# Listeners' knowledge of phonological universals: evidence from nasal clusters 

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## Supplementary materials

## Appendix A: Formal presentation of the proposition in (5)

In this analysis we assume that for any underlying form /uf/ there is a set of surface forms faithful to /uf/. There may be multiple such forms because /uf/ may be underspecified to some degree; e.g. prosodic structure may not be present in /uf/. We assume a surface form [sf] to be fully specified in the sense that it uniquely determines a phonetic form $|\varphi f|$ which is the fully faithful phonetic interpretation of $[\mathrm{sf}] ;|\varphi \mathrm{f}|$ in turn determines a unique auditory form $\{\mathrm{af}\}$, which results from the fully faithful execution of $|\varphi f|$. Conversely, we assume that if $\left[\mathrm{sf}_{1}\right] \neq\left[\mathrm{sf}_{2}\right]$ then the phonetic forms that are fully faithful to them, $\left|\mathrm{sf}_{1}\right|$ and $\left|\mathrm{sf}_{2}\right|$, are not equal, and analogously, if $\left|\mathrm{sf}_{1}\right| \neq\left|\mathrm{sf}_{2}\right|$ then the acoustic forms fully faithful to them, $\left\{\mathrm{af}_{1}\right\}$ and $\left\{\mathrm{af}_{2}\right\}$, are not equal. Assumption 1 spells these out in the notation of the text.

Assumption 1. Faithful forms: $f$
For any underlying form /uf/ there is a set $f(/ \mathrm{uf} /)$ of surface forms faithful to /uf/: for each $[\mathrm{sf}] \in f(/ \mathrm{uf} /)$, the pair (/uf $/,[\mathrm{sf}])$ satisfies the faithfulness constraint $\mathrm{F}_{/ \mathrm{uf} /[\text { [sf] }]}$.
For any surface form [sf] there is a unique phonetic form $f([\mathrm{sf}])$ that is faithful to $[\mathrm{sf}]:([\mathrm{sf}], f([\mathrm{sf}]))$ satisfies $\mathrm{F}_{[\mathrm{sf}],|\varphi \mathrm{f}|}$. This function $f$ is one-to-one. $f([\mathrm{sf}])$ is also said to be the phonetic form corresponding to [sf].
For any phonetic form $|\varphi f|$ there is a unique auditory form $f(|\varphi f|)$ that is faithful to $|\varphi f|:(|\varphi f|, f(|\varphi f|))$ satisfies $\mathrm{F}_{|\varphi f|,\{a f\}}$. This function $f$ is one-to-one. $f(|\varphi f|)$ is also said to be the acoustic form corresponding to $|\varphi f|$.

Def. Grammatical. With respect to a given grammar $\mathcal{G}$ :
A surface form [sf] is grammatical iff there is an underlying form /uf/ such that the pair (/uf/, [sf]) is optimal: for any other surface form [sf'], the grammar assigns lower harmony to (/uf/, [sf']) than to (/uf/, [sf]): (/uf/, $\left.\left[\mathrm{sf}^{\prime}\right]\right) \prec_{G}(/ \mathrm{uf} /,[\mathrm{sf}])$.
A phonetic form is grammatical iff it corresponds to a grammatical surface form.
An acoustic form is grammatical iff it corresponds to a grammatical phonetic form.

Suppose we are given a phonetic form $\left|\varphi f_{\mathrm{u}}\right|$ and its corresponding acoustic form $\left\{\mathrm{af}_{\mathrm{u}}\right\}=f\left(\left|\varphi \mathrm{f}_{\mathrm{u}}\right|\right)$. We are interested in cases where $\left|\varphi \mathrm{f}_{\mathrm{u}}\right|$ is ungrammatical for a hearer, that is, where there is no surface form [sf ${ }_{g}$ ] that is grammatical for the hearer such that $\left|\varphi f_{u}\right|$ is the faithful phonetic interpretation of $\left[\mathrm{sf}_{\mathrm{g}}\right]$. In this case, we assume that among the grammatical surface forms, one, call it [ $\mathrm{sf}_{\mathrm{g}}$ ], is most faithful to $\left|\varphi f_{\mathrm{u}}\right|$. That is, given any other grammatical surface form [ $\mathrm{sf}_{\mathrm{g}}{ }^{\prime}$ ], the pair ( $\left[\right.$ sf $\left.f_{\mathrm{g}}\right],\left|\varphi f_{\mathrm{u}}\right|$ ) better satisfies faithfulness $\mathrm{F}_{\text {[sf], }|\varphi \mathrm{f}|}$ than does the pair $\left(\left[\mathrm{sf}_{\mathrm{g}}{ }^{\prime}\right],\left|\varphi f_{\mathrm{u}}\right|\right.$ ); we write this:

$$
\mathrm{F}_{[\mathrm{sf}],|\varphi f|}\left(\left[\mathrm{sf} \mathrm{sf}_{\mathrm{g}}\right],\left|\varphi f_{\mathrm{u}}\right|\right) \succ \mathrm{F}_{[\mathrm{sf}],|\varphi f|}\left(\left[\mathrm{sf}_{\mathrm{g}}{ }^{\prime}\right],\left|\varphi \mathrm{f}_{\mathrm{u}}\right|\right)
$$

We assume that the phonetic interpretation of this surface form, $\left|\varphi \mathrm{f}_{\mathrm{g}}\right|=f\left(\left[\mathrm{sf}_{\mathrm{g}}\right]\right)$, is the grammatical phonetic form that is most faithful to the ungrammatical acoustic form $\left\{\mathrm{af}_{\mathrm{u}}\right\}$ corresponding to $\left|\varphi \mathrm{f}_{\mathrm{u}}\right|$.
Assumption 2. Given any phonetic form $\left|\varphi \mathrm{f}_{0}\right|$ and its corresponding acoustic form $\left\{\mathrm{af}_{0}\right\}=f\left(\left|\varphi \mathrm{f}_{0}\right|\right)$ :

Among grammatical surface forms, one, $\left[\mathrm{sf}_{\mathrm{g}}\right]$, is most faithful to $\left|\varphi f_{0}\right|$; i.e. $\left[\mathrm{sf}_{\mathrm{g}}\right]$ best satisfies $\mathrm{F}_{[\mathrm{sf}],|\varphi f|}\left([\mathrm{sf}],\left|\varphi f_{0}\right|\right)$ among grammatical [sf].
The phonetic form corresponding to $\left[\mathrm{sf}_{\mathrm{g}}\right],\left|\varphi \mathrm{f}_{\mathrm{g}}\right|=f\left(\left[\mathrm{sf}_{\mathrm{g}}\right]\right)$, is the grammatical phonetic form that is most faithful to the given acoustic form $\left\{\mathrm{af}_{0}\right\}:\left|\varphi \mathrm{f}_{\mathrm{g}}\right|$ best satisfies $\mathrm{F}_{|\varphi \mathrm{f}|,\{\mathrm{af}\}}\left(|\varphi f|,\left\{\mathrm{af}_{0}\right\}\right)$ among grammatical $|\varphi f|$.
We assume given an OT grammar $\mathcal{G}$ for the mapping between underlying and surface forms. The harmonic ordering of forms determined by $\mathcal{G}$ (Prince $\&$ Smolensky 1993: ch. 5) will be written as follows: when $G$ assigns higher harmony to $(/ \mathrm{x} /,[\mathrm{x}])$ than to $(/ \mathrm{y} /,[\mathrm{y}])$ we write $(/ \mathrm{x} /,[\mathrm{x}]) \succ_{G}(\mid \mathrm{y} /,[\mathrm{y}])$ or equivalently $H_{1}(/ \mathrm{x} /$, $[\mathrm{x}]) \succ H_{1}(/ \mathrm{y} /,[\mathrm{y}])$, since this is harmony in the first component. Harmony in the second component, linking surface and phonetic forms, is determined solely by the faithfulness constraint $\mathrm{F}_{[\mathrm{sf}],|\varphi \mathrm{f}|}$, so $H_{2}([\mathrm{x}],|\mathrm{x}|) \succ H_{2}([\mathrm{y}],|\mathrm{y}|)$ means that $([\mathrm{x}]$, $|\mathrm{x}|)$ better satisfies faithfulness than does $\left.([\mathrm{y}],|\mathrm{y}|): \mathrm{F}_{[\mathrm{sf}],|\varphi f|}(\mathrm{y}],|\mathrm{y}|\right) \succ \mathrm{F}_{[\mathrm{sf}],|\varphi \mathrm{f}|}(\mathrm{yy}]$, $|y|)$. In the same way, harmony in the third component, linking phonetic and acoustic forms, is determined by $\mathrm{F}_{|\varphi \mathrm{f}|,\{\mathrm{aff}} ; H_{3}(|\mathrm{x}|,\{\mathrm{x}\}) \succ H_{3}(|\mathrm{y}|,\{\mathrm{y}\})$ means that $\mathrm{F}_{|\varphi \mathrm{f}|,\{a \mathrm{f}\}}(|\mathrm{x}|,\{\mathrm{x}\}) \succ \mathrm{F}_{|\varphi \mathrm{f}|,\{\mathrm{af}\}}(|\mathrm{y}|,\{\mathrm{y}\})$. All the analogous definitions apply to $\prec$ and to $\approx$ (equal harmony), as well as to $\succ$ and $\prec$.

Optimality in component $n$ is the obvious generalisation of the component- 1 notion of optimality in OT.
Def. Optimal in a component $n$
( $x_{n}, x_{n+1}$ ) is optimal in component $n$ (' $n$-opt') iff for all level- $n+1$ forms $x_{n+1}{ }^{\prime} \neq x_{n+1}, H_{n}\left(x_{n}, x_{n+1}\right) \succ H_{n}\left(x_{n}, x_{n+1}{ }^{\prime}\right)$

Def. Harmonic ordering w.r.t. component $n$
Given two representations $\mathrm{X}=(/ \mathrm{x} /,[\mathrm{x}],|\mathrm{x}|,\{\mathrm{x}\}) \equiv\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ and $\mathrm{Y}=\left(y_{1}\right.$, $y_{2}, y_{3}, y_{4}$ ):
$\mathrm{X} \succ_{n} \mathrm{Y}$ iff either
i. $\left(x_{n}, x_{n+1}\right)$ is optimal and $\left(y_{n}, y_{n+1}\right)$ is not, or
ii. $\left(x_{n}, x_{n+1}\right)$ and $\left(y_{n}, y_{n+1}\right)$ are both suboptimal and $H_{n}\left(x_{n}, x_{n+1}\right) \succ H_{n}\left(y_{n}, y_{n+1}\right)$
$\mathrm{X} \approx_{n} \mathrm{Y}$ iff either
i. $\left(x_{n}, x_{n+1}\right)$ and $\left(y_{n}, y_{n+1}\right)$ are both optimal, or
ii. $\left(x_{n}, x_{n+1}\right)$ and $\left(y_{n}, y_{n+1}\right)$ are both suboptimal and $H_{n}\left(x_{n}, x_{n+1}\right) \approx H_{n}\left(y_{n}, y_{n+1}\right)$
$\mathrm{X} \succ_{n} \mathrm{Y}$ iff either $\mathrm{X} \succ_{n} \mathrm{Y}$ or $\mathrm{X} \approx_{n} \mathrm{Y}$
To combine harmonic ordering across all components, we assume that no component has priority over any others. Thus for a full (four-level) representation X to be preferred to Y , written $\mathrm{X} \succ \mathrm{Y}$, there must be some component in which X is preferred and no component in which Y is preferred. The ordering is partial in that for many pairs $\mathrm{X}, \mathrm{Y}$ that are not harmonically equivalent, neither $\mathrm{X} \succ \mathrm{Y}$ nor $\mathrm{Y} \succ \mathrm{X}$. Our ordering is defined for use in perception: it compares X and Y only if they have the same acoustic form \{af\}, and are therefore both potential full perceptual representations for $\{a f\}$.
Def. Harmonic ordering of full representations: $\succ$
Given two representations $X=(/ x /,[x],|x|,\{x\}), Y=(/ y /,[y],|y|,\{y\})$ :
$\mathrm{X} \succ \mathrm{Y}$ iff $\{\mathrm{y}\}=\{\mathrm{x}\}$ and either
(a) i. $\forall k[\mathrm{Y}$ is $k$-opt $\Rightarrow \mathrm{X}$ is $k$-opt $]$
ii. $\exists n$ [ X is $n$-opt and Y is not $n$-opt]
or
(b) i. $\forall k$ [Y is $k$-opt $\Leftrightarrow \mathrm{X}$ is $k$-opt $] \&$
ii. $\forall k\left[\mathrm{X}\right.$ is not $k$-opt $\left.\Rightarrow \mathrm{X} \succ_{k} \mathrm{Y}\right] \&$
iii. $\exists n\left[\mathrm{X} \succ{ }_{n} \mathrm{Y}\right]$

In case (a), optimality makes the decision: X is optimal in some component in which Y is not (a.ii), and there is no component in which Y is optimal but X is not (a.i). In case (b), $X$ and $Y$ tie with respect to optimality, in that they are optimal in exactly the same components (b.i): then relative harmony among suboptimal representations decides. Now X is preferred to Y only if there is some component in which X is suboptimal but higher-harmony than Y (b.iii), and there is no component in which Y is suboptimal but higher-harmony than X (b.ii).

Henceforth references such as (b.ii) will always refer to the correspondingly labelled condition in the definition of $\succ$.
Lemma 1. Suppose X is optimal in every component. Then
$(\alpha)$ there is no Y such that $\mathrm{Y} \succ \mathrm{X}$, and
$(\beta)$ for every $Z$ such that $\{z\}=\{x\}, X \succ Z$ unless $Z$ is also optimal in every component.

## 4

Proof of Lemma 1. Suppose there were a Y such that $\mathrm{Y} \succ \mathrm{X}$. Then either (a.ii) $\exists n$ [ Y is $n$-opt and X is not $n$-opt], which is impossible because X is $k$-opt $\forall k$, or (b.iii) $\exists n\left[\mathrm{Y} \succ_{n} \mathrm{X}\right]$, which is also impossible because X is $k$-opt $k$. This establishes $(\alpha)$. Now suppose given $Z$ such that $\{z\}=\{x\}$. Suppose $Z$ is not optimal in every component: $\exists n$ [ $Z n$ is subopt]. Then (a) of the definition of $\mathrm{X} \succ \mathrm{Z}$ is satisfied, establishing ( $\beta$ ). QED

Lemma 2. Suppose X is suboptimal in component $k$ and optimal in the other components. Let Y be a candidate with $\{\mathrm{y}\}=\{\mathrm{x}\}$. Then

1. $\mathrm{X} \succ \mathrm{Y}$ iff either
a. Y is (i) suboptimal in some non- $k$ component and (ii) not $k$-optimal or b. Y is (ii) optimal in all non- $k$ components and (ii)-(iii) $\mathrm{X} \succ_{k} \mathrm{Y}$,
2. $\mathrm{Y} \succ \mathrm{X}$ iff [ Y is optimal in all non- $k$ components and $\mathrm{Y} \succ_{k} \mathrm{X}$ ]
3. neither $\mathrm{X} \succ \mathrm{Y}$ nor $\mathrm{Y} \succ \mathrm{X}$ if [ Y is optimal in component $k$ and suboptimal in some non- $k$ component].

Proof of Lemma 2. Suppose X is suboptimal in component $k$ and optimal in the other components, and $Y$ is a candidate with $\{y\}=\{x\}$.

Part 1. Under these conditions, conditions (1a) and (1b) of Lemma 2 are equivalent to (a) and (b) respectively of the definition of $\mathrm{X} \succ \mathrm{Y}$.
Part 2. Exchanging $X$ and $Y$ in (1), we see that (1a) cannot be met and condition (1b) becomes (2) of Lemma 2.
Part 3. Suppose now that Y is optimal in component $k$ and suboptimal in some non- $k$ component. Then in the definition of $\mathrm{X} \succ \mathrm{Y}$, condition (i) cannot hold in either (a) or (b), and the same is true for the definition of $\mathrm{Y} \succ \mathrm{X}$, because in both (a) and (b), (i) requires that the set of components in which one of the candidates X or Y is optimal is a (not necessarily strict) subset of the components in which the other candidate is optimal. This establishes result (3). QED
Now, as in (4) in the text, we define a possible percept as follows.
Def. Perceptual principle
Let $\mathrm{X}=(|\mathrm{x} /,[\mathrm{x}],|\mathrm{x}|,\{\mathrm{x}\})$ be a globally faithful representation and consider
the auditory form $\{x\}$. A representation $Y$ is a possible percept for $\{x\}$ iff
the acoustic form of $Y$ is $\{x\}$, and there is no Z with acoustic form $\{\mathrm{x}\}$ such that $\mathrm{Z} \succ \mathrm{Y}$.

The main result, (5) in the text, follows.

## Proposition:

Let $\mathrm{X}=(/ \mathrm{x} /,[\mathrm{x}],|\mathrm{x}|,\{\mathrm{x}\})$ be a globally faithful representation. For the auditory input $\{\mathrm{x}\}$, there are two possibilities.
a. If $[\mathrm{x}]$ is grammatical then the only possible percept type is
$A^{\prime}=\left(\left|x^{\prime}\right|,[x],|x|,\{x\}\right)$
where $/ \mathrm{x}^{\prime} /$ is any underlying form for which $\left(/ \mathrm{x}^{\prime} /,[\mathrm{x}]\right)$ is optimal.
b. If $[\mathrm{x}]$ is not grammatical, there are three possible percept types:

$$
\begin{aligned}
& \mathrm{A}^{\prime}=\left(\left|\mathrm{x}^{\prime}\right|,[\mathrm{x}],|\mathrm{x}|,\{\mathrm{x}\}\right) \\
& \mathrm{C}^{\prime}=\left(\left|\mathrm{y}^{\prime}\right|,[\mathrm{y}],|\mathrm{x}|,\{\mathrm{x}\}\right) \\
& \mathrm{D}^{\prime}=\left(\left|\mathrm{y}^{\prime}\right|,[\mathrm{y}],|\mathrm{y}|,\{\mathrm{x}\}\right)
\end{aligned}
$$

where
[y] is the grammatical surface form most faithful to $|\mathrm{x}|$, $|\mathrm{y}|=f([\mathrm{y}])$ is the phonetic form faithful to $[\mathrm{y}]$,
$\mid \mathrm{y}^{\prime} /$ is any underlying form for which $\left(\mid \mathrm{y}^{\prime} /,[\mathrm{y}]\right)$ is optimal, and $\left|\mathrm{x}^{\prime}\right| \in f([\mathrm{x}])$ is any underlying form faithful to $[\mathrm{x}]$.

Proof. Part a: grammatical case
That $\mathrm{A}^{\prime}$ (as given in the Proposition) is a possible percept follows immediately from $(\alpha)$ of Lemma 1 , since $A^{\prime}$ is optimal in every component: $\left(/ x^{\prime} /,[x]\right)$ is optimal by assumption, and in the other two components, $\mathrm{A}^{\prime}$ is faithful, hence optimal.

Now consider any possible percept $\mathrm{U}=(/ \mathrm{u}|,[\mathrm{u}],|\mathrm{u}|,\{\mathrm{u}\})$; we must show it is of type $A^{\prime}$. As a possible percept of $\{x\}, U$ must have $\{u\}=\{x\}$ and we must not have $A \succ U$, where $A$ is a candidate of type $A^{\prime}$. Now according to Lemma $1(\beta)$, since $A$ is optimal in every component, we would have $A \succ U$ unless $U$ is also globally optimal. To be optimal in the third component, since $\{\mathrm{u}\}=\{\mathrm{x}\},|\mathrm{u}|$ must be faithful to $\{\mathrm{x}\}$, so, by Assumption 1, we must have $|\mathrm{u}|=$ $|\mathrm{x}|$, as $|\mathrm{x}|$ is the unique phonetic form faithful to $\{\mathrm{x}\}:\{\mathrm{x}\}=f(|\mathrm{x}|)$. By identical reasoning, to be optimal in the second component, [u] must be faithful to $|\mathrm{u}|=|\mathrm{x}|$, so we must have $[\mathrm{u}]=[\mathrm{x}]$. To be optimal in the first component, $/ \mathrm{u} \mid$ must be optimal when paired with [x]; this means U is a candidate of type $\mathrm{A}^{\prime}$.

Part b: ungrammatical case
$\mathrm{U}=(/ \mathrm{u}|,[\mathrm{u}],|\mathrm{u}|,\{\mathrm{u}\})$ is a possible percept only if $\{\mathrm{u}\}=\{\mathrm{x}\}$. There are three cases to consider. In case $1,|\mathrm{u}|=|\mathrm{x}|$ and $[\mathrm{u}]=[\mathrm{x}]$; in case $2,|\mathrm{u}|=|\mathrm{x}|$ and $[\mathrm{u}]$ $\neq[\mathrm{x}]$; in case $3,|\mathrm{u}| \neq|\mathrm{x}|$. We show that in case $1, \mathrm{U}$ is a possible percept iff it is of type $\mathrm{A}^{\prime}$; in case 2 , type $\mathrm{C}^{\prime}$; in case 3 , type $\mathrm{D}^{\prime}$. Note that the globally faithful candidate X is optimal in components 2 and 3 (as it is faithful there), but cannot be optimal in component 1 because $[\mathrm{x}]$ is ungrammatical: there is no $/ \mathrm{x}^{\prime} /$ such that $\left(\mid \mathrm{x}^{\prime} /,[\mathrm{x}]\right)$ is optimal. X thus meets the conditions of Lemma 2 , with $k=1$. While not optimal in component $1, \mathrm{X}$ is however faithful in component 1, by definition.
Case 1. Here, $\mathrm{U}=(/ \mathrm{u} /,[\mathrm{x}],|\mathrm{x}|,\{\mathrm{x}\}) . \mathrm{U}$ is a possible percept iff there is no Z with acoustic form $\{\mathrm{x}\}$ such that $\mathrm{Z} \succ \mathrm{U}$. Like $\mathrm{X}, \mathrm{U}$ is optimal in components 2 and 3 but cannot be optimal in component 1 . So U satisfies the conditions of Lemma 2, with $k=1$. By Part 2 of Lemma 2, for a candidate $Z$ with acoustic form $\{x\}, Z \succ U$ iff $Z$ is optimal in components 2 and 3 and $Z \succ_{1} U$. This is true iff $Z=(|z|,[x],|x|,\{x\})$ and $(|z|,[x]) \succ_{1}(/ u /,[x])$. In this harmony comparison, the two pairs tie on markedness since they have the same surface form $[\mathrm{x}] ;(\mid \mathrm{z} /,[\mathrm{x}]) \succ_{1}(/ \mathrm{u} /,[\mathrm{x}])$ will hold iff $(\mid \mathrm{z} /,[\mathrm{x}])$ satisfies faithfulness better than does $(/ \mathrm{u} /,[\mathrm{x}])$. For no such Z to exist, it is necessary and sufficient that $/ \mathrm{u} /$ is fully faithful to $[\mathrm{x}]$ : if $/ \mathrm{u} /$ is fully faithful to $[\mathrm{x}]$, no $/ \mathrm{z} /$ can be more faithful, and if $/ \mathrm{u} /$ is not faithful to $[\mathrm{x}]$, then any $/ \mathrm{z} /$ that is faithful to [ x$]$ will satisfy $(/ \mathrm{z} /,[\mathrm{x}]) \succ_{1}(/ \mathrm{u} /,[\mathrm{x}])$. Thus, under the conditions of Case $1, \mathrm{U}$ is a possible percept iff it is of the form $\mathrm{A}^{\prime}$ as defined in Part 2 of the Proposition.

Case 2. Now, $\mathrm{U}=(/ \mathrm{u} /,[\mathrm{u}],|\mathrm{x}|,\{\mathrm{x}\})$, where $[\mathrm{u}] \neq[\mathrm{x}] . \mathrm{U}$ is optimal (faithful) in component 3 but is suboptimal in component 2 , since (by Assumption 1) [ x$]$ is the unique surface form faithful to $|\mathrm{x}|$. Since X meets the conditions of Lemma 2 with $k=1$, and U is 2 -suboptimal, by part 1 of Lemma 2, we will have $\mathrm{X} \succ \mathrm{U}$ iff U is not 1 -optimal (this is (1a); (1b cannot be met by U ). So to be a possible percept it is necessary that $(/ \mathrm{u} /,[\mathrm{u}])$ be optimal, i.e. [u] must be a grammatical surface form and $/ \mathrm{u} /$ a possible underlying form for $[\mathrm{u}]$. This means that U satisfies Lemma 2 with $k=2$. Now U is a possible percept for $\{\mathrm{x}\}$ iff there is no candidate Z with acoustic form $\{\mathrm{x}\}$ such that $\mathrm{Z} \succ \mathrm{U}$. By part 2 of Lemma 2, such a $Z \succ U$ iff $Z$ is optimal in components 1 and 3 and $Z \succ_{2} U$, which is true iff $Z \succ_{2} U$ and $Z=(|z|,[z],|x|,\{x\})$ with $(|z|,[z])$ optimal. This in turn is true iff $[z]$ is grammatical and $([z],|x|)$ is more faithful than ( $[u]$, $|u|)$. If we choose $[u]$ to be the grammatical surface form [y] that is most faithful to $|\mathrm{x}|$ (unique by Assumption 2) then no such [z] exists, otherwise, choosing $[z]=[y]$ entails $Z \succ \mathrm{U}$. Thus, under the conditions of Case 2, U is a possible percept for $\{\mathrm{x}\}$ iff $[\mathrm{u}]=[\mathrm{y}]$ and $/ \mathrm{u} /$ is an underlying form $/ \mathrm{y} /$ for which $(/ y /,[y])$ is optimal, that is, iff $U$ is a candidate of type $\mathrm{C}^{\prime}$.
Case 3. Now, $\mathrm{U}=(/ \mathrm{u}|,[\mathrm{u}],|\mathrm{u}|,\{\mathrm{x}\})$, where $|\mathrm{u}| \neq|\mathrm{x}| . \mathrm{U}$ is suboptimal in component 3 since (by Assumption 1) $|\mathrm{x}|$ is the only phonetic form that is faithful to $\{\mathrm{x}\}$. Since X satisfies Lemma 2 with $k=1$ and U is 3 -suboptimal, by part 1 of Lemma 2 we will have $\mathrm{X} \succ \mathrm{U}$ iff U is not 1 -optimal (this is (1a); (1b) cannot be satisfied by such a U). So, as in case 2 , for U to be a possible percept of $\{x\}$ we must have $(/ u /,[u])$ optimal, so $[u]$ must be a grammatical surface form. Let W be a candidate of type $\mathrm{C}^{\prime}$. W satisfies Lemma 2 with $k=2$, so again by part 1 of Lemma $2, \mathrm{~W} \leftharpoonup \mathrm{U}$ iff U is not 2-optimal; U must be 2optimal to be a possible percept, which means ( $[u],|u|$ ) must be faithful. This means U satisfies Lemma 2 with $\mathrm{k}=3$. So by part 2 of that lemma, there exists a candidate Z with acoustic form $\{\mathrm{x}\}$ such that $\mathrm{Z} \succ \mathrm{U}$ iff there exists Z optimal in components 1 and 2 and $Z \succ{ }_{3} U$, i.e. a $Z=(|z|,[z],|z|,\{x\})$ such that $[z]$ is a grammatical surface form with underlying form $|z|,|z|=f([z])$, and $(|z|$, $\{x\})$ is more faithful than $(|u|,\{x\})$. Such a $Z$ does not exist if $|u|$ is the (unique) grammatical phonetic form $|y|$ most faithful to $\{x\}$, otherwise, such a $Z$ does exist, namely, where $|\mathrm{z}|=|\mathrm{y}|$. By Assumption 2, this $|\mathrm{y}|=f([\mathrm{y}])$, where $[\mathrm{y}]$ is as defined in the Proposition. Thus, under the conditions of Case 3, U is a possible percept for $\{\mathrm{x}\}$ iff U is of type $\mathrm{D}^{\prime}$. QED
Note that the second part of Assumption 2 can be dropped if $\mathrm{D}^{\prime}$ in the Proposition is set to $\left(\left|z^{\prime}\right|,[z],|z|,\{x\}\right)$, where $|z|$ is the grammatical phonetic form most faithful to $\{x\},[z]$ is the surface form faithful to $|z|$, which is grammatical by definition of 'grammatical phonetic form', and $/ z^{\prime} \mid$ is an underlying form for which ( $\left|z^{\prime}\right|,[z]$ ) is optimal, which exists since $[z]$ is grammatical. Now the $[z]$ of $D^{\prime}$ may not be the same as the $[y]$ of $B^{\prime}$.

## Appendix B: Measures of segmental lexical statistics

To evaluate the possibility that the preference of items like $m l_{l} f$ reflects only the co-occurrence of their segments in the English lexicon, we calculated several statistical measures of our materials. These measures correspond to factors which have been reported in the literature as modulating perceptual accuracy and speed. The means of these measures, provided in Table I, reflect averages computed over the twelve items representing each onset type (these means were not used in our analyses; they are presented merely as descriptive statistics). A brief description of these measures is found in the text - below we offer a more detailed description of these measures, their calculation and their expected effects on behaviour.

The statistical properties included neighbourhood measures and measures of segment or letter co-occurrence (for auditory and printed materials respectively). A final measure concerned the identity of the initial consonants.

## 1 Neighbourhood measures

A target's lexical neighbourhood comprises all words obtained by adding, deleting or substituting one of a target's phonemes (or letters, for printed words). Previous research suggests that words with a large neighbourhood consisting of frequent words are recognised more readily in naming and AX tasks (e.g. Carreiras et al. 1997, Perea \& Carreiras 1998, Vitevitch \& Luce 1998, 1999). The better recognition of the items with rising-sonority onsets might thus be due to the structure of their lexical neighbourhoods, rather than sonority per se. We evaluated this possibility using two neighbourhood measures:
(1) a. Neighbourhood count

The number of lexical neighbours
b. Neighbourhood frequency

The summed frequency of a target's neighbours
For example, the item [mlıf] has one phonological neighbour, /klif/, whose frequency is 11 per million. Both measures were calculated from the Speech and Hearing Lab Neighborhood Database. ${ }^{1}$

## 2 Measures of segment/letter co-occurrence

Words whose segments or letters co-occur frequently are better recognised (e.g. Perea \& Carreiras 1998, Vitevitch \& Luce 1998, 1999). To determine whether the advantage of sonority rises is due to these properties, we computed two sets of measures of co-occurrence, one for the whole word and one for the onset specifically.

[^0](2) a. Whole-word measures

The co-occurrence of elements in the word as a whole was estimated using either segment- or letter co-occurrence, for auditory and printed words respectively. Segment co-occurrence in auditory words was captured by the following measures based on the Phonotactic Probability Calculator (Vitevitch \& Luce 2004):

Position-sensitive phoneme probability
The probability that any given phoneme in the target occurs at the same string position (first through fourth), averaged across the target's four phonemes.
Position-sensitive biphone probability
The probability that any adjacent phoneme pair in the target occurs at the same string position (averaged across the target's three biphones).
For printed words (in Experiment 6), we used two measures of letter cooccurrence:

## Bigram count

The number of words sharing each of the target's adjacent letter pairs in the same string position (calculated based on Solso \& Juel 1980). Bigram frequency
The summed frequency of the words sharing the target's bigrams (from Kučera \& Francis 1967).
For example, the phoneme probability of the auditory [mlıf] is 0.0382 (averaged across the positional probability of its four phonemes: 0.0572 , $0 \cdot 0447,0 \cdot 0350,0 \cdot 0159$ ). In its printed form, $m l_{l} f$ shares its second bigram $\left(l_{I}\right)$ with ten words, whose summed frequency is 85 per million (the initial and final bigrams are not shared with any words), so its bigram count is 10 , and its bigram frequency is 85 .
b. Onset-probability measures

To examine the possibility that the advantage of sonority rises might be due to the individual probabilities of occurrence of their onset consonants in their string positions, we computed the log of the product of the positionspecific probabilities of each consonant (i.e. the summed frequency of words sharing that consonant in its position relative to the total summed frequency of the sample) in CCVC words listed in the Speech and Hearing Lab Neighborhood Database. Consider the onset $m l$. The database includes a total of 757 CCVC words, three of which begin with an $m$ (mews, mule, mute) and 199 words that include an $l$ in the second position. ${ }^{2}$ The probabilities of $m$-first and $l$-second words are respectively 0.003963012 and 0.262879789 and what we call the 'log cluster probability' for $m l$ is the log of their product ( $0 \cdot 001041796$ ): $2 \cdot 982217428$.

[^1]
## 3 The onset's initial consonant

Previous research suggests that $n$-initial onsets might be less perceptible than $m$ initial onsets for reasons unrelated to sonority (Byrd 1992, Surprenant \& Goldstein 1998). Because failure to register the acoustic input as an onset cluster (e.g. registering $n b_{I f}$ as $b_{I f} f$ reduces its markedness, we also entered the identity of the initial consonant into the analysis of auditory stimuli.

|  | onset type |  |
| :--- | :---: | :---: |
|  | sonority rise | sonority fall |
| statistical property | auditory items |  |
| number of neighbours | 1.58 | 0.50 |
| neighbours' frequency (summed) | 581 | 390 |
| mean phoneme probability | 0.0316 | 0.0262 |
| mean biphone probability | 0.0016 | 0.0006 |
| log cluster probability | -5.4097 | -10.7296 |
| statistical property | printed items |  |
| number of neighbours | 0.33 | 0.33 |
| neighbours' frequency (summed) | 1.25 | 2.00 |
| bigram count (summed) | 13.67 | 6.25 |
| bigram frequency (summed) | 316.92 | 224.50 |
| log cluster probability | -5.4097 | -10.7296 |

Table I
Some statistical properties of the auditory and printed stimuli. The figures provided in the table reflect means per onset type (averaged across the 12 items representing each onset type).

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[^0]:    1 Available (March 2009) at http://128.252.27.56/Neighborhood/Home.asp. At present, the Speech and Hearing Lab Neighborhood Database incorrectly collapses casedistinctions in phonological inputs (e.g. it fails to distinguish [lif] and [lif]). In view of this problem, we manually inspected the output of the database and corrected them as necessary.

[^1]:    2 To avoid biasing the calculations by theory-internal structural assumptions, we based frequency calculations on the segment sequences as indicated in the database (see also §4.2).

