Supplementary Material

On Complex Economic Scenario Generators: Is Less More?*

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Abstract

This article proposes a complex economic scenario generator that nests versions of well-known actuarial frameworks. The generator estimation relies on the Bayesian paradigm and accounts for both model and parameter uncertainty via Markov chain Monte Carlo methods.

So, to the question *is less more?*, we answer maybe, but it depends on your criteria. From an in-sample fit perspective, on the one hand, a complex economic scenario generator seems better. From the conservatism, forecasting, and coverage perspectives, on the other hand, the situation is less clear: having more complex models for the short rate, term structure, and stock index returns is clearly beneficial. However, that is not the case for inflation and the dividend yield.

Keywords: Economic scenario generators; Long-term forecast; Risk management; Scenario simulation.

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SM.A Other Economic Scenario Generators

This section introduces three ESGs nested in our more general framework of Section 2: a Wilkie-like model, an extended Wilkie-like model based on his extension of 1995, and an Ahlgrim, D'Arcy, and Gorvett-like model (note that this latter model is also known as the CAS-SOA model).

SM.A.1 A Wilkie-Like Model

The Wilkie (1986) model is considered the first comprehensive *open access* ESG in the actuarial academic literature. Since its publication in 1986, the framework has been the subject of extensive study and debate (e.g., Geoghegan et al., 1992; Wilkie, 1995; Huber, 1997; Şahin et al., 2008). This section presents the Wilkie-like model used in this study and describes its similarities and differences with the actual Wilkie model.

The inflation rate and the transformed short rate are modelled by AR(1) dynamics defined as follows:

$$
\begin{aligned}
q_t &= \mu_q + a_q\,(q_{t-1} - \mu_q) + \sigma_q\,\varepsilon_{q,t},\\
\tilde{r}_t &= \mu_r + a_r\,(\tilde{r}_{t-1} - \mu_r) + \sigma_r\,\varepsilon_{r,t}.
\end{aligned}
$$

Instead of having the contemporaneous inflation rate as well as the dividend yield innovation in the short rate equation, we assume that the standardized innovation $\varepsilon_{r,t}$ is correlated with $\varepsilon_{q,t}$ and $\varepsilon_{d,t}$ in our Wilkie-like model; this makes the Wilkie-like model comparable to that of Ahlgrim et al. (2005). Moreover, it has no impact on the fit of the model.

We assume that each entry in matrix $\boldsymbol{A}_f$ is set to zero, meaning that the transformed forward rates are given by:

$$
\tilde{\boldsymbol{f}}_t - \mathbf{1}_n\,\tilde{r}_t = \boldsymbol{\mu}_f + \boldsymbol{\Sigma}_f\,\boldsymbol{\varepsilon}_{f,t}.
$$

In other words, we assume that each transformed forward rate is a noisy translated version of the short rate. This is also consistent with Wilkie (1986) that uses only one interest rate factor to describe the whole yield curve—in this case, the short rate.

The dividend yield is given by the following equation:

$$
\log(d_t) = \log(\mu_d) + a_d\left(\log(d_{t-1}) - \log(\mu_d)\right) + \sigma_d\,\varepsilon_{d,t}.
$$

Again, we assume dependence between the standardized innovations instead of having functionals of the inflation rate in our dynamics (i.e., parameter YW of the original paper is set to zero).

Finally, the stock index return in period t is given by the following dynamics:

$$
y_t = r_t + \mu_y + \sigma_y\,\varepsilon_{y,t}.
$$

Again, for the sake of comparability, we include the short rate in our dynamics instead of the inflation rate—similar to Ahlgrim et al. (2005). Also, parameters DY and DB of the original paper are set to zero.

SM.A.2 An Extended Wilkie-Like Model

Wilkie (1995) extends the classic Wilkie model and includes an earnings index, short-term interest rates, and property prices. For the sake of consistency with Wilkie's (1986) model and the framework introduced in Section 2, we only consider inflation, nominal interest rates, the dividend yield, and stock index returns.

We focus on a variant of Wilkie's (1995) that is nested within the model presented in Section 2. Specifically, the inflation rate, short rate, dividend yield, and stock index return models are exactly the same as those proposed above for the Wilkie (1986) model. The only difference with the 1986 framework comes from the addition of long rate dynamics that are modelled via a second equation. In the 1995 paper, Wilkie models the spread between the long- and short-rate interest rates, similar to our approach. Indeed, this spread is called the slope in the present study. Our version of Wilkie (1995) thus relies on the following transformed forward rate dynamics:

$$\tilde{\boldsymbol{f}}_t - \mathbf{1}_n \tilde{r}_t = \boldsymbol{\mu}_f + \boldsymbol{A}_{f,1} F_{1,t} + \boldsymbol{\Sigma}_f \boldsymbol{\varepsilon}_{f,t},$$

where $\boldsymbol{A}_{f,1}$ is the first column of matrix $\boldsymbol{A}_f$.

SM.A.3 An Ahlgrim, D'Arcy, and Gorvett-Like Model

The Ahlgrim et al. (2005) framework is the end-product of a research project on *Modelling of Economic Series Coordinated with Interest Rate Scenarios* initiated by a joint request from the Society of Actuaries and the Casualty Actuarial Society. Their original model considers inflation, nominal and real interest rates, dividend yield, stock index returns, real estate returns, and unemployment. Again, for the sake of consistency with Wilkie's (1986) model and the framework introduced in Section 2, we consider only inflation, nominal interest rates, dividend yield, and stock index returns.

Ahlgrim et al. (2005) express their ESG via continuous-time diffusions. To make our analysis consistent with the rest of this study, we use a discrete-time equivalent of their model. As their model is estimated based on discrete-time observations, this should not impact the model's behaviour (additional details are available in Ahlgrim et al., 2005).

Similar to Wilkie's (1986) framework, the inflation rate is modelled by an AR(1) process defined as follows:

$$q_t = \mu_q + a_q \left(q_{t-1} - \mu_q\right) + \sigma_q \, \varepsilon_{q,t}.$$

The transformed short rate is also modelled by an AR(1) process defined as follows:

$$\tilde{r}_t = \mu_r + a_r \left(\tilde{r}_{t-1} - \mu_r\right) + \sigma_r \, \varepsilon_{r,t}.$$

In the original paper, the authors use a second—long rate—factor to model the yield curve. Our approach in this study—capturing the yield curve by using two factors—is slightly different, but ultimately equivalent; i.e., short and long rates in Ahlgrim et al. (2005) and short rate and average forward rate in excess of the short rate in our variant of their model. Our version relies on the following transformed forward rate dynamics:

$$\tilde{\boldsymbol{f}}_t - \mathbf{1}_n \tilde{r}_t = \boldsymbol{\mu}_f + \boldsymbol{A}_{f,1} F_{1,t} + \boldsymbol{\Sigma}_f \boldsymbol{\varepsilon}_{f,t}.$$

Again, similar to Wilkie, the following equations give the dividend yield:

$$\log(d_t) = \log(\mu_d) + a_d \left(\log(d_{t-1}) - \log(\mu_d)\right) + \sigma_d \, \varepsilon_{d,t}.$$

Finally, the stock index return in period t is given by the following dynamics:

$$y_t = r_t + \mu_{y,m_t} + \sigma_{y,m_t} \, \varepsilon_{y,t},$$

where $\sigma^2_{y,\mathrm{u}}$, $\sigma^2_{y,\mathrm{s}}$, and $\sigma^2_{y,\mathrm{d}}$ are regime-dependent return variances. This is obviously slightly different from Ahlgrim et al.'s model as we consider three regimes instead of two, and we use observable regimes based on the monetary policy instead of latent ones. Note that these two modifications to the original framework do not negatively impact the fit.

As mentioned above, this framework also considers real estate returns and unemployment. Adding more variables is helpful under some circumstances but should not materially impact this study's results.

SM.B More on the Estimation

SM.B.1 Likelihood Function

Whether one uses maximum likelihood or Bayesian inference, the likelihood function serves as an important building block to estimate the model parameters. Note that the following equations are all based on the new ESG and, for the sake of conciseness, we do not explicitly include $\mathcal{M}_4$ in the conditioning set. Also, we only show the likelihoods for $\mathcal{M}_4$, but the likelihoods for frameworks $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ are obtained in a similar fashion.

For the monetary policy model, the likelihood function is given by

$$\begin{aligned}\mathcal{L}_m\left(m_{1:T} \mid p_{\text{uu}}, p_{\text{su}}, p_{\text{sd}}, p_{\text{dd}}\right) &= f\left(m_1 \mid p_{\text{uu}}, p_{\text{su}}, p_{\text{sd}}, p_{\text{dd}}\right) \prod_{t=2}^{T} f\left(m_t \mid m_{t-1}, p_{\text{uu}}, p_{\text{su}}, p_{\text{sd}}, p_{\text{dd}}\right) \\ &= f\left(m_1 \mid p_{\text{uu}}, p_{\text{su}}, p_{\text{sd}}, p_{\text{dd}}\right) \prod_{t=2}^{T} p_{m_{t-1}m_t}.\end{aligned}$$

The probability mass function related to the first term m_1 is different since we do not know m_0—there is no preceding observation on which to condition. To circumvent this issue, we condition on m_0 and treat it as an additional parameter:

$$\mathcal{L}_m\left(m_{1:T} \mid \mathbf{\Theta}_m\right) = \prod_{t=1}^{T} p_{m_{t-1}m_t},$$

where $\mathbf{\Theta}_m = \begin{bmatrix} p_{\text{uu}} & p_{\text{su}} & p_{\text{sd}} & p_{\text{dd}} & m_0 \end{bmatrix}$.

The inflation rate, short rate, and dividend yield innovations are related through linear correlations:

$$\operatorname{Corr}\left(\begin{bmatrix} \varepsilon_{q,t} & \varepsilon_{r,t} & \varepsilon_{d,t} \end{bmatrix}\right) = \begin{bmatrix} 1 & \rho_{q,r} & \rho_{q,d} \\ \rho_{q,r} & 1 & \rho_{r,d} \\ \rho_{q,d} & \rho_{r,d} & 1 \end{bmatrix},$$

To make the derivation below more direct, we define the Cholesky decomposition of the above matrix and use it to obtain the following relationship:

$$\begin{bmatrix} \varepsilon_{q,t} \\ \varepsilon_{r,t} \\ \varepsilon_{d,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{12} & l_{22} & 0 \\ l_{13} & l_{23} & l_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}, \quad \text{where} \quad \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \sim \mathcal{N}_3(\mathbf{0}_3, \mathbf{I}_3)$$

In the case of the inflation rate model, the likelihood function is given by

$$\mathcal{L}_q\left(q_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, m_{1:T}\right) = \prod_{t=1}^{T} f\left(q_t \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, m_{1:T}, q_{1:t-1}\right)$$

$$= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\,\sigma_{q,t}} \exp\left(-\frac{1}{2}\left(\frac{q_t - \mu_{q,m_t} - a_q\left(q_{t-1} - \mu_{q,m_t}\right)}{\sigma_{q,t}}\right)^2\right)$$

if we treat the initial inflation rate q_0 and the time-1 conditional variance $\sigma^2_{q,1}$ as additional parameters, where $\boldsymbol{\Theta}_q = [\ \mu_{q,\mathrm{u}} \quad \mu_{q,\mathrm{s}} \quad \mu_{q,\mathrm{d}} \quad a_q \quad \sigma^2_q \quad \alpha_q \quad \beta_q \quad \gamma_q \quad q_0 \quad \sigma^2_{q,1}\ ]$.

Similar to the inflation model, the likelihood function of the short rate model is given by

$$\begin{aligned}
&\mathcal{L}_r\left(\tilde{r}_{1:T} \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, m_{1:T}, q_{1:T}\right) \\
&= \prod_{t=1}^{T} f\left(\tilde{r}_t \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, m_{1:T}, q_{1:T}, \tilde{r}_{1:t-1}\right) \\
&= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\, l_{22}\,\sigma_{r,t}} \exp\left(-\frac{1}{2}\left(\frac{\tilde{r}_t - \mu_{r,m_t} - a_r\left(\tilde{r}_{t-1} - \mu_{r,m_t}\right) - l_{12}\,\sigma_{r,t}\,\varepsilon_{1,t}}{l_{22}\,\sigma_{r,t}}\right)^2\right),
\end{aligned}$$

where $\boldsymbol{\Theta}_r = [\ \mu_{r,\mathrm{u}} \quad \mu_{r,\mathrm{s}} \quad \mu_{r,\mathrm{d}} \quad a_r \quad \rho_{q,r} \quad \sigma^2_r \quad \alpha_r \quad \beta_r \quad \gamma_r \quad r_0 \quad \sigma^2_{r,1}\ ]$.

The forward rate model is comprised of two components: (1) the factor dynamics, and (2) the link between the factors and the transformed forward rates. For the former, the likelihood associated with the factor dynamics is given by:

$$\begin{aligned}
&\mathcal{L}_{\boldsymbol{F}}\left(\boldsymbol{F}_{1:T} \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}\right) \\
&= \prod_{t=1}^{T} f\left(\boldsymbol{F}_t \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, m_{1:T}, q_{1:T}, \tilde{r}_{1:t-1}, \boldsymbol{F}_{1:t-1}\right) \\
&= \prod_{t=1}^{T}\prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\,\sigma_{F_i}} \exp\left(-\frac{1}{2}\left(\frac{F_{i,t} - \mu_{F_i} - a_{F_i}\left(F_{i,t-1} - \mu_{F_i}\right)}{\sigma_{F_i}}\right)^2\right),
\end{aligned}$$

where $\boldsymbol{\Theta}_F = [\ \mu_{F_1} \quad \mu_{F_2} \quad a_{F_1} \quad a_{F_2} \quad \sigma^2_{F_1} \quad \sigma^2_{F_2} \quad F_{1,0} \quad F_{2,0}\ ]^\top$. For the latter, the likelihood function is given by:

$$\begin{aligned}
&\mathcal{L}_f\left(\tilde{\boldsymbol{f}}_{1:T} \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, \boldsymbol{\Theta}_f, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}\right) \\
&= \prod_{t=1}^{T} f\left(\tilde{\boldsymbol{f}}_t \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, \boldsymbol{\Theta}_f, m_{1:T}, q_{1:T}, \tilde{r}_{1:t-1}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:t-1}\right) \\
&= \prod_{t=1}^{T}\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma_{f_i}} \exp\left(-\frac{1}{2}\left(\frac{\tilde{f}_{i,t} - \tilde{r}_t - \mu_{f_i} - a_{f_i,1}\,F_{1,t} - a_{f_i,2}\,F_{2,t}}{\sigma_{f_i}}\right)^2\right),
\end{aligned}$$

where $\boldsymbol{\Theta}_f = [\ \mu_{f_1} \quad \dots \quad \mu_{f_n} \quad a_{f_1,1} \quad \dots \quad a_{f_n,1} \quad a_{f_1,2} \quad \dots \quad a_{f_n,2} \quad \sigma^2_{f_1} \quad \dots \quad \sigma^2_{f_n}\ ]$.

In the case of the dividend yield model, the likelihood function is given by:

$$\begin{aligned}
&\mathcal{L}_d\left(d_{1:T} \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, \boldsymbol{\Theta}_f, \boldsymbol{\Theta}_d, m_{1:T}, q_{1:T}, \tilde{r}_{1:T},, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}\right) \\
&= \prod_{t=1}^{T} f\left(d_t \,\middle|\, \boldsymbol{\Theta}_m, \boldsymbol{\Theta}_q, \boldsymbol{\Theta}_r, \boldsymbol{\Theta}_F, \boldsymbol{\Theta}_f, \boldsymbol{\Theta}_d, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}, d_{1:t-1}\right)
\end{aligned}$$

$$= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\, l_{33}\, \sigma_{d,t}} \exp\left(-\frac{1}{2}\left(\frac{\log(d_t) - \mu_{d,m_t} - a_d\left(\log(d_{t-1}) - \mu_{d,m_t}\right) - l_{13}\, \sigma_{d,t}\, \varepsilon_{1,t} - l_{23}\, \sigma_{d,t}\, \varepsilon_{2,t}}{l_{33}\, \sigma_{d,t}}\right)^2\right),$$

where $\mathbf{\Theta}_d = [\ \mu_{d,\mathrm{u}} \quad \mu_{d,\mathrm{s}} \quad \mu_{d,\mathrm{d}} \quad a_d \quad \rho_{q,d} \quad \rho_{r,d} \quad \sigma_d^2 \quad \alpha_d \quad \beta_d \quad \gamma_d \quad d_0 \quad \sigma_{d,1}^2\ ]$.

Then, the likelihood associated with the stock index returns is given by:

$$\begin{aligned}
&\mathcal{L}_y\left(y_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, \mathbf{\Theta}_f, \mathbf{\Theta}_d, \mathbf{\Theta}_y, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}, d_{1:T}\right) \\
&= \prod_{t=1}^{T} f\left(y_t \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, \mathbf{\Theta}_f, \mathbf{\Theta}_d, \mathbf{\Theta}_y, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}, d_{1:T}, y_{1:t-1}\right) \\
&= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\, \sigma_{y,t}} \exp\left(-\frac{1}{2}\left(\frac{y_t - r_t - \mu_{y,m_t}}{\sigma_{y,t}}\right)^2\right),
\end{aligned}$$

where $\mathbf{\Theta}_y = [\ \mu_{y,\mathrm{u}} \quad \mu_{y,\mathrm{s}} \quad \mu_{y,\mathrm{d}} \quad \sigma_y^2 \quad \alpha_y \quad \beta_y \quad \gamma_y \quad \sigma_{y,1}^2\ ]$.

Finally, the total likelihood is simply the product of the seven previous likelihoods:

$$\begin{aligned}
\mathcal{L}(X \,|\, \mathbf{\Theta}) = {} & \mathcal{L}_m\left(m_{1:T} \,|\, \mathbf{\Theta}_m\right) \times \mathcal{L}_q\left(q_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, m_{1:T}\right) \times \mathcal{L}_r\left(\tilde{r}_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, m_{1:T}, q_{1:T}\right) \\
& \times \mathcal{L}_{\boldsymbol{F}}\left(\boldsymbol{F}_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}\right) \\
& \times \mathcal{L}_{\boldsymbol{f}}\left(\tilde{\boldsymbol{f}}_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, \mathbf{\Theta}_f, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}\right) \\
& \times \mathcal{L}_d\left(d_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, \mathbf{\Theta}_f, \mathbf{\Theta}_d, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}\right) \\
& \times \mathcal{L}_y\left(y_{1:T} \,\middle|\, \mathbf{\Theta}_m, \mathbf{\Theta}_q, \mathbf{\Theta}_r, \mathbf{\Theta}_F, \mathbf{\Theta}_f, \mathbf{\Theta}_d, \mathbf{\Theta}_y, m_{1:T}, q_{1:T}, \tilde{r}_{1:T}, \boldsymbol{F}_{1:T}, \tilde{\boldsymbol{f}}_{1:T}, d_{1:T}\right).
\end{aligned} \tag{SM.1}$$

SM.B.2 Implementation and Convergence Diagnostics

This study utilizes a version of the adaptive Metropolis algorithm of Haario et al. (2001) (for a review on adaptive MCMC algorithms, refer to Roberts and Rosenthal, 2009). We rely on blocking to divide our framework into seven smaller components, representing the seven likelihoods of Equation (SM.1): monetary policy (m), inflation (q), short rate (r), interest rate factors ($\boldsymbol{F}$), term structure ($\boldsymbol{f}$), dividend yield (d), and stock index returns (y). Specifically, we perform Metropolis steps with proposal distributions based on the following mixture at step j:

$$\mathbf{\Theta}_i^* \sim \begin{cases} \mathcal{N}\left(\mathbf{\Theta}_i^{(j-1)}, (2.38)^2\, \mathbf{\Sigma}_{i,n}/d_i\right) & \text{with probability } 1-\beta \\ \mathcal{N}\left(\mathbf{\Theta}_i^{(j-1)}, \boldsymbol{K}_i\, \mathbf{I}_{d_i}/d_i\right) & \text{with probability } \beta \end{cases}, \tag{SM.2}$$

where $i \in \{m, q, r, \boldsymbol{F}, \boldsymbol{f}, d, y\}$, β is a small positive constant (we use $\beta = 0.05$ in this study), $\mathbf{\Sigma}_{i,n}$ is the covariance structure of the target density of the i^{th} block, d_i is size of $\mathbf{\Theta}_i$, and $\boldsymbol{K}_i$ is a $d_i \times d_i$ diagonal matrix containing scaling factors. The samples are accepted as usual, using the Hastings acceptance ratio:

$$\alpha_{\mathcal{M}}\left(\mathbf{\Theta}_i^*, \mathbf{\Theta}_i^{(j-1)}\right) = \min\left(1, \frac{\mathcal{L}(X \,|\, \mathbf{\Theta}^*, \mathcal{M})\, \pi(\mathbf{\Theta}^* \,|\, \mathcal{M})}{\mathcal{L}\left(X \,\middle|\, \mathbf{\Theta}^{(j/j-1)}, \mathcal{M}\right) \pi\left(\mathbf{\Theta}^{(j/j-1)} \,\middle|\, \mathcal{M}\right)}\right), \tag{SM.3}$$

where $\mathbf{\Theta}^* = [\ \mathbf{\Theta}_m^{(j)} \ \ \ldots \ \ \mathbf{\Theta}_i^* \ \ \ldots \ \ \mathbf{\Theta}_y^{(j-1)}\ ]$ and $\mathbf{\Theta}^{(j/j-1)} = [\ \mathbf{\Theta}_m^{(j)} \ \ \ldots \ \ \mathbf{\Theta}_i^{(j-1)} \ \ \ldots \ \ \mathbf{\Theta}_y^{(j-1)}\ ]$. The details of the method are given in Algorithm 1.

To cope with potential slow convergence issues, we employ a long Markov chain; that is, $M =$

Algorithm 1 Adaptive Metropolis Algorithm

set $\boldsymbol{\Theta}_i^{(0)}$ for each block $i \in \{m, q, r, \boldsymbol{F}, \boldsymbol{f}, d, y\}$
for each $j \in \{1, ..., M\}$ **do**
  for each $i \in \{m, q, r, \boldsymbol{F}, \boldsymbol{f}, d, y\}$ **do**
    generate a candidate $\boldsymbol{\Theta}_i^*$ from Equation (SM.2)
    calculate the Hastings acceptance ratio $\alpha_{\mathcal{M}}\left(\boldsymbol{\Theta}_i^*, \boldsymbol{\Theta}_i^{(j-1)}\right)$ using Equation (SM.3)
    accept of reject this new sample using a uniform random number
  end for
end for

300,000 observations. The first 100,000 observations are burned and are therefore removed from the sample. Thereafter, every second simulation is recorded for posterior analysis (i.e., thinning). This process yields a final Markov chain of size 100,000 for each model that is used for empirical purposes.

It is very difficult to verify the convergence from a theoretical standpoint in high-dimensional problems. We therefore rely on practical tests to verify that our chains converged: the Gelman and Rubin (1992) diagnostic assesses the convergence by comparing the estimated between-chains and within-chain variances for each model parameter. All the potential scale reduction factors (PSRFs) range between 1 and 1.025 for all the parameters and models, meaning that the chains seem to have converged to their target posterior distribution. Indeed, Brooks and Gelman (1998) suggested that one can be confident that convergence has been reached if the PSRFs are below 1.2—which is the case in this study. Also, for all our models and parameters, the trace plots exhibit a *hairy caterpillar* behaviour, meaning that the mixing seems adequate. They are available in Figures SM.1 to SM.6 for $\mathcal{M}_4$ and upon request for $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$.

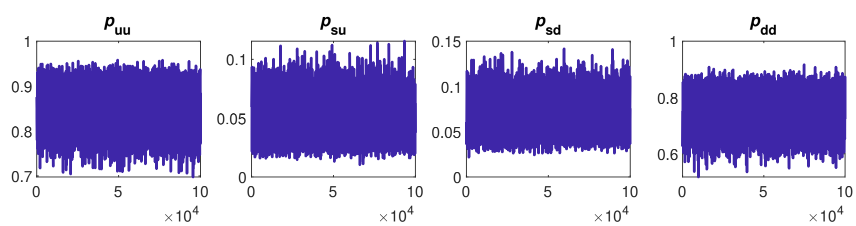

Figure SM.1: **Trace Plots of Monetary Policy Parameters.**
This figure reports the trace plots for the parameters of the monetary policy model.

SM.B.3 Reversible Jump MCMC

Most MCMC schemes create samples from a target density with a fixed number of dimensions. The reversible jump MCMC algorithm of Green (1995) is different in that respect because it allows for dimension jumping moves. Specifically, it enables us to move from one model and one parameter realization to another model and a new parameter realization consequential in size.

Assume that you want to move from $\mathcal{M}_k$ to $\mathcal{M}_{k'}$ (or from $\boldsymbol{\Theta}_k \in \mathbb{R}^{d_k}$ to $\boldsymbol{\Theta}_{k'} \in \mathbb{R}^{d_{k'}}$). Green's idea is to pad out $\boldsymbol{\Theta}_k$ and $\boldsymbol{\Theta}_{k'}$ with random auxiliary variables, say, $\boldsymbol{W}_k$ and $\boldsymbol{W}_{k'}$, so that the final size of $\boldsymbol{Z}_k = [\ \boldsymbol{\Theta}_k \quad \boldsymbol{W}_k\ ]$ is equal to that of $\boldsymbol{Z}_{k'} = [\ \boldsymbol{\Theta}_{k'} \quad \boldsymbol{W}_{k'}\ ]$ (i.e., dimension matching requirement). Then, we need to build a smooth continuous bijective mapping $h_{k,k'}$ with a smooth continuous inverse such that $h_{k,k'}(\boldsymbol{Z}_k) = \boldsymbol{Z}_{k'}$ to move from one parameter space to another.

Once we have these two additional components, we can apply the rationale of the Metropolis algorithm as achieved in the previous section. To do so, we need a proposal distribution for both model jumps

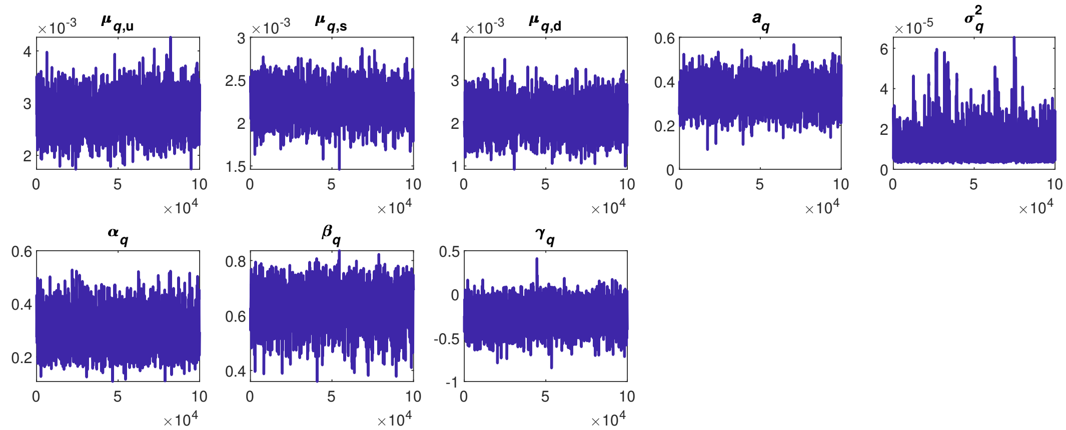

Figure SM.2: **Trace Plots of Inflation Parameters.**
This figure reports the trace plots for the parameters of the inflation model.

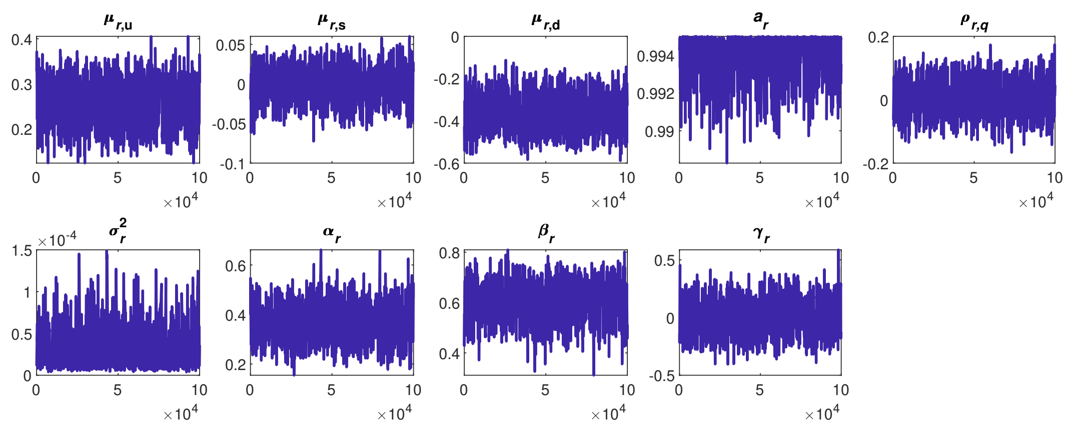

Figure SM.3: **Trace Plots of Short Rate Parameters.**
This figure reports the trace plots for the parameters of the short rate model.

and parameter values. To jump from $\mathcal{M}_k$ to $\mathcal{M}_{k'}$, on the one hand, we employ a proposal that assumes equally likely moves:

$$q(\mathcal{M}_{k'} \,|\, \mathcal{M}_k) = \tfrac{1}{4}, \qquad \text{where } k' = 1 \in \{1,2,3,4\} \text{ and } k \in \{1,2,3,4\}.$$

The proposal for $\boldsymbol{Z}_{k'}$, on the other hand, is obtained in two parts: $\boldsymbol{\Theta}_{k'}$ is obtained using the adaptive Metropolis method explained in Appendix SM.B.2, and the auxiliary variables $\boldsymbol{W}_{k'}$ from a distribution with density, say, $g_{k'}$. We use a multivariate normal distribution in our application as we can easily generate samples from this distribution, and the density exists in closed form.

Finally, because we can jump from one model to the other, the Hastings ratio needs to be adjusted in the following way:

$$\min\left(1, \frac{\mathcal{L}(\boldsymbol{X} \,|\, \boldsymbol{\Theta}_{k'}, \mathcal{M}_{k'})\, \pi(\boldsymbol{\Theta}_{k'} \,|\, \mathcal{M}_{k'})\, \pi(\mathcal{M}_{k'})\, g_{k'}(\boldsymbol{W}_{k'})\, q(\mathcal{M}_{k'} \,|\, \mathcal{M}_k)}{\mathcal{L}(\boldsymbol{X} \,|\, \boldsymbol{\Theta}_{k}, \mathcal{M}_{k})\, \pi(\boldsymbol{\Theta}_{k} \,|\, \mathcal{M}_{k})\, \pi(\mathcal{M}_{k})\, g_{k}(\boldsymbol{W}_{k})\, q(\mathcal{M}_{k} \,|\, \mathcal{M}_{k'})} \cdot \left|\frac{\partial h_{k,k'}(\boldsymbol{Z}_k)}{\partial \boldsymbol{Z}_k}\right|\right),$$

where the last component is the determinant of the Jacobian matrix related to the bijection $h_{k,k'}$.

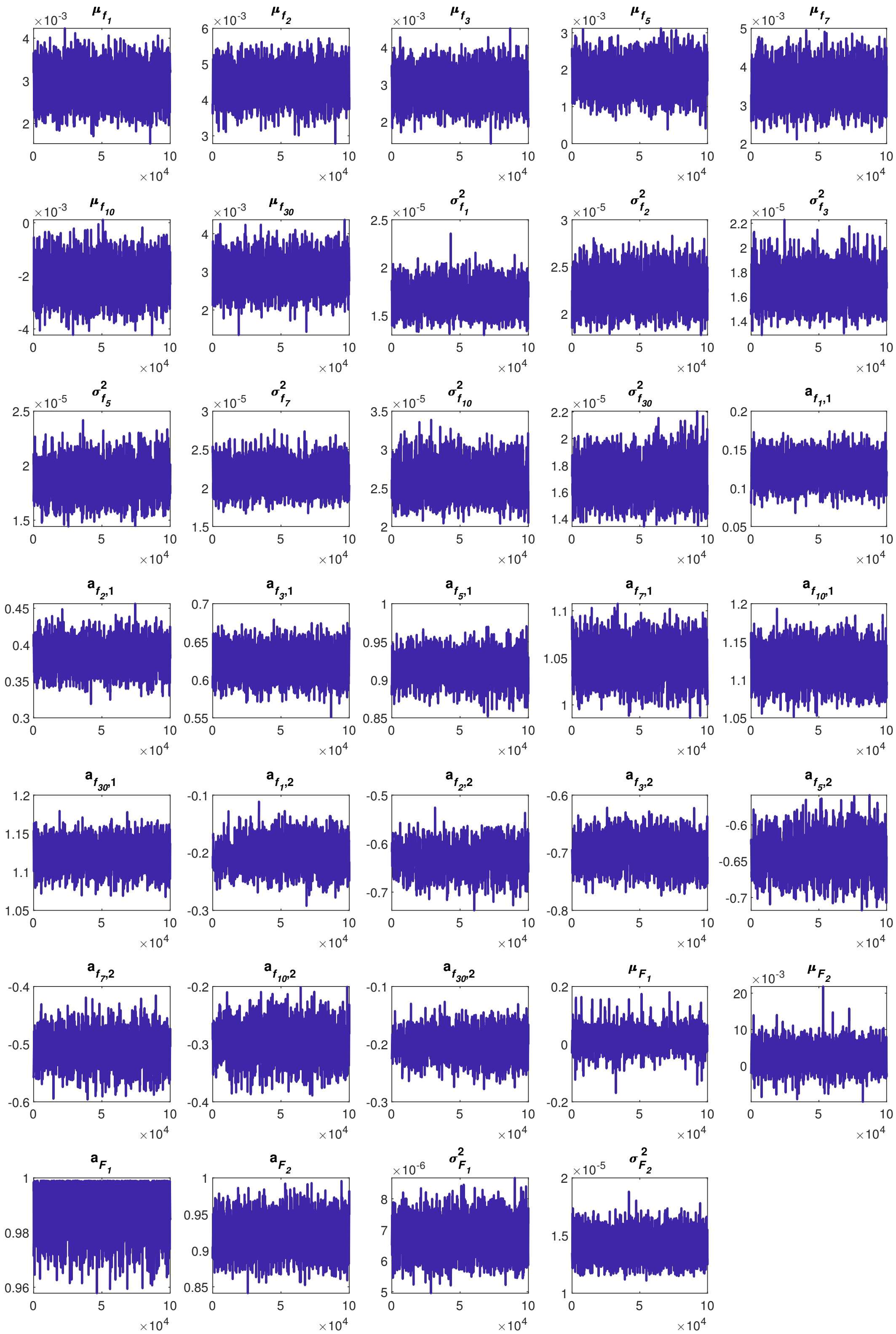

Figure SM.4: **Trace Plots of Term Structure Parameters.**
This figure reports the trace plots for the parameters of the term structure model.

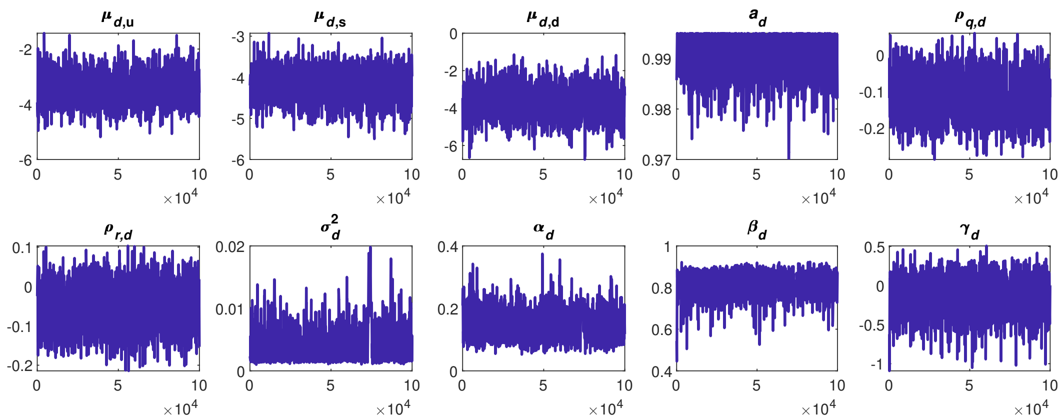

Figure SM.5: **Trace Plots of Dividend Yield Parameters.**
This figure reports the trace plots for the parameters of the dividend yield model.

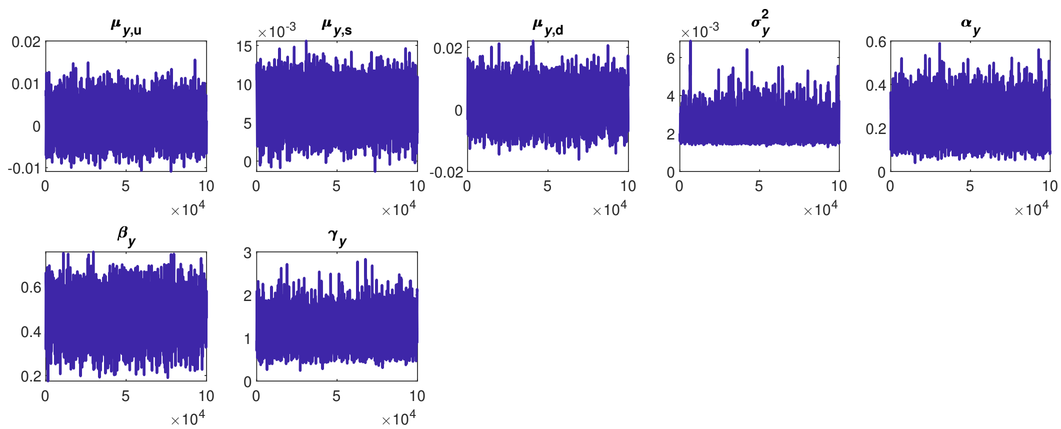

Figure SM.6: **Trace Plots of Stock Index Returns Parameters.**
This figure reports the trace plots for the parameters of the stock index returns model.

SM.C Empirical Results

SM.C.1 Term Structure Factors

The term structure factors—the slope and the curvature—are constructed from the (transformed) forward rates. The former is computed as the difference between the 30-year transformed forward rate and the one-year transformed forward rate. The latter is obtained by summing the 30-year transformed forward rate and the one-year transformed forward rate, and subtracting twice the three-year transformed forward rate. Figure SM.7 reports the time series of the two factors.

SM.C.2 Results and Discussion

Table SM.1 shows the average parameters and their standard deviations. Section 4 of the article discusses these parameters in more detail.

Figure SM.8 shows an example of one realization obtained from $\mathcal{M}_4$. Overall, the simulated series yield a similar behaviour to those observed empirically.

SM.D An Ad Hoc Ensemble Model Averaging

This supplementary section gives preliminary results on ensemble models. Based on the results of

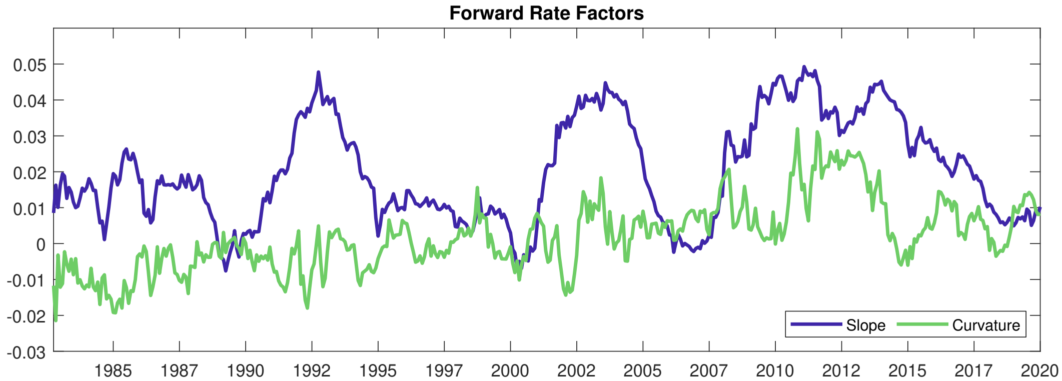

Figure SM.7: **Term Structure Factors.**
This figure shows the two term structure factors—the slope and the curvature. The former is computed as the difference between the 30-year transformed forward rate and the one-year transformed forward rate. The latter is obtained by summing the 30-year transformed forward rate and the one-year transformed forward rate, and subtracting twice the three-year transformed forward rate.

Sections 4, 5, and 6, there are clearly some benefits and disadvantages for each of the four models considered. From an in-sample perspective, the most complex ESG seems better for most components. From the conservatism, forecasting, and coverage perspectives, on the other hand, the issue is less clear: having more complex models for the short rate, term structure, and stock index returns is clearly beneficial. However, this is not the case for inflation and the dividend yield. Moreover, as the saying goes, it is difficult to make predictions, especially about the future. Perhaps a better way to construct ESGs is by combining them. The idea is actually quite simple: we use the scenarios we have and blend them to create an ensemble model.

This strategy has been used extensively in data-heavy domains such as weather forecasting (e.g., Gneiting and Raftery, 2005). A range of methodology exists to combine models; it goes from simple unweighted averages to more sophisticated methods such as principal component-based methods, trimmed means, performance-based weighting, optimal least squared estimates, and Bayesian shrinkage (Graefe et al., 2015). One drawback regarding the advanced methods is that they involve a level of complexity that can often be unnecessary.

In this study, we use simple unweighted average models in lieu of sophisticated combination methods to create better ESGs. Table SM.2 reports ensemble model out-of-sample forecast RMSEs (Panel A) and coverage errors (Panel B) for all series and horizons considered in Tables 5 and 6. As expected, the ensemble model RMSEs are better than those of the worst model but marginally worse than the best model. For instance, the inflation RMSE is 2.2% for $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$ and is 2.5% for $\mathcal{M}_4$; the ensemble model for inflation yields an RMSE of 2.3%. We reach a similar conclusion for every other economic and financial variable.

We obtain a similar story for coverage errors: the ensemble model improves the performance of the worst model but slightly deteriorates the best model's performance. Overall, the average coverage errors are close to zero; that is, between 2.4% and 16.1%.

In sum, the ensemble model is a good compromise; it allows for correcting overly or insufficiently conservative estimates. Moreover, if one model is off at times, the other models compensate and bring the ensemble to decent levels. This ultimately reduces the potential for model risk.

Table SM.1: **Parameters Statistics for the Four Models (1986–2020).**

Panel A: Monetary Policy.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
p_{uu}					0.856	(0.035)	0.856	(0.035)
p_{su}					0.050	(0.013)	0.050	(0.013)
p_{sd}					0.068	(0.015)	0.068	(0.015)
p_{dd}					0.753	(0.049)	0.753	(0.049)
Panel B: Inflation.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$\mu_{q,\mathrm{u}} \times 100$							0.281	(0.030)
$\mu_{q,\mathrm{s}} \times 100$ / $\mu_q \times 100$	0.216	(0.019)	0.216	(0.019)	0.216	(0.019)	0.222	(0.016)
$\mu_{q,\mathrm{d}} \times 100$							0.215	(0.032)
a_q	0.433	(0.043)	0.433	(0.043)	0.433	(0.043)	0.341	(0.057)
$\sigma_q \times 100$	0.225	(0.008)	0.225	(0.008)	0.225	(0.008)	0.318	(0.117)
α_q							0.290	(0.063)
β_q							0.613	(0.066)
γ_q							−0.250	(0.131)
Panel C: Short Rate.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$\mu_{r,\mathrm{u}} \times 10$							2.611	(0.423)
$\mu_{r,\mathrm{s}} \times 10$ / $\mu_r \times 10$	0.196	(0.194)	0.196	(0.194)	0.196	(0.194)	−0.015	(0.187)
$\mu_{r,\mathrm{d}} \times 10$							−3.687	(0.770)
a_r	0.992	(0.003)	0.992	(0.003)	0.992	(0.003)	0.994	(0.001)
$\rho_{q,r}$	0.065	(0.048)	0.065	(0.048)	0.065	(0.048)	0.007	(0.047)
$\sigma_r \times 100$	0.290	(0.010)	0.290	(0.010)	0.290	(0.010)	0.557	(0.228)
α_r							0.361	(0.071)
β_r							0.601	(0.070)
γ_r							0.002	(0.128)
Panel D: Term Structure.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$\mu_{f_1} \times 100$	0.500	(0.021)	0.342	(0.036)	0.341	(0.036)	0.292	(0.034)
$\mu_{f_2} \times 100$	1.095	(0.039)	0.596	(0.060)	0.596	(0.061)	0.446	(0.038)
$\mu_{f_3} \times 100$	1.401	(0.048)	0.456	(0.061)	0.457	(0.061)	0.295	(0.034)
$\mu_{f_5} \times 100$	1.884	(0.061)	0.333	(0.060)	0.332	(0.059)	0.183	(0.036)
$\mu_{f_7} \times 100$	2.339	(0.069)	0.470	(0.053)	0.470	(0.052)	0.352	(0.038)
$\mu_{f_{10}} \times 100$	2.305	(0.073)	0.214	(0.047)	0.213	(0.047)	0.142	(0.043)
$\mu_{f_{30}} \times 100$	2.484	(0.076)	0.341	(0.036)	0.343	(0.037)	0.294	(0.034)
$a_{f_1,1}$			0.080	(0.015)	0.081	(0.015)	0.123	(0.014)
$a_{f_2,1}$			0.252	(0.025)	0.252	(0.025)	0.384	(0.016)
$a_{f_3,1}$			0.477	(0.025)	0.476	(0.025)	0.622	(0.015)
$a_{f_5,1}$			0.780	(0.024)	0.780	(0.024)	0.915	(0.015)
$a_{f_7,1}$			0.940	(0.021)	0.941	(0.022)	1.047	(0.016)
$a_{f_{10},1}$			1.054	(0.019)	1.054	(0.019)	1.118	(0.019)
$a_{f_{30},1}$			1.081	(0.014)	1.080	(0.015)	1.122	(0.015)

This table reports the average and standard deviation (in parentheses) for each of the parameters out of a sample of 100,000 values. This is done using the adaptive Metropolis algorithm of Section 3 with the three models of Section 2.6: the Wilkie-like framework ($\mathcal{M}_1$), the extended Wilkie-like framework ($\mathcal{M}_2$), the Ahlgrim, D'Arcy, and Gorvett-like framework ($\mathcal{M}_3$), and the new ESG ($\mathcal{M}_4$).

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Table SM.1: **Parameters Statistics for the Four Models (1986–2020), continued.**

Panel D: Term Structure, continued.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$a_{f_1,2}$							−0.202	(0.022)
$a_{f_2,2}$							−0.633	(0.025)
$a_{f_3,2}$							−0.701	(0.021)
$a_{f_5,2}$							−0.641	(0.022)
$a_{f_7,2}$							−0.509	(0.024)
$a_{f_{10},2}$							−0.298	(0.026)
$a_{f_{30},2}$							−0.202	(0.020)
$\sigma_{f_1} \times 100$	0.463	(0.016)	0.449	(0.015)	0.449	(0.015)	0.410	(0.014)
$\sigma_{f_2} \times 100$	0.826	(0.027)	0.744	(0.025)	0.744	(0.025)	0.474	(0.015)
$\sigma_{f_3} \times 100$	1.016	(0.034)	0.756	(0.025)	0.758	(0.025)	0.410	(0.014)
$\sigma_{f_5} \times 100$	1.328	(0.044)	0.724	(0.025)	0.724	(0.024)	0.432	(0.014)
$\sigma_{f_7} \times 100$	1.490	(0.048)	0.655	(0.023)	0.656	(0.022)	0.463	(0.016)
$\sigma_{f_{10}} \times 100$	1.606	(0.054)	0.577	(0.019)	0.578	(0.020)	0.511	(0.017)
$\sigma_{f_{30}} \times 100$	1.601	(0.053)	0.449	(0.015)	0.449	(0.015)	0.411	(0.014)
μ_{F_1}			2.057	(2.771)	2.057	(2.771)	2.057	(2.771)
μ_{F_2}							0.245	(0.251)
a_{F_1}			0.988	(0.008)	0.988	(0.008)	0.988	(0.008)
a_{F_2}							0.921	(0.019)
σ_{F_1}			0.259	(0.009)	0.259	(0.009)	0.259	(0.009)
σ_{F_2}							0.375	(0.012)
Panel E: Dividend Yield.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$\mu_{d,\text{u}} \times 100$							3.651	(2.058)
$\mu_{d,\text{s}} \times 100 \,/\, \mu_d \times 100$	1.809	(0.389)	1.809	(0.389)	1.809	(0.389)	1.594	(0.522)
$\mu_{d,3} \times 100$							3.238	(2.876)
a_d	0.988	(0.005)	0.988	(0.005)	0.988	(0.005)	0.991	(0.004)
$\rho_{q,d}$	−0.129	(0.046)	−0.129	(0.046)	−0.129	(0.046)	−0.113	(0.049)
$\rho_{r,d}$	−0.102	(0.047)	−0.102	(0.047)	−0.102	(0.047)	−0.056	(0.047)
$\sigma_d \times 100$	4.436	(0.150)	4.436	(0.150)	4.436	(0.150)	6.323	(1.907)
α_d							0.146	(0.037)
β_d							0.811	(0.058)
γ_d							−0.143	(0.193)
Panel F: Stock Index Returns.								
	$\mathcal{M}_1$		$\mathcal{M}_2$		$\mathcal{M}_3$		$\mathcal{M}_4$	
$\mu_{y,\text{u}} \times 100$					0.486	(0.359)	0.104	(0.337)
$\mu_{y,\text{s}} \times 100 \,/\, \mu_y \times 100$	0.511	(0.200)	0.511	(0.200)	0.637	(0.264)	0.674	(0.217)
$\mu_{y,\text{d}} \times 100$					−0.150	(0.517)	0.157	(0.473)
$\sigma_{y,\text{u}} \times 100$					3.446	(0.257)		
$\sigma_{y,\text{s}} \times 100 \,/\, \sigma_y \times 100$	4.234	(0.143)	4.234	(0.143)	4.440	(0.191)	4.773	(0.623)
$\sigma_{y,\text{d}} \times 100$					4.516	(0.380)		
α_y							0.223	(0.079)
β_y							0.176	(0.112)
γ_y							1.197	(0.358)

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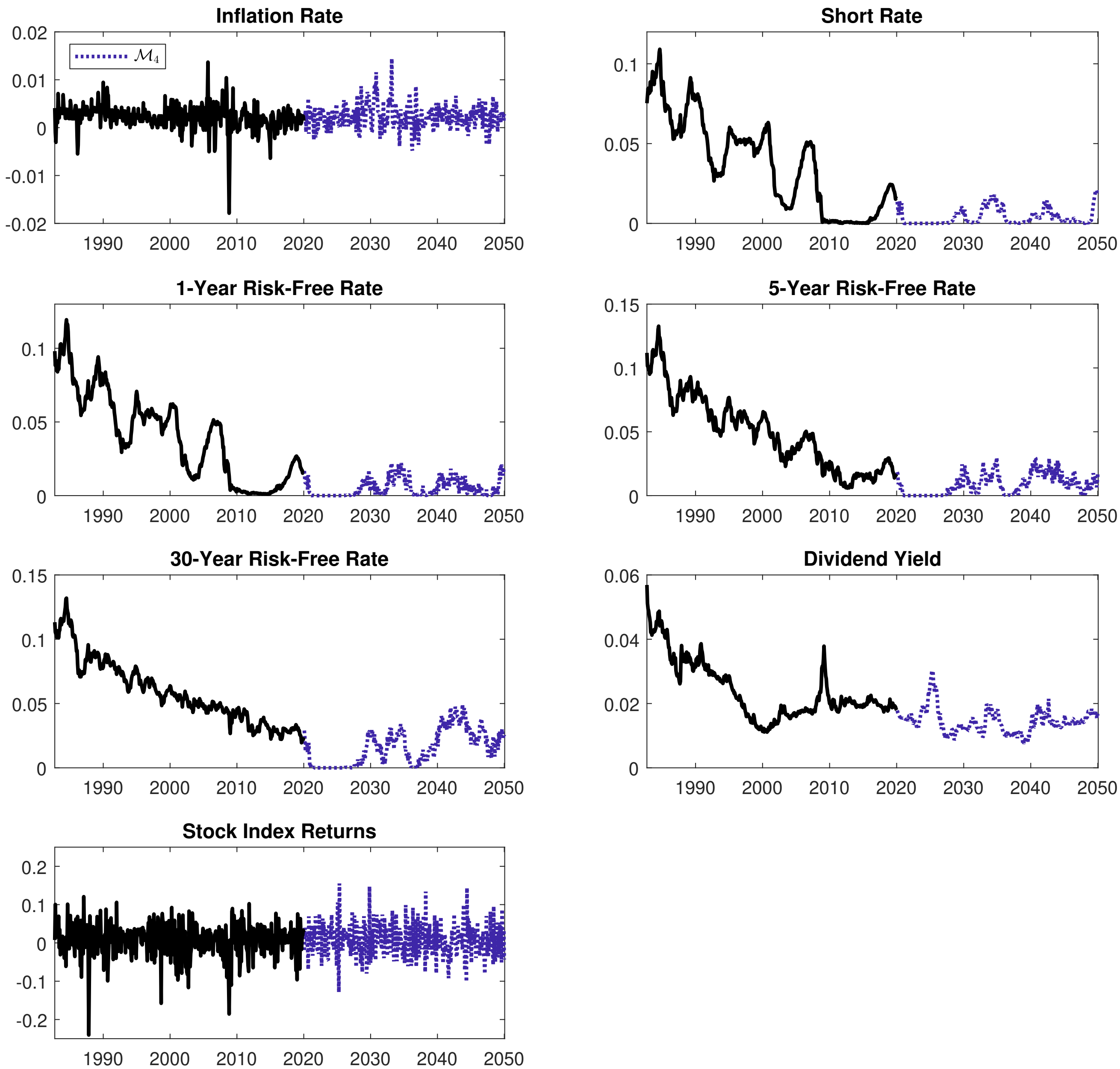

Figure SM.8: **Example of Simulated Series for the New Economic Scenario Generator.**
This figure shows one scenario generated from our new ESG.

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Table SM.2: **Out-of-Sample Forecast Root-Mean-Square Errors and Coverage Errors Over Different Horizons for Ensemble Model.**

Panel A: Out-of-Sample Forecast Root-Mean-Square Errors.

	1-month	3-month	6-month	1-year	2-year	3-year	4-year	5-year	Average
Inflation	0.257	0.612	0.936	1.426	2.350	3.252	4.256	5.294	2.298
Short Rate	0.202	0.405	0.691	1.233	2.014	2.444	2.599	2.566	1.519
Dividend Yield	0.092	0.171	0.265	0.365	0.448	0.446	0.455	0.480	0.340
Stock Index Returns	4.298	7.737	11.763	17.916	28.648	36.270	42.635	48.316	24.698
1-Year Risk-Free Interest Rate	0.354	0.545	0.803	1.278	2.011	2.448	2.629	2.636	1.588
2-Year Risk-Free Interest Rate	0.435	0.608	0.827	1.224	1.874	2.281	2.461	2.513	1.528
3-Year Risk-Free Interest Rate	0.497	0.640	0.820	1.148	1.721	2.090	2.274	2.360	1.444
5-Year Risk-Free Interest Rate	0.714	0.763	0.858	1.041	1.448	1.722	1.896	2.025	1.308
7-Year Risk-Free Interest Rate	0.929	0.934	0.972	1.055	1.302	1.483	1.632	1.781	1.261
10-Year Risk-Free Interest Rate	1.156	1.126	1.121	1.129	1.241	1.315	1.418	1.548	1.257
30-Year Risk-Free Interest Rate	1.734	1.639	1.555	1.453	1.347	1.220	1.177	1.233	1.420

Panel B: Out-of-Sample Coverage Errors.

	1-month	3-month	6-month	1-year	2-year	3-year	4-year	5-year	Average
Inflation	0.016	0.020	0.014	0.069	0.160	0.247	0.335*	0.428*	0.161
Short Rate	0.106	0.075	0.029	0.013	0.132	0.152	0.109	0.001	0.077
Dividend Yield	0.013	0.002	0.001	0.005	0.046	0.013	0.057	0.059	0.024
Stock Index Returns	0.031	0.009	0.012	0.032	0.069	0.098	0.078	0.074	0.051
1-Year Risk-Free Interest Rate	0.035	0.042	0.015	0.013	0.124	0.102	0.100	0.019	0.056
2-Year Risk-Free Interest Rate	0.035	0.024	0.026	0.009	0.085	0.086	0.083	0.024	0.046
3-Year Risk-Free Interest Rate	0.024	0.005	0.034	0.010	0.054	0.078	0.048	0.008	0.033
5-Year Risk-Free Interest Rate	0.028	0.035	0.056	0.051	0.029	0.009	0.065	0.036	0.039
7-Year Risk-Free Interest Rate	0.021	0.031	0.060	0.070*	0.057	0.075	0.100	0.082*	0.062
10-Year Risk-Free Interest Rate	0.013	0.038	0.071*	0.089*	0.080	0.100	0.100	0.100	0.074
30-Year Risk-Free Interest Rate	0.039	0.064*	0.089*	0.089*	0.100	0.100	0.100	0.100	0.085

This table reports ensemble model out-of-sample forecast RMSEs (Panel A) and coverage errors (Panel B) for all series and horizons considered in Tables 5 and 6. We use simple unweighted averages in lieu of sophisticated combination methods to create better forecasts and models. An asterisk indicates a coverage error that is statistically different from zero at a significance level of 5%. The significance is assessed with robust Newey-West t-statistics as given in Equation (38) of Engle et al. (2017). The RMSEs are multiplied by 100.

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