APPENDIX 1

On Chang's Account of Parfit's Notion of Imprecise Equality

Chang claims that Parfit would deny something like *Condition* 2, but rather start from precise betterness (e.g. '– is better by amount *x* than –') and equality relations under *Condition* 1, from which the imprecise betterness and equality relations then would have to be derived. When Parfit talks about precision, he seems in one aspect to have precise comparison of value differences in mind. Precise relations would thus be relations between *differences* in value. So far, this would support Chang's account. However, Parfit goes on to acknowledge that, in most cases, all we need is to rank objects and that can be done without precision, i.e. without assumptions about direct comparisons of differences in value. Thus, in practice, he does *not* assume betterness relations between differences in value; rather, he sticks to a betterness relation between objects and merely distinguishes between precise and imprecise equality, just as I have done. Hence, he clearly seems to accept *Condition* 2. If so, Parfit *in practice* adopts a view very much like Chang's own and *not* like her account of his (intended wider) view.

APPENDIX 2

Proofs

Observation 1: Given that self-concatenation is value increasing, *Condition* 5 is implied by *Condition* 3.

Proof:

Assume, for *reductio*, that *Condition* 3 is true, but *Condition* 5 is false. The latter implies

(1) ($t\boldsymbol{x}$ is better than $\boldsymbol{y}$) and (there is n: $\boldsymbol{y}\circ n\boldsymbol{x}$ is better than $t\boldsymbol{x}\circ n\boldsymbol{x}$)

(2) Let $\boldsymbol{x}$ be better than $\boldsymbol{y}$

(3) There is n: $\boldsymbol{x}\circ n\boldsymbol{x}$ is better than $t\boldsymbol{x}\circ n\boldsymbol{x}$ (from (1), (2) and *Condition* 3)

(4) (3) violates that concatenation is value increasing *q.e.d.*

Observation 4: Consider a sequence of objects ***b***, …, ***q*** of decreasing value, where ***b*** is lexically better than ***q***. Assume that '– is better than –' is transitive and that *Condition* 1 holds. Let ***x*** belong to the set {***x*** | ***b*** is better than ***x*** and ***x*** is better than ***q***}. For any ***x***: Assume that ***b*** is not lexically better than ***x***, i.e. is exchangeable with ***x***. Then ***x*** is lexically better than ***q***.

Proof:

(1) ***b*** is lexically better than ***q***: there is *m*, for all *n*: *m**b*** is better than *n**q*** (assumption)

(2) ***b*** is not lexically better than ***x***: for all *m*, there is *n*: *m**b*** is not better than *n**x*** (assumption, from *Definition* 3)

(3) There is *n*, such that *m*'***b*** is not better than *n**x*** (from (2), *m*' from (1))

(4) There is *n*: *n**x*** is at least as good as *m*'***b*** (from (3) and *Condition* 1)

(5) There is *n*, for all *t*: *n**x*** is better than *t**q*** (from (4), (1) and transitivity of 'better than'), i.e. ***x*** is lexically better than ***q*** *q.e.d.*

Observation 5: If, for some discrete sequence ***a***, …, ***z***, *Premise* (A) holds, and transitivity holds for betterness, then ***a*** is exchangeable with ***z***.

Proof

a is exchangeable with ***b***, i.e. for all m_a, there is some n_b such that n_b***b*** is better than m_a***a***. And ***b*** is exchangeable with ***c***, i.e. for all m_b, there is some n_c, such that n_c***c*** is better than m_b***b***. But since there is some n_b such that n_b***b*** is better than m_a***a***, it follows by transitivity, that there is some n_c, such that, for all m_a, n_c***c*** is better than m_a***a***. This procedure can be repeated all the way down to ***z***, such that we reach the conclusion that ***a*** is exchangeable with ***z***.

Condition α: if ***x*** is lexically better than ***y***, then *Condition* 1 applies for all *m**x*** and *n**y***, i.e. for all *m* and *n*: either *m**x*** is better than *n**y***, or *n**y*** is better than *m**x***, or *m**x*** and *n**y*** are precisely equally good.

Observation 6: Assume ***x*** is better than ***y***. *Condition* 3 and *Condition* 5 (*Non-diminishing Marginal Value*) and *Condition α* implies that, if ***x*** is lexically better than ***y***, then ***x*** is also strictly lexically better than ***y***, i.e. if for some *m* and all *n*, *m**x*** is better than *n**y***, then ***x*** is better than *n**y***.

Proof:

Assume, for *reductio*, that, for some *m* and all *n*, *m**x*** is better than *n**y***, but for some *t*, ***x*** is not better than *t**y***.

(1) For some *m* and all *n*: *m**x*** is better than *n**y*** (assumption)

(2) For some *t*: ***x*** is not better than *t**y*** (assumption)

(3) For some *t*: *t**y*** is at least as good as ***x*** (from (2) and *Condition α*)

(4) For some *t*’ = *t*+1: *t*’***y*** is better than ***x*** (from (3), the assumption that concatenation is value increasing and transitivity of ‘better than’)

(5) Let (4) be the basis of a mathematical deduction, for *n* = 2

(6) Hypothesis of the induction: (*n*-1)*t*’***y*** is better than (*n*-1)***x***

It shall be established that, if the hypothesis holds for (*n*-1), it also holds for *n*.

(7) (*n*-1)*t*’***y***◦***x*** is better than *n**x*** (from (6) and *Condition* 3)

(8) (*n*-1)*t*’***y***◦*t*’***y*** is better than (*n*-1)*t*’***y***◦***x*** (from (4) and *Condition* 5)

(9) *nt*’***y*** is better than *n**x*** (from (7) and (8) and transitivity of ‘better than’; conclusion of the mathematical induction)

(10) For *n* = *mt*’: *n**y*** is better than *m**x*** (mathematical induction from (4), *m* from (1))

(11) (10) contradicts (1) *q.e.d.*

Observation 7: Given that concatenation is value-increasing, $\boldsymbol{x}$ is both minimally lexically better than and minimally exchangeable with $\boldsymbol{y}$ is equivalent with $\boldsymbol{x}$ is radically imprecisely equal to $\boldsymbol{y}$.

Proof of $\boldsymbol{x}$ *is both minimally lexically better than and minimally exchangeable with* $\boldsymbol{y}$ *implies that* $\boldsymbol{x}$ *is radically imprecisely equal to* $\boldsymbol{y}$*.*

(1) The definition of minimal lexical betterness identifies some m', such that for all n, $n\boldsymbol{y}$ is not better than $m'\boldsymbol{x}$ (from *Definition* 13)

(2) for all $m \geq m'$ and for all n, $n\boldsymbol{y}$ is not better than $m\boldsymbol{x}$ (from 1 and (Concatenation is value-increasing))

(3) for all $m \geq m'$ there is n_m, such that $m\boldsymbol{x}$ is not better than $n_m\boldsymbol{y}$ (from *Definition* 12)

(4) for all $m \geq m'$ there is n_m, such that for all $n \geq n_m$, $m\boldsymbol{x}$ is not better than $n\boldsymbol{y}$ (from 3 and (Concatenation is value increasing))

(5) for all $m \geq m'$ there is n_m, such that for all $n \geq n_m$, $n\boldsymbol{y}$ is not better than $m\boldsymbol{x}$ and $m\boldsymbol{x}$ is not better than $n\boldsymbol{y}$ (from (2) and (4))

(6) for all $m > m'$ there is n_m, such that for all $n > n_m$, $m\boldsymbol{x}$ is imprecisely equal to $n\boldsymbol{y}$ *q.e.d.*

Proof of $\boldsymbol{x}$ *is radically imprecisely equal to* $\boldsymbol{y}$ *implies that* $\boldsymbol{x}$ *is minimally lexically better than* $\boldsymbol{y}$*.*

(7) There is m', such that for all n, $m'\boldsymbol{x}$ is imprecisely equal to $n\boldsymbol{y}$ (from *Definition* 15)

(8) There is m', such that for all n, $n\boldsymbol{y}$ is not better than $m'\boldsymbol{x}$ (from (7)) *q.e.d.*

Proof of $\boldsymbol{x}$ *is radically imprecisely equal to* $\boldsymbol{y}$ *implies that* $\boldsymbol{x}$ *is minimally exchangeable with* $\boldsymbol{y}$*.*

(9) For all $m \geq m'$, there is n_m, such that for all $n \geq n_m$, $m\boldsymbol{x}$ is imprecisely equal to $n\boldsymbol{y}$ (from *Definition* 15)

(10) For all $m \geq m'$, there is n, such that $m\boldsymbol{x}$ is not better than $n\boldsymbol{y}$ (from (9))

(11) For all $m < m'$, $m'\boldsymbol{x}$ is better than $m\boldsymbol{x}$ (from (Concatenation is value-increasing))

(12) For all m, there is n, such that $m\boldsymbol{x}$ is not better than $n\boldsymbol{y}$ (from (11), (10) and transitivity of '– is better than –') *q.e.d.*

Observation 9: Consider a (possibly order-dense) set of objects ***b***, …, ***q*** of decreasing value, where ***b*** is lexically better than ***q***. Assume that '– is better than –' is transitive and that *Condition* 2 holds. Let ***x*** belong to the set {***x*** | ***b*** is better than ***x*** and ***x*** is better than ***q***}.

(9.1) For any ***x***: Assume that ***b*** is not lexically better than ***x***. Then ***x*** is minimally lexically better than ***q***.

(9.2) For any ***x***: Assume that ***x*** is not lexically better than ***q***. Then ***b*** is minimally lexically better than ***x***.

Proof of (9.1)

(1) For *reductio*, assume that ***x*** is not minimally lexically better than ***q***: for all t, there is n: $n\boldsymbol{q}$ is better than $t\boldsymbol{x}$.

(2) For some m and all n: $m\boldsymbol{b}$ is better than $n\boldsymbol{q}$ (assumption)

(3) $m\boldsymbol{b}$ is better than $n\boldsymbol{q}$ (from (2), n from (1))

(4) For all t: $m\boldsymbol{b}$ is better than $t\boldsymbol{x}$ (from (1), (3) and transitivity of '– is better than –', m from (2))

(5) (4) contradicts the assumption that ***b*** is not lexically better than ***x*** *q.e.d.*

Proof of (9.2)

(1) For *reductio*, assume that ***b*** is not minimally lexically better than ***x***: for all m, there is n: $n\boldsymbol{x}$ is better than $m\boldsymbol{b}$.

(2) ***b*** is lexically better than ***q***, i.e. there is m, for all n, $m\boldsymbol{b}$ is better than $n\boldsymbol{q}$ (assumption)

(3) There is *n*: *n**x*** is better than *m**b*** (from (2) and (1), *m* from (2))

(4) There is *n*, for all *t*, *n**x*** is better than *t**q*** (from (2), (3) and transitivity of '– is better than –')

(5) (4) contradicts the assumption that ***x*** is not lexically better than ***q*** *q.e.d.*

Observation 10: For some finite discrete sequence ***a***, …, ***z***, where each member is exchangeable with its successor except for one neighboring pair (***i***, ***j***) ***i***≠***z*** and ***j***≠***a***, where ***i*** is radically imprecisely equal to ***j***, and transitivity holds for betterness, ***a*** cannot be lexically better than ***z***.

Proof: Assume, for *reductio*, that ***a*** is lexically better than ***z***. Then we have

(1) ***j*** is exchangeable with ***z*** (from transitivity of exchangeability)

(2) ***a*** is exchangeable with ***i*** (from transitivity of exchangeability)

Lemma 1: if ***x*** is lexically better than ***y***, then **y** is exchangeable with ***x***.

Proof: for any *n*, *m**x*** as determined by ***x*** being lexically better than ***y*** will always be better than *n**y***.

(3) ***z*** is exchangeable with ***a*** (from *Lemma* 1)

(4) ***j*** is exchangeable with ***i*** (from (1)-(3) and transitivity of exchangeability)

Lemma 2: If ***x*** is minimally lexically better than ***y***, then ***y*** is minimally exchangeable with ***x***.

Proof: for any *n*, *n**y*** will not be better than *m**x*** as determined by ***x*** being minimally lexically better than ***y***

(5) ***j*** is minimally exchangeable with ***i*** (from assumption and Lemma 2)

(6) ***j*** is minimally lexically better than ***i*** (from *Observation* 8)

(7) ***j*** is not exchangeable with ***i***, which contradicts (4) (from *Lemma* 2 and *Definition* 12) *q.e.d.*

Note that the proof breaks down (1) if ***i*** is lexically better than ***j***; and (2) if radical imprecise equality obtains between 2 pairs of neighboring objects.

Observation 11: Assume ***x*** is better than ***y***. *Condition* 3 and *Condition* 5 implies that, if ***x*** is minimally lexically better than ***y***, then ***x*** is also strictly minimally lexically better than ***y***, i.e. if for some *m* and all *n*, *n**y*** is not better than *m**x***, then *n**y*** is not better than ***x***.

Proof

(1) Assume, for *reductio*, that, for some *m* and all *n*, *n**y*** is not better than *m**x***, but for some *t*, *t**y*** is better than ***x***.

(2) For some *m* and all *n*: *n**y*** is not better than *m**x*** (assumption)

(3) For some *t*: *t**y*** is better than ***x*** (assumption)

(3) Let (2) be the basis of a mathematical deduction, for *n*=2

(4) Hypothesis of the induction: (*n*-1)*t**y*** is better than (*n*-1)***x***

It shall be established that, if the hypothesis holds for (*n*-1), it also holds for *n*.

(5) (*n*-1)*t**y***◦***x*** is better than *n**x*** (from (4) and *Condition* 3)

(6) *nt**y*** is better than (*n*-1)*t**y***◦***x*** (from (2) and *Condition* 5)

(7) *nt**y*** is better than *n**x*** (from (5) and (6) and transitivity of 'better than'; conclusion of the mathematical induction)

(8 For *n*=*mt*: *n**y*** is better than *m**x*** (mathematical induction from (2), *m* from (2))

(9) (8) contradicts (1) *q.e.d.*

APPENDIX 3

How the Lower Bound for Exchangeability View Violates the Non-Elitism Condition

According to Arrhenius,[1] *the Non-Elitism Condition* implies what he calls *Condition β*. Thus, if *Condition β* does not hold, *the Non-Elitism Condition* does not hold either. I shall first demonstrate that *the Lower Bound for Exchangeability View* violates *Condition β*.

Arrhenius' framework assumes that welfare levels are discrete. Let $\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$ designate a population of lives with welfare ranging from the welfare of ***z*** to one discrete welfare level above the welfare of ***y***. In my terminology, *Condition β* can be stated thus:

> For any triplet of lives ***x***, ***y***, ***z*** of descending value and any positive integer *n*, there is a number $m > n$ such that, if we have $n\boldsymbol{x}$, $m\boldsymbol{z}$ and $(m+n)\boldsymbol{y}$, then for any population $\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$, $(m+n)\boldsymbol{y}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$ is at least as good as $n\boldsymbol{x}\circ m\boldsymbol{z}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$.[2]

Let me specify the negation of this condition (the condition might be compelling, but certainly not intuitively so):

> For any triplet of lives ***x***, ***y***, ***z*** of descending value, there is a positive integer *n*, such that for all numbers $m > n$, if we have $n\boldsymbol{x}$, $m\boldsymbol{z}$ and $(m+n)\boldsymbol{y}$, then for some population $\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$, $(m+n)\boldsymbol{y}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$ is *not* at least as good as $n\boldsymbol{x}\circ m\boldsymbol{z}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$.

Now, assume that ***x*** has 'some high quality of life' whereas ***z*** is 'much less good'.[3] Then Parfit would claim that ***x*** is lexically better than ***z***. ***x*** could moreover be lexically better than ***y+***. This means that there is some *n*, such that for all *m*', $n\boldsymbol{x}$ is better than $m'\boldsymbol{y}+$. Further, there must be some *m*', such that $m'\boldsymbol{y}+$ is better than $(m+n)\boldsymbol{y}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$. By transitivity, it follows that there is some *n*, such that $n\boldsymbol{x}$ is better than $(m+n)\boldsymbol{y}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$. Since adding lives with positive welfare is value-increasing, it follows that for all *n* and *m*, $n\boldsymbol{x}\circ m\boldsymbol{z}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$ is better than $n\boldsymbol{x}$. By transitivity, there is some *n*, such that for all *m*, $n\boldsymbol{x}\circ m\boldsymbol{z}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$ is better than $(m+n)\boldsymbol{y}\circ\boldsymbol{D}_{[\mathbf{z},\,\mathbf{y}+]}$. The fact that there could be a zone of radical imprecise equality between ***x*** and ***y+*** does not change anything. Hence, I conclude that *Condition β* has been violated by

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[1] Gustaf Arrhenius, 'The Impossibility', at p. 10.

[2] ***y+*** designates a life with a quality of life one level about ***y***.

[3] Parfit, 'Can We Avoid', p. 112.

the Lower Bound for Exchangeability View. I now demonstrate how it violates *the Non-Elitism Condition* itself:

> For any lives $\boldsymbol{x}$, $\boldsymbol{y}$, where $\boldsymbol{x}_{-1}$ is a life at the level just below $\boldsymbol{x}$ and above the level of $\boldsymbol{y}$, there is a positive number n such that, if we have $\boldsymbol{x}$, $n\boldsymbol{y}$, and $(n+1)\boldsymbol{x}_{-1}$, then for any population $\boldsymbol{D}_{[\mathbf{y},\, x]}$, $(n+1)\boldsymbol{x}_{-1}\circ\boldsymbol{D}_{[\mathbf{y},\, x]}$ is at least as good as $\boldsymbol{x}\circ n\boldsymbol{y}\circ \boldsymbol{D}_{[\mathbf{y},\, x]}$.

Suppose that $\boldsymbol{x}$ is lexically better than $\boldsymbol{x}_{-1}$. Under *Condition* 1, this would be implied by *the Lower Bound for Exchangeability View* for some $\boldsymbol{x}$. Then for some m, $m\boldsymbol{x}$ is better than any number of $\boldsymbol{x}_{-1}$s. But for $(m\text{-}1)\boldsymbol{x}$, there is some number of $\boldsymbol{x}_{-1}$s which is better. Suppose $\boldsymbol{x}_{-1}$ is exchangeable with $\boldsymbol{y}$. Then, by transitivity, there is some number n, such that $n\boldsymbol{y}$ is better than $(m\text{-}1)\boldsymbol{x}$.

Now compare $m\boldsymbol{x}\circ n\boldsymbol{y}$ with $(m\text{-}1)\boldsymbol{x}\circ(n+1)\boldsymbol{x}_{-1}$. According to *The Non-Elitism Condition*, the latter should be at least as good as the former. But from the above, we get that $m\boldsymbol{x}$ is better than $(n+1)\boldsymbol{x}_{-1}$, and ny is better than $(m\text{-}1)x$; But then $(m\text{-}1)\boldsymbol{x}\circ(n+1)\boldsymbol{x}_{-1}$ is *not* at least as good as $m\boldsymbol{x}\circ n\boldsymbol{y}$. The fact that $\boldsymbol{x}$ could radically imprecisely equal to $\boldsymbol{x}_{-1}$, which again could be radically imprecisely equal to $\boldsymbol{y}$ will merely imply that $(m\text{-}1)\boldsymbol{x}\circ(n+1)\boldsymbol{x}_{-1}$ is imprecisely equal to $m\boldsymbol{x}\circ n\boldsymbol{y}$; but that still means that $(m\text{-}1)\boldsymbol{x}\circ(n+1)\boldsymbol{x}_{-1}$ is *not* at least as good as $m\boldsymbol{x}\circ n\boldsymbol{y}$. Hence, *the Lower Bound for Exchangeability View* violates *the Non-Elitism Condition*.