Lattice Studies of Gerrymandering Strategies:

Supplementary Material

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A. Flowcharts

In this Appendix we present flowcharts for each of the algorithms proposed here, in order:

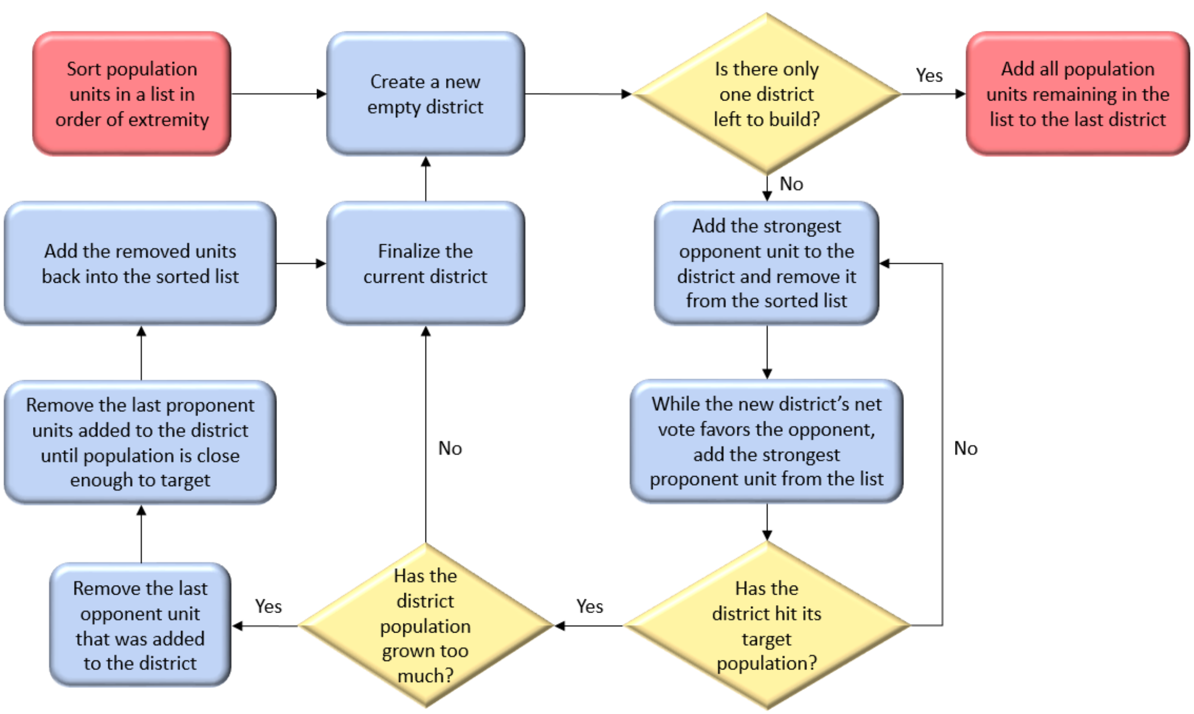

Figure 1: Flow chart of our implementation of the Friedman-Holden method.

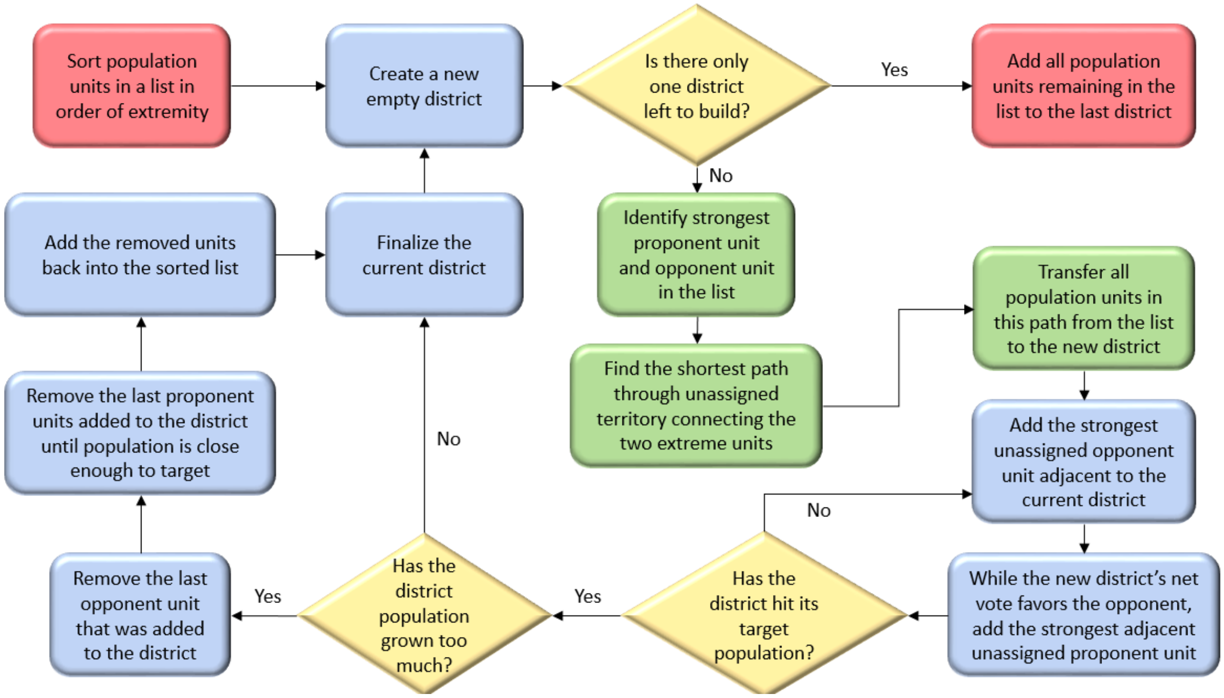

Figure 2: Flow chat of Spatial Restricted Friedman-Holden Packing of Section 3.1.

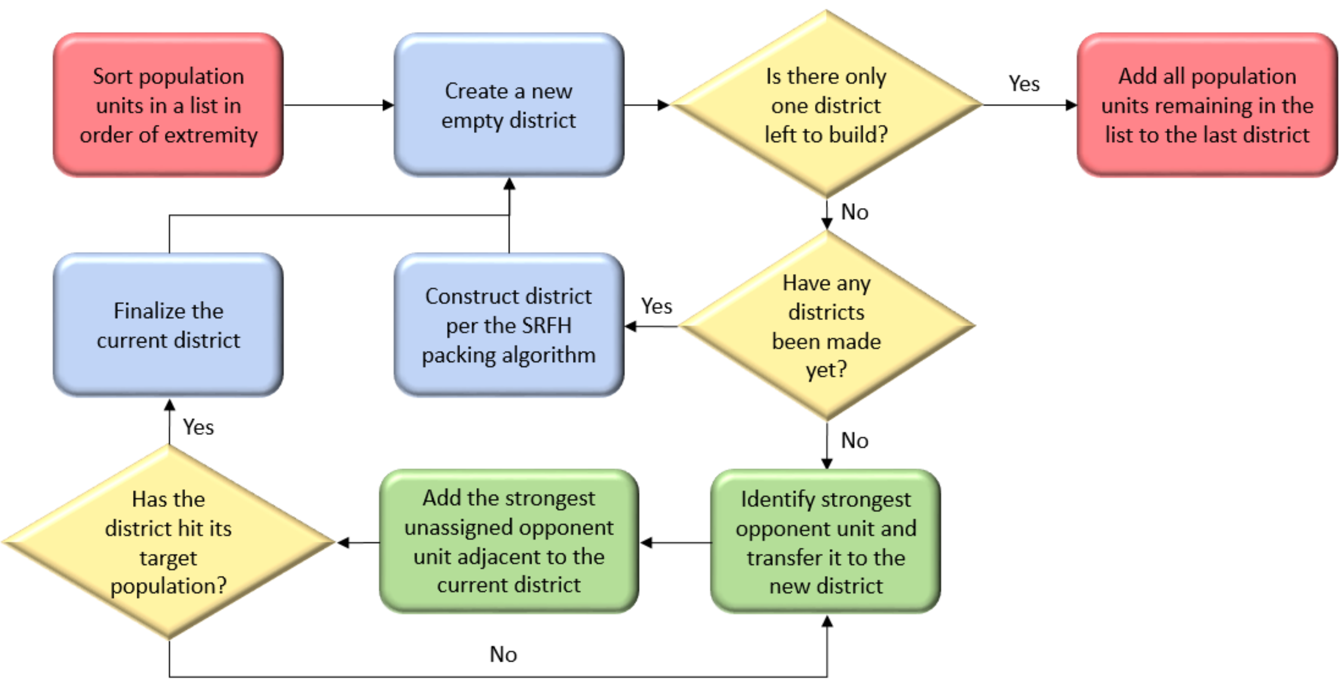

Figure 3: Flow chat of Saturation Packing detailed in Section 3.2.

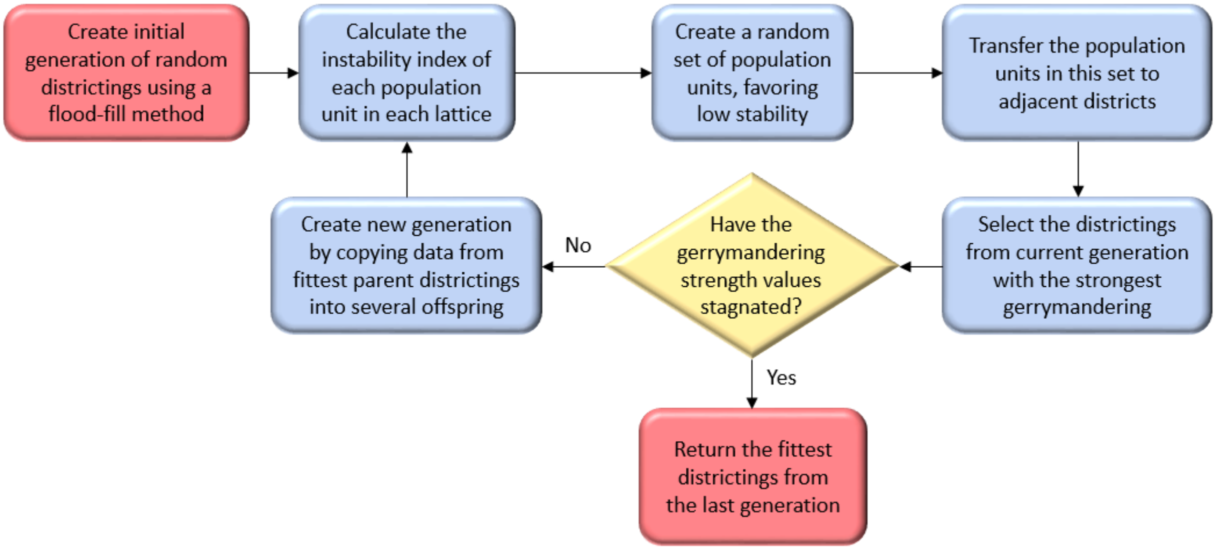

Figure 4: Flow chat of Genetic Gerrymandering algorithm of Section 4.

B. Genetic Algorithm for Fair Redistricting

While fairness is a subjective concept, one can construct a reasonable fitness index via a heuristic formula that takes into account:

i) The district population balance;

ii) Accuracy of state representation in Congress;

iii) Representation accuracy in each district.

We quantify these three characteristics, respectively, via fitness functions f_P, f_S, f_D:

$$f_P := 1 - g; \qquad f_S := 1 - \left| \frac{1}{2}\left(1 + \frac{N_S}{P_S}\right) - \frac{W}{n} \right|; \qquad f_D := \frac{1}{n} \sum_{i=1}^{n} \frac{Q_{D_k}}{P_{D_k}}, \tag{1}$$

where W and g are defined in eq. (7) & (8), and we denote by Q_D a measure of the proportion of the population in a given district $D = \cup_{(i,j)\in I} T_{i,j}$ (for an index set I) whose vote preference $v_{i,j}$ aligns with the net district vote N_D, defined by:

$$Q_D := \sum_{(i,j)\in I} P_{i,j} \cdot \frac{1}{2}\left(1 + \text{sign}[v_{i,j}] \cdot \text{sign}[N_D]\right). \tag{2}$$

Sample execution in of Modified-GG for Fair Redistricting

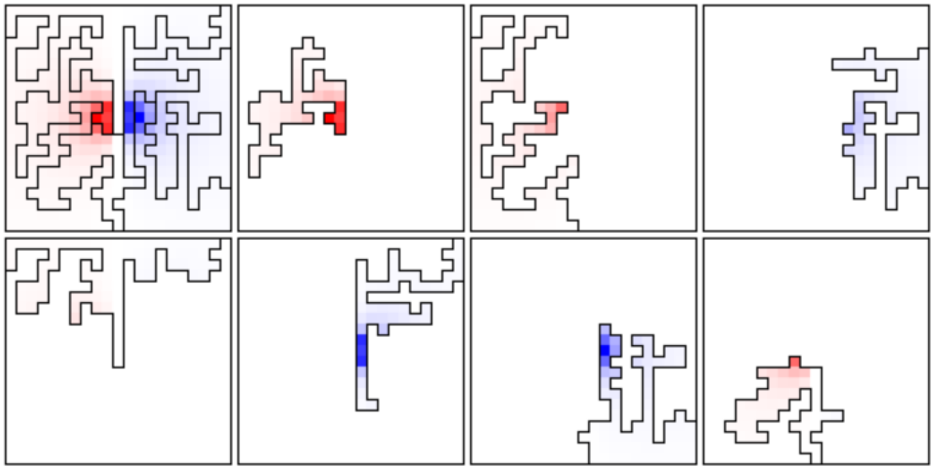

Figure 5: Construction of 7 fair districts via the Modified GG for Fair Redistricting applied to a 21×21 territory with balanced vote and two source points (with placement as in model #1 of Table 1). Presentation is analogous to Figure 3.

Whereas g measured the population spread, here f_P measures the population balance.

Using the partial fitness functions, we define the overall "fair" fitness F to be

$$F := \frac{2}{5}\left(f_P + f_S + f_D - \frac{1}{2}\right), \tag{3}$$

whose values lie in the interval $[0,1]$. Districts that are deemed "fairest" should have the highest fitness index. Typically for our trails we have found that the final set of fair districts tends to have fitness index values in excess of $F > 0.9$.

We will refer to the Genetic Gerrymandering (GG) algorithm equipped with the index F (rather than G) as the "Modified-GG for Fair Redistricting". Following the same methodology of Section 4.4 we execute an example run using $N = 100$ seed configurations and taking 30 iterations. The results of this example run are shown in Figure 15 for seven districts. In particular, By design, the equitable district configurations produced by the Modified GG for Fair Redistricting support a $50\% - 50\%$ vote split for balanced territories, accurately representing voter preference across the whole territory. Moreover, a majority of the districts avoid including oppositely polarized voters, ensuring that each elected district representative reflects the values of their constituents.