STABILITY OF POINT ESTIMATES IN THE UNFOLDING ANALYSIS OF STATE POLICY SPENDING

A report to accompany "A New Measure of Policy Spending Priorities in the American States"

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It is generally useful to determine the stability of parameter estimates obtained when a formal model is applied to empirical data. With statistical models, such as a regression equation, stability is usually conceptualized in terms of sampling error. However, that is not applicable to the spatial proximity model which is employed in "A New Measure of Policy Spending Priorities in the American States." For present purposes, we regard the unfolding analysis as a strictly descriptive procedure. It generates two sets of point coordinates (i.e., for states and policies, respectively) which summarize the yearly reflected proportionate policy spending data. And, the data themselves are regarded as given quantities for the time period under investigation. Therefore, the traditional idea of inferential stability is not really appropriate for the present situation (Gifi 1981).

Nevertheless, it is still important to determine the degree to which the estimated point locations are "tightly constrained" by the transformation from reflected spending values to interpoint distances. We can lay out the problem a bit differently, as follows: Let us assume that the input dataset is subjected to some small change. If the original estimated parameters in the spatial proximity model are stable, then the relative positions of state points and the policy points should not change very much when an unfolding analysis is carried out on the altered data. And, the amount of change that does occur can be interpreted as an estimate of the stability in the unfolding solution.

We operationalize this strategy by adapting a jackknifing procedure which was developed by de Leeuw and Meuman (1986). Specifically, we replicate the unfolding analysis nine times. On each replication, we remove one policy area, and estimate the state and policy point locations using only the remaining eight policy areas. This produces nine jackknife replicate values for each state point in each year, and eight jackknife replicate values for each of the policy points. Note each jackknife replication removes one-ninth of the original dataset (i.e., one column of the data matrix that is input to the unfolding analysis). In fact, this is a fairly large proportion of the data, so the jackknife procedure is a very rigorous test of the stability in the point estimates.

As explained in “A New Measure ...”, the unfolded point locations constitute interval-level data. This implies that the *relative* distances between the points are fixed, but that the origin and measurement units can be changed. Or, more succinctly, the unfolding solution is unique up to a linear transformation. Therefore, there are at least two ways to use the jackknife replications to assess stability in the estimates. First, we can correlate the jackknife replications with the point coordinates from the “real” unfolding results (i.e., those obtained from the complete data matrix, with nine policy areas and 1200 state/year observations). Second, we can use the procedures originally developed by Schönemann and Carroll (1970) to (a) translate and dilate the bootstrap replicates as necessary in order to bring the specific numeric values into maximum least-squares conformity with the “real” unfolding results, and then (b) examine the dispersion in the overall set of bootstrap replications. In this report, we discuss the results obtained by employing both of these strategies. Stability results are presented separately, for state points and policy points.

STABILITY OF STATE POINTS

Each jackknife replication removes one of the nine policy areas, and performs the unfolding analysis using the remaining eight policies. We will first consider the stability of the estimated state points, across the replications. Figure 1 shows a dot plot of the correlations between the original state points, and the state points obtained from each of the nine jackknife replications. Most of the correlations are very large: The mean coefficient across the replications is 0.908 So, the state points in the jackknife replications generally reproduce the original state points very closely.

However, the figure also reveals that the unfolding solutions depart fairly seriously from the original results in two of the replications: When education is omitted, the correlation between the replicated state points and the original state points is 0.875. And, when welfare is omitted, the correlation drops even farther, to 0.365. It is probably not too surprising that these two replications exhibit the lowest degree of conformity to the original results: Education and welfare comprise the two largest spending categories for all states, in all years, by a substantial margin. Leaving either of these two categories out of the dataset magnifies the sizes of the *relative* differences among the remaining spending categories, even though the actual amounts of spending remain unchanged, in absolute terms. This, in turn, produces unfolding results which differ more substantially from the results obtained in the original analysis of the full dataset. Note that all of the remaining eight correlations are much higher, with values greater than 0.965. Given the two unusually small correlations, it is probably better to use the median correlation as a summary of the values: The median of the coefficients is 0.989, and this does reflect the stability in the state point estimates which occurs in most of the jackknife replications.

The jackknife procedure produces an enormous amount of output, with nine replications of 50 state points, in each of the 24 years from 1982 through 2005. We are interested mainly in the dispersion of the yearly state points across the jackknife replications. And, we will use the inter-quartile range (the difference between the values of the third and first quartiles of the data, usually abbreviated the "IQR") as a measure of dispersion, since it is relatively resistant to outliers. Rather than trying to interpret the IQR's in absolute terms, we will express each state's IQR for each year as a percentage of the range in the original state points for that year (i.e., the unfolded state points obtained from the complete dataset). Since this still leaves 24 distinct IQR's for each state (one for each year), we will take each state's median IQR, across the years. These successive summaries of the dispersion in the yearly state point estimates are used simply to reduce the amount of information to a manageable and comprehensible size. The complete output from the jackknife procedure is available from the authors, upon request.

Figure 2 is a dot plot showing the median state IQR's, expressed as percentages of the yearly ranges in the state point values. The horizontal axis in the figure ranges from zero to 100, in order to show clearly that the IQR's for the states are all very small, relative to the overall ranges of the state points in each year. This can be discerned very easily, since the plotted points all fall close to the left-hand side of the plotting region. But, it is difficult to say anything more about the specific values of the IQR's from Figure 2, alone, since the points are all crammed into a small physical range.

In order to see more detail in the results, Figure 3 presents the same information as Figure 2, but with a much smaller range of values on the horizontal axis. The state with the smallest dispersion is Kentucky, with a median IQR that covers only 1.54% of the range. At the other extreme, Nevada has the greatest dispersion, with a median IQR of 10.02%. Connecticut also has a fairly large IQR, at 8.06%. But, the latter two states are actually outliers, with much larger values than the remaining states. In fact, the median IQR for all 50 states is very small, at only 3.18%. This provides very strong evidence that the unfolded state point locations are stable and quite resistant to small changes in the data used to produce those point locations.

STABILITY OF POLICY POINTS

The unfolded policy points have the most extreme coordinate values, occupying the two opposing ends of the continuum recovered from the scaling procedure. In assessing stability, we are probably less concerned with the specific locations of the individual state points, so long as the jackknife replications consistently produce the two contrasting subsets that appeared in the original unfolding solution. And, it is important that the latter continue to fall above and below the the distribution of unfolded state points, just as they do in the "real" scaling solution, obtained from the complete dataset.

The jackknife procedure does, for the most part, reproduce the particularized benefit and collective goods subsets of state points. However, the points for two of the policy areas "switch sides" on a few of the replications. Specifically, the point representing health care moves from the particularized benefits cluster to the collective goods side on the jackknife replication that omits education spending. And, the point representing corrections moves from the collective goods side of the continuum over to the particularized benefits on the three jackknife replications which omit welfare, health care, and hospitals, respectively.

These "switches" in location occur on fewer than half of the jackknife replications. Both health care and corrections represent relatively small proportions of state spending. And, since only one policy changes relative positions on any given replication, the movements do not really compromise the basic nature of the two groupings. For these reasons, we are not unduly troubled by these movements in a few of the policy points across the jackknife replications.

Turning to the summary measures of stability, Figure 4 shows a dot plot of the correlations between the eight policy points estimated on each jackknife replication, and the coordinates of those same eight policy points in the original unfolding results. All nine of the correlations are quite high. But, they are also divided clearly into two subsets: Five of the correlations are extremely large, approaching 1.00. Four correlations are lower, at around 0.70; these occur on the jackknife replications where one of the points "switched sides." The latter, of course, pull the value of the mean correlation down a bit, to 0.867. The median correlation downplays the effect of the four lowest coefficients; its value is much higher than the mean, at 0.999. Thus, apart from the location changes that occur for the health care and corrections points, the estimated point locations in the jackknife replications are almost perfectly linearly related to the original unfolded policy points.

Figure 5 shows a plot which summarizes the dispersion in the jackknife replications of the nine separate policy points. Specifically, the horizontal line segments run from the first to the third quartiles of the jackknife replications for each policy; thus, the lengths of the segments represent the IQR's. The vertical, dashed, lines represent the range of the original unfolded policy points (i.e., the most extreme policy points obtained from the "real" scaling procedure, using all of the data). For eight of the nine policy areas, the point locations hardly vary at all across the replications; in fact, the line segments are so short that they are a bit difficult to see in the figure. The exception, of course, is the segment for corrections, which stretches across nearly the full range of the original policy points. Once again, this results from the "switching" that occurs in the location of this policy's point, across the jackknife replications. The latter is definitely an exception to the more general finding: For most of the policies (and, in fact, for most of the replications of the corrections policy point, too), the estimated point locations are extremely stable.

In conclusion, this report has presented evidence about the stability of the point coordinates estimated for the spatial proximity model of policy spending in the American states. The unfolding technique employed in “A New Measure of Policy Spending Priorities in the American States” is a data-analytic, rather than inferential, procedure. But, it is still important to demonstrate that the empirical results are *not* due to the impact of unusual observations or some unique combination of numeric values which simply happens to arise in our particular dataset. The jackknife resampling procedure laid out in this report demonstrates that the latter problems do not occur. The relative positions of the unfolded points— and, particularly, those representing the states— are generally very stable. These results provide even greater confidence that the state points can be used as a measure of yearly policy spending priorities.

REFERENCES

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Figure 1: Dot plot showing correlations between original unfolded state points and the state points obtained on each of the jackknife replications.

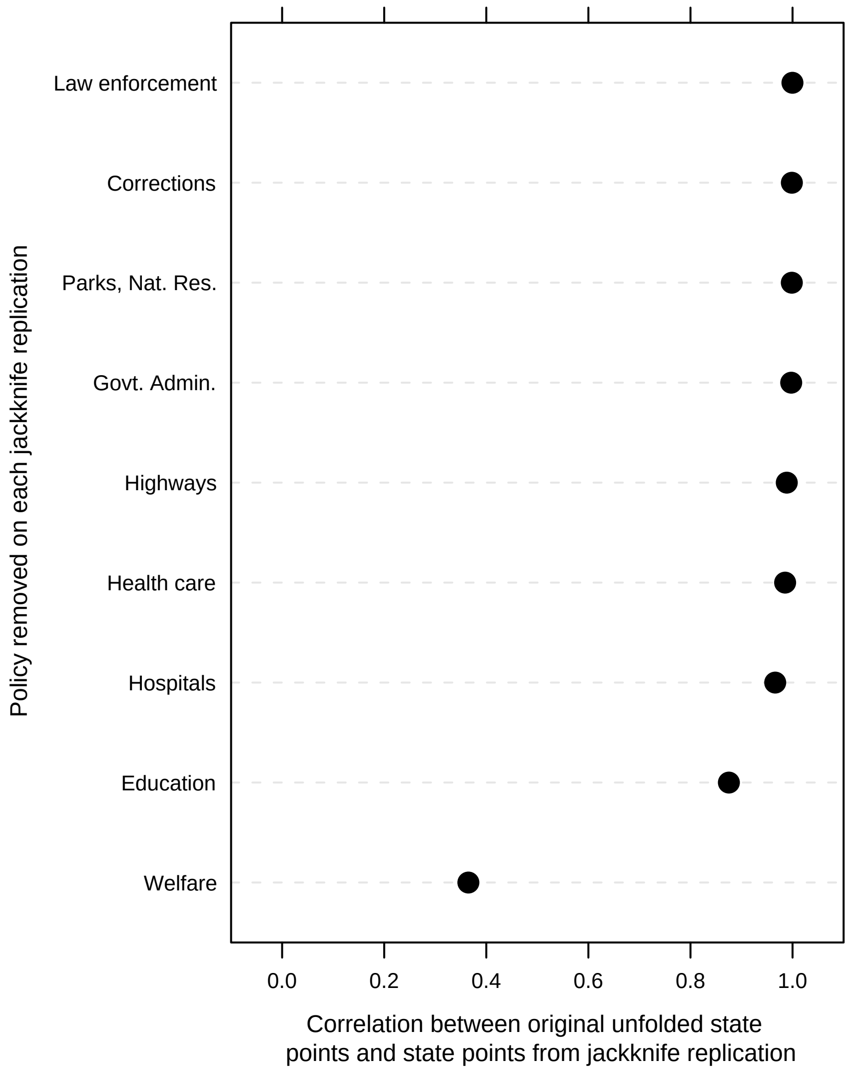

Figure 2: Dot plot showing median inter-quartile range (across the 24 years) of the jackknife replications for each state point. Values are expressed as a percentage of the yearly range in the unfolded state points (obtained in the original unfolding analysis, using all of the data). Horizontal axis is set to range from zero to 100.

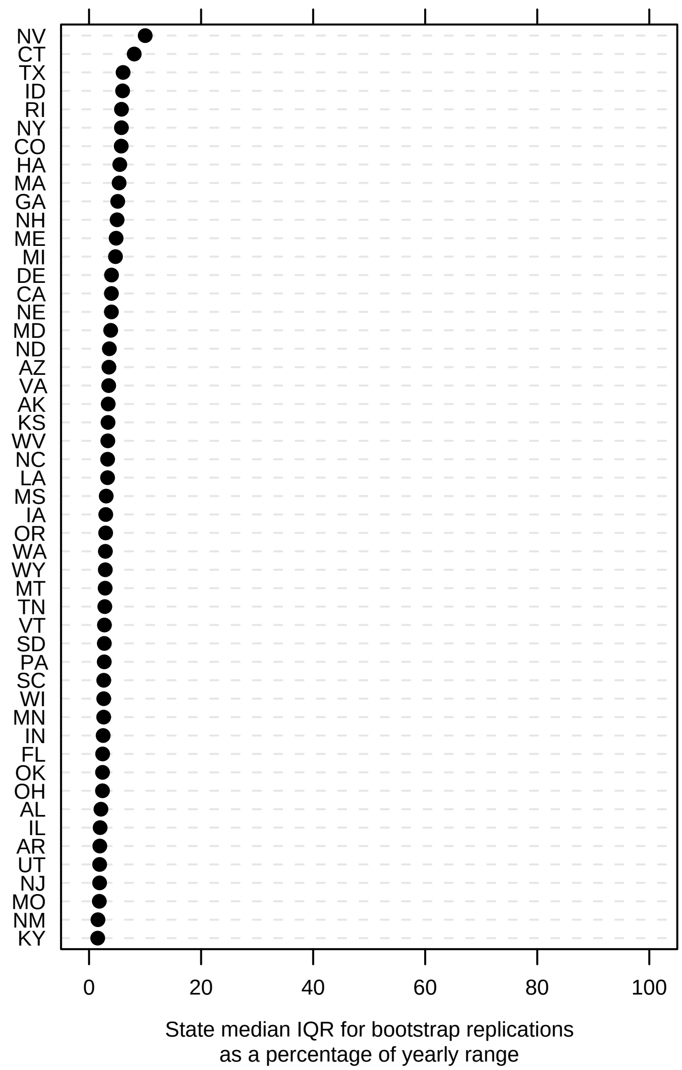

Figure 3: Dot plot showing median inter-quartile range (across the 24 years) of the jackknife replications for each state point. Values are expressed as a percentage of the yearly range in the unfolded state points (obtained in the original unfolding analysis, using all of the data). Horizontal axis is set to just contain the points.

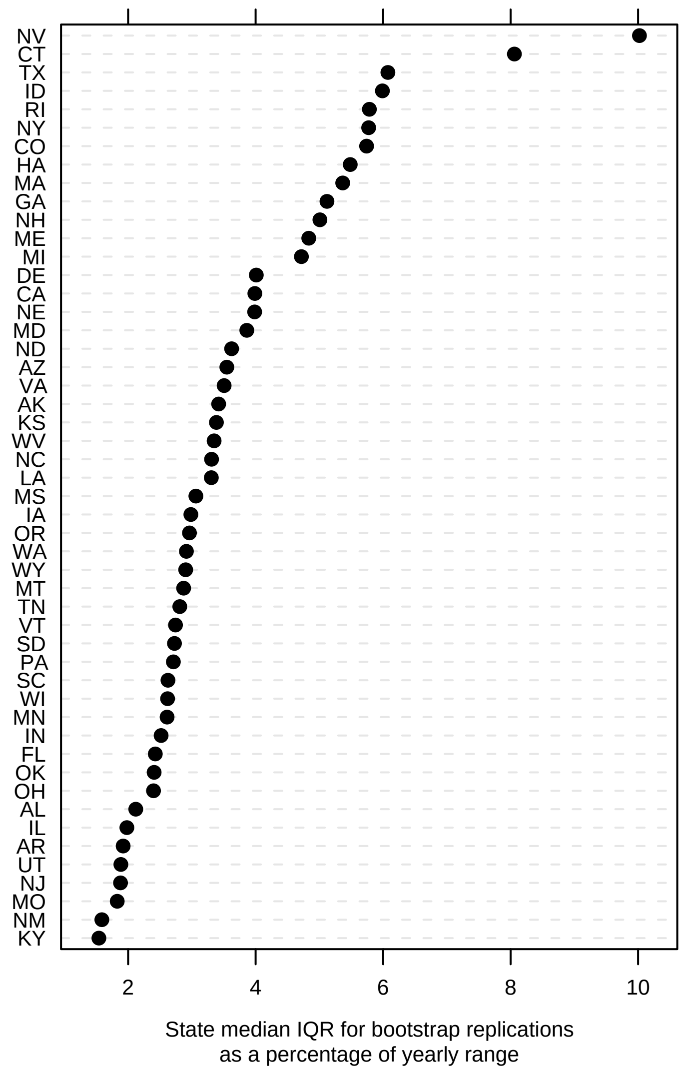

Figure 4: Dot plot showing correlations between original unfolded policy points and the policy points obtained on each of the jackknife replications.

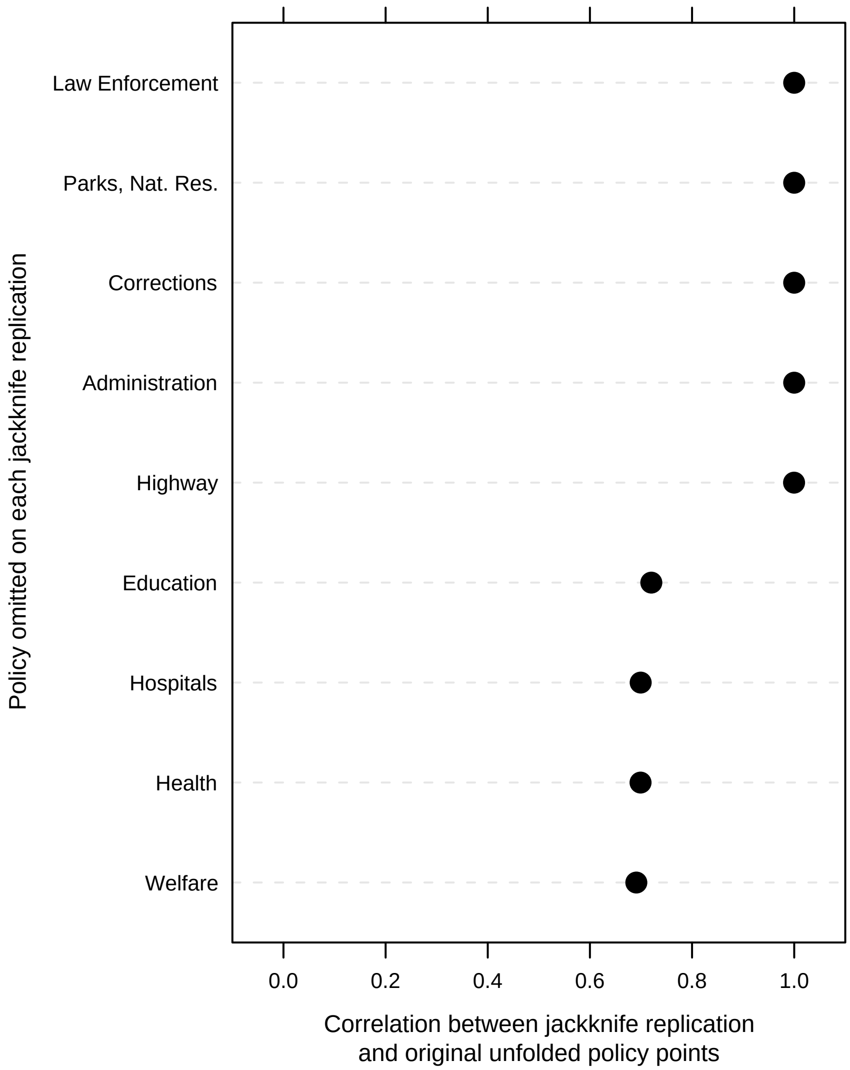

Figure 5: Plot showing the interval containing the central 50% of policy points obtained on jackknife replications. Vertical dashed lines show the range of the policy points from the original unfolding analysis (i.e., based upon all of the data).