**Virtual machine concept applied to uncertainties estimation in instrumented indentation testing**

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**Supplementary material**

**Expressions for uncertainties definition in the virtual machine program**

*Model for the uncertainties of the displacement sensor*

The metrological behavior of the displacement sensor was modeled by a non-linear effect with a parabolic model. The displacement *h* is modified by $\frac{4\*val}{h\_{ch}^{2}}\*h\*(h-h\_{ch})$, being $h\_{ch}$ a characteristic length equal to 2000 nm, this value is zero for $h=0$ and $h=h\_{ch}$. For $h=\frac{h\_{ch}}{2}$, $\frac{4\*val}{h\_{ch}^{2}}\*h\*\left(h-h\_{ch}\right)=val$. The value *val* is given by a normal distribution centered at zero. In addition, between $h=0$ and $h=h\_{ch}$, the slope of $\frac{4\*val}{h\_{ch}^{2}}\*h\*(h-h\_{ch})$ lies between 0 and $\frac{4\*val}{h\_{ch}}$ , so we add the effect of an average slope $\frac{2\*val}{h\_{ch}}$ .

*Model for asymmetrical uncertainty distributions for β and ε*

The best estimate of the coefficient *ε* is 0.75 with an uncertainty between 0.74 and 0.79 [1,2]. For a normal distribution, the distribution would not be centered at 0.75. Consequently, we choose the following expression to represent the interval, $ε=0.75+0.02(-1+e\left[val\right])$, where *val* is a Gaussian variable with an average of 0 and a standard deviation of 0.3. Similarly, the coefficient *β* is defined as $β=1.034+(-0.017+0.02e\left[val\right])$

*Normal distribution and uniform distribution*

The probability density of a normal distribution is given by $f\left(x\right)=\frac{1}{\sqrt{2πu^{2}}}exp⁡(-\frac{\left(x-μ\right)^{2}}{2u^{2}})$, where $μ$ is the average and $u$ the standard deviation. Usually *u* is called the standard uncertainty with or without a Gaussian variable. The Gaussian variable is the limit described by the central limit theorem for the cumulative effect of many phenomena and is a good model for repeatability or to express the confidence in a result around the average.

The probability density of the uniform or rectangular distribution is 0 outside of $\left[a\_{-},a\_{+}\right]$ and $\frac{1}{2(a\_{+}-a\_{-})}$ in the interval. The uniform distribution is the most appropriate model for resolution force of displacement for example. The standard variance associate to this law is $\frac{(a\_{+}-a\_{-})^{2}}{12}$ .

*References*

[1] W. C. Oliver and G. M. Pharr: Measurement of hardness and elastic modulus by instrumented indentation: Advances in understanding and refinements to methodology. *J. Mater. Res*. **19**, 3 (2004).

[2] G. M. Pharr and A. Bolshakov: Understanding nanoindentation unloading curves. *J. Mater. Res*. **17**, 2660 (2002).