

Supplemental Materials: Description of Model

The model predicts $\eta(x, y, t)$, the ice-shelf's vertical displacement field, in response to $F(x, y, t)$, a specified surface load, using the following equations:

$$\frac{\partial \Phi}{\partial t} - \mathbf{V}^{-1} \mathbf{M} = \mathbf{0} \quad (1)$$

$$-\frac{\partial^2 M_{xx}}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial y \partial x} - \frac{\partial^2 M_{yy}}{\partial y^2} + \rho_{sw} g \eta = F \quad (2)$$

$$\Phi + \mathbf{D}^{-1} \mathbf{M} - \mathbf{H} = \mathbf{0} \quad (3)$$

where x , y , and t are horizontal spatial coordinates and time, respectively, g is the acceleration of gravity, ρ_{sw} is the density of sea water, \mathbf{M} is a vector of bending moment components (see below), Φ is a temporary vector-valued variable defined by the third of the above equations, \mathbf{H} is a vector containing the components of displacement curvature, and \mathbf{D}^{-1} and \mathbf{V}^{-1} represent elastic and viscous material property operators, respectively:

$$\mathbf{H}^T = \left[\frac{\partial^2 \eta}{\partial x^2} \quad \frac{\partial^2 \eta}{\partial y^2} \quad \frac{\partial^2 \eta}{\partial x \partial y} \right] \quad (4)$$

$$\mathbf{M}^T = [M_{xx} \quad M_{yy} \quad M_{xy}] \quad (5)$$

$$\mathbf{D}^{-1} = -\frac{12}{EH^3} \begin{bmatrix} 1 & -\mu & 0 \\ -\mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu^2)}{1-\mu} \end{bmatrix}, \quad \mathbf{V}^{-1} = -\frac{1}{\nu H^3} \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad (6)$$

where E is the Young modulus, μ is the Poisson ratio, H is ice thickness and ν is the Newtonian viscosity.

Modifying the above treatment of viscoelastic ice-shelf flexure to account for the non-Newtonian creep behavior of ice presents a challenge. According to the above treatment, horizontal strain and strain rate components vary linearly with vertical distance through the ice shelf. Under the assumption of linear elasticity and viscous flow, this implies that variation of horizontal stress components within the ice shelf (i.e., T_{xx} , T_{yy} and $T_{xy} = T_{yx}$) is also linear in vertical distance through the ice shelf. With Glen's flow law, the assumed linear variation of horizontal strain rates with vertical distance then implies that stress varies non-linearly. This may seem like an insurmountable dilemma. Its resolution, however, comes from the thin-plate assumption. According to the assumption, the bending moment \mathbf{M} and η are the principle variables of the problem. We thus proceed with the analysis of viscoelastic creep flexure by eliminating stress as a variable in favor of the bending moment \mathbf{M} .

Glen's law is expressed using a viscosity ν that is a function of the second invariant of the strain rate tensor $\dot{\mathbf{e}}$:

$$T'_{ij} = 2\nu \dot{e}_{ij} \quad (7)$$

where \mathbf{T}' is the deviatoric stress tensor. The viscosity is a function of the second invariant of the strain rate \dot{e}_{II} :

$$\nu = \frac{B}{2\dot{e}_{II}^{1-\frac{1}{n}}} \quad (8)$$

where $B(\zeta)$ is the flow rate constant, n is the flow-law exponent, and

$$\dot{e}_{II}^2 = \dot{e}_{xx}^2 + \dot{e}_{yy}^2 + \dot{e}_{xx} \dot{e}_{yy} + \dot{e}_{xy}^2 \quad (9)$$

Adhering to the simplification associated with the thin-plate approximation, and taking $B(\zeta) = \bar{B}$ to be a constant, where ζ is the vertical distance coordinate taken to be zero at the ice-shelf's mid plane (alternative expressions when B is a function of ζ require evaluation of an integral), we obtain the relation between \mathbf{M} and the time-derivative of \mathbf{H} :

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = -\frac{\bar{\nu}H^3}{2n+1} \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \cdot \frac{\partial \mathbf{H}}{\partial t} \quad (10)$$

where, the effective viscosity, $\bar{\nu}$, is

$$\bar{\nu} = \frac{n}{2} \left(\frac{2n}{2n+1} \right)^{n-1} \bar{B}^n \left(\frac{H}{2} \right)^{2(n-1)} \left[\frac{1}{3} (M_{xx}^2 + M_{yy}^2 - M_{xx}M_{yy}) + M_{xy}^2 \right]^{\frac{1-n}{2}} \quad (11)$$

With the definition for \mathbf{M} given by eqn. (10) substituted into eqn. (1), the expression for \mathbf{V} becomes:

$$\mathbf{V} = -\frac{\bar{\nu}H^3}{2n+1} \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \quad (12)$$

which reduces to the expression for constant viscosity given when $n = 1$.

The viscoelastic flexure problem considered in this study is axisymmetric and thus uses polar coordinates r and θ instead of x and y . The study further assumes that θ derivatives of η are zero, thus rendering η a function of r and t only. The expressions given above in Cartesian coordinates reduce to the following:

$$\frac{\partial \Phi}{\partial t} - \mathbf{V}^{-1} \mathbf{M} = \mathbf{0} \quad (13)$$

$$-\frac{\partial^2 M_{rr}}{\partial r^2} - \frac{2}{r} \frac{\partial M_{rr}}{\partial r} + \frac{1}{r} \frac{\partial M_{\theta\theta}}{\partial r} + \rho_{sw} g \eta = F \quad (14)$$

$$\Phi + \mathbf{D}^{-1} \mathbf{M} - \mathbf{H} = \mathbf{0} \quad (15)$$

where,

$$\mathbf{M} = \begin{bmatrix} M_{rr} \\ M_{\theta\theta} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \frac{\partial^2 \eta}{\partial r^2} \\ \frac{1}{r} \frac{\partial \eta}{\partial r} \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi_r \\ \Phi_\theta \end{bmatrix}, \quad \mathbf{V} = -\frac{\nu H^3}{3} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}, \quad \mathbf{D} = -\frac{EH^3}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix} \quad (16)$$

in the case of constant viscosity. Treatment of Glen's law requires,

$$\begin{bmatrix} M_{rr} \\ M_{\theta\theta} \end{bmatrix} = -\frac{\bar{\nu}H^3}{2n+1} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \dot{\eta}_f}{\partial r^2} \\ \frac{1}{r} \frac{\partial \dot{\eta}_f}{\partial r} \end{bmatrix} \quad (17)$$

where,

$$\bar{\nu} = \frac{n}{2} \left(\frac{2n}{2n+1} \right)^{n-1} \bar{B}^n \left(\frac{H}{2} \right)^{2(n-1)} \left[\frac{1}{3} (M_{rr}^2 + M_{\theta\theta}^2 - M_{rr}M_{\theta\theta}) \right]^{\frac{1-n}{2}} \quad (18)$$