**Virtual machine concept applied to uncertainties estimation in instrumented indentation testing**

Thierry Coorevits1, Stephania Kossman1, Didier Chicot2, François Hennebelle3, Alex Montagne1, Alain Iost1

1Arts et Métiers ParisTech, Mécanique, Surface, Matériaux et Procédés, MSMP-EA7350, F-59800 Lille, France

2 Université de Lille, Laboratoire de Génie Civil et géo-Environnement, LGCgE-EA4515, Villeneuve d’Ascq F-59650, France

3 Université Bourgogne Franche-Comté, F-21000 Dijon, France

**Supplementary material**

**Expressions for uncertainties definition in the virtual machine program**

*Model for the uncertainties of the displacement sensor*

The metrological behavior of the displacement sensor was modeled by a non-linear effect with a parabolic model. The displacement *h* is modified by , being a characteristic length equal to 2000 nm, this value is zero for and . For , . The value *val* is given by a normal distribution centered at zero. In addition, between and , the slope of lies between 0 and , so we add the effect of an average slope .

*Model for asymmetrical uncertainty distributions for β and ε*

The best estimate of the coefficient *ε* is 0.75 with an uncertainty between 0.74 and 0.79 [1,2]. For a normal distribution, the distribution would not be centered at 0.75. Consequently, we choose the following expression to represent the interval, , where *val* is a Gaussian variable with an average of 0 and a standard deviation of 0.3. Similarly, the coefficient *β* is defined as

*Normal distribution and uniform distribution*

The probability density of a normal distribution is given by , where is the average and the standard deviation. Usually *u* is called the standard uncertainty with or without a Gaussian variable. The Gaussian variable is the limit described by the central limit theorem for the cumulative effect of many phenomena and is a good model for repeatability or to express the confidence in a result around the average.

The probability density of the uniform or rectangular distribution is 0 outside of and in the interval. The uniform distribution is the most appropriate model for resolution force of displacement for example. The standard variance associate to this law is .

*References*

[1] W. C. Oliver and G. M. Pharr: Measurement of hardness and elastic modulus by instrumented indentation: Advances in understanding and refinements to methodology. *J. Mater. Res*. **19**, 3 (2004).

[2] G. M. Pharr and A. Bolshakov: Understanding nanoindentation unloading curves. *J. Mater. Res*. **17**, 2660 (2002).