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Linking speech errors and phonological grammars: insights from Harmonic Grammar networks

Matthew Goldrick
Robert Daland
Northwestern University

Supplementary materials

These materials provide resources relating to the proposal in the paper for linking Harmonic Grammars and speech errors, in the form of two further appendices (Appendix B and Appendix C; see below), as well as links to computational functions instantiating the proposal.

Computational functions

These are instantiated within the R statistical software environment and Microsoft Excel.

Monte Carlo estimation of error probabilities assuming normally distributed disruption
R function: errorProb.fnc
Microsoft Excel worksheet (note: be sure to enable macros when opening this worksheet)

Calculation of error probabilities assuming normally distributed disruption
R function: normErrProb.fnc
Microsoft Excel worksheet

Using error probabilities to estimate standard deviation of normally distributed disruption
R function: normErrSD.fnc
Microsoft Excel worksheet

Appendix B: Monte Carlo estimation of error probabilities

The notation below follows that of the Appendix in the paper.

1 Focus of simulation: effect of disruption on harmony differences

As shown in Corollary 2 from the Appendix, the criterion for an error outcome z is dependent on the relationship between the harmony advantage of the target y in the intact network $\Delta H_x^G(y,z)$ and the harmony difference following disruption $\Delta H_x^{\delta G}(y,z)$. Here we show that the latter term can be reduced to a small number of components of the overall network.

By definition of ΔH:

$$\Delta H_x^{\delta G}(y,z) = H^{\delta G}(x,y) - H^{\delta G}(x,z)$$

That is, the difference between the change in target harmony following disruption and the change in error harmony.

By definition of H:

$$\Delta H_x^{\delta G}(y,z) = \left(x\delta G y^T + y\delta G y^T\right) - \left(x\delta G z^T + z\delta G z^T\right)$$

Recall that the first term of the target and error harmonies represents the contribution of FAITHFULNESS constraints, and the second the contribution of MARKEDNESS constraints (see Definition 1 of the Appendix in the paper). Since the MARKEDNESS constraints' weight matrix M is constrained to be symmetric and there are no self-connections:

$$\Delta H_x^{\delta G}(y,z) = \left(\sum x_b \delta G_{ba} y_a + 2\sum_{e<a} y_e \delta G_{ea} y_a\right) - \left(\sum x_d \delta G_{dc} z_c + 2\sum_{f<c} z_f \delta G_{fc} z_c\right)$$

Since x is $\{0,1\}^m$ and y, z are $\{0,1\}^n$:

$$\Delta H_x^{\delta G}(y,z) = \left(\sum_{x_b=1, y_a=1} x_b \delta G_{ba} y_a + 2\sum_{y_{a,e}=1, e<a} y_e \delta G_{ea} y_a\right) - \left(\sum_{x_b=1, z_c=1} x_d \delta G_{dc} z_c + 2\sum_{z_{f,c}=1, f<c} z_f \delta G_{fc} z_c\right)$$

For those elements where the surface representation of the target and error are identical, their harmony contributions to the first terms of the target and error harmonies are identical and can be eliminated:

$$\Delta H_x^{\delta G}(y,z) = \left(\sum_{x_b=1, y_a=1\neq z_a} x_b \delta G_{ba} y_a + 2\sum_{y_{a,e}=1, e<a} y_e \delta G_{ea} y_a\right) - \left(\sum_{x_b=1, z_c=1\neq y_c} x_d \delta G_{dc} z_c + 2\sum_{z_{f,c}=1, f<c} z_f \delta G_{fc} z_c\right)$$

Similarly, for the second terms, we can eliminate the harmony contribution for instances where both representational elements are identical across target and error:

(Equation B1)

$$\Delta H_x^{\delta G}(y,z) =$$

$$\left(\sum_{x_b=1, y_a=1\neq z_a} x_b \delta G_{ba} y_a + 2\left(\sum_{y_a\neq z_a, y_e\neq z_e, y_{a,e}=1, e<a,} y_e \delta G_{ea} y_a + \sum_{y_a=z_a, y_e\neq z_e, y_{a,e}=1, e<a} y_e \delta G_{ea} y_a + \sum_{, y_a\neq z_a, y_e=z_e, y_{a,e}=1, e<a} y_e \delta G_{ea} y_a \right)\right) -$$

$$\left(\sum_{x_b=1, z_c=1\neq y_c} x_d \delta G_{dc} z_c + 2\left(\sum_{z_c\neq y_c, z_f\neq y_f, z_{c,f}=1, f<c} z_f \delta G_{fc} z_c + \sum_{z_c=y_c, z_f\neq y_f, z_{c,f}=1, f<c} z_f \delta G_{fc} z_c + \sum_{z_c\neq y_c, z_f=y_f, z_{c,f}=1, f<c} z_f \delta G_{fc} z_c \right)\right)$$

In other words, for the target we can focus on those network weights that involve elements of its surface representation that are both non-zero and distinct from the error. Similarly, for the error we need only focus on weights involving surface elements that are non-zero and distinct from the target. As discussed in the text, it is only these elements that make distinct contributions to harmony of each representation (and, as such, lead to differences in harmony between the target and error).

2 Monte Carlo estimation of error probabilities

If we assume that the elements of the harmonic disruption δG are independent, then error probabilities can be approximated by the following procedure:

Given:

sampleNumber: Number of samples for Monte Carlo procedure

underlying: Number of (non-zero) representational elements in the target's underlying representation x (i.e. number of elements where $x_i = 1$)

uniqueTarget: Number of non-zero elements in the target surface representation y that are distinct from the error surface representation z (i.e. number of elements where $y_i = 1 \neq z_i$)

uniqueError: Number of non-zero elements in the target surface representation y that are distinct from the error surface representation z (i.e. number of elements where $z_i = 1 \neq y_i$)

common: Number of non-zero elements in the target surface representation y that are identical to the error surface representation z (i.e. number of elements where $y_i = 1 = z_i$)

harmonyAdvantage: Harmony advantage of y over z for input x: $\Delta H_x^G(y,z)$

P: Function returning a random number from the specified probability distribution for elements of δG

Initialisation:

1. Set an error counter to 0.
2. Calculate the number of FAITHFULNESS weights that make a unique, non-zero contribution to harmony.

   $nFaithTarget = underlying*uniqueTarget$

   $nFaithError = underlying*uniqueError$

   Note: See first terms of target and error harmonies in equation (B1) above.
3. Calculate the number of MARKEDNESS weights that make a unique, non-zero contribution to harmony.

$$nMarkTarget = \binom{uniqueTarget + common}{2} - \binom{common}{2}$$

$$nMarkError = \binom{uniqueError + common}{2} - \binom{common}{2}$$

Note: Let $\binom{1}{2}, \binom{0}{2} = 0$

Note: As shown in (B1) above, to focus on weights making a unique contribution to harmony we need to exclude those MARKEDNESS weights where both elements connected by the weight are common to the target and error. The number of these weights is the number of pairs of non-zero representational elements shared by the target and error, i.e. $\binom{common}{2}$. As the total number of weights making *any* non-zero contribution is given by the first term in each of the equations above, the total number of weights making a *unique* contribution is therefore the difference between these two numbers.

Repeat the following for *sampleNumber* times:

1. For each weight that makes a unique, non-zero contribution to harmony, generate a random disruption using the function *P*. Calculate the contribution to target and error harmony following (B1).

$$harmonyTarget = \sum_i^{nFaithTarget} P + 2 \sum_i^{nMarkTarget} P$$

$$harmonyError = \sum_i^{nFaithError} P + 2 \sum_i^{nMarkError} P$$

2. The harmony advantage of the target following disruption is given by:

   $\Delta H_x^{\delta G}(y,z) \;=\; harmonyTarget - harmonyError$

3. By Corollary 2 from the Appendix, if $\Delta H_x^{G}(y,z) < -\Delta H_x^{\delta G}(y,z)$, an error is produced. If this is true, increment the error counter.

Yields:

The number of samples where an error occurred. The estimated error proportion is simply this divided by the total number of samples.

With respect to the Appendix in the paper, it is not difficult to show that if a random variable is spherically distributed and each dimension is independent it must be normally distributed.

We have implemented this function, assuming that P generates random numbers from a normal probability distribution with mean 0 and variable standard deviation σ. Two implementations are available at the URLs given on page 1 of these supplementary materials.

Appendix C: Error probabilities assuming normally distributed disruption

The notation below follows that of the Appendix in the paper and Appendix B above.

1 Computation of error probabilities, assuming normal distributions

Assume a constant probability distribution P for all elements of the harmonic disruption δG. Assume P is a normal distribution with mean 0 and a variable variance σ^2: $N\left(0,\sigma^2\right)$.

Recall from the section above that the influence of disruption on harmony values is simply the sum of disruption values over all the relevant weights. Since the sum of m normal distributions is also a normal distribution $N\left(0,m\sigma^2\right)$, and multiplying a normally distributed random variable by a constant c is equivalent to increasing its standard deviation by the same factor (i.e. $cN\left(0,\sigma^2\right) = N\left(0,c^2\sigma^2\right)$, the harmony advantage of the target is distributed according to:

$$N\left(0,\left(nFaithTarget + 4nMarkTarget\right)\sigma^2\right) - N\left(0,\left(nFaithError + 4nMarkError\right)\sigma^2\right)$$

$$= N\left(0,\left(nFaithTarget + 4nMarkTarget + nFaithError + 4nMarkError\right)\sigma^2\right)$$

In other words, the harmony advantage of the target is normally distributed, with a variance proportional to the number of weights making unique harmony contributions to the target and error. Let:

$$nVarElements = nFaithTarget + 4nMarkTarget + nFaithError + 4nMarkError$$

and write this distribution as $N\left(0,nVarElements\sigma^2\right)$.

The probability of an error is therefore the cumulative distribution function for this normal distribution evaluated at the negative of the harmony advantage of the target $-\Delta H_x^G(y,z)$. In other words, the probability of an error is the probability that disruption will reduce the target's harmony advantage so as to make the error more harmonic.

We have a function that will compute this probability, given the arguments specified above as well as the standard deviation for the normal disruption function. Two implementations are available at the URLs given on page 1 of these supplementary materials.

2 Setting the standard deviation parameter to model data

Following the above, assume P is a normal distribution with mean 0 and a variable variance σ^2: $N\left(0,\sigma^2\right)$. Given some error rate e we need to find the σ^2 that best fits the data. In other words, we need to ensure that a sufficient amount of disruption is produced such that disruption will overcome the harmony advantage of the target on the observed proportion of trials.

Let $\Phi_{0,nVarElements\sigma^2}\left(x\right)$ be the cumulative distribution function of the distribution of the harmony advantage of the target (following above). We desire to solve $\Phi_{0,nVarElement\sigma^2}\left(-\Delta H_x^G(y,z)\right)=e$ for σ^2.

Since $\Phi_{0,nVarElements\sigma^2}\left(x\right)=\Phi_{0,1}\left(\frac{x}{\sqrt{nVarElements\sigma^2}}\right)$, if we can find w such that

$\Phi_{0,1}\left(w\right)=e$, then $\sqrt{\sigma^2}=\frac{-\Delta H_x^G(y,z)}{w\sqrt{nVarElements}}$

This is given by $\Phi_{0,1}^{-1}\left(e\right)=w$ (i.e. the inverse cumulative distribution function, defined in many software packages).

We have a function that will compute this standard deviation, given the arguments specified in Appendix B as well as the observed error rate. Two implementations are available at the URLs given on page 1 of these supplementary materials.