Supplementary Information

The weight function has been defined in terms of the displacement field near the crack tip

$$m(x,b)=\frac{\partial V(x,b)}{\partial b} \tag{1-1}$$

However, the displacement function and weight influence may be not known. Therefore, we make use of the methodology and solutions given by Wu and Carlson[1] to determine the displacement function and the weight effect. So, the function V and $m(\kappa,a)$ is determined by the method of (in the scaled coordinates, $\kappa=\frac{x}{a}, b=d/a$)

$$V=\frac{\sigma_0 b\sqrt{1-\left(\frac{\kappa}{b}\right)}}{\frac{\sqrt{2}E}{\left(1-v^2\right)}}\cdot\sum_{j=1}^{4}F_j(b)\left[1-\left(\frac{\kappa}{b}\right)\right]^{j-1} \tag{1-2}$$

$$m(\kappa,b)=\frac{1}{\sqrt{2\pi b}}\sum_{n=1}^{5}\beta_n(b)\left[1-\left(\frac{\kappa}{b}\right)\right]^{n-3/2} \tag{1-3}$$

where the coefficients $F_n(b)$ and $\beta_i(b)$ requird for the evaluation of normalized crack opening displacement are given in referance[2] and reproduced below

$$F_1(b)=4f_1 \tag{1-4}$$

$$F_2(b)=\frac{1}{12\sqrt{2}}\left[315\pi\psi(b)-105\zeta-208\sqrt{2}f_1\right] \tag{1-5}$$

$$F_3(b)=\frac{1}{30\sqrt{2}}\left[-1260\pi\psi(b)-525\zeta+616\sqrt{2}f_1\right] \tag{1-6}$$

$$F_4(b)=\sqrt{2}\zeta-\left[F_1(b)+F_2(b)+F_3(b)\right] \tag{1-7}$$

$$f_1=1.1215\cdot\frac{1}{(1-b)^{3/2}} \tag{1-8}$$

$$\zeta=\frac{2.9086+1.0429b}{(1-b)^2} \tag{1-9}$$

$$\psi(b)=\frac{1}{2}\left(\frac{1.1215}{1-R}\right)^2 \tag{1-10}$$

$$\beta_1(b)=2.0 \tag{1-11}$$

$$\beta_2(b)=\left[bF_1^{'}+\frac{1}{2}(3F_2+F_1)\right]\Big/ f_1 \tag{1-12}$$

$$\beta_3(b)=\left[bF_2^{'}+\frac{1}{2}(5F_3-F_2)\right]\Big/ f_1 \tag{1-13}$$

$$\beta_4(b)=\left[bF_3^{'}+\frac{1}{2}(7F_3-3F_2)\right]\Big/ f_1 \tag{1-14}$$

$$\beta_5(b)=\left[bF_4^{'}+\frac{5}{2}F_4\right]\Big/ f_1 \tag{1-15}$$

To sum up the above, we can get the expression

$$K_I=\sqrt{\frac{R}{\pi b}}\int_0^b\sum_{n=1}^{5}\sigma_\theta^{'}(x,t)\beta_n(b)\left(1-\frac{\psi}{b}\right)^{n-3/2}dx \tag{1-16}$$

References:

1. W. Xu, X.R. Wu and Y. Yu: Weight function, stress intensity factor and crack opening displacement solutions to periodic collinear edge hole cracks *Fatigue Fract Eng M.* **40**(12), 2068 (2017).
2. M. Pharr, Z. Suo and J.J. Vlassak: Measurements of the Fracture Energy of Lithiated Silicon Electrodes of Li-Ion Batteries *Nano Lett.* **13**(11), 5570 (2013).