**Supplementary material**

Parameters used in the model are follows:

= observed risk of all-cause pneumonia (ACP) in unvaccinated individuals.

= observed risk of ACP in vaccinated individuals.

= observed risk of pneumococcal pneumonia (PP) in unvaccinated individuals.

= observed risk of PP in vaccinated individuals.

= true risk of PP in unvaccinated individuals.

= true risk of PP in vaccinated individuals.

= test sensitivity for diagnosing PP.

= test specificity for diagnosing PP.

Followings are our assumptions:

Assumption 1 (A1): the misclassification in the diagnosis of ACP is non-differential.

Assumption 2 (A2): the pneumococcal vaccine does not change the risk of non-pneumococcal pneumonia.

Assumption 3 (A3): the directions of and are identical, and the value of is equal to or greater than that of (i.e., or ).

Assumption 4 (A4): the pneumococcal vaccine does not affect and .

The observed VE against ACP (, observed VE against PP (), and true VE against PP ( are given as follows:

,

,

*.*

According to the assumptions A1 and A2, the risk differences for ACP and true PP are identical [[25](#_ENREF_25)].

. (1)

Thus, the true proportion of PP among ACP in unvaccinated individuals is given as:

*.* (2)

According to the supplementary table, the proportion of tested positive is:

. (3)

Thus,

. (4)

By (1), (2), and (3), is rewritten as follows:

*.* (5)

Thus,

Because , .

By (2) and (5), is given as follows:

. (6)

By (4),

,

.

Thus, is given as follows:

(7)

(8)

As other values are given as observed values in trials, is a linear rational function of (see supplementary figure).

If , becomes minimum at =1. By (7),

.

If , becomes minimum at . By (8),

.

**Supplementary table**

True and observed risks of pneumococcal pneumonia and diagnostic test accuracy.

|  |  |  |  |
| --- | --- | --- | --- |
|  | True pneumococcal pneumonia | True non-pneumococcal pneumonia | Total |
| Tested positive |  |  |  |
| Tested negative |  |  |  |
| Total |  |  |  |

, true risk of pneumococcal pneumonia (PP) in total individuals; , observed risk of PP in total individuals; , observed risk of all-cause pneumonia in total individuals; , test sensitivity for diagnosing PP; , test specificity for diagnosing PP.

**Supplementary figure**

Association between specificity and true vaccine efficacy for pneumococcal pneumonia.

, observed vaccine efficacy (VE) against all-cause pneumonia (ACP); , observed VE against pneumococcal pneumonia (PP); , true VE against PP;, sensitivity; , specificity;, observed risk of ACP in unvaccinated individuals; , observed risk of ACP in vaccinated individuals; , observed risk of PP in unvaccinated individuals; , observed risk of PP in vaccinated individuals.

The x-axis shows , and the y-axis shows . is given as a linear rational function of (solid line). A. When , the function has branches in the upper right and lower left. Because can take values between and 1 (unshaded area), becomes minimum at =1. B. When , the function has branches in the upper left and lower right. Because can take values between and (unshaded area), becomes minimum at .

