**Supplementary information**

**Estimates of the effective properties of the tows and homogenised matrix**

The 3D noobed composite comprised Toray T700S 12k carbon fibre tows in a NM FW3070 epoxy matrix. The properties of these two constituents as given by the fibre and matrix manufacturers are:

(i) The fibre Young’s modulus and Poisson’s ratio are $E\_{f}=210 GPa$ and $ν\_{f}=0.25$, respectively while the tensile strength of the fibres, $σ\_{f}=4 GPa$.

(ii) The matrix Young’s modulus and Poisson’s ratio are $E\_{m}=3 GPa$ and $ν\_{m}=0.25$, respectively while the matrix tensile yield strength, $σ\_{m}=140 MPa$.

Here we summarise the procedure developed by Das et al.S1 to use these properties of the constituents to derive estimates of the effective properties of the different phases in the 3D noobed composite comprised of four phases. The $Z$-direction tows have a significantly larger cross-sectional area compared to the $X$ and $Y$-direction tows. Thus, in the finite element (FE) calculations in Section 4 we explicitly considered the $Z$-direction tows but modelled the $X$ and $Y$-direction tows and the matrix pockets that surround the $Z$-direction tows as a single effective medium referred to as the homogenised matrix. We shall thus first derive effective properties for the tows and then use them to estimate properties of the homogenised matrix. All the relevant anisotropic properties will be stated using the global co-ordinate system. For example, $E\_{Z}^{Z}$ and $E\_{X}^{Z}$ denote the longitudinal and transverse moduli, respectively of the $Z$-direction tow (the superscript specifies that these properties relate to the $Z$-direction tow while the subscripts specify the direction of the property). Similarly, $E\_{X}^{X}$ and $E\_{Z}^{X}$ are the longitudinal and transverse moduli, respectively of the $X$-direction tow while $E\_{X}^{h}$ and $E\_{Z}^{h}$ are the moduli of the homogenised matrix in the $X$ and $Z$-directions, respectively.

*S1. Elastic properties*

The tows are assumed to be transversely isotropic with the fibre direction normal to the plane of isotropy. We first consider the $Z$-direction tows. The longitudinal modulus $E\_{Z}^{Z}$ is given by the Voigt bound as $E\_{Z}^{Z}=f\_{Z}E\_{f}+\left(1-f\_{Z}\right)E\_{m}$ while the transverse moduli $E\_{X}^{Z}=E\_{Y}^{Z}$ are given by the equivalent Reuss bound. Since the Poisson’s ratios of the matrix and fibres are assumed equal, we take $ν\_{ZX}^{Z}=ν\_{XY}^{Z}=ν\_{m}$ and the shear modulus $G\_{XZ}^{Z}$ is estimated from a Reuss bound such that

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|  | $$\frac{1}{G\_{XZ}^{Z}}=\frac{2(1+ν\_{f})f\_{Z}}{E\_{f}}+\frac{2(1+ν\_{m})(1-f\_{Z})}{E\_{m}}.$$ | (S1.1) |

The five independent elastic constants required to describe the elastic properties of the transversely isotropic $Z$-direction tows are listed in Table S1. Equivalent estimates can be evaluated for the $X$ (or $Y$)-direction tows with $f\_{Z}$ replaced by $f\_{X}$. These properties are also listed in Table 1 for the $X$-direction tow. Note that the $X$-direction is normal to the plane of isotropy for the $X$-direction tow and hence the components of the elasticity tensor listed in Table 1 differ for the $X$ and $Z$-direction tows.

We proceed to calculate the properties of the homogenised matrix that surrounds the $Z$-direction tows. From the unit cell sketched in Fig. 1c it is clear that this homogenised matrix is an orthotropic effective material with Young’s moduli equal in the $X$ and $Y$-directions. Thus, in order to simplify the constitutive description it is reasonable to assume that this homogenised matrix is also transversely isotropic with the $Z$-direction being normal to the plane of isotropy. Again, since all the constituents have equal Poisson’s ratios it is reasonable to take $ν\_{ZX}^{h}=ν\_{XY}^{h}=ν\_{m}$. The Voigt estimate for the moduli $E\_{X}^{h}=E\_{Y}^{h}$ is given as

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|  | $$E\_{X}^{h}=\frac{v\_{X}\left(E\_{X}^{X}+E\_{Y}^{X}\right)+v\_{m}E\_{m}}{2v\_{X}+v\_{m}},$$ | (S1.2) |

while that for modulus $E\_{Z}^{h}$ is

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|  | $$E\_{Z}^{h}=\frac{2v\_{X}E\_{Y}^{X}+v\_{m}E\_{m}}{2v\_{X}+v\_{m}}.$$ | (S1.3) |

Similarly, the shear modulus $G\_{XZ}^{h}$ is given by the Voigt bound as

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|  | $$G\_{XZ}^{h} =\frac{v\_{X}\left(G\_{YZ}^{X}+G\_{XZ}^{X}\right)+v\_{m}\frac{E\_{m}}{2(1+ν\_{m})}}{2v\_{X}+v\_{m}},$$ | (S1.4) |

where $G\_{YZ}^{X}=0.5E\_{Y}^{X}/(1+ν\_{ZY}^{X})$. The five independent elastic constants for this effective medium are listed in Table S1.



**Fig. S1**: (a) Sketch of the homogenised matrix within the unit cell with the constituents of the homogenised matrix also indicated. The three regions A, B and C into which the homogenised matrix within the unit cell is divided for the analysis of the effective properties are also indicated. (b) Sketch of the indirect tension mechanism operative during the compression of region B in the $Z$-direction.

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| $Z$-direction tow | $$E\_{Z}^{Z}=65$$ | $$E\_{X}^{Z}=E\_{Y}^{Z}=4.2$$ | $$ν\_{ZX}^{Z}=0.25$$ | $$ν\_{XY}^{Z}=0.25$$ | $$G\_{XZ}^{Z}=G\_{YZ}^{Z}=1.7$$ |
| $X$-direction tow | $$E\_{Z}^{X}=E\_{Y}^{X}=8.8$$ | $$E\_{X}^{X}=142$$ | $$ν\_{XZ}^{X}=0.25$$ | $$ν\_{ZY}^{X}=0.25$$ | $$G\_{XZ}^{X}=G\_{XY}^{X}=3.6$$ |
| homogenised matrix | $$E\_{Z}^{h}=7.1$$ | $$E\_{X}^{h}=E\_{Y}^{h}=54$$ | $$ν\_{ZX}^{h}=0.25$$ | $$ν\_{XY}^{h}=0.25$$ | $$G\_{XZ}^{h}=G\_{YZ}^{h}=2.8$$ |

**Table S1:** The elastic properties of the transversely isotropic tows and the homogenised matrix in the 3D noobed composite. The $X$ and $Y$ -direction tows have identical properties with the super/subscript $X$ replaced by $Y$. All the moduli are given in GPa.

*S1.2 Plastic/failure strengths*

In estimating the plastic/failure strengths of the different phases we note that the strength for tensile loading along the fibre direction is limited by the failure strength $σ\_{f}$ of the fibres while loading in other directions (e.g. transverse or shear loading) is limited by flow of the matrix around fibres. Since the fibre strength is significantly greater than the matrix strength, the fibres may be assumed to be rigid for the purposes of analysis of strength in matrix flow governed regimes. With this understanding we proceed to develop estimates for the anisotropic strengths of the tows and the homogenised matrix.

First consider the $Z$-direction tow. The longitudinal tensile strength is limited by fibre fracture and directly given by a Voigt estimate as $Y\_{Z}^{Z}=f\_{Z}σ\_{f}+\left(1-f\_{Z}\right)σ\_{m}$. The calculation of the transverse strength is more complex. A Reuss estimate assuming rigid fibres will specify that the transverse strength is equal to that of the matrix which is a poor estimate as the rigid fibres constrain the flow of the matrix and enhance the strength. Bele and DeshpandeS2 provided a simple analytical estimate (verified via FE calculations) for the transverse strength of a composite comprising rigid cylinders dispersed in a plastic matrix. Here we use that prescription to estimate the transverse and shear strengths of the tow. The HashinS3 lower bound for the Young’s modulus $E$ of a composite comprising a volume fraction $f\_{Z}$ of rigid inclusions in an incompressible matrix of modulus $E\_{m}$ is

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|  | $$\frac{E}{E\_{m}}=1+\frac{5f\_{Z}}{2(1-f\_{Z})}.$$ | (S2.1) |

This linear bound can be transformed to an estimate of the strength using the method proposed by SuquetS4 by using the bound (S2.1) as the properties of a fictitious linear comparison composite. The transverse strength is then given as

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|  | $$Y\_{X}^{Z}=Y\_{Y}^{Z}=σ\_{m}\sqrt{\frac{E}{E\_{m}}(1-f\_{Z})},$$ | (S2.2) |

with $E/E\_{m}$ given by Eq. (S2.1). The shear strengths are assumed to be related to the transverse strength via a Tresca yield criterion such that $Y\_{XY}^{Z}=Y\_{XZ}^{Z}=Y\_{ZY}^{Z}=Y\_{X}^{Z}/2$. These properties of the $Z$-direction tow are listed in Table S2. The plastic/failures strengths for the $X$-direction tow can also be estimated in an analogous manner and these predictions are also listed in Table S2.

Next consider the homogenised matrix sketched in Fig. S1a. Uniaxial loading in the $X$-direction results in longitudinal and transverse loading of the $X$-direction and $Y$-direction tows, respectively as well as loading of the matrix pockets. The average stress sustained by this homogenised material at failure then follows as

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|  | $$Y\_{X}^{h}=\overbar{A}\_{ZY}^{X}(Y\_{X}^{X}+σ\_{m})+0.5\overbar{A}\_{ZY}^{Z}\left(σ\_{m}+Y\_{Y}^{X}\right),$$ | (S2.3) |

with $Y\_{Y}^{h}=Y\_{X}^{h}$. In order to calculate the strength $Y\_{Z}^{h}$ it is convenient to divide the $X-Y$ plane of the homogenised matrix into three regions A, B and C as shown in Fig. S1a. The uniaxial stress in the $Z$-direction over regions A and C is limited to the matrix yield strength $σ\_{m}$ while compression of region B is equivalent to the compression of a cross-ply laminate. The out-of-plane compression of a cross-ply laminate results in the development of tensile stresses in the fibres due to the anisotropic Poisson expansion of the cross-plies; see Fig. S1b. This so-called indirect tension mechanism was analysed by Attwood et al.S5 who showed that the compressive strength of cross-ply laminates approximately equals the in-plane tensile strength $Y\_{X}^{X}$ of each lamina. The strength $Y\_{Z}^{h}$ then is given by the average over the three regions such that

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|  | $$Y\_{Z}^{h}=\frac{2\overbar{A}\_{XY}^{m}σ\_{m}+\left(1-\overbar{A}\_{XY}^{Z}-2\overbar{A}\_{XY}^{m}\right)Y\_{X}^{X}}{1-\overbar{A}\_{XY}^{Z}},$$ | (S2.4) |

where $\overbar{A}\_{XY}^{m}$ is the area fraction that the matrix pockets occupy in the $X-Y$ plane on the surface of the unit cell (it is equal to the ratio of the area of region A to the area $\left(1.81+1.25\right)^{2} mm^{2}$ of the unit cell projected on the $X-Y$ plane). We assume all shear strengths to be equal ($Y\_{ZX}^{h}=Y\_{ZY}^{h}=Y\_{XY}^{h})$ and given by a Voigt bound such that

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|  | $$Y\_{ZX}^{h}=\frac{2v\_{X}Y\_{XY}^{X}+v\_{m}σ\_{m}/2}{2v\_{X}+v\_{m}},$$ | (S2.5) |

where we have assumed that the matrix shear strength is $σ\_{m}/2$ (Tresca yield criterion). These plastic collapse and failure strengths are listed in Table S2.

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| $Z$-direction tow | $$Y\_{Z}^{Z}=1300$$ | $$Y\_{X}^{Z}=Y\_{Y}^{Z}=170$$ | $$Y\_{XY}^{Z}=Y\_{XZ}^{Z}=Y\_{ZY}^{Z}=85$$ |
| $X$-direction tow | $$Y\_{X}^{X}=2800$$ | $$Y\_{Z}^{X}=Y\_{Y}^{X}=200$$ | $$Y\_{XY}^{X}=Y\_{XZ}^{X}=Y\_{ZY}^{X}=100$$ |
| Homogenised matrix | $$Y\_{Z}^{h}=1260$$ | $$Y\_{X}^{h}=Y\_{Y}^{h}=940$$ | $$Y\_{ZX}^{h}=Y\_{ZY}^{h}=Y\_{XY}^{h}=92$$ |

**Table S2:** The plastic/failure strengths of the tows and the homogenised matrix in the 3D noobed composite. In this table all the strengths are in MPa.

**References**

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