Web Appendix

Revisiting a Signaling Game of
Legislative-Judiciary Interaction

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A Full Game-Theoretical Model with Correct Payoff Values

There are two players: Legislature (L) and Court (Ct). L doesn't know whether the preferences of Ct converge with or diverge from the preferences of L. L also does not know the true state of the world – whether the legislation is appropriate for the true state or not. On the other hand, Ct knows both in advance. So, the game has two kinds of uncertainty.

The sequence of the game is as follows:

1. Nature (N) chooses the convergence of Ct's preferences (C denotes the convergence, and D denotes the divergence) with $Pr(C) = r$.
2. N also chooses the true state (A denotes that the legislation is appropriate for the true state, and I denotes that the legislation is inappropriate for the true state) with $Pr(A) = q$.
3. L chooses whether to enact the legislation or not (E denotes the enactment, and $\bar{E}$ denotes no enactment).
4. Ct reviews the legislation and chooses whether to strike it down or uphold it (V denotes the veto, and $\bar{V}$ denotes no veto).
5. L chooses whether to punish Ct or not (Δ denotes the punishment, and $\bar{\Delta}$ denotes no punishment).
6. Payoff is realized, and the game ends.

The payoffs of two players are as follows:[1]

1. L gets P if the legislation is appropriate for the true state (A), $-P$ if inappropriate for the true state (I), and 0 if not enacted or turned down (and not overruled).
2. Ct gets the same payoff as L's if it is convergent (C), the payoff of opposite sign of L's if it is divergent (D), and 0 if the legislation is not enacted or turned down (and not overruled).
3. Ct additionally gets K if it is not punished by L and $-K$ if punished by L (here, let's assume that $K > P > 0$).

The set of types is $T = \{CA, CI, DA, DI\}$. The action set of L for Period 1 is $A_L^1 = \{E, \bar{E}\}$. The action set of Ct's message for Period 2 is $M_{Ct} = \{V, \bar{V}\}$ if $a_L^1 = E$ and $M_{Ct} = \emptyset$ otherwise. The action set of L for Period 3 is $A_L^3 = \{\Delta, \bar{\Delta}\}$ if $m = V$ and $A_L^3 = \emptyset$ otherwise. Therefore, the strategy set of L is $S_L = S_L^1 \times S_L^3 = \{(E, \Delta|V), (E, \bar{\Delta}|V), (\bar{E}, \Delta|V), (\bar{E}, \bar{\Delta}|V)\}$. And, the strategy set of Ct is $S_{Ct} : T \to M_{Ct}$.

The solution concept here, as well as in the original model of Rogers (2001), is Perfect Bayesian Equilibrium. Among others, we are interested in three equilibria.

[1] Note that the payoff scheme here is same as that in the original model of Rogers (2001), but the payoff values have been corrected based on the discussion in the research memo.

Equilibrium 1 (Separation of Power): The equilibrium strategy profile is

$$((E, \bar{\Delta}|V), (\bar{V}|CA, V|CI, V|DA, \bar{V}|DI)).$$

L's beliefs in period 3 are $\beta_1 = \beta_4 = 0, \beta_2 = \frac{r-rq}{r+q-2rq}, \beta_3 = \frac{q-rq}{r+q-2rq} \leq \frac{1}{2}$ (i.e. $r \geq q$), and the prior probabilities r and q are constrained to be $r + q \geq 1$.

Proof. Let's start from Period 3. Since L has an information set, the beliefs of L should be consistent with the equilibrium strategy, which requires $\beta_1 = 0$, $\beta_2 = \frac{r(1-q)}{r(1-q)+(1-r)q} = \frac{r-rq}{r+q-2rq}$, $\beta_3 = \frac{(1-r)q}{r(1-q)+(1-r)q} = \frac{q-rq}{r+q-2rq}$, and $\beta_4 = 0$. The equilibrium strategy of L should also be sequentially rational (for her beliefs), which requires $\text{EV}(\bar{\Delta}) \geq \text{EV}(\Delta)$. Since $\text{EV}(\bar{\Delta}) = 0$ and $\text{EV}(\Delta) = \beta_2(-P) + \beta_3 P$, the condition is $\beta_3 \leq \frac{1}{2}$, which is equivalent to $r \geq q$.

In Period 2, observe that the equilibrium strategy of Ct is sequentially rational because $\text{EV}(\bar{V}|CA) = P + K > K = \text{EV}(V|CA)$, $\text{EV}(V|CI) = K > -P + K = \text{EV}(\bar{V}|CI)$, $\text{EV}(V|DA) = K > -P + K = \text{EV}(\bar{V}|DA)$, and $\text{EV}(\bar{V}|DI) = P + K > K = \text{EV}(V|DI)$.

In Period 1, the equilibrium strategy of L should be sequentially rational, which requires $\text{EV}(E) \geq \text{EV}(\bar{E})$. Since $\text{EV}(E) = rqP + (1-r)(1-q)(-P)$ and $\text{EV}(\bar{E}) = 0$, the condition is $r + q \geq 1$. Observe also that the Bayes rule applies to every node in L's information set, which gives L's beliefs consistent with the equilibrium strategy. □

Equilibrium 2 (Legislative Supremacy w/ Enactment): The equilibrium strategy profile is

$$((E, \Delta|V), (\bar{V}|CA, \bar{V}|CI, \bar{V}|DA, \bar{V}|DI))$$

L's belief in Period 3 is any set of beliefs, $\beta_1, \cdots, \beta_4$, such that $\beta_1 + \beta_3 \geq \frac{1}{2}$ and $0 \leq \beta_2 + \beta_4 < \frac{1}{2}$, and the prior probability q is constrained to be $q \geq \frac{1}{2}$.

Proof. Let's start from Period 3. Since the information set of L is not reached in the equilibrium, no Bayes rule applies. But, the equilibrium strategy of L should be sequentially rational, which requires $\text{EV}(\Delta) \geq \text{EV}(\bar{\Delta})$. Since $\text{EV}(\Delta) = \beta_1 P + \beta_2(-P) + \beta_3 P + (1-\beta_1-\beta_2-\beta_3)(-P)$ and $\text{EV}(\bar{\Delta}) = 0$, the condition is $\beta_1 + \beta_3 \geq \frac{1}{2}$ and $0 \leq \beta_2 + \beta_4 = 1 - \beta_1 - \beta_3 < \frac{1}{2}$.

In Period 2, observe that the equilibrium strategy of Ct is sequentially rational because $\text{EV}(\bar{V}|CA) = P + K > P - K = \text{EV}(V|CA)$, $\text{EV}(\bar{V}|CI) = -P + K > -P - K = \text{EV}(V|CI)$, $\text{EV}(\bar{V}|DA) = -P + K > -P - K = \text{EV}(V|DA)$, and $\text{EV}(\bar{V}|DI) = P + K > P - K = \text{EV}(V|DI)$.

In Period 1, the equilibrium strategy of L should be sequentially rational, which requires $\text{EV}(E) \geq \text{EV}(\bar{E})$. Since $\text{EV}(E) = rqP + r(1-q)(-P) + (1-r)qP + (1-r)(1-q)(-P)$ and $\text{EV}(\bar{E}) = 0$, the condition is $q \geq \frac{1}{2}$. Observe also that the Bayes rule applies to every node in L's information set, which gives L's beliefs consistent with the equilibrium strategy. □

Equilibrium 3 (Legislative Supremacy w/o Enactment): The equilibrium strategy profile is

$$((\bar{E}, \Delta|V), (\bar{V}|CA, \bar{V}|CI, \bar{V}|DA, \bar{V}|DI)).$$

L's belief in Period 3 is any set of beliefs, $\beta_1, \cdots, \beta_4$, such that $\beta_1 + \beta_3 \geq \frac{1}{2}$ and $0 \leq \beta_2 + \beta_4 < \frac{1}{2}$, and the prior probability q is constrained to be $q \leq \frac{1}{2}$.

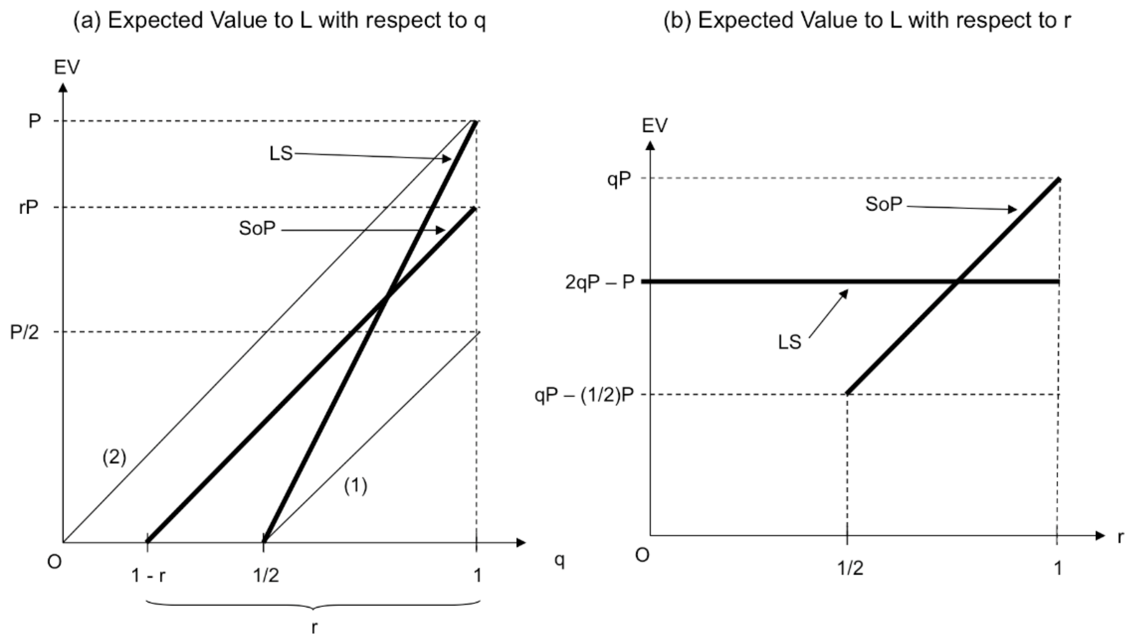


Figure A.1: Expected Value of Enactments to Legislature

Proof. The proof for Period 3 and 2 is exactly same as in Equilibrium 2. In Period 1, the sequential rationality requires EV($\bar{E}$) $\geq$ EV(E), which gives the condition of $q \leq \frac{1}{2}$. $\square$

Without loss of generality, assume that both q and r are uniformly distributed for $[0, 1]$. Additionally, following Rogers (2001), let's define the following:

Definition: The informational quality of a law, l^1, is higher than that of another law, l^2, when the probability q^1 that l^1 is appropriate in the true state of the world is greater than the probability q^2 that l^2 is appropriate in the true state of the world.

Consider the expected value to L from the equilibrium enactments. EV*(E) under SoP is $qP + rP - P$ (with the requirement of $r \geq \frac{1}{2}$)[2] and EV*(E) under LS is $2qP - P$ (with the requirement of $q \geq \frac{1}{2}$). Figure A.1 plots them with respect to q and r.

Proposition 1 The number of enactments under SoP is always greater than that under LS. And, the quality of legislations under SoP is always inferior to that under LS. *Furthermore, under SoP, the number of enactments becomes larger and the quality of enactments becomes lower as Ct becomes more convergent.*[3]

[2]This directly follows from the condition $r + q \geq 1$ and $r \geq q$.

[3]This second part of the proposition (emphasized in italic) is a new addition that Rogers (2001) does not discuss. However, it makes an intuitive sense. If the Legislature believes that the Court has similar preferences, then it will write more bills with a hope that some of inappropriate laws will be corrected by the convergent Court.

Proof. Under LS, the legislation whose q is greater than $\frac{1}{2}$ will be enacted. And, under SoP, the legislation whose q is greater than $1-r$ will be enacted. So, $\frac{1}{2}$ and $1-r$ are the cut points. Since q is uniformly distributed, the length between the cut point and 1 represents the quantity of equilibrium enactments and the average value from the cut point to 1 represents the quality of equilibrium enactments. From the conditions for the equilibrium 1 (SoP), we have $r \geq \frac{1}{2}$. Therefore, the number of enactments under SoP ($= r$) is always greater than that under LS ($= \frac{1}{2}$) and the quality of legislations under SoP ($= \frac{2-r}{2}$) is always lower than that under LS ($= \frac{3}{4}$). Furthermore, as r becomes bigger, EV*(E) under SoP moves from (1) to (2) in Figure A.1(a), which completes the proof of later part of the proposition. □

Proposition 2 For all possible values of q which induce the SoP equilibrium, L prefers SoP to LS when $r \geq \frac{\sqrt{2}}{2}$. Additionally, for all possible values of r which induce the SoP equilibrium, L prefers SoP to LS when $q \leq \frac{7}{12}$.[4]

Proof. Above all, all the conditions which induce the enactment of legislations under the SoP equilibrium applies here: $p+q \geq 1$ and $r \geq q$. The first part of the proposition comes from Figure A.1(a). The total legislative payoff under SoP is $\frac{1}{2} \times r \times rP$ and the total legislative payoff under LS is $\frac{1}{2} \times \frac{1}{2} \times P$. Therefore, when $\frac{r^2}{2}P \geq \frac{1}{4}P$ (i.e. $r \geq \frac{\sqrt{2}}{2}$), L is better off under SoP. The second part of the proposition comes from Figure A.1(b). The total legislative payoff under SoP is $\frac{1}{2} \times ((qP - \frac{1}{2}P) + qP) \times \frac{1}{2}$ and the total legislative payoff under LS is $1 \times (2qP - P)$. Therefore, when $q \leq \frac{7}{12}$, L is better off under SoP. □

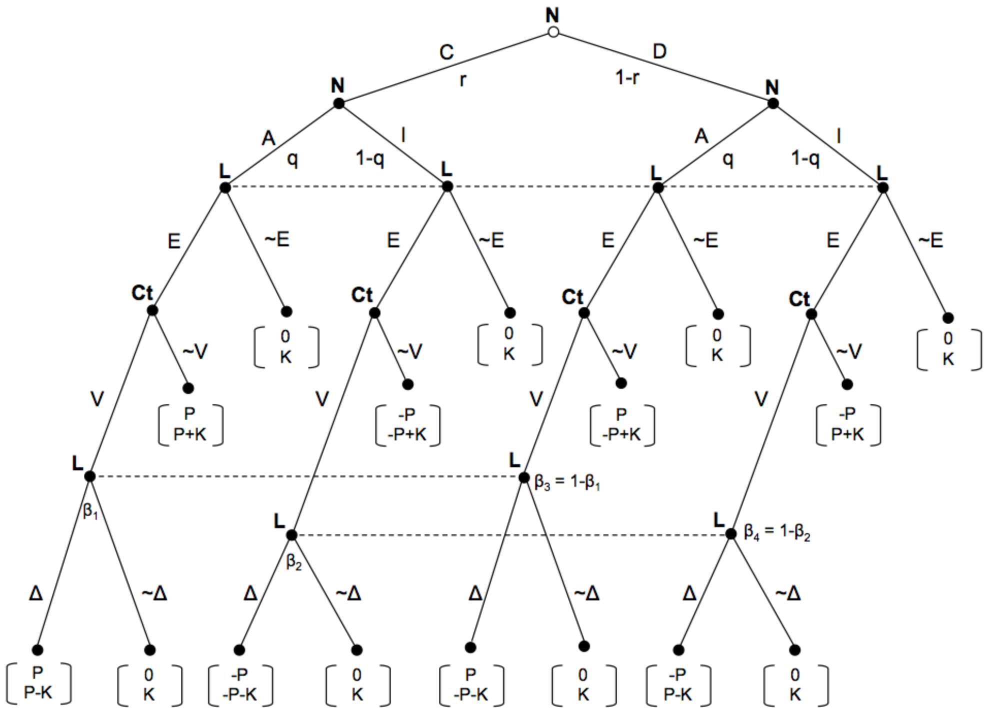
B Game-Theoretical Model with Alternative Information Sets

An alternative modeling specification is possible with respect to information sets. In the original model, information is exogenously given only to the court but not to the legislature, throughout the whole period of time. But, what if information is exogenously given depending *both* on players *and* on timing? According to Rogers (2001), the court enjoys informational advantage mainly because the judicial review occurs *after* the legislation is actually implemented. Even when we follow his own logic, the legislature is more likely to acquire information on the appropriateness of its legislation during Period 3, when (a) it is *after* the legislation has been implemented; and (b) the discussions during the judicial review have been *revealed* to the public and ultimately to the legislature. As such, considering alternative information sets in Period 3 could not only better accords with the strategic setting but also more closely fits the original article's intent. Figure B.1 shows the revised game tree with alternative information sets.

The sets of types are $T_1 = \{C, D\}$ and $T_2 = \{A, I\}$. The action set of L in Period 1 is $A_L^1 = \{E, \bar{E}\}$. The action set of Ct in Period 2 is $A_{Ct} = \{V, \bar{V}\}$ if $a_L^1 = E$ and $A_{Ct} = \emptyset$ otherwise. The action set of L in Period 3 is $A_L^3 = \{\Delta, \bar{\Delta}\}$ if $a_{Ct} = V$ and $A_L^3 = \emptyset$ otherwise.

[4] This proposition is a new addition that Rogers (2001) does not discuss. It also well fits to the reality. If the legislature believes that the court has similar preferences, the legislature can easily rely on the court. Also, when the state of the world is very pessimistic, then it is better for the legislature to rely more on the information that the court will acquire in the future.

Figure B.1: Game Tree with Alternative Information Sets



Therefore, the strategy set of L is $S_L = S_L^1 \times S_L^3$ where $S_L^1 = A_L^1$ and $S_L^3 : T_2 \to A_L^3$. And, the strategy set of Ct is $S_{Ct} : T_1 \times T_2 \to A_{Ct}$.

The solution concept, again, is Perfect Bayesian Equilibrium. Among others, we are interested in two equilibria.

Equilibrium 1 (With Enactments): The equilibrium strategy profile is

$$((E, (\Delta|A, \bar{\Delta}|I)), (\bar{V}|CA, V|CI, \bar{V}|DA, \bar{V}|DI)).$$

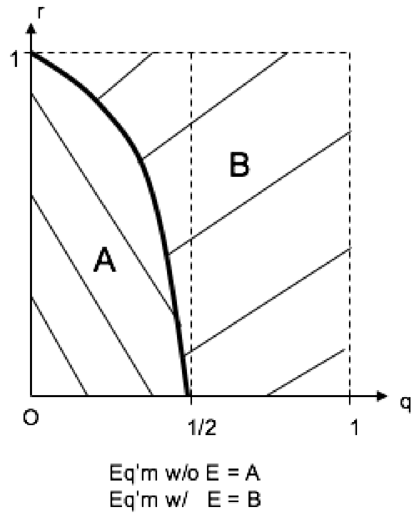
L's beliefs in Period 3 are $\beta_2 = 1, \beta_4 = 0$, and any set of β_1 and β_3 such that $\beta_1 + \beta_3 = 1$. The prior probabilities r and q are constrained to be $r + 2q - rq - 1 \geq 0$.

Proof. In Period 3, observe that the equilibrium strategy of L is sequentially rational because $\text{EV}(\Delta|A) = P > 0 = \text{EV}(\bar{\Delta}|A)$ and $\text{EV}(\bar{\Delta}|I) = 0 > -P = \text{EV}(\Delta|I)$. The beliefs of L should be consistent with the equilibrium strategy, which requires $\beta_2 = 1$ and $\beta_4 = 0$. Since β_1 and β_3 are not reached in the equilibrium, no Bayes rule applies, which just requires $\beta_1 + \beta_3 = 1$.

In Period 2, observe that the equilibrium strategy of Ct is sequentially rational because $\text{EV}(\bar{V}|CA) = P + K > P - K = \text{EV}(V|CA)$, $\text{EV}(V|CI) = K > -P + K = \text{EV}(\bar{V}|CI)$, $\text{EV}(\bar{V}|DA) = -P + K > -P - K = \text{EV}(V|DA)$, and $\text{EV}(\bar{V}|DI) = P + K > K = \text{EV}(V|DI)$.

In Period 1, the equilibrium strategy of L should be sequentially rational, which requires $\text{EV}(E) > \text{EV}(\bar{E})$. Since $\text{EV}(E) = rqP + (1-r)qP + (1-r)(1-q)(-P)$ and $\text{EV}(\bar{E}) = 0$, the condition is $r + 2q - rq - 1 > 0$. Observe also that the Bayes rule applies to every node in L's information set, which gives L's beliefs consistent with the equilibrium strategy. □

Figure B.2: Probability Combinations of Types under Equilibria, with Alternative Information Sets



Equilibrium 2 (Without Enactments): The equilibrium strategy profile is

$$((\bar{E}, (\Delta|A, \bar{\Delta}|I)), (\bar{V}|CA, V|CI, \bar{V}|DA, \bar{V}|DI)).$$

L's beliefs in Period 3 are $\beta_2 = 1, \beta_4 = 0$, and any set of β_1 and β_3 such that $\beta_1 + \beta_3 = 1$. The prior probabilities r and q are constrained to be $r + 2q - rq - 1 \leq 0$.

Proof. The proof for Period 3 and 2 is same as in Equilibrium 1. In Period 1, the sequential rationality requires the opposite condition of Equilibrium 1, which is $r + 2q - rq - 1 \leq 0$. □

It is important to note that this is not a signaling game anymore, and we now have neither Ct's independency nor L's supremacy in equilibrium. Instead, Ct's role is somewhere in the middle: Ct is constrained by L, but it also enjoys some levels of autonomy. This is because L does not suffer fully from the informational asymmetry.

For the perspective of Ct, there is one strategic situation: divergent Ct must choose not to veto the appropriate statute in order to avoid L's punishment, even though the veto is consistent with Ct's sincere policy preferences. In other three cases, Ct follows its sincere preferences. This is a mixture of sincere and strategic choices, which Rogers (2001)'s original "Independent Judiciary" equilibrium also includes in a similar fashion. Previously, it gave additional informational benefits to L, but it is not the case anymore. Ct is now constrained to become strategic, but not in a way that gives additional benefits to L.

In Figure B.2, we can evaluate the probability combination of types under the two equilibria. When $q \geq \frac{1}{2}$ (i.e. when the state is more likely to be appropriate for the legislation), L always enact the legislation. If the true state is indeed inappropriate, then convergent Ct will save L. Even though Ct is divergent, q is large enough that a positive payoff is expected. On the other hand, when $q \leq \frac{1}{2}$ (i.e. when the state is more likely to be inappropriate), L increases the enactments only when L believes Ct is more likely to be convergent (i.e. when r become larger).