

Supporting Information for Guessing and Forgetting: A Latent Class Model
for Measuring Learning

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Appendix A A Latent Class Model For Forced Choice Items

Many closed-ended knowledge items do not include an explicit ‘Don’t know’ option. Since rates of guessing are perforce equal or higher when a ‘Don’t know’ option is not offered than when it is, the threat to inference when not accounting for guessing is yet greater on items without a ‘Don’t know’ option. Here we develop a latent class model for such items.

Using the notation in the paper, responses to such closed-ended multiple choice items across two measurement waves can be represented by the following two-way table:

Table A1: Two-way Table of Manifest Pre-Post Response Patterns for a Single Closed-Ended Question

		Post	
Pre	0	1	
0	x_{00}	x_{01}	
1	x_{10}	x_{11}	

Thus, the traditional estimate of the effect for any single item is: $(x_{01} + x_{11}) - (x_{10} + x_{11})$, or simply, $(x_{01} - x_{10})$. Next, we specify the latent class model for a single wave. We once again assume that there is no partial knowledge, and that either an individual knows the answer or doesn’t— so knowledge and ignorance are the only two latent states. We again assume that the conditional probability of answering correctly given a respondent knows the answer is 1 or $P(u = 1|\text{Knowing}) = 1$ and $P(u = 0|\text{Knowing}) = 0$. Only the ignorant guess and when they guess they are lucky γ number of times or $P(u = 1|\text{guess}) = \gamma$. The following table summarizes the:

Table A2: Latent Class Model for A Closed-ended Question

Observed Response (u)	Latent Class	
	Guesser	Knower
u=0	$(1 - \gamma)$	0
u=1	γ	1

The probability of different response patterns across two waves can be described by multiplying the latent class models for each wave -

$$\otimes \begin{pmatrix} & G & K \\ u = 0 & (1 - \gamma) & 0 \\ u = 1 & \gamma & 1 \end{pmatrix} \begin{pmatrix} & G & K \\ u = 0 & (1 - \gamma) & 0 \\ u = 1 & \gamma & 1 \end{pmatrix} =$$

$$\begin{pmatrix} & GG & GK & KG & KK \\ u = 00 & (1 - \gamma)^2 & 0 & 0 & 0 \\ u = 01 & (1 - \gamma)\gamma & (1 - \gamma) & 0 & 0 \\ u = 10 & \gamma(1 - \gamma) & 0 & (1 - \gamma) & 0 \\ u = 11 & \gamma^2 & \gamma & (1 - \gamma) & 1 \end{pmatrix}$$

Next, in order to define the system of equations that describe the underlying multinomial distribution of observed response patterns, we multiply the latent class transition matrix by a vector of latent class transition parameters. If we let, G stand for Guessing, and K stand for Knowledgeable, there are four possible latent class transitions that could hypothetically occur: GG , GK , KG , and KK .

Assuming the respondent who knows the right answer doesn't forget it over the course of the experiment, and that she doesn't become misinformed, the proportion of people moving from knowing to guessing (KG) must be zero.

Multiplying the 4×4 matrix from equation 1 by the latent class transition vector, λ , and setting λ_{KG} to zero, we get the following system of equations that defines the vector of expected proportions, π :

$$\begin{aligned}
& \begin{pmatrix} & GG & GK & KG & KK \\ u=00 & (1-\gamma)^2 & 0 & 0 & 0 \\ u=01 & (1-\gamma)\gamma & (1-\gamma) & 0 & 0 \\ u=10 & \gamma(1-\gamma) & 0 & (1-\gamma) & 0 \\ u=11 & \gamma^2 & \gamma & (1-\gamma) & 1 \end{pmatrix} \begin{pmatrix} \lambda_{GG} \\ \lambda_{GK} \\ \lambda_{KG} = 0 \\ \lambda_{KK} \end{pmatrix} \\
& = \begin{pmatrix} (1-\gamma)(1-\gamma)\lambda_{GG} \\ (1-\gamma)\gamma\lambda_{GG} + (1-\gamma)\lambda_{GK} \\ \gamma(1-\gamma)\lambda_{GG} \\ \gamma^2\lambda_{GG} + \gamma\lambda_{GK} + \lambda_{KK} \end{pmatrix} = \begin{pmatrix} \pi_{00} \\ \pi_{01} \\ \pi_{10} \\ \pi_{11} \end{pmatrix}
\end{aligned}$$

Appendix B Accounting for Hidden Knowledge in Don't Know Responses

Some research suggests that roughly 3% of the people who know a piece of information still mark ‘don’t know’ on a closed-ended item (Luskin and Bullock 2011; Luskin and Sood 2012; Sanchez and Morchio 1992; Sturgis et al. 2008). In this section, we describe a version of the latent class model developed in the paper that accounts for this: the model takes the probability of a respondent who knows the answer offering a Don’t Know response to be .03.

In the notation developed in the main body of the paper, responses to such closed-ended multiple choice items that offer a ‘Don’t Know’ across two measurement waves can be represented by the following two-way table:

Table B1: Relationship Between Manifest Responses Across Two Waves

		Post		
		0	1	c
Pre	0	x_{00}	x_{01}	x_{0c}
	1	x_{10}	x_{11}	x_{1c}
	c	x_{c0}	x_{c1}	x_{cc}

Thus, the traditional estimate of the effect for any single item is: $(x_{01} + x_{c1} - x_{10} - x_{1c})$

Next, we specify the latent class model for a single wave. We once again assume that there is no ‘partial knowledge’, that either an individual knows the answer or doesn’t. And those who don’t know the answer can choose to guess or confess. We now assume that the conditional probability of answering correctly given a respondent knows the answer is .97 or $P(u = 1|K) = .97$, $P(u = 0|K) = 0$ and the $P(u = c|K) = .03$. Only the ignorant guess and when they guess they are lucky γ number of times or $P(u = 1|G) = \gamma$. The following table summarizes the model:

Table B2: Relationship Between Manifest Responses and Latent Classes when DK can Indicate Knowledge

		Latent Classes		
		<i>G</i>	<i>K</i>	<i>C</i>
Observed	$u = 0$	$(1 - \gamma)$	0	0
	$u = 1$	γ	.97	0
	$u = c$	0	.03	1

The probability of different response patterns across two waves can be described by multiplying the latent class models for each wave:

$$\begin{pmatrix} & G & K & C \\ u = 0 & 1 - \gamma & 0 & 0 \\ u = 1 & \gamma & .97 & 0 \\ u = c & 0 & .03 & 1 \end{pmatrix} \otimes \begin{pmatrix} & G & K & C \\ u = 0 & 1 - \gamma & 0 & 0 \\ u = 1 & \gamma & .97 & 0 \\ u = c & 0 & .03 & 1 \end{pmatrix} = \quad (1)$$

$$\begin{pmatrix} & GG & GK & GC & KG & KK & KC & CG & CK & CC \\ u = 00 & (1 - \gamma)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 01 & (1 - \gamma)\gamma & .97(1 - \gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 0c & 0 & 0 & .03(1 - \gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 10 & (1 - \gamma)\gamma & 0 & 0 & .97(1 - \gamma) & 0 & 0 & 0 & 0 & 0 \\ u = 11 & \gamma^2 & .97\gamma & 0 & .97\gamma & (.97)^2 & 0 & 0 & 0 & 0 \\ u = 1c & 0 & .03\gamma & \gamma & 0 & .03(.97) & .97 & 0 & 0 & 0 \\ u = c0 & 0 & 0 & 0 & .03(1 - \gamma) & 0 & 0 & (1 - \gamma) & 0 & 0 \\ u = c1 & 0 & 0 & 0 & .03\gamma & .03(.97) & 0 & \gamma & .97 & 0 \\ u = cc & 0 & 0 & 0 & 0 & (.03)^2 & .03 & 0 & .03 & 1 \end{pmatrix}$$

Next, the latent class transition matrix is multiplied by a vector of unknown latent class transition parameters to define the system of equations that describe the multinomial distribution of observed response patterns. There are nine latent class transitions that could hypothetically occur: *GG*, *GK*, *GC*, *KG*, *KK*, *KC*, *CG*, *CK*, and *CC*.

To identify the item level conditional probability of getting an item correct if the respondent chooses to guess, we assume that the respondent who knows the right answer doesn't forget it over the course of the experiment, and that she doesn't become misinformed. Under these assumptions, the proportion of people moving from knowing to guessing (*KG*) or confessing to ignorance (*KC*) must be zero.

Multiplying the 9×9 matrix from equation 1 by the latent class transition vector, λ , and setting λ_{KG} and λ_{KC} to zero, we get the following system of equations that defines the vector of expected response pattern proportions, π :

$$\begin{pmatrix} & GG & GK & GC & KG & KK & KC & CG & CK & CC \\ u = 00 & (1-\gamma)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 01 & (1-\gamma)\gamma & .97(1-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 0c & 0 & 0 & .03(1-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 10 & (1-\gamma)\gamma & 0 & 0 & .97(1-\gamma) & 0 & 0 & 0 & 0 & 0 \\ u = 11 & \gamma^2 & .97\gamma & 0 & .97\gamma & (.97)^2 & 0 & 0 & 0 & 0 \\ u = 1c & 0 & .03\gamma & \gamma & 0 & .03(.97) & .97 & 0 & 0 & 0 \\ u = c0 & 0 & 0 & 0 & .03(1-\gamma) & 0 & 0 & (1-\gamma) & 0 & 0 \\ u = c1 & 0 & 0 & 0 & .03\gamma & .03(.97) & 0 & \gamma & .97 & 0 \\ u = cc & 0 & 0 & 0 & 0 & (.03)^2 & .03 & 0 & .03 & 1 \end{pmatrix} \begin{pmatrix} \lambda_{GG} \\ \lambda_{GK} \\ \lambda_{GC} \\ \lambda_{KG} = 0 \\ \lambda_{KK} \\ \lambda_{KC} = 0 \\ \lambda_{CG} \\ \lambda_{CK} \\ \lambda_{CC} \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} (1-\gamma)^2\lambda_{GG} \\ (1-\gamma)\gamma\lambda_{GG} + .97(1-\gamma)\lambda_{GK} \\ .03(1-\gamma)\lambda_{GG} + (1-\gamma)\lambda_{GC} \\ \gamma(1-\gamma)\lambda_{GG} \\ \gamma^2\lambda_{GG} + .97\gamma\lambda_{GK} + (.97)^2\lambda_{KK} \\ .03\gamma\lambda_{GK} + \gamma\lambda_{GC} + .03(.97)\lambda_{KK} \\ (1-\gamma)\lambda_{CG} \\ .03(.97)\lambda_{KK} + \gamma\lambda_{CG} + .97\lambda_{CK} \\ (.03)^2\lambda_{KK} + .03\lambda_{CK} + \lambda_{CC} \end{pmatrix} = \begin{pmatrix} \pi_{00} \\ \pi_{01} \\ \pi_{0c} \\ \pi_{10} \\ \pi_{11} \\ \pi_{1c} \\ \pi_{c0} \\ \pi_{c1} \\ \pi_{cc} \end{pmatrix}$$

Appendix C Details of Deliberative Polls

The polls were conducted by James Fishkin in collaboration with other scholars. More information about Deliberative Polling is available at <http://cdd.stanford.edu>. The references in the reference column in the table below are to other papers, research notes, and reports based on the data from the relevant polls. (Note: While we limit ourselves to citing a single paper, research note or report, often enough there are multiple papers and research notes that utilize data from the poll.)

Table C1: Details of Deliberative Polls

Poll Name	Year	Region	Country	Mode	Issue(s)	Reference
UK EU	1995		U.K.	Face-to-Face	Future of Britain in Europe	Sturgis et al. (2005)
UK Health	1998		U.K.	Face-to-Face	National Health Service	Park et al. (1998)
UK Monarchy	1996		U.K.	Face-to-Face	Future of Monarchy	Sturgis et al. (2005)
UK BGE	1997		U.K.	Face-to-Face	Electoral Choice	Luskin et al. (1999)
AU Monarchy	1999		Australia	Face-to-Face	Referendum on becoming a Republic	Luskin et al. (2000)
UK Crime	1994		U.K.	Face-to-Face	Crime	Luskin et al. (2002)
EU 2007	2007		EU-Wide	Face-to-Face	Pension systems, Globalization, and Enlargement	Luskin et al. (2008)
CPL	1996	Texas	U.S.	Face-to-Face	Energy Choices	Luskin et al. (1999)
WTU	1996	Texas	U.S.	Face-to-Face	Energy Choices	Luskin et al. (1999)
SWEPKO	1996	Texas	U.S.	Face-to-Face	Energy Choices	Luskin et al. (1999)
Bulgaria	2007		Bulgaria	Face-to-Face	Roma: Housing, Crime and Education	Luskin et al. (1999)
NIC	1996		U.S.	Face-to-Face	Foreign and Economic Policy	CDD ^a
Denmark	2000		Denmark	Face-to-Face	Adoption of Euro	Luskin and Fishkin (1998)
N. Ireland	2007	Omagh	N. Ireland	Face-to-Face	Local Education	Hansen (2004)
Vermont	2007	Vermont	U.S.	Face-to-Face	Energy Choices	Luskin et al. (2014)
Ca. Ref.	2011	California	U.S.	Face-to-Face	Government Reform	Luskin et al. (2008)
San Mateo	2008	California	U.S.	Face-to-Face	Housing	Fishkin et al. (2015)
EU 2009	2009		EU-Wide	Face-to-Face	Environment and EU-Reform	Weiksner (2008)
BTP 2004 Primaries	2004		U.S.	Online	Online Primaries	Fishkin et al. (2014)
BTP 2004 GE	2004		U.S.	Online	General Elections	Iyengar et al. (2004)
BTP 2005	2005		U.S.	Online	Healthcare and Education	Luskin et al. (2006)
BTP 2007	2007		U.S.	Online	Electoral Reform	PBS ^b
Michigan	2009	Michigan	U.S.	Face-to-Face	Economy and Budget Choices	Siu et al. (2011)

^a<http://cdd.stanford.edu/polls/bulgaria/2007/bulgaria-results.pdf>

^bhttp://www.pbs.org/newshour/spc/btp/articles/events_deliberation.html

Appendix D Question Text and Response Options

Correct answers are in bold text.

AU Monarchy

- Issue Related Items: Which one of these best describes the current role of the Queen in relation to the Australian Governor General? - She chooses the Governor General, **She appoints the Governor General only on the advice of the Prime Minister**, She appoints the Governor General only on the recommendation of parliament, She has no role in choosing or appointing the Governor General
- Which one of these is true of the current role of the Governor-General in Australia - **He performs only ceremonial duties**, He can decide whether or not to dismiss the government, He acts only on the Queen's instructions, He controls the government
- If the referendum on the republic is passed, which one of these would the role of the President be more like? - **The current Governor General**, The American President, The British Prime Minister, the current Australian Prime Minister
- If the referendum on the republic is passed, which one of these best describes how the Australian Prime Minister in the Republic can remove the President? - At any time without reporting to parliament, **At any time but must later obtain approval from the House of Representatives**, Only after a fair trial in parliament, The Prime Minister could not remove president.
- Under the proposed change to a Republic, which of these, as far as you know, would definitely change.
 - The Australian Flag
 - The National Anthem

- **The word ‘Royal’ in the names of the Royal Australian Navy and the Royal Australian Air Force**
- Australia’s participation in the Commonwealth Games
- On the whole would you say the Liberal Party, or the Labor Party, is more in favour of Australia becoming a Republic?
- Which one of these best describes Aden Ridgeway - Leader of the federal opposition, Leader of the Australian Democrats, **An aboriginal senator in parliament**, A justice of the high court, Or is he none of these
- Which one of these best describes Jennie George - A Labor member of federal parliament, Secretary of the Teacher’s Federation, **President of the ACTU**, Leader of the Australian Worker’s Union, Or is she none of these
- And now about social and welfare issues. On the whole would you say the Liberal Party, or the Labor Party, is more concerned about social and welfare issues, they are equally concerned, or do you not have much of an im-pression about that? - the Liberal Party, **the Labor Party**, equally close
- And now thinking about business. On the whole would you say the Liberal Party, or the Labor Party, is closer to business, they are equally close or do you not have much of an impression about that? - **the Liberal Party**, the Labor Party, equally close

UK EU

- EU has recently expanded to 15 members - **True** or False.
- Switzerland is to join EU - True or **False**.
- Britain’s income tax rates are decided in Brussels - True or **False**.
- Elections to European Parliament are held every 5 years - **True** or False.

- Of the three British parties, Liberal Democrats least in favour of EU - True or **False**.

UK Health

- Standard charge for NHS prescription is 10 pounds - True or **False**.
- Proportion of old people in Britain getting larger - **True** or False.
- Even with inflation taken into account, spending on NHS doubled over last 20 years - **True** or False.
- Most people use private medical treatment instead of NHS - True or **False**.
- All British women get free breast cancer screening on NHS - True or **False**.
- Cosmetic surgery is never available on NHS - True or **False**.

UK Crime

- British courts are allowed to sentence a murderer to death - True or **False**.
- Britain has largest prison population in Western Europe - **True** or False.
- Britain has more people serving life sentences than rest of European Community put together - True or **False**.
- Possible to be tried by jury in local magistrate court - True or **False**.

Politics Items -

- Number of MPs about 100 - True or **False**.
- Longest time allowed between general elections is 4 years - True or **False**.
- Britain's electoral system is based on proportional representation - True or **False**.

UK Monarchy

- Princess Anne next in line to throne - True or **False**.
- PM, not the queen, is Britain's Head of State - True or **False**.

- Queen is head of Commonwealth - True or **False**.
- Queen's duty to decide election date - True or **False**.
- No PM takes office without being asked by the Queen - True or **False**.
- Queen is head of Church of England - **True** or False.
- Queen is head of Church of Scotland - True or **False**.
- Britain has unwritten constitution - **True** or False.

UK BGE

- Inflation has been less than 5
- Interest rates are decided by Bank of England - True or **False**.
- Unemployment in UK is higher than in Germany - True or **False**.
- (Conservative, Labor, Liberal Democrat) Party's View on Government role in making incomes equal: 7 point semantic scale going from "Government should do nothing to make incomes in Britain more equal" to "Government should try much harder to make incomes in Britain more equal."
- (Conservative, Labor, Liberal Democrat) Party's View on Spending: 7 point semantic scale ranging from "Government should spend much less on services like education and health in order to cut taxes a lot" to "Government should spend a lot more on services like education and health, even if it means putting up taxes a lot."
- (Conservative, Labor, Liberal Democrat) Party's View on minimum wage: 7 point semantic scale ranging from "Government should definitely not introduce a minimum wage because too many low paid workers would then lose their jobs." to "Government should definitely introduce a minimum wage so that no employer can pay their workers too little."
- (Conservative, Labor, Liberal Democrat) Party's View on European Union: 7 point semantic scale ranging from "Britain should do much more to keep its distance

from the European Union.” to “Britain should do much more to unite fully with the European Union.”

CPL

- What energy source do you think produces the greatest amount of CPL’s electricity; coal, wind, **natural gas**, fuel oil, nuclear, or solar?
- Overall, what group of CPL customers consume the most kilowatt hours of electricity; residential, commercial, or **industrial**?
- Which customer group pays the highest electric rates; residential, **commercial**, or industrial?
- Which of the following resources used to produce electricity produces the most air emissions; nuclear, **coal**, or natural gas?
- Roughly, how much profit do Texas electric utilities make on fuel; **0%**, 10%, 20%, or 30%?
- Which agency sets electric rates in Texas? - **PUC**, Texas Utility Commission, Government, CPL, WT, SWEPCO, Other.
- How much of your electric bill goes for fuel; less than 20%, **21 to 30%**, 31 to 40%, 41 to 50%, or more than 50%?

SWEPCO

- What energy source do you think produces the greatest amount of SWEPCO’s electricity; **coal**, wind, natural gas, fuel oil, nuclear, or solar?
- Overall, what group of SWEPCO customers consume the most kilowatt hours of electricity; residential, commercial, or **industrial**?
- Which customer group pays the highest electric rates; **residential**, commercial, or industrial?

- Which of the following resources used to produce electricity produces the most air emissions; nuclear, **coal**, or natural gas?
- Which agency sets electric rates in Texas? - **PUC**, Texas Utility Commission, Government, CPL, WT, SWEPCO, Other.

WTU

- What energy source do you think produces the greatest amount of WTU's electricity; coal, wind, **natural gas**, fuel oil, nuclear, or solar?
- Overall, what group of WTU customers consume the most kilowatt hours of electricity; **residential**, commercial, or industrial?
- Which customer group pays the highest electric rates; **residential**, commercial, or industrial?
- Which of the following resources used to produce electricity produces the most air emissions; nuclear, **coal**, or natural gas?
- Which agency sets electric rates in Texas? - **PUC**, Texas Utility Commission, Government, CPL, WT, SWEPCO, Other.

NIC

- Has the United States sent ground troops to Iraq in the past ten years? – **Yes**, No
- Has the United States sent ground troops to Somalia in the past ten years? – **Yes**, No
- Has the United States sent ground troops to Rwanda in the past ten years? – **Yes**, No
- Has the United States sent ground troops to Bosnia in the past ten years? – **Yes**, No
- On which of the following does the federal government spend more money? – The space program, Foreign Aid, **National defense**, Agriculture subsidies

- With which of the following countries does the United States conduct the most international trade? **Canada**, Japan, Germany, Mexico
- Where would you put the Republican party on the following scale: Extremely liberal, Liberal, Slightly liberal, Moderate middle of road, **Slightly conservative**, **Conservative**, **Extremely conservative**
- Where would you put the Democratic party on the following scale: **Extremely liberal**, **Liberal**, **Slightly liberal**, Moderate middle of road, Slightly conservative, Conservative, Extremely conservative

Bulgaria

- If a policeman wants to detain a citizen for questioning at any time of the day or night, the citizen does not have the right to refuse – **True** or False.
- The Court has the right to detain suspects as long as needed to prove if they are guilty or not – True or **False**.
- The prosecution has the right to detain suspects, until the crime is solved – True or **False**.
- The Chief Prosecutor is responsible and accountable to Parliament – **True** or False.
- The police do not have the right to use violence against a detainee even if proven that he/she has committed a crime – True or **False**.
- The police has the right to keep you in detention for three days – True or **False**.
- A confession is not enough to find the defendant guilty – True or **False**.

Northern Ireland

- What percentage of majority-Protestant or majority-Catholic schools in Northern Ireland have at least 10% of the other religion in their enrolment? – 40 - 50%, 20 - 30%, **5 - 10%**, less than 1%

- By approximately what percentage has the number of children entering Omagh schools increased or decreased over the past five years – Increased by 20%, Increased by 10%, Stayed about the same, **Decreased by 10%**, Decreased by 20%
- The new entitlement framework requires that ... – **Every school provides all 14-year-olds with a choice of at least 24**, subjects, Every child has the right to attend any school his or her parents wish, Every child be provided a free school meal every school day, Every child receives free textbooks
- The new entitlement framework requires that ... – Every child receives tuition in the language of his or her parents' choice, Every child receives free transportation to and from school, Every denominational group has the right to run its own schools, **One-third of all the subjects offered must be applied**
- Which of the following is true of what pupils in Northern Ireland do after they leave school? - **About one-quarter go directly into employment**, About one-quarter leave school to be unemployed, About three-quarters of grammar school pupils attend university, About three-quarters of secondary school students attend Further Education College
- Which of the following is true of current school funding? – **Schools receive more funding for older pupils**, Schools receive the same funding for all pupils, regardless of age, Schools receive more funding for younger pupils
- Which of the following is true of the employing authority in the schools? – The official employer for all teachers is the Education and Library Board, The official employer for all teachers is the Department of Education, The official employer for all teachers in Catholic schools is the CCMS (Council for Catholic Maintained Schools), **The official employer for all teachers in voluntary grammar schools is the school's Board of Governors**

Denmark

- As a member of the single currency Denmark will, in case of a large public budget deficit, be sanctioned economically by the other European member-states: **Yes**, No, It has not been decided.

- Denmark can have an independent currency policy even if it decides to join the single currency. (Currency policy is the interest rate, currency rate of number of money in circulation) – Yes, **No**, It has not been decided.

- The value of the euro in Danish kroner is – 3,80 kr., **7,50 kr.**, 8,50 kr., 12,00 kr.

- If we vote yes at the referendum on the 28th September, the new currency will be in circulation in Denmark on – January 1, 2001, **January 1 2004**, January 1, 2007, January 1, 2010

- If Denmark joins the single currency, the Danish National Bank will be closed down, continue to operate as now, **become part of the European Central Bank**, or be a museum of Danish currency through times

- Will the single currency have a national part on its coins and bills? – **Yes**, No, It has not been decided

- Is Denmark already involved in a monetary union where the member states help each other in situations of an unstable foreign exchange market? – **Yes**, No

- I will now read some of the movements and parties up names. Please answer whether you think that these parties and movements recommend a yes or no on the euro in the coming referendum.

- People’s Movement against union – Recommend a yes, **Recommend a no**

- June Movement – Recommend a yes, **Recommend a no**

- Social Democrats – Recommend a yes, **Recommend a no**

- Social Liberal – **Recommend a yes**, Recommend a no

- The Conservative Party – **Recommend a yes**, Recommend a no

- Centre Democrats – **Recommend a yes**, Recommend a no
- Socialist People’s Party – Recommend a yes, **Recommend a no**
- Danish People’s Party – Recommend a yes, **Recommend a no**
- Christian Party – Recommend a yes, **Recommend a no**
- Left Denmark’s Liberal Party – **Recommend a yes**, Recommend a no
- Progress Party – Recommend a yes, **Recommend a no**
- Unity – Recommend a yes, **Recommend a no**
- Freedom 2000 – Recommend a yes, **Recommend a no**

EU 2007

- Which of the following countries is an official candidate to join the EU? Romania, Montenegro, **Croatia**, or Morocco.
- Are the members of the European Parliament ...? Answer options: **directly elected by the citizens of their country**, elected for three year terms, elected by the parliament of their country, or appointed by their national head of government
- Are new EU laws in the field of employment adopted by ...? the European Commission and in some cases with the European Parliament, **the Council of Ministers and in some cases with the European Parliament**, the European Parliament by itself or the European commission by itself
- Is the EU’s role regarding unemployment benefits to ...? Finance the member states’ unemployment benefit systems, decide the level and length of unemployment benefits in the member states, require that the member states merge their unemployment benefit systems by 2010 or **guarantee that all EU citizens have access to unemployment benefits where they live**
- Roughly a third of the EU’s budget is devoted to one of the following. Is it ...? Answer options: **Helping the EU’s less prosperous regions**, subsidizing the EU’s

fishing industry, financing diplomatic missions abroad or maintaining the EU's administration and bureaucracy

- By 2050, is the percentage of the adult EU population that is 65 or older projected to be ... Answer options: About one quarter of what it is now, about half of what it is now, about the same as what it is now, **about twice what it is now**, or about four times what it is now
- Which of the following is true of the amount of foreign aid given by the EU and its member states, combined, versus the amount given by the US? The EU and its member states give roughly four times as much, **the EU and its member states give roughly twice as much**, the EU and its member states and the US give about the same amount, the US gives roughly twice as much, or the US gives roughly four times as much
- On average, what percentage of the total spending by the governments of the EU member states for foreign aid? Answer options: **About 1%**, About 5%, About 9%, About 13%, or About 17%
- Which of the following countries does NOT possess nuclear weapons? Pakistan, India, North Korea or **Japan**
- In politics, people often talk of "left" and "right." On a scale from 0 to 10, where 0 means as far left as possible, 10 means as far right as possible, and 5 is exactly in the middle, where would you place the views of Nicholas Sarkozy (**6 to 10**)
- In politics, people often talk of "left" and "right." On a scale from 0 to 10, where 0 means as far left as possible, 10 means as far right as possible, and 5 is exactly in the middle, where would you place the views of Gordon Brown (**0 to 5**)

EU 2009

- Is the main decision-making body of the European Union the ...? European

Commission, **Council of Ministers**, European Parliament, European Court of Auditors, Couldn't say about that

- Only one of the following statements about the European Parliament is false.

Which one is it? **It passes all EU laws**, It can dismiss the European Commission, It can reject the budget proposed by the Council of Ministers, It is involved in decisions about the admission of new Member States, Couldn't say about that

- Is the European Union represented on the international stage by the ...?

European Commission, Council of Ministers, European Parliament, European Court of Auditors, Couldn't say about that

- Which of the following is true of Blue card workers? They can work anywhere in the EU, **They must have a university education**, They cannot bring family members to join them any faster than other immigrants, They are subject to the Returns Directive, Couldn't say about that

- Which of the following is true about the ways in which immigration policy is currently made? **The EU sets the basic rules about entry and residency requirements**, The EU decides how many immigrants can be admitted to each country, Work permits for immigrants must be approved by the EU, The EU plays no role in immigration policy, Couldn't say about that

- Which of the following is true of the EU's immigrants ...? **Most illegal immigrants enter the EU legally but outstay their visas**, Roughly 9% of the EU's total population was born outside the EU, Because of immigration, the EU's total population will increase by about 30 million by 2050, There are more illegal than legal immigrants in the EU, Couldn't say about that

- The percentage of the EU's total energy consumption that comes from fossil fuels (coal, gas or oil) is about ...? 50%, 60%, 70%, **80%**, Couldn't say about that, Couldn't say about that

- Which of the following produces the most greenhouse gases? **China**, The European Union, The United States, India, Couldn't say about that
- Which of the following is true about wind power in the European Union? Most of the EU's wind power is currently produced off-shore, **Wind power's share of EU energy consumption is increasing by about roughly 30% a year**, Wind power's share of the EU energy consumption is about three times that of solar power, Wind power's share of the EU energy consumption is currently about 5%, Couldn't say about that

Vermont

- Is the surcharge Vermonters pay on their electric bills for programs to reduce the need for electricity currently ...? Zero -there is no surcharge, **About half a cent per kilowatt hour**, About two cents per kilowatt hour, About five cents per kilowatt hour, About seven-and-a-half cents per kilowatt hour, Couldn't say
- What effect has Vermont's energy efficiency program had on the annual increase in the amount of electricity used by Vermonters? Has it - Had almost no impact on the increase, Reduced it by 20%, **Reduced it by 50%**, Reduced it by 70%, Reduced it by 90%, Couldn't say
- Excluding Hydro Quebec, about what percentage of its electricity does Vermont currently get from renewable resources? 5%, **15%**, **25%**, 40%, Couldn't say
- And about what percentage of its electricity Does Vermont currently get from the Vermont Yankee nuclear plant? 5%, 10%, 20%, **33%**, Couldn't say
- About what percentage of its electricity does Vermont currently get from Hydro Quebec? 15%, **33%**, 45%, 60%, Couldn't say
- Roughly what percentage of Vermont's electricity is currently generated within Vermont? 12%, 33%, **55%**, 72%, Couldn't say

- How do Vermont's electricity rates compare with those of the rest of New England? Are they, on average, ... Roughly 20% higher, Roughly 10% higher, About the same, **Roughly 10% lower**, Roughly 20% lower, Couldn't say
- What is the average electric bill for the typical Vermonter? Is it ... Roughly \$60, **Roughly \$80**, Roughly \$120, Roughly \$180, Couldn't say
- Does Vermont's contract with the Vermont Yankee nuclear power plant expire in ... 2010, **2012**, 2018, 2025, Couldn't say

San Mateo

- According to the Association of Bay Area Governments, between 1999 and 2006, what was the gap each year between the number of new homes needed and the number actually produced in San Mateo County? about 200, about 500, **about 1,000**, about 5,000, about 8,000, Or couldn't you say about that?
- As of September 2007, about how much was the median price of a single-family home in San Mateo County? \$550,000, \$680,000, \$750,000, \$830,000, **\$940,000**, Or couldn't you say about that?
- As of 2006, about what percentage of County households could afford a median-priced house in San Mateo County? **12%**, 23%, 37%, 43%, 55%, Or couldn't you say about that?
- As of September 2007, what was the average rent for a 2-bedroom apartment in San Mateo County? \$1,100, \$1,300, **\$1,700**, \$2,200, \$3,000, Or couldn't you say about that?
- According to the 2006 San Mateo County Housing Needs Study, by about how many housing units is the demand for housing in San Mateo County projected to grow between 2005 and 2025? 20,000, 40,000, 60,000, **70,000**, 90,000, Or couldn't you say about that?

- According to the same Housing Needs Study, among the new households that are expected to form in the County between 2005 and 2025, about what percentage, combined, would be households with low, very low and extremely low-incomes? less than one-third, less than one-half, **about half**, more than one-half, more than two-thirds, Or couldn't you say about that?
- According to the 2006 San Mateo County Housing Needs Study, roughly how many new homes (including houses, townhomes, condos, and apartments) will be built between 2005 and 2025 at the current rate of housing development? **25,000**, 50,000, 75,000, 100,000, Or couldn't you say about that?
- What percentage of San Mateo County's land is agricultural use, watershed, open space, wetlands, or parks? less than 25%, 33%, 58%, 67%, **more than 75%**, Or couldn't you say about that?

Michigan

- Which political party holds the majority in the Michigan State Senate? Or couldn't you say about that? Democrats, **Republicans**, Other, Or couldn't you say about that?
- How about the Michigan State House of Representatives? Which political party holds the majority there? Democrats, **Republicans**, Other, Or couldn't you say about that?
- Which of the following states has unemployment rates similar to Michigan's? **Oregon**, West Virginia, Ohio, Louisiana, Or, couldn't you say about that?
- People who have reached the 48-month limit can stay eligible for the Family Independence Program if...? **they live in a county with a high unemployment rate**, they were fired from a job while on the Family Self-Sufficiency Plan, they are

non-citizens on the Refugee Assistance Program, they have a small child who is active in the ChildFirst program, Or couldn't you say about that?

- About what percentage of African American children in Michigan live in poverty? Under 20%, About 30%, **About 40%**, Over 50%, Or couldn't you say about that?

California Referendum

- Which political party holds the majority in the California State Senate? **The Democratic Party**, Independents, The Republican Party, Couldn't say
- How about the California State Assembly? **The Democratic Party**, Independents, The Republican Party, Couldn't say
- How large a majority of the State Legislature is needed to approve a proposed constitutional amendment? A simple majority of both houses, a simple majority of the Assembly and two-thirds of Senate, **two-thirds of both houses**, three-fourths of both houses, Couldn't say
- How large a majority of the State Legislature is needed to increase taxes? A simple majority of both houses, a simple majority of the Assembly and two-thirds of Senate, **two-thirds of both houses**, three-fourths of both houses, Couldn't say
- Ballot measures can be signed by ...? anyone over 18 years of age, any citizen over 18 years of age, **only registered voters**, only registered voters who voted in last election, Couldn't say

BTP 2004 Primaries

- As far as you know, what did President Bush's tax cut do to the tax rate on income from investments such as dividends and capital gains? Did it ... **Lower it**, Keep it the same, Raise it, Don't know

- As far as you know, what was Wesley Clark's most recent position in government?
—Senator from Arkansas, Secretary of the Air Force, **Supreme Allied Commander of NATO**, National Security Advisor to the President, Don't Know
- As far as you know, did President Bush end U.S. tariffs on imported steel because . . . The steel industry and steelworkers unions objected, **He wanted to avoid increased tariffs by the European Union on American exports**, Domestic steel production is insufficient to meet demand, thus raising the price of steel, He has never supported tariffs, Don't Know
- Which one of the Democratic presidential candidates served as Majority Leader in the House of Representatives? John Edwards, Howard Dean, Joe Lieberman, **Richard Gephardt**, Don't know
- Since President Bush took office, would you say that the rate of unemployment in the US has —decreased, stayed about the same, **increased**, don't know
- So far the number of Americans killed in action in Iraq can be numbered as: fewer than a hundred, **several hundred**, several thousand, more than twenty thousand, don't know
- As far as you know, what is the Bush administration's position on creating a new Free Trade Area of the Americas? Does the Bush administration . . . **Support it**, Neither support nor oppose it, Oppose it, Don't know

BTP 2004 GE

- How did John Kerry vote in the Senate on the resolution authorizing President Bush to go to war with Iraq? Did he . . . **Vote for it**, Vote against it, Not vote on it, Don't Know
- Which of the following countries now harbors the most Al Qaeda and Taliban fighters? India, **Pakistan**, Sri Lanka, Indonesia, Don't Know

- Iraq was directly involved in the attacks on the WTC and the Pentagon on 9-11 2001.
True, **False**, Don't Know
- Large quantities of weapons of mass destruction have been found in Iraq. True, **False**, Don't Know
- On average, prescription drugs cost more in Canada than in the US. True, **False**, Don't Know
- Which of the following was true of George W. Bush during the Vietnam War? He was drafted but never went to Vietnam, He was a decorated officer serving in Vietnam, He was ineligible to serve in the military because of a medical deferment, **He served in the Texas Air National Guard**, Don't Know
- Which of the following was true of John Kerry during the Vietnam War? He was drafted but never went to Vietnam, **He was a decorated officer serving in Vietnam**, He was ineligible to serve in the military because of a medical deferment, He served in the Massachusetts Air National Guard, Don't Know
- A major destination of white collar jobs that have gone to other countries is? South Africa, Japan, Brazil, **India**, Don't Know
- Which of the following is closest to the number of Americans killed in Iraq since the war began? 100, 500, **1000**, 10,000, Don't Know

BTP 2005

- Among 29 wealthy industrialized nations, does the US rank top 3, top 10, **bottom 10**, or bottom 3 in students' math skills?
- In the last few years, it has become **harder**, easier or about the same to obtain US work permits for foreigners.

- In the last few years, the gap between minority and white students in math and reading tests at the elementary school level has **increased**, decreased, or stayed about the same.
- Number of uninsured Americans. 1 million, 25 million, **45 million**, 65 million or 85 million.
- Over the next fifty years, do you think the number of Americans over age 65 will ...? tripled, **doubled**, increased by half, or stayed about the same.
- The percentage of uninsured low income families. a quarter, fifty percent, **two-thirds**, or ninety percent.

BTP 2007

- About how many Americans are prevented from voting because of criminal convictions, or couldn't you say about that? 50,000, 500,000, **5,000,000**, 15,000,000, Couldn't say
- Does "gerrymandering" reshape districts to...? Increase electoral competition, **Ensure a majority for one party over another**, Include independents in the primary process, Make them much smaller, Couldn't say
- Approximately what percentage of eligible American voters actually vote in a given presidential general election, or couldn't you say about that? 10%, 30%, **50%**, 70%, Couldn't say
- Which of the following countries has a system of "compulsory voting," or couldn't you say about that? Ireland, United Kingdom, **Australia**, South Korea, Couldn't say
- At present, who must register with the selective service system in the US, or couldn't you say about that? Nobody, **Men between 18 and 25 years old**, Both men and

women between 18 and 25 years old, Men and women who receive federal grants for education, Couldn't say

- A candidate is elected President of the United States if he or she . . . Gets more votes than any other candidate, Gets a majority of all the votes cast, Gets more Electoral College votes than any other candidate, **Gets a majority of Electoral College votes**, Couldn't say
- How often are congressional districts redrawn, or couldn't you say about that? Every 5 years, **Every 10 years**, Every 15 years, Every 20 years, Couldn't say
- Which two states have traditionally had the earliest presidential primary events for determining their delegates to the parties' national conventions, or couldn't you say about that? Nevada and South Carolina, **Iowa and New Hampshire**, Indiana and New Mexico, Wyoming and Delaware, Michigan and Florida, Couldn't say

Appendix E Unaccounted Gains in Related Knowledge?

As we note above, the latent class model implicitly accounts for gains in related knowledge as most such gains are already reflected in increases in proportion correct. However, it is possible that some gains in related knowledge are not reflected in increases in proportion correct. For instance, consider the following tendentious case: an individual learns that a particular response option isn't correct, and also by some accident chooses only between the other remaining incorrect options. Such growth in related knowledge paired with that particular pattern of responding would not increase probability of marking the right answer. To address worries that our model fails to account for gains in related knowledge, we present estimates from closed-ended responses that offer just two response options. On the mild assumption that people choose the option that they are most confident about (for empirical support, see [Luskin and Sood 2012](#)), all respondents who are more than 50% sure about the correct answer will choose the correct answer. If we assume confidence in correct answer to be uniformly distributed across the entire number line, and if the process only creates an intercept shift in certainty, increase in proportion correct ought to completely reflect all growth in knowledge.

Thus we compare differences in learning between the 51 items that offer only two options, and 124 items that offer more than two options using various estimators. For identification, we assume that items with two options and items with more than two options are exchangeable. The mean difference in learning between items with more than two options and items with just two options was .060 based on the conventional estimator. If there are greater unaccounted for gains in items with two-options (items in which fewer, if not no, gains in related knowledge go unmeasured) than more than two-options, one ought to see this difference shrink when we account for guessing. But adjusting for guessing using the LCA, rather than decrease this difference, slightly increases the difference to .067 ($p < .01$). In all, worries about unaccounted for gains in related knowledge are not

supported by the data.

Appendix F Estimates of Learning over Deliberative Polls using Different Estimators

Table F1: Estimates of Learning over Deliberative Polls

Poll	<i>n</i>	<i>n (items)</i>	Raw	LCA	Std.
AU Monarchy	347	10	.284 (.014)	.439 (.018)	.345 (.023)
BTP 2004 Primaries	328	7	.027 (.016)	.06 (.017)	.034 (.014)
BTP 2004 GE	250	9	.071 (.01)	.089 (.013)	.084 (.021)
BTP 2005	454	6	.035 (.011)	.054 (.015)	.039 (.011)
BTP 2007	301	8	.112 (.01)	.126 (.012)	.122 (.022)
Bulgaria	278	7	.093 (.017)	.082 (.026)	.075 (.017)
Ca. Ref.	401	5	.211 (.015)	.286 (.016)	.231 (.024)
CPL	216	7	.161 (.016)	.19 (.02)	.155 (.016)
Denmark	363	9	.211 (.011)	.359 (.014)	.293 (.023)
EU 2007	335	11	.182 (.01)	.229 (.013)	.206 (.015)
EU 2009	348	6	.166 (.014)	.182 (.018)	.175 (.016)
N. Ireland	124	7	.286 (.022)	.323 (.028)	.337 (.023)
Michigan	310	9	.098 (.013)	.088 (.022)	.082 (.013)
NIC	466	8	.091 (.009)	.193 (.013)	.116 (.015)
San Mateo	239	8	.113 (.012)	.121 (.014)	.096 (.015)
SWEPCO	232	5	.203 (.02)	.233 (.026)	.222 (.032)
UK BGE	275	15	.103 (.012)	.191 (.018)	.157 (.018)
UK Crime	299	7	.095 (.013)	.142 (.02)	.102 (.016)
UK EU	224	5	.203 (.019)	.275 (.025)	.244 (.032)
UK Health	230	6	.079 (.014)	.122 (.02)	.134 (.025)
UK Monarchy	258	8	.152 (.01)	.212 (.015)	.209 (.022)
Vermont	146	9	.397 (.018)	.441 (.021)	.4 (.027)
WTU	230	5	.288 (.017)	.347 (.021)	.329 (.023)
Inverse Variance Weighted			.14 (.003)	.17 (.004)	.168 (.004)

Appendix G Three-wave Latent Class Model of the Relationship between Observed Response Patterns and Underlying Latent Transition Classes

The system of equations defining the multinomial distribution for response patterns across three waves of testing is gotten by taking the Kronecker product of a wave 1 classification matrix (see Table 1 in the main text and the Kronecker product of the classification matrices for two additional waves of testing. This result is then multiplied by a vector of latent class transition parameters, (λ) . The sequence of steps and resulting system of 27 equations are shown below.

$$\begin{pmatrix} & G & K & C \\ u = 0 & 1 - \gamma & 0 & 0 \\ u = 1 & \gamma & 1 & 0 \\ u = c & 0 & 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} & GG & GK & GC & KG & KK & KC & CG & CK & CC \\ u = 00 & (1 - \gamma)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 01 & (1 - \gamma)\gamma & (1 - \gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 0c & 0 & 0 & (1 - \gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ u = 10 & (1 - \gamma)\gamma & 0 & 0 & (1 - \gamma) & 0 & 0 & 0 & 0 & 0 \\ u = 11 & \gamma^2 & \gamma & 0 & \gamma & 1 & 0 & 0 & 0 & 0 \\ u = 1c & 0 & 0 & \gamma & 0 & 0 & 1 & 0 & 0 & 0 \\ u = c0 & 0 & 0 & 0 & 0 & 0 & 0 & (1 - \gamma) & 0 & 0 \\ u = c1 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & 1 & 0 \\ u = cc & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

This Kronecker product results in a 27×27 latent class transition matrix (not shown here), similar to one produced using equation 1 in the main text. The matrix describes the conditional probabilities of the possible three-wave response patterns given each possible latent class transition. For example, given the latent transition from guess to guess to guess (GGG), equations defining the probability of the 27 possible response patterns (000, 001, 00c, ..., cc0, cc1, ccc) are produced.

To get the final system of equations, the 27×27 latent class transition matrix is multiplied by the vector of possible latent class transition parameters, λ . Since we assume

that participants cannot transition from knowing to guessing or confessing, we assign those transition parameters as 0. The resulting vector of transition parameters is as follows:

$$\begin{pmatrix} \lambda_{GGG} \\ \lambda_{GGK} \\ \lambda_{GGC} \\ \lambda_{GKG} = 0 \\ \lambda_{GKK} \\ \lambda_{GKC} = 0 \\ \lambda_{GCG} \\ \lambda_{GCK} \\ \lambda_{GCC} \\ \lambda_{KGG} = 0 \\ \lambda_{K GK} = 0 \\ \lambda_{KGC} = 0 \\ \lambda_{KKG} = 0 \\ \lambda_{KKK} \\ \lambda_{KKC} = 0 \\ \lambda_{KCG} = 0 \\ \lambda_{KCK} = 0 \\ \lambda_{KCC} = 0 \\ \lambda_{CGG} \\ \lambda_{CGK} \\ \lambda_{CGC} \\ \lambda_{CKG} = 0 \\ \lambda_{CKK} \\ \lambda_{CKC} = 0 \\ \lambda_{CCG} \\ \lambda_{CCK} \\ \lambda_{CCC} \end{pmatrix}$$

The final result is the system of 27 equations defining π which identifies the underlying multinomial distribution of response patterns shown below.

$$\begin{pmatrix}
(1 - \gamma)^3 \lambda_{GGG} \\
(1 - \gamma)^2 \gamma \lambda_{GGG} + (1 - \gamma)^2 \lambda_{GGK} \\
(1 - \gamma)^2 \lambda_{GGC} \\
(1 - \gamma)^2 \gamma \lambda_{GGG} \\
(1 - \gamma) \gamma^2 \lambda_{GGG} + (1 - \gamma) \gamma \lambda_{GGK} + (1 - \gamma) \lambda_{KKK} \\
(1 - \gamma) \gamma \lambda_{GGC} \\
(1 - \gamma)^2 \lambda_{GCG} \\
(1 - \gamma) \gamma \lambda_{GCG} + (1 - \gamma) \gamma \lambda_{GCK} \\
(1 - \gamma) \lambda_{GCC} \\
(1 - \gamma)^2 \gamma \lambda_{GGG} \\
(1 - \gamma) \gamma^2 \lambda_{GGG} + (1 - \gamma) \gamma \lambda_{GGK} \\
(1 - \gamma) \gamma \lambda_{GGC} \\
(1 - \gamma) \gamma^2 \lambda_{GGG} \\
\gamma^3 \lambda_{GGG} + \gamma^2 \lambda_{GGK} + \lambda_{KKK} \\
\gamma^2 \lambda_{GCG} \\
(1 - \gamma) \gamma \lambda_{GCG} \\
\gamma^2 \lambda_{GCG} + \gamma \lambda_{GCK} \\
\gamma \lambda_{GCC} \\
(1 - \gamma)^2 \gamma \lambda_{CGG} \\
(1 - \gamma) \gamma \lambda_{CGG} + (1 - \gamma) \lambda_{CGK} \\
(1 - \gamma) \lambda_{CGC} \\
(1 - \gamma) \gamma \lambda_{CGG} \\
\gamma^2 \lambda_{CGG} + \gamma \lambda_{CGK} + \lambda_{CKK} \\
\gamma \lambda_{CGC} \\
(1 - \gamma) \lambda_{CCG} \\
\gamma \lambda_{CCG} + \lambda_{CCK} \\
\lambda_{CCC}
\end{pmatrix}
=
\begin{pmatrix}
\pi_{000} \\
\pi_{001} \\
\pi_{00c} \\
\pi_{010} \\
\pi_{011} \\
\pi_{01c} \\
\pi_{0c0} \\
\pi_{0c1} \\
\pi_{0cc} \\
\pi_{100} \\
\pi_{101} \\
\pi_{10c} \\
\pi_{110} \\
\pi_{111} \\
\pi_{11c} \\
\pi_{1c0} \\
\pi_{1c1} \\
\pi_{1cc} \\
\pi_{c00} \\
\pi_{c01} \\
\pi_{c0c} \\
\pi_{c10} \\
\pi_{c11} \\
\pi_{c1c} \\
\pi_{cc0} \\
\pi_{cc1} \\
\pi_{ccc}
\end{pmatrix}$$

The model that defines the three-wave multinomial distribution of response patterns affords some additional opportunities beyond the ones available in the two-wave model. In particular, the model allows for more accurate estimates of γ given that the number of lucky guesses at wave one and two are identified further with the addition of the third wave. The only ambiguous response patterns that remain are ‘111’ response patterns that represent three lucky guesses which are inherently highly unlikely. The three-wave model can also be used to calculate guessing adjusted estimates of learning between waves 1 and 2, and waves 2 and 3.

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