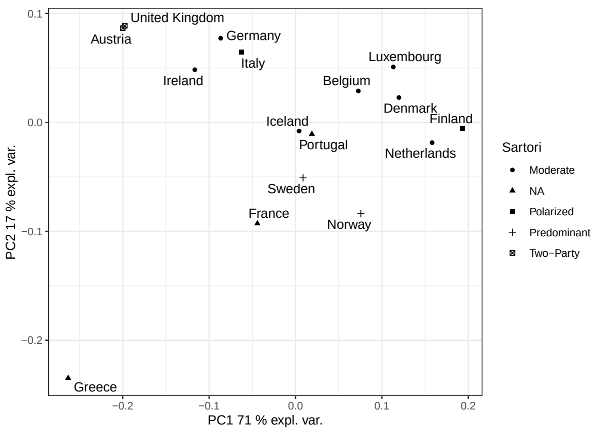
(b) Rokkan PC1-PC3



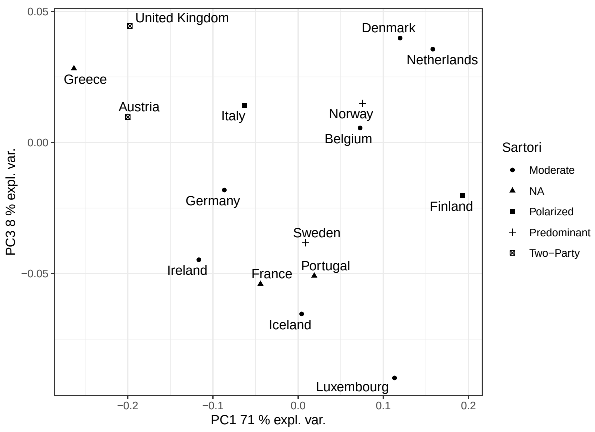
(c) Rokkan PC2-PC3

Figure G.19: Rokkan's classification on three dimensions

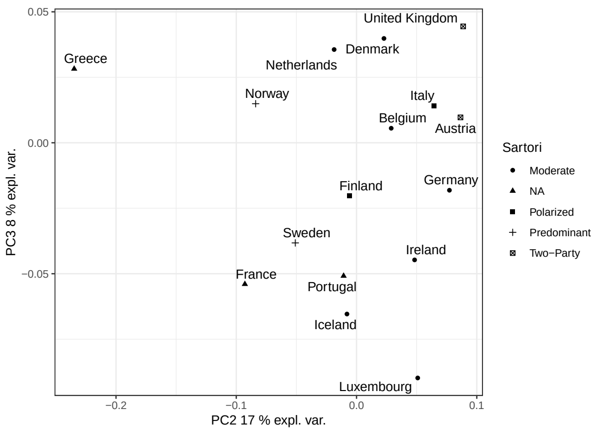
Sartori



(a) Sartori PC1-PC2



(b) Sartori PC1-PC3



(c) Sartori PC2-PC3

Figure G.20: Sartori's classification on three dimensions

G.6. Other Indices and the Third Dimension

Table G.2: Correlation Table with the Third PC Dimension

	Fraction.	HH	Entropy	ENP	P.in.Gov	Shap..ENP	Big.Party	ENOP	OPOP	Bopp	Competition
HH	-1.00										
	(0.00)										
Entropy	0.96	-0.96									
	(0.01)	(0.01)									
ENP	0.91	-0.91	0.95								
	(0.02)	(0.02)	(0.01)								
P.in.Gov	0.70	-0.70	0.73	0.75							
	(0.03)	(0.03)	(0.03)	(0.03)							
Shap.ENP	0.87	-0.87	0.87	0.91	0.76						
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)						
Big.Party	-0.94	0.94	-0.89	-0.90	-0.71	-0.91					
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.02)					
ENOP	0.66	-0.66	0.73	0.68	0.21	0.52	-0.50				
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)				
OPOP	-0.62	0.62	-0.63	-0.58	-0.33	-0.46	0.46	-0.74			
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)			
Bopp	-0.58	0.58	-0.61	-0.58	-0.38	-0.47	0.45	-0.68	0.94		
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.01)		
Competition	-0.47	0.47	-0.36	-0.43	-0.40	-0.58	0.71	0.03	-0.13	-0.09	
	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	
PC1	0.98	-0.98	0.94	0.93	0.70	0.87	-0.94	0.64	-0.66	-0.62	-0.43
	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.02)	(0.01)	(0.03)	(0.03)	(0.03)	(0.03)
PC2	0.06	-0.06	-0.04	0.04	0.12	0.24	-0.34	-0.33	0.46	0.39	-0.90
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.02)
PC3	0.05	-0.05	0.23	0.30	0.16	0.22	-0.04	0.33	0.04	-0.01	0.01
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

Notes: Computed correlation using Pearson-method with pairwise deletion. Standard Errors are in parentheses. Fraction: Fractionalization Index,HH:Herfindahl–Hirschman Index, Entropy: Entropies, ENP: Effective Number of Parties, P.in Gov: Number of parties in the Government, Shap.ENP: Effective Number of Parties(Shapley), Big Party: Size of the biggest party over the size of the legislature, P.s in Gov: Parties in Government, *ENOP*: Effective Number of Opposition Parties, *OPOP*: The difference between the first and the second biggest opposition parties over the size of the legislature, *BOPP*: Size of the biggest opposition party over the size of the legislature.

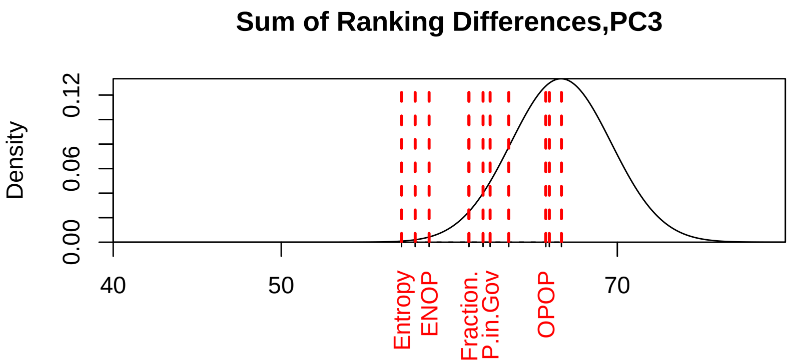


Figure G.21: Sum of Ranking Differences

References

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