

A Bayesian joint model for population and portfolio-specific mortality

Online appendix

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Abstract

In this online appendix we present convergence diagnostics for the mortality models discussed in [van Berkum et al. \(2017\)](#).

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1 England & Wales population: POP(B)

Parameter estimates for POP(f) and POP(B).

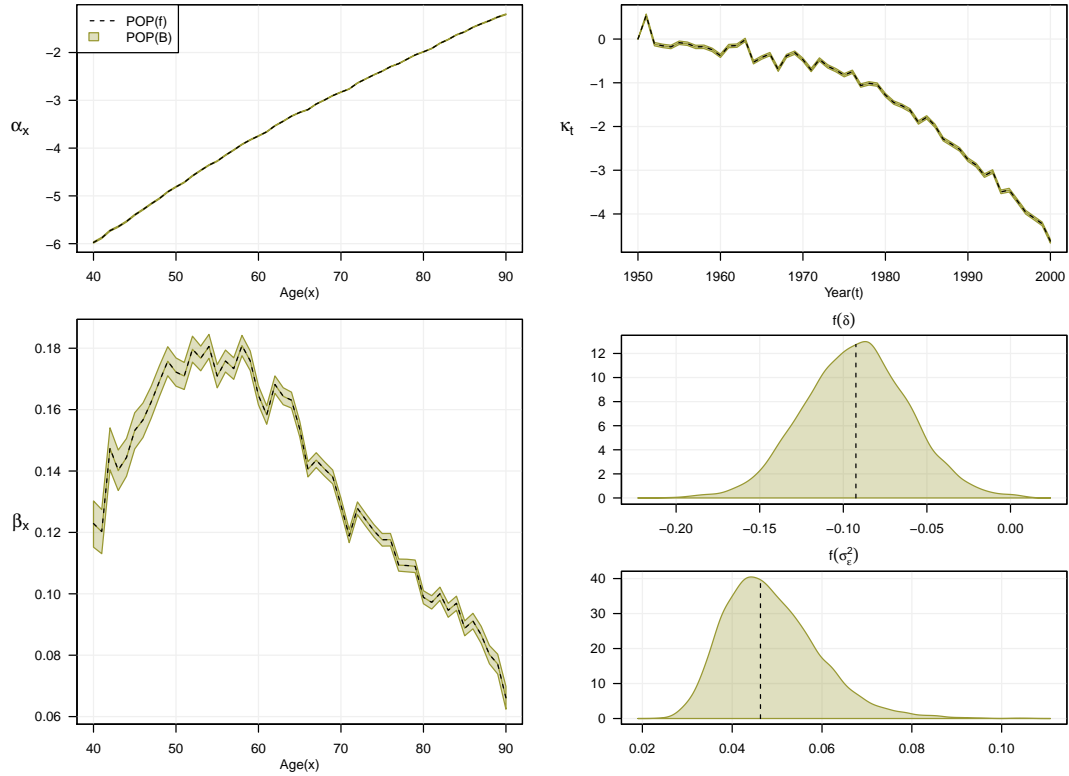


Figure 1: Parameter estimates for α_x , β_x , κ_t , δ and σ_ϵ^2 using the England & Wales population. (Colored versions of the figures can be found online.)

Convergence diagnostics for α_x in POP(B).

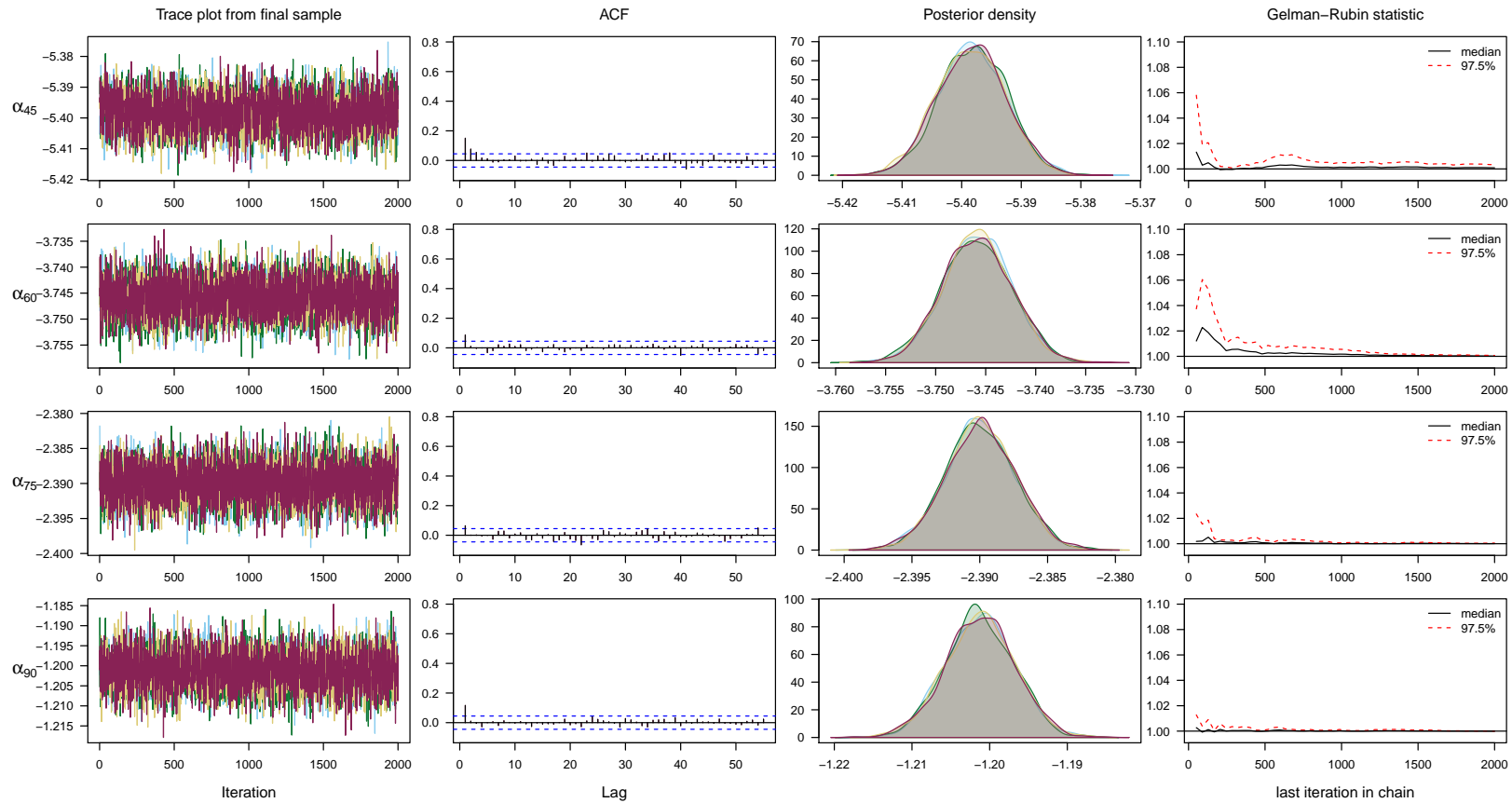


Figure 2: Convergence diagnostics for α_x for selected x . First column: traceplot for the final sample from the MCMC procedure. Second column: autocorrelation function for the final sample from the first chain. Third column: density plots from the final sample for all four chains. Fourth column: Gelman-Rubin statistic showing the convergence between the different chains, see [Gelman and Rubin \(1992\)](#) for more information.

Convergence diagnostics for β_x in POP(B).

4

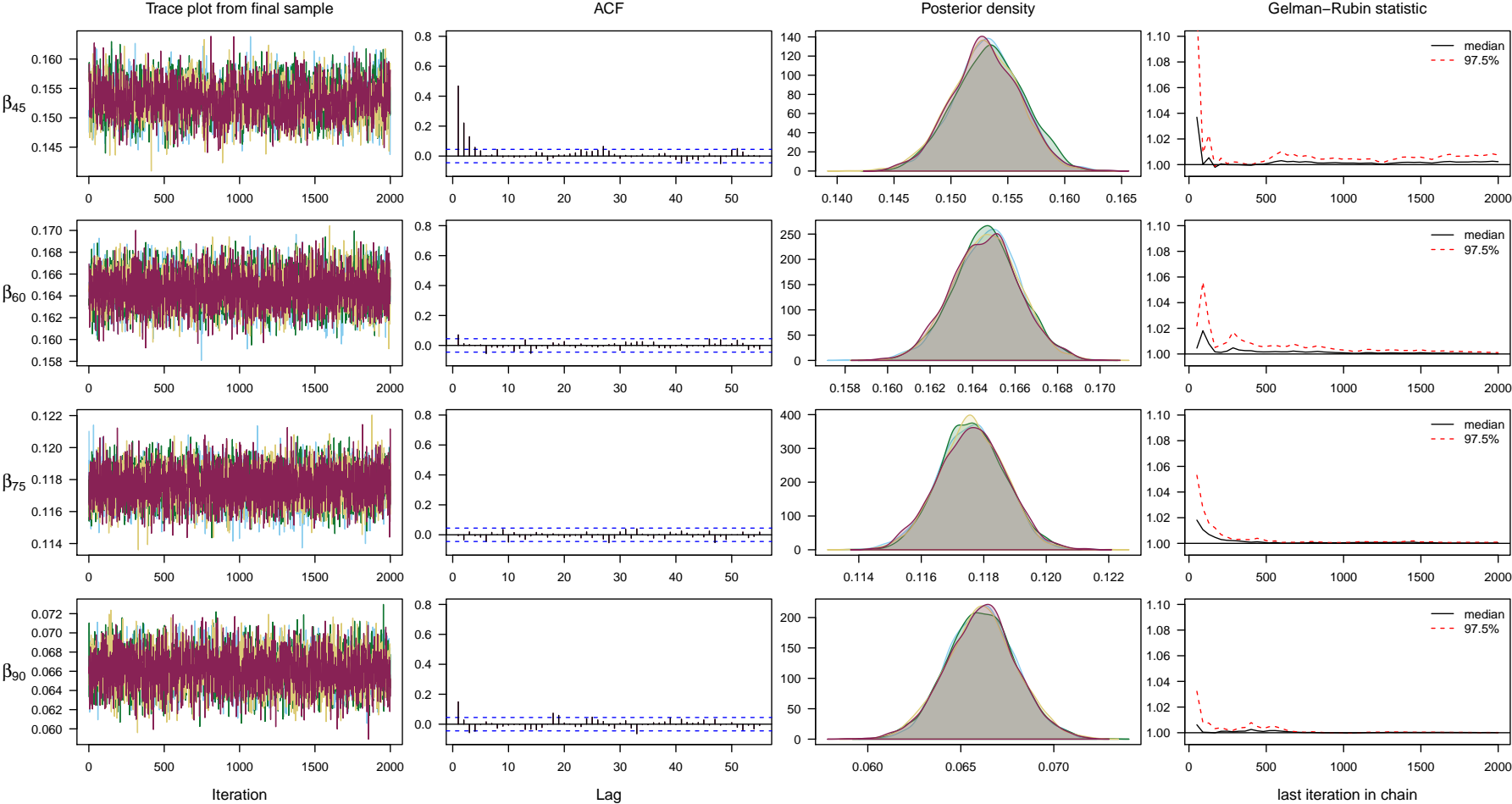


Figure 3: For comments: see Figure 2.

Convergence diagnostics for κ_t in POP(B).

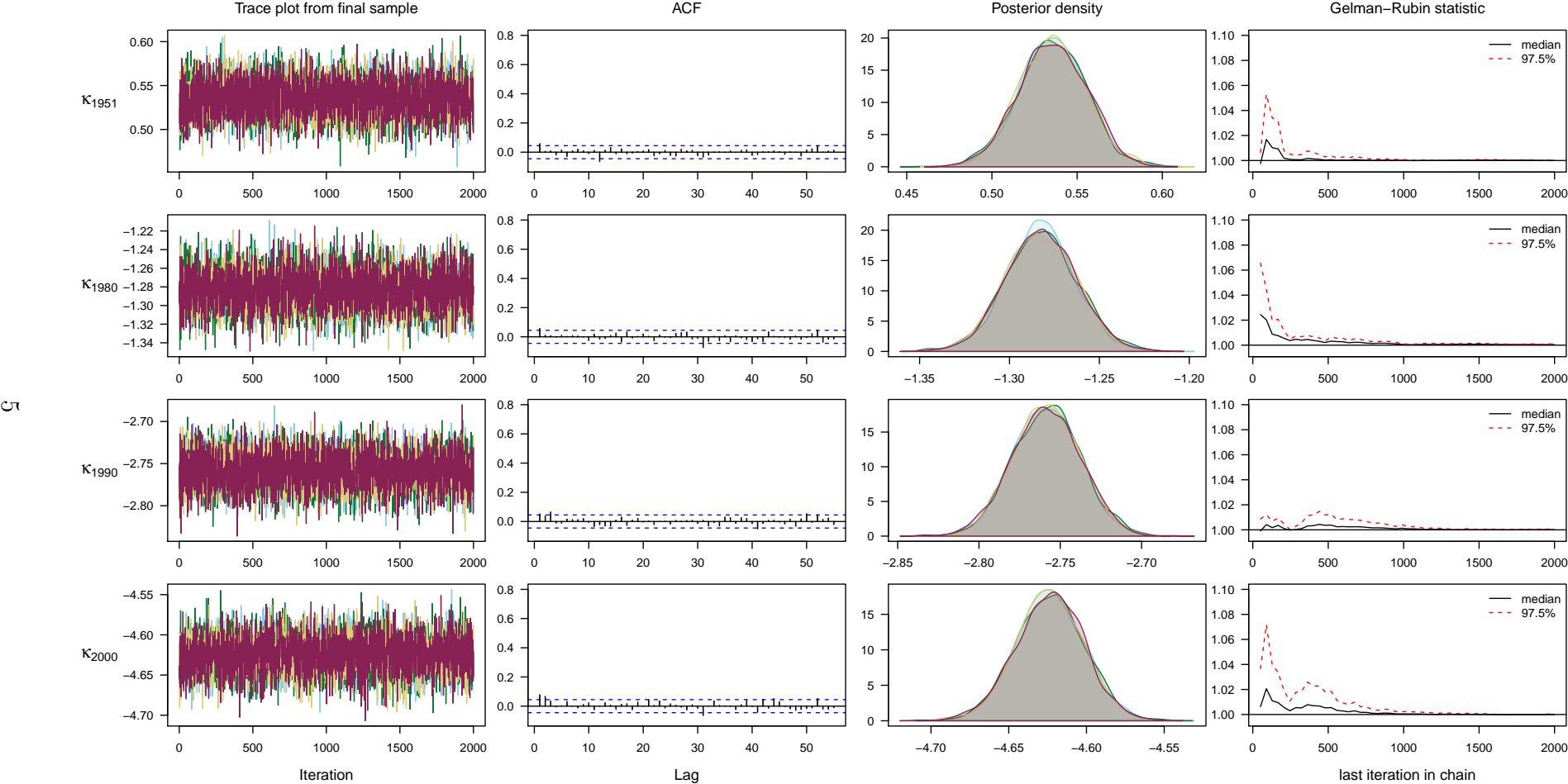


Figure 4: For comments: see Figure 2.

Convergence diagnostics for δ and σ_ε^2 in POP(B).

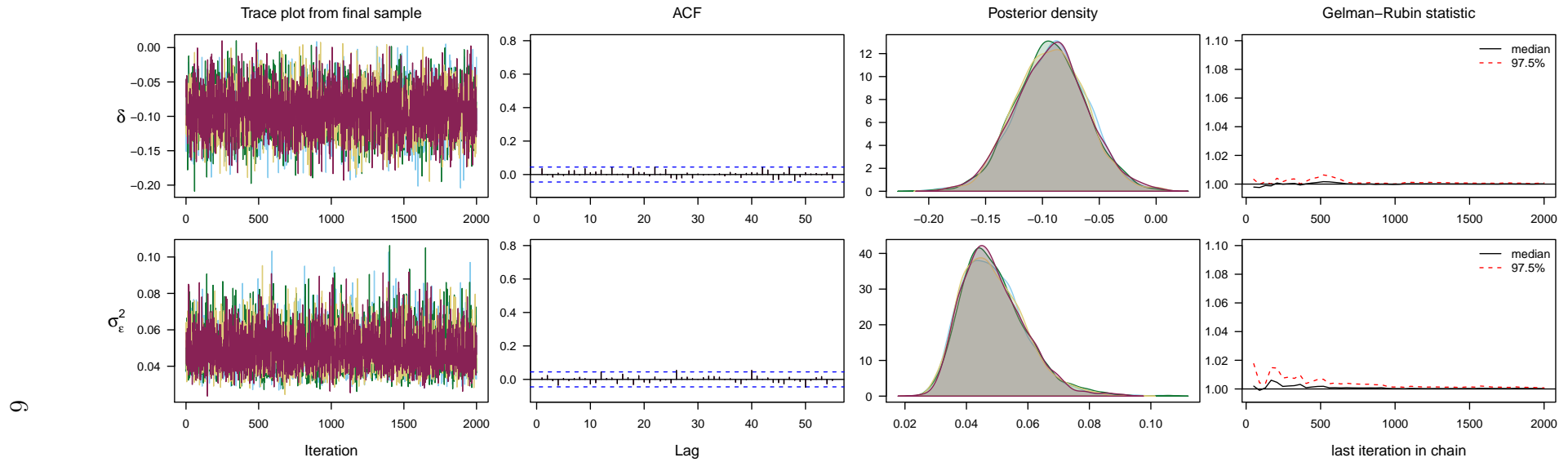


Figure 5: For comments: see Figure 2.

MH-sampling variances and acceptance probabilities in POP(B).

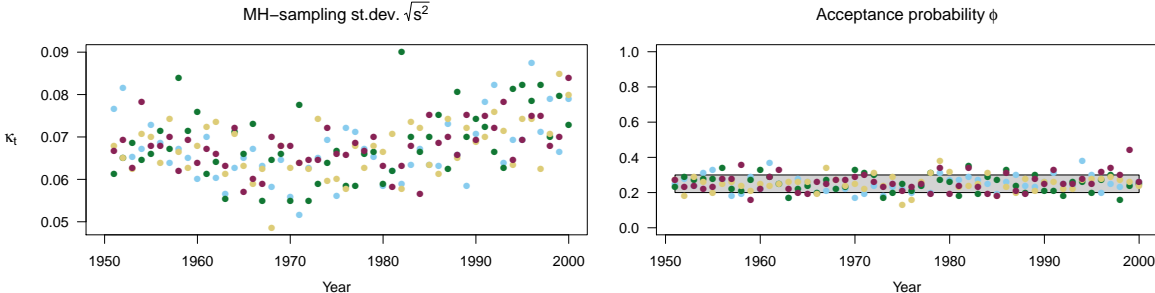


Figure 6: Metropolis(-Hastings) sampling variances used during the final sample phase and the acceptance probabilities from the last sample.

2 CMI original dataset: PF(B-G) and PF(B-logN)

Parameter estimates for PF(B-G) and PF(B-logN) (original portfolio size).

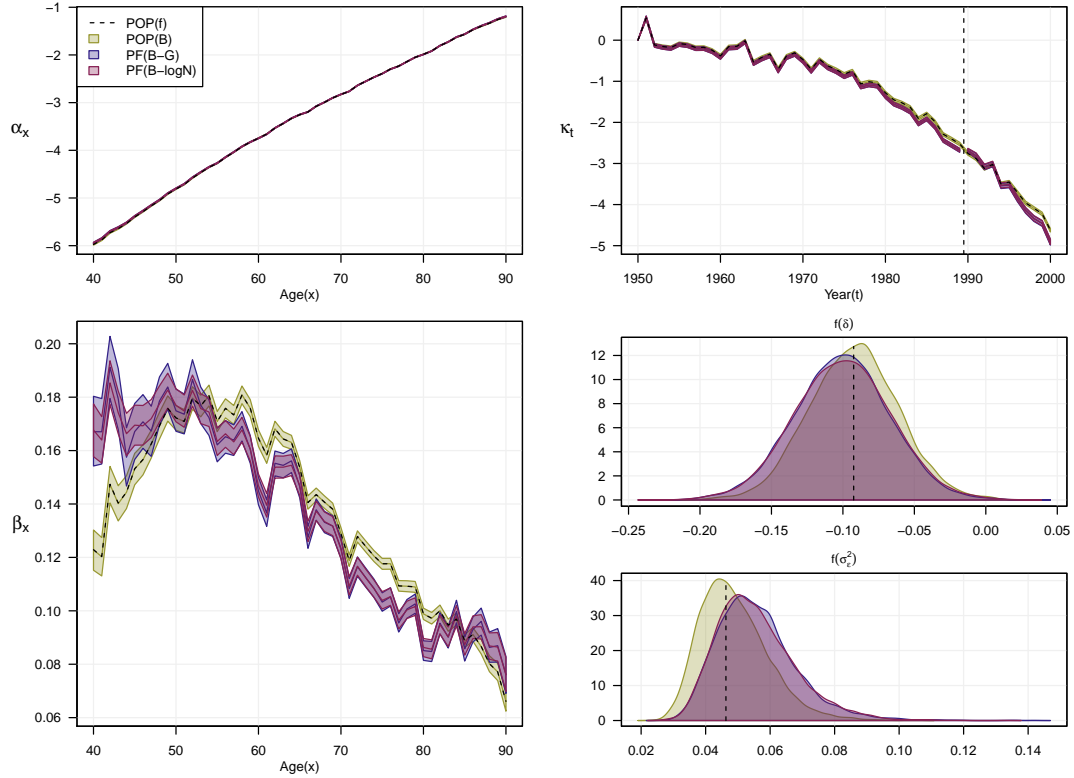


Figure 7: Parameter estimates for α_x , β_x , κ_t , δ and σ_ε^2 using the original CMI portfolio.

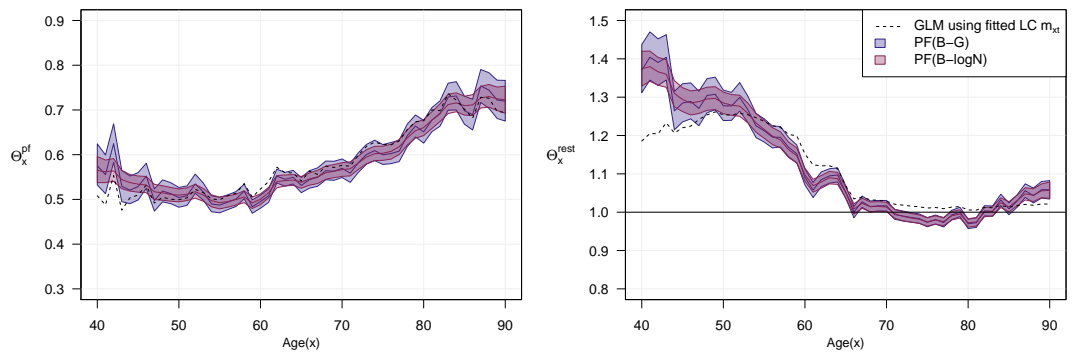


Figure 8: Parameter estimates for Θ_x^{pf} and Θ_x^{rest} using the original CMI portfolio.

Convergence diagnostics for α_x in PF(B-G) (original portfolio size).

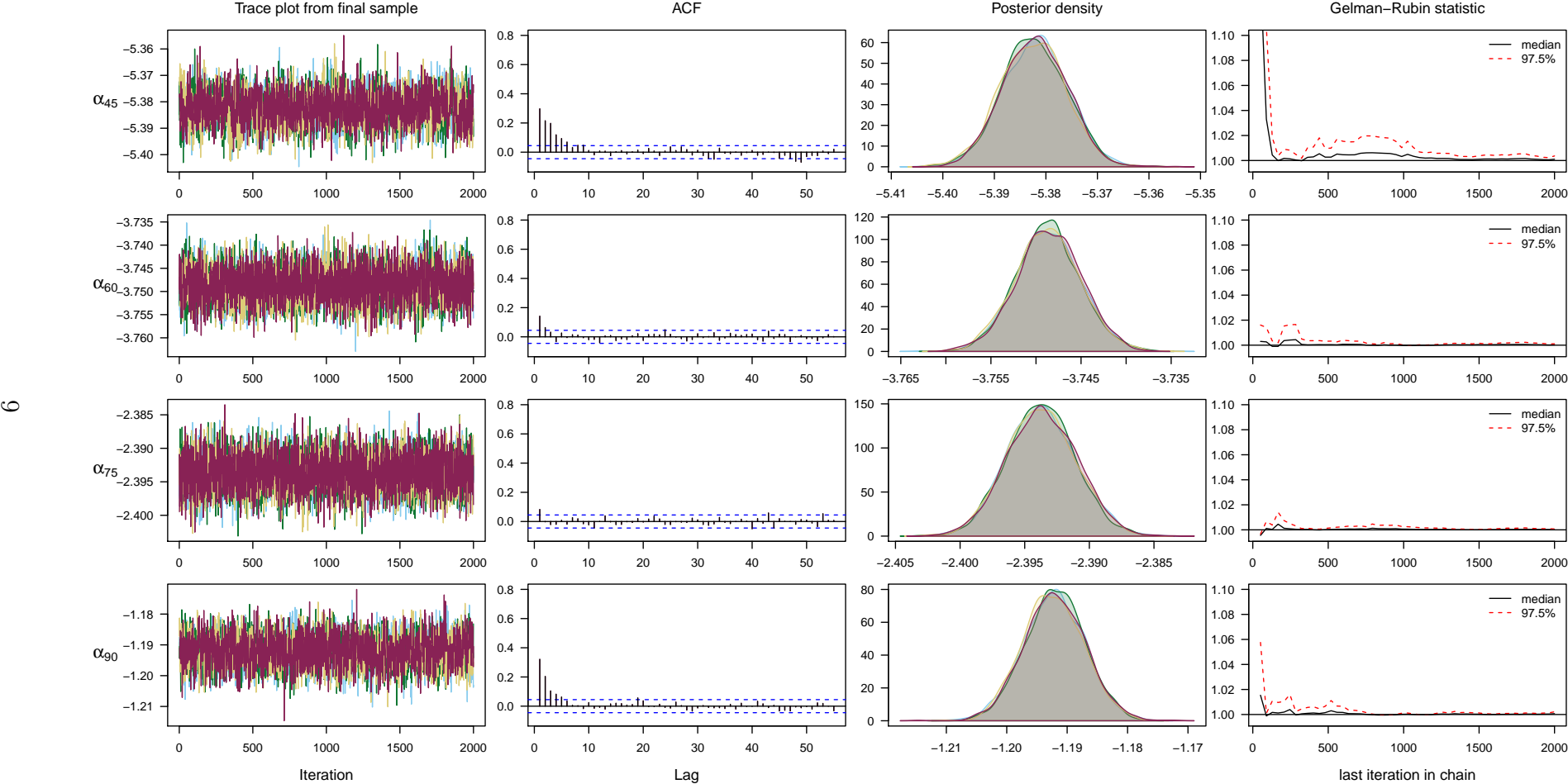


Figure 9: For comments: see Figure 2.

Convergence diagnostics for β_x in PF(B-G) (original portfolio size).

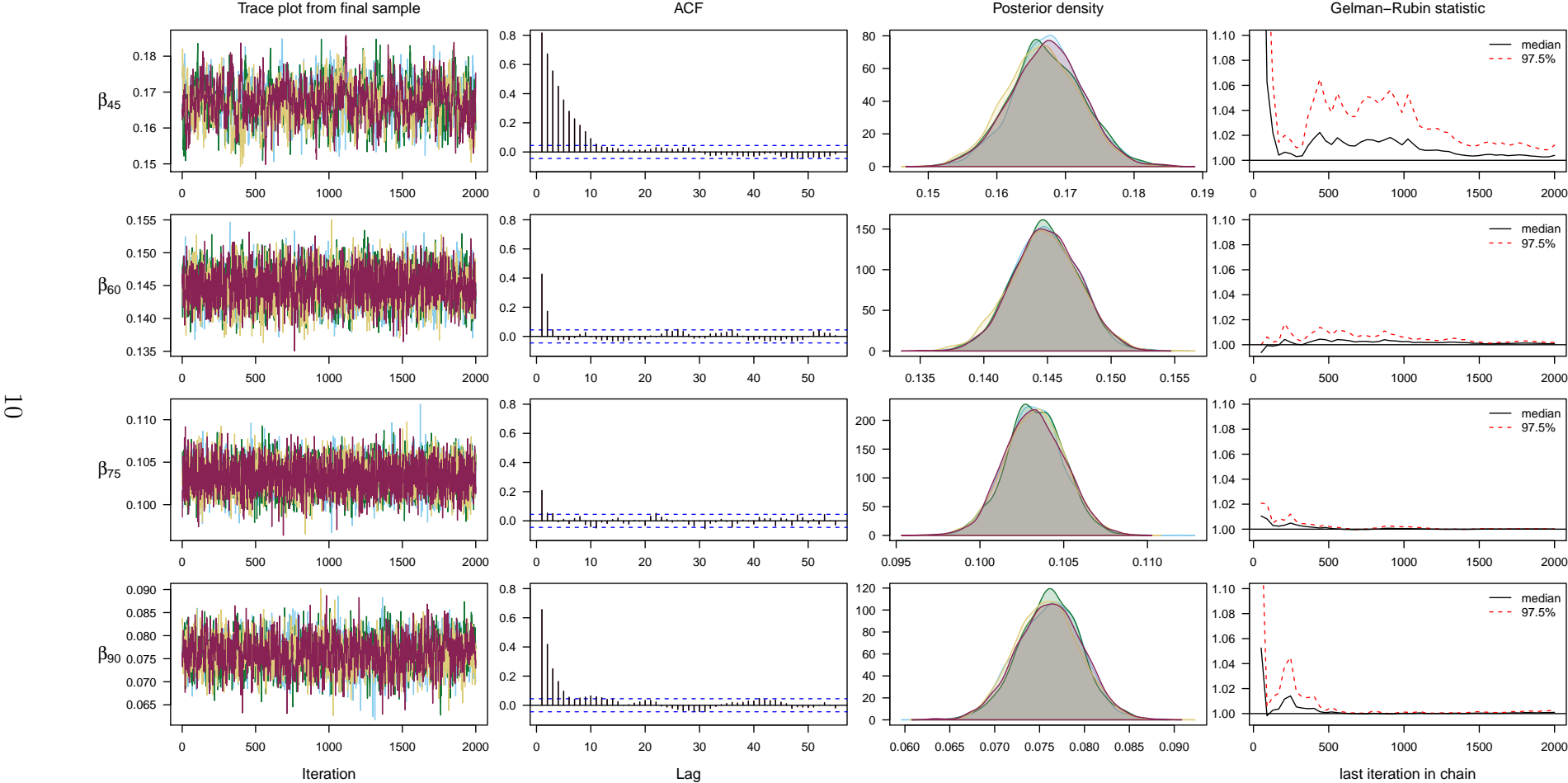
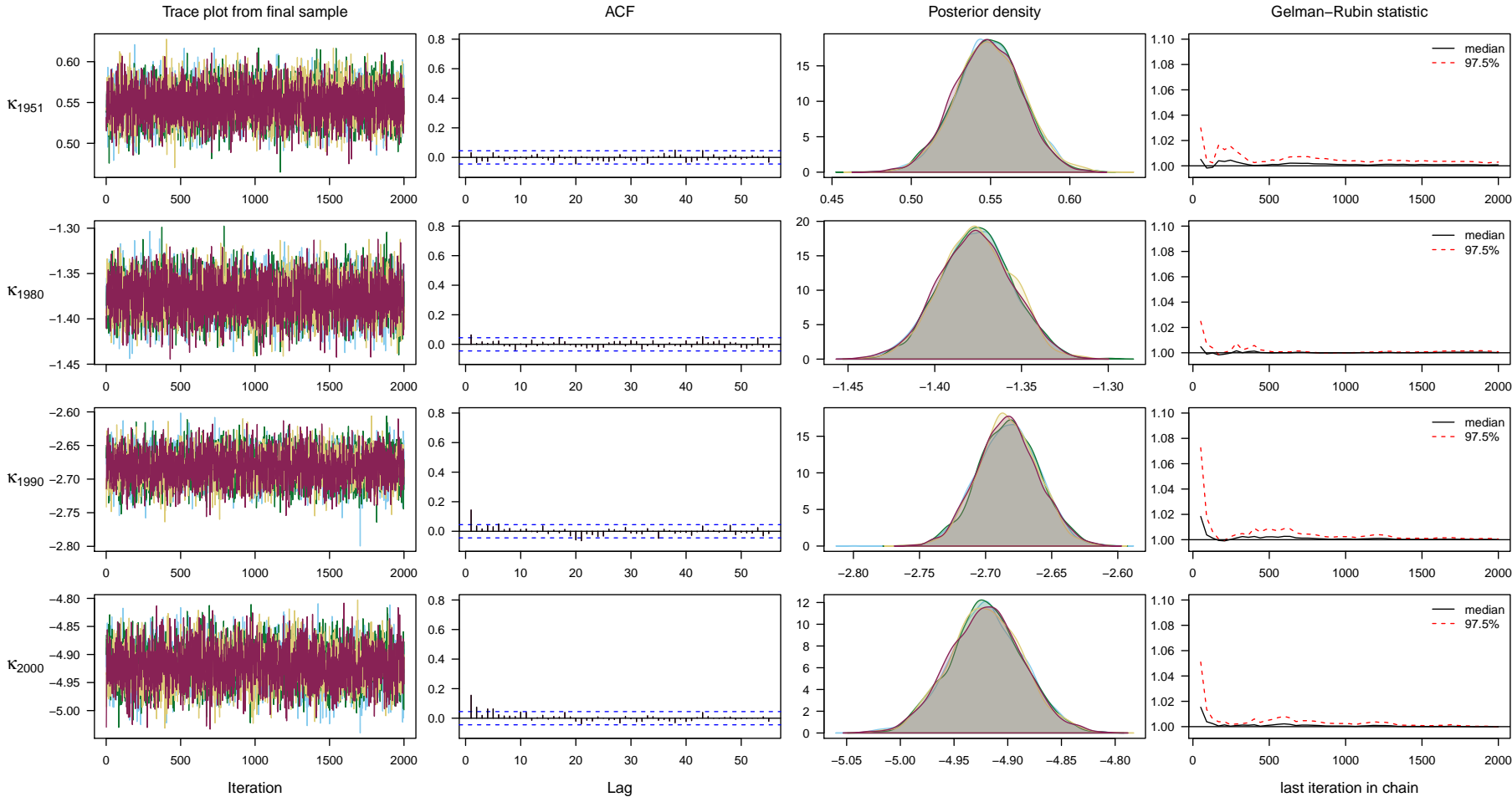


Figure 10: For comments: see Figure 2.

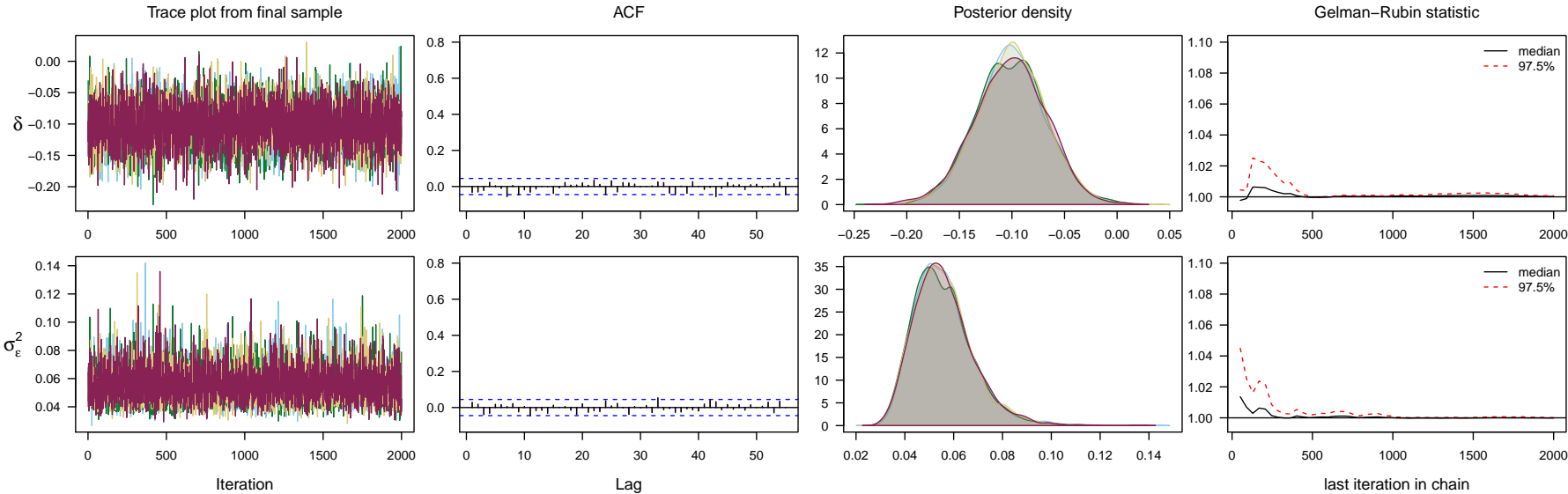
Convergence diagnostics for κ_t in PF(B-G) (original portfolio size).



11

Figure 11: For comments: see Figure 2.

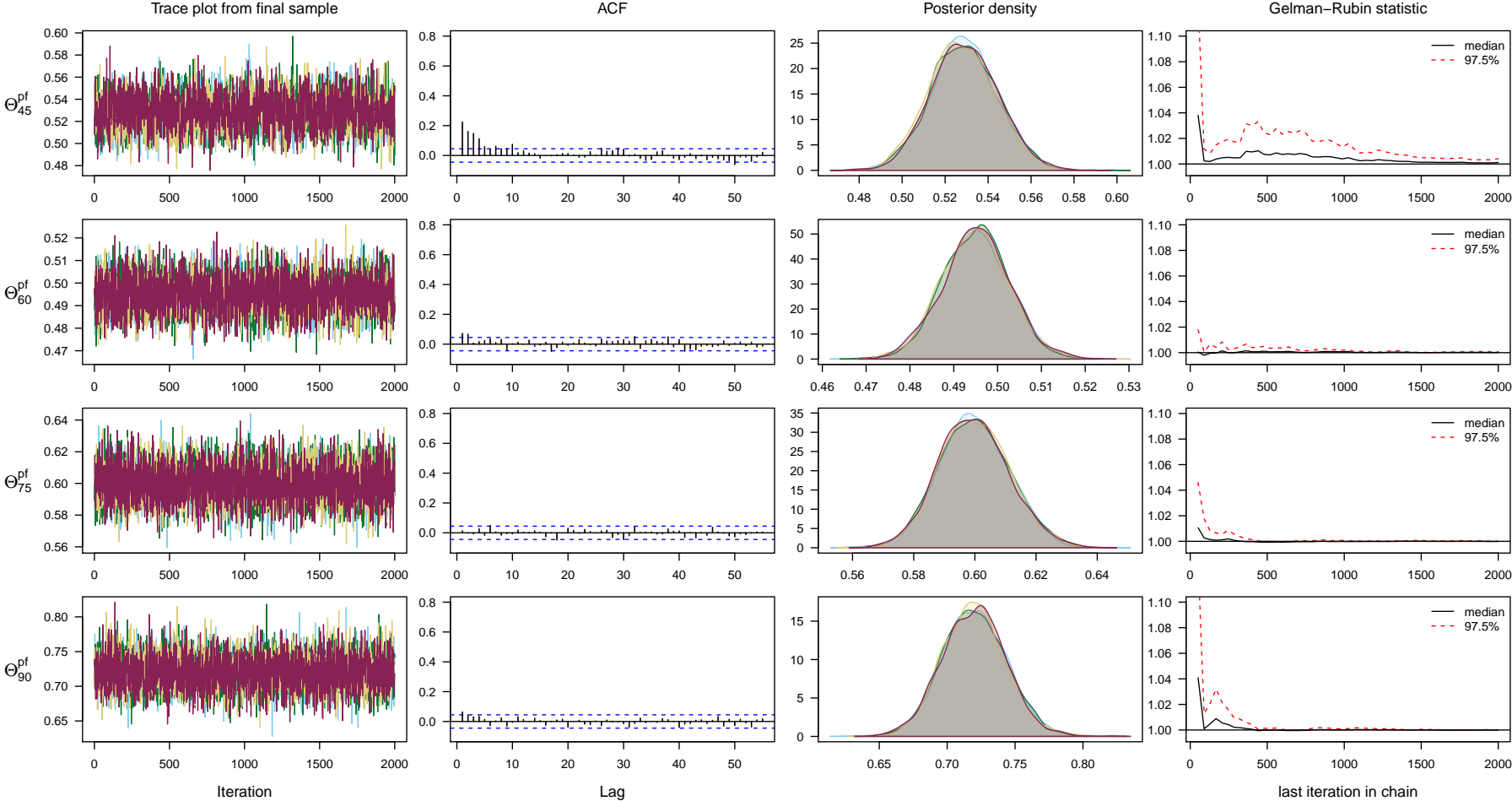
Convergence diagnostics for δ and σ_ε^2 in PF(B-G) (original portfolio size).



12

Figure 12: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{pf} in PF(B-G) (original portfolio size).



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Figure 13: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{rest} in PF(B-G) (original portfolio size).

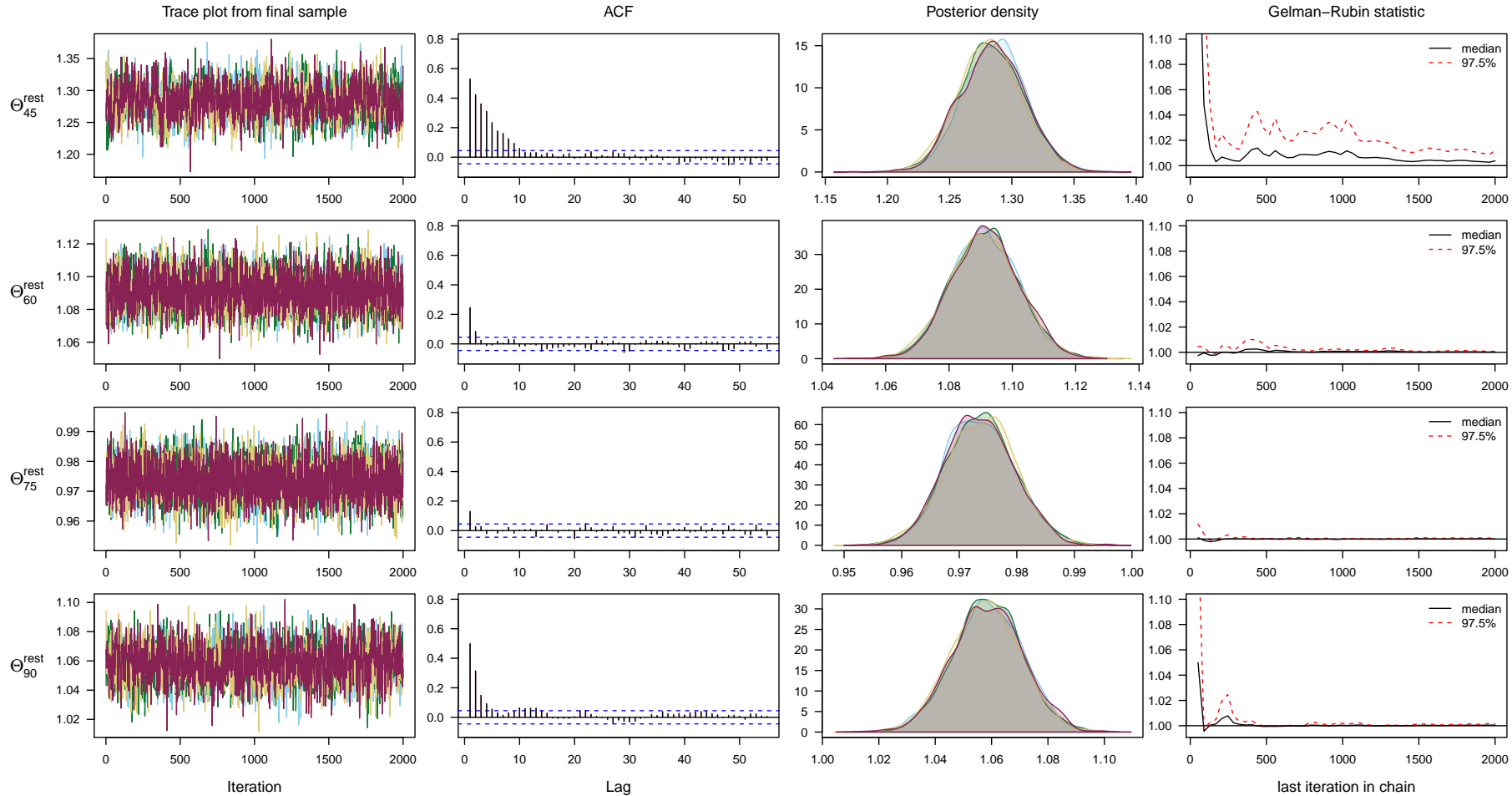


Figure 14: For comments: see Figure 2.

MH-sampling variances and acceptance probabilities in PF(B-G) (original portfolio size).

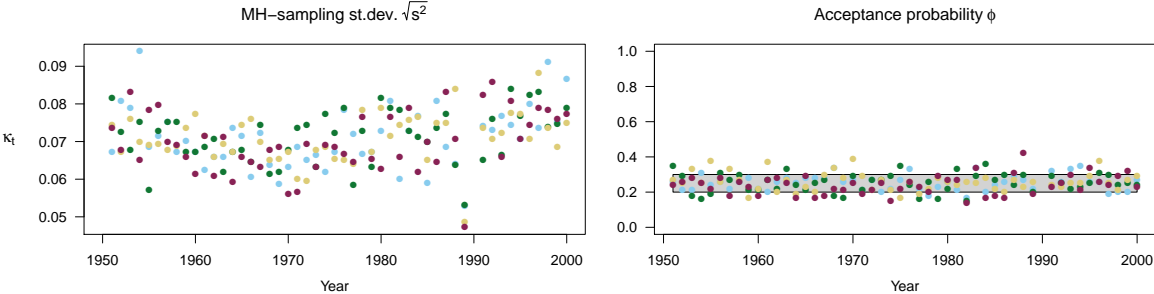


Figure 15: Metropolis(-Hastings) sampling variances used during the final sample phase and the acceptance probabilities from the last sample.

Convergence diagnostics for α_x in PF(B-logN) (original portfolio size).

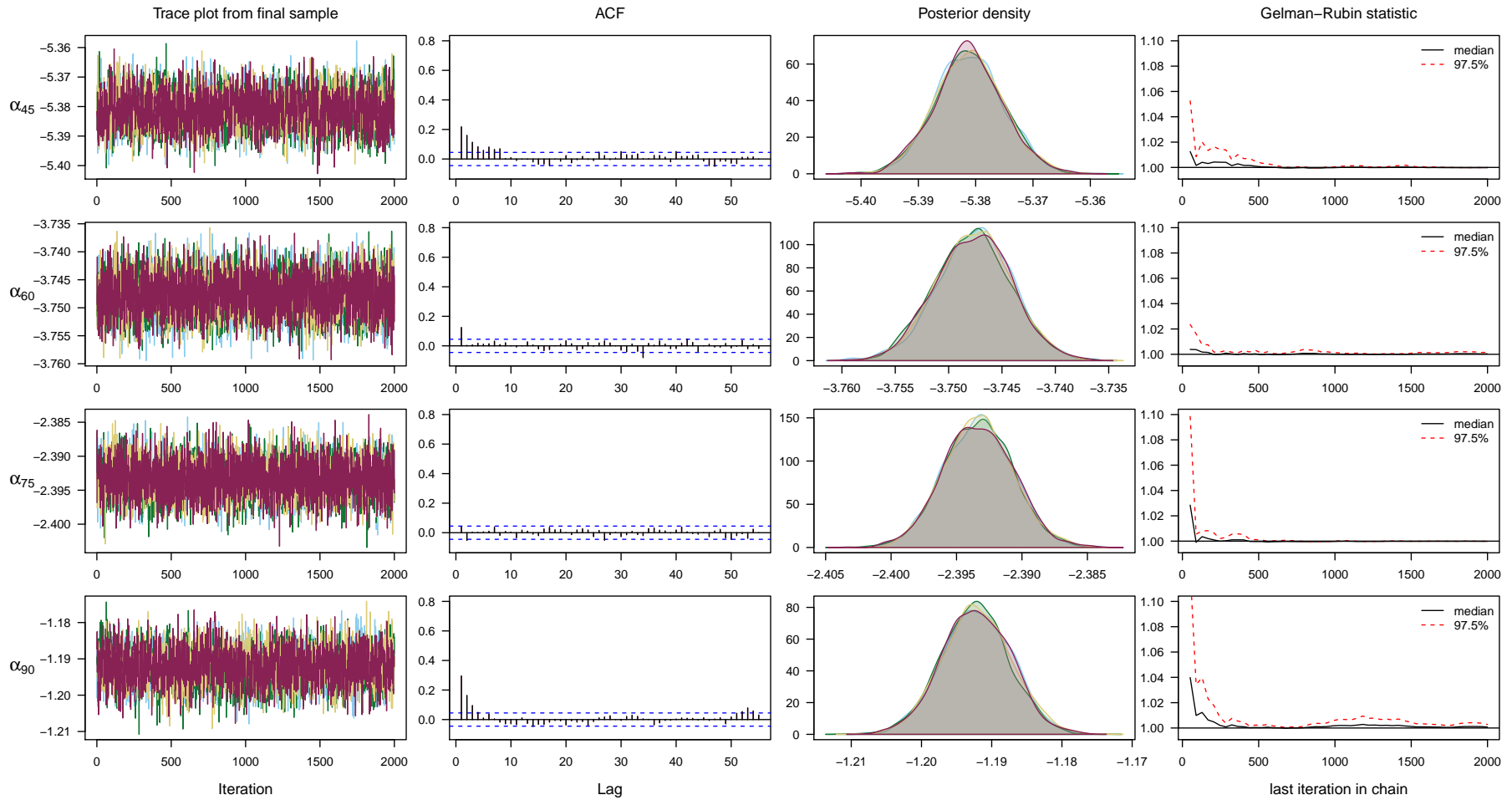


Figure 16: For comments: see Figure 2.

Convergence diagnostics for β_x in PF(B-logN) (original portfolio size).

17

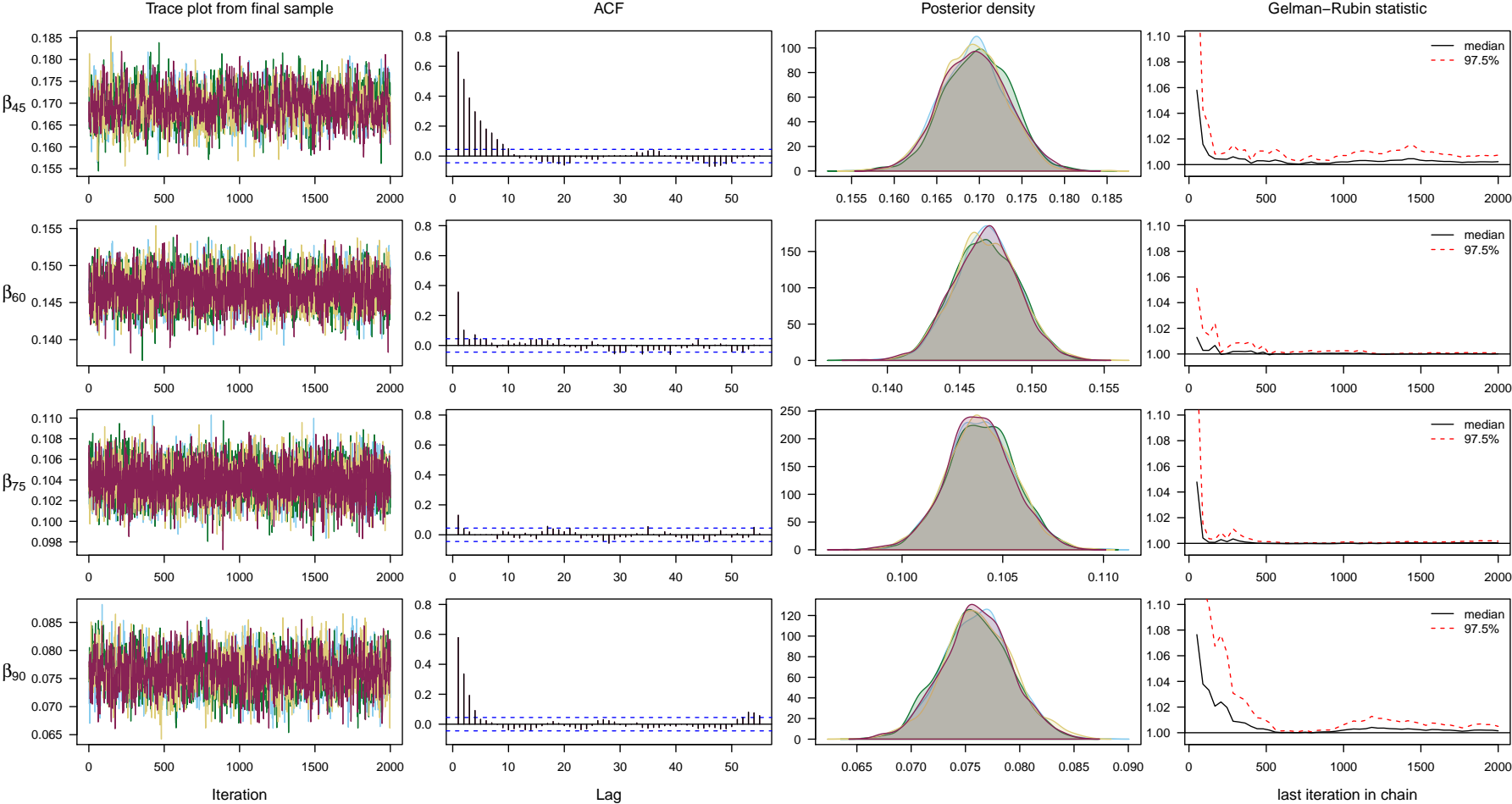


Figure 17: For comments: see Figure 2.

Convergence diagnostics for κ_t in PF(B-logN) (original portfolio size).

18

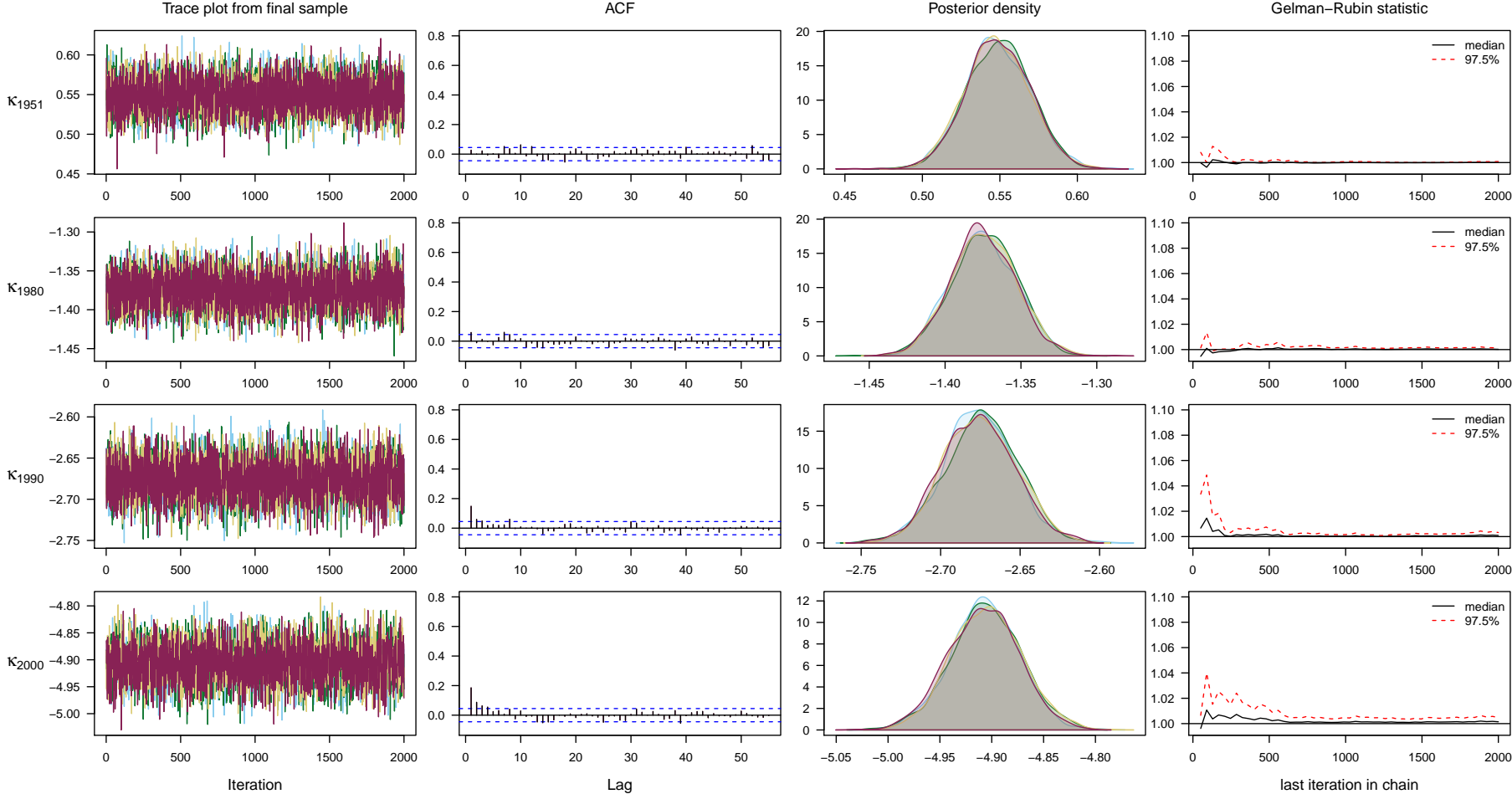


Figure 18: For comments: see Figure 2.

Convergence diagnostics for δ and σ_ε^2 in PF(B-logN) (original portfolio size).

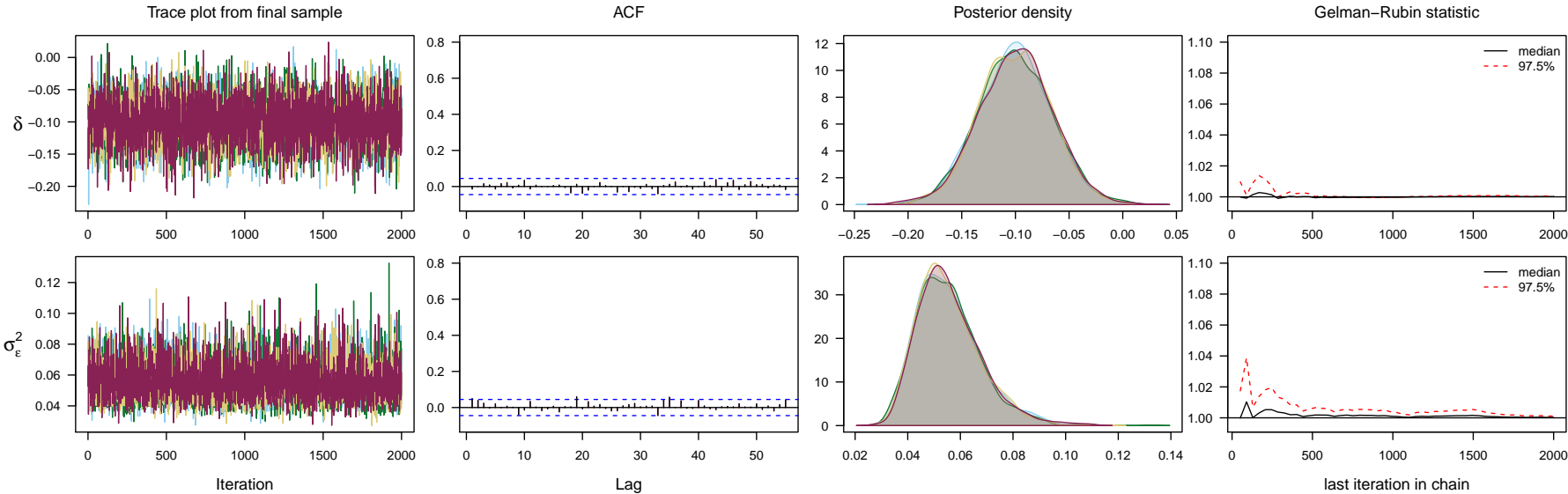
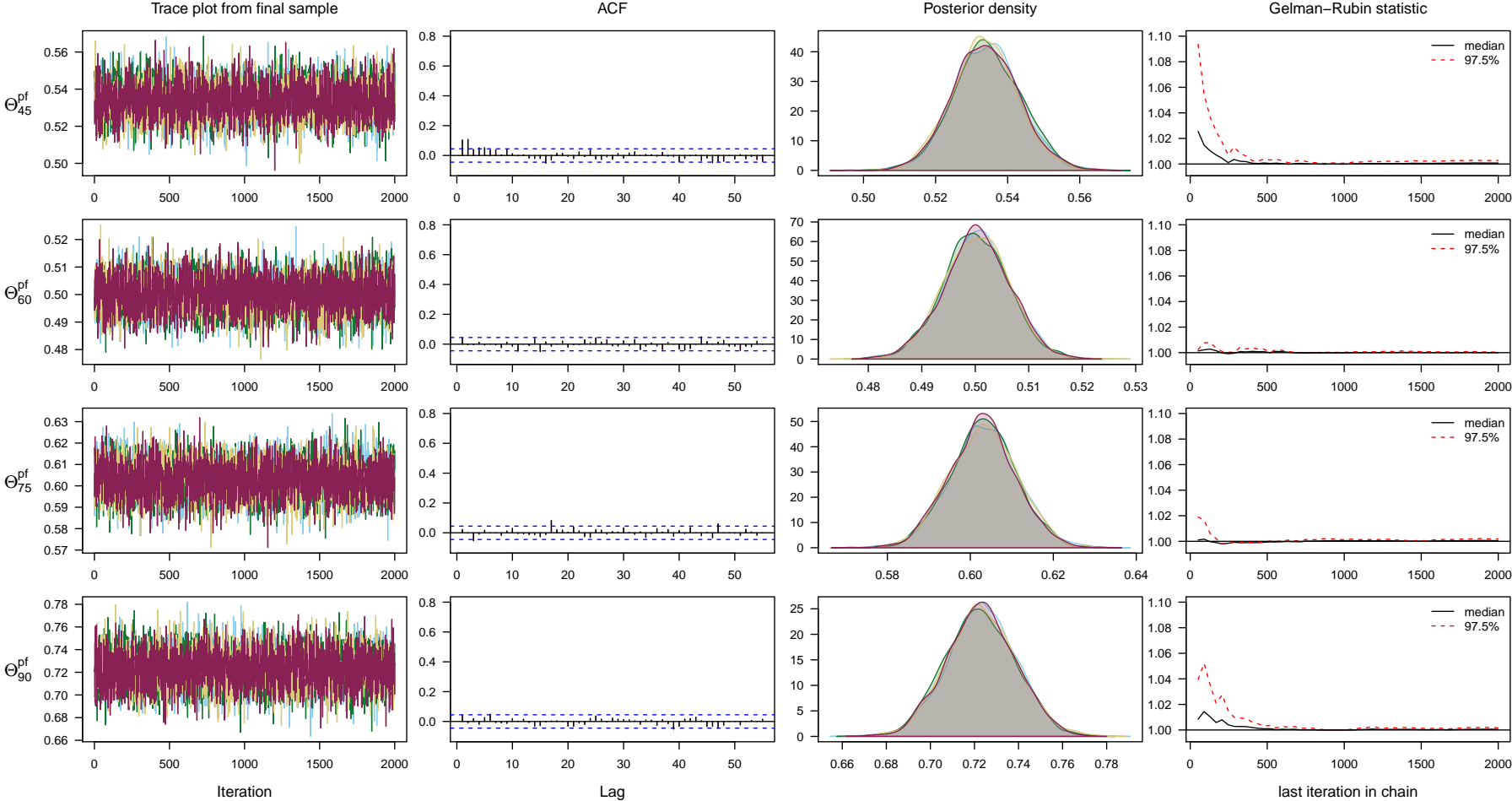


Figure 19: For comments: see Figure 2.

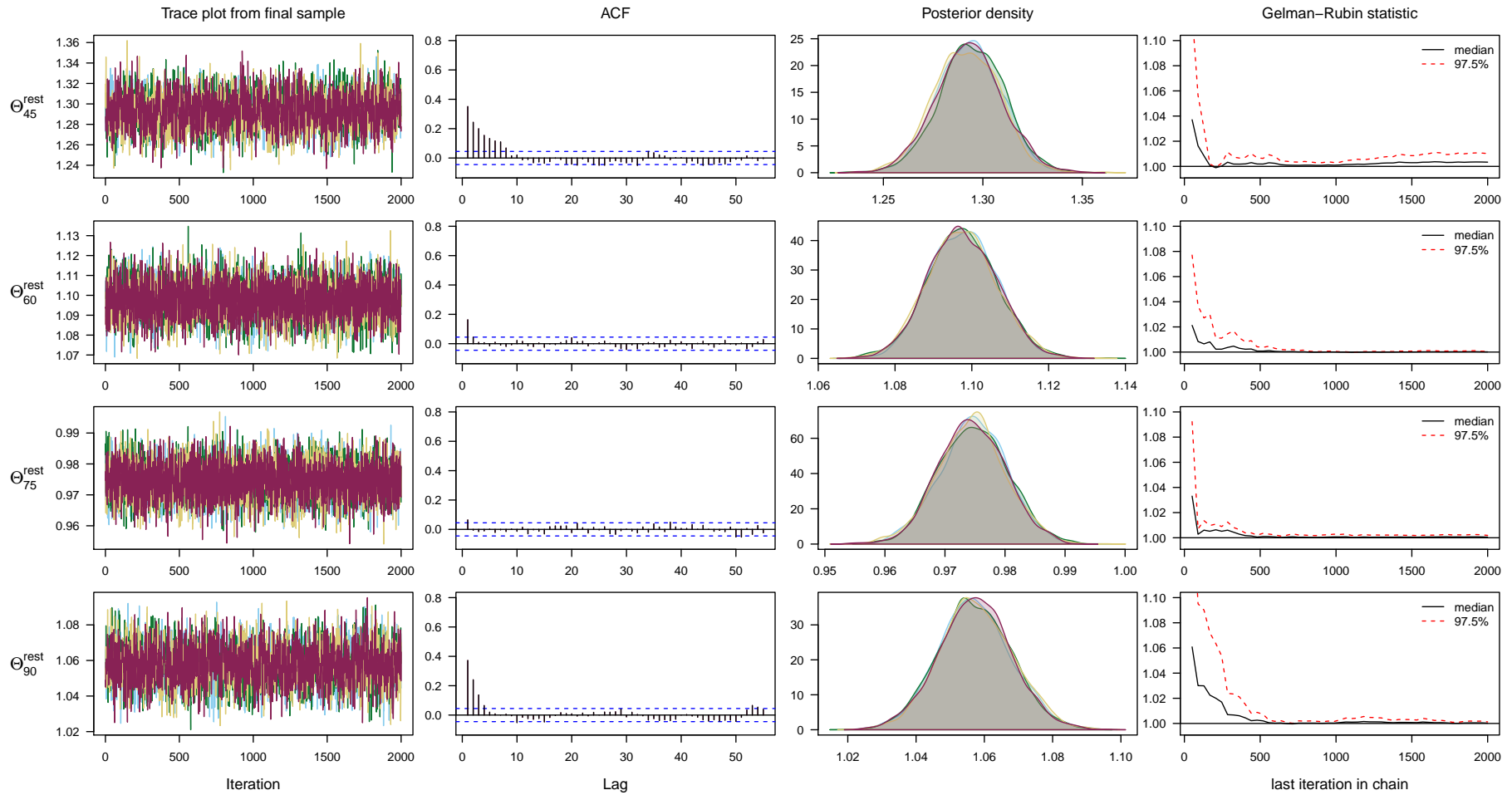
Convergence diagnostics for Θ_x^{pf} in PF(B-logN) (original portfolio size).



20

Figure 20: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{rest} in PF(B-logN) (original portfolio size).



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Figure 21: For comments: see Figure 2.

Convergence diagnostics for $\sigma_{\Theta^i}^2$ and ρ_{Θ^i} in PF(B-logN) (original portfolio size).

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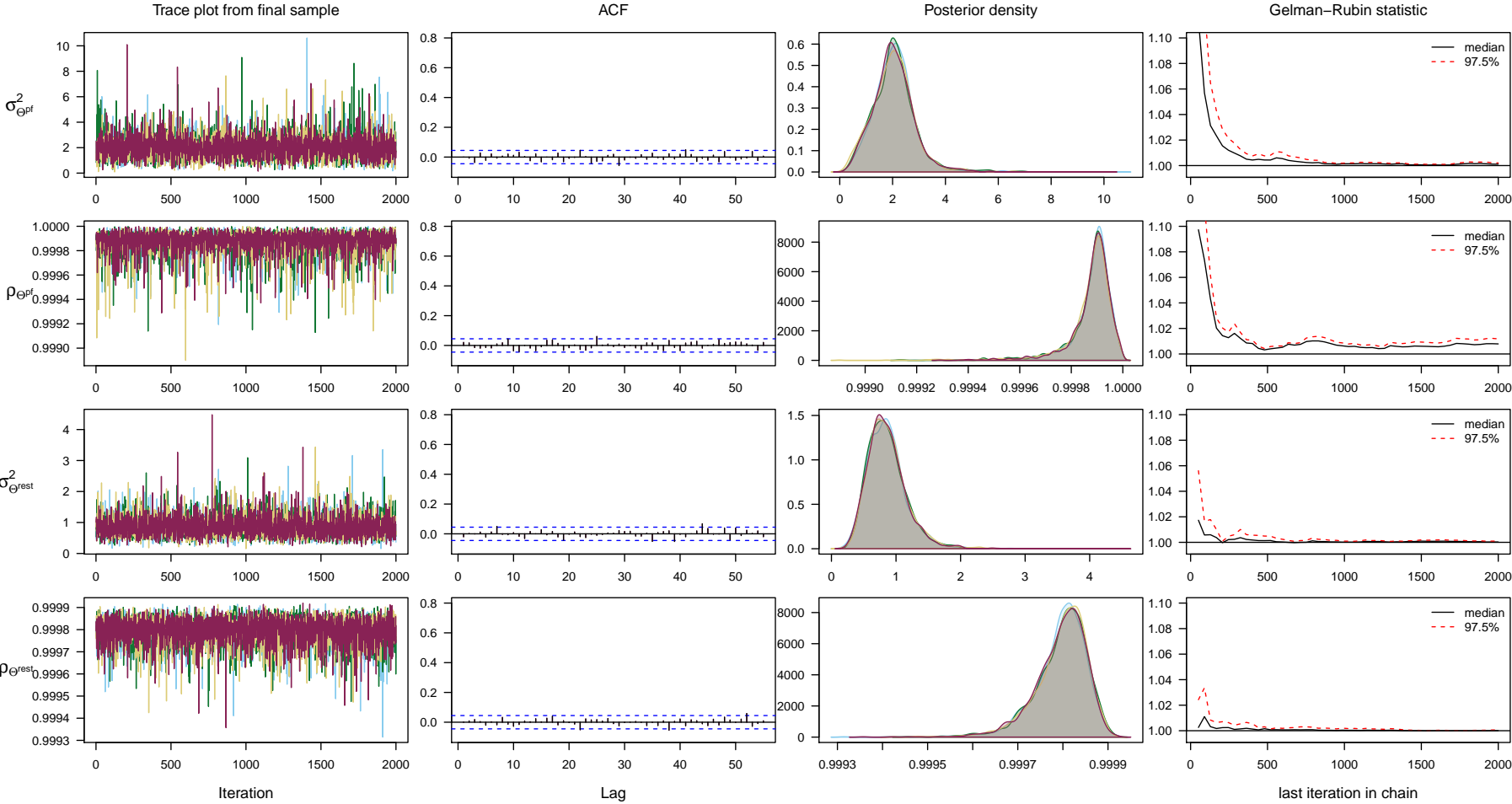


Figure 22: For comments: see Figure 2.

MH-sampling variances and acceptance probabilities in PF(B-logN) (original portfolio size).

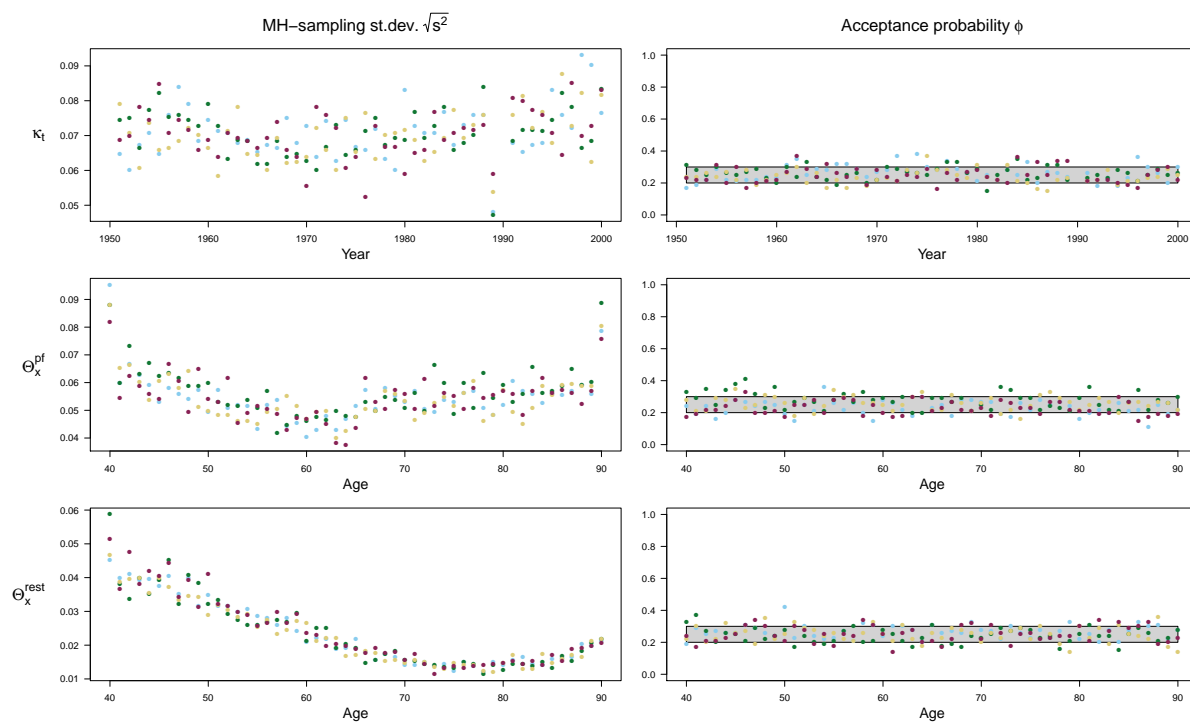


Figure 23: Metropolis(-Hastings) sampling variances used during the final sample phase and the acceptance probabilities from the last sample.

3 CMI reduced portfolio size: PF(B-G) and PF(B-logN)

Parameter estimates for PF(B-G) and PF(B-logN) (reduced portfolio size).

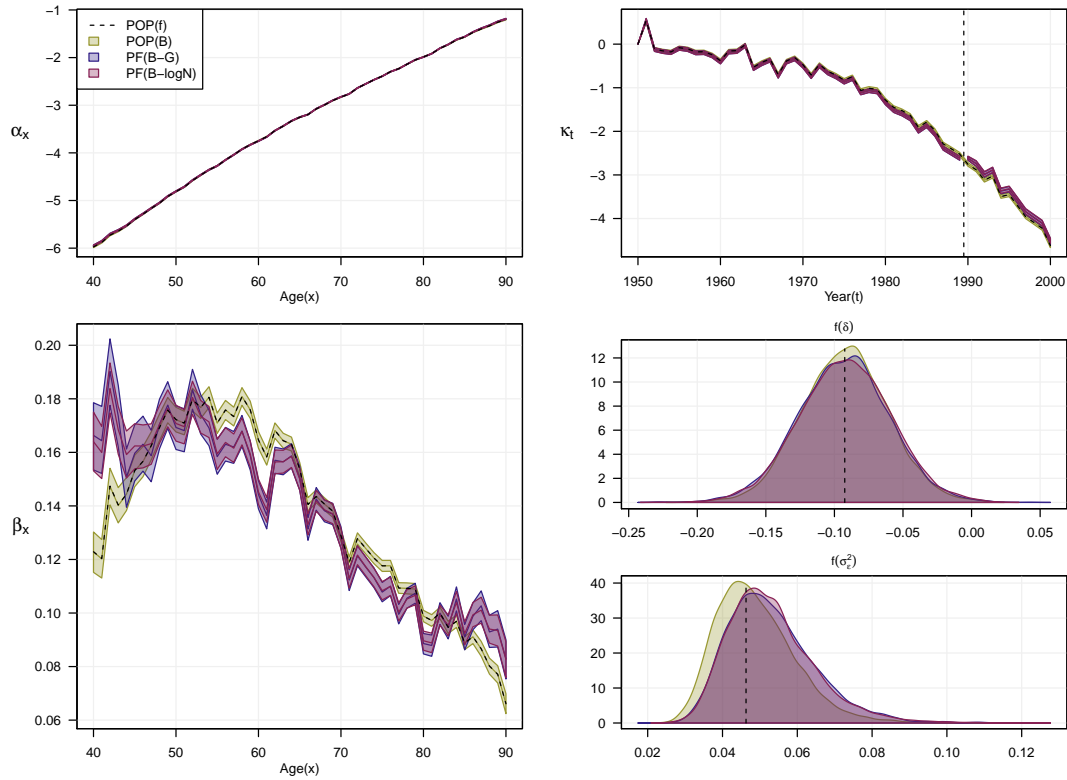


Figure 24: Parameter estimates for Θ_x^{pf} and Θ_x^{rest} using the reduced CMI portfolio.

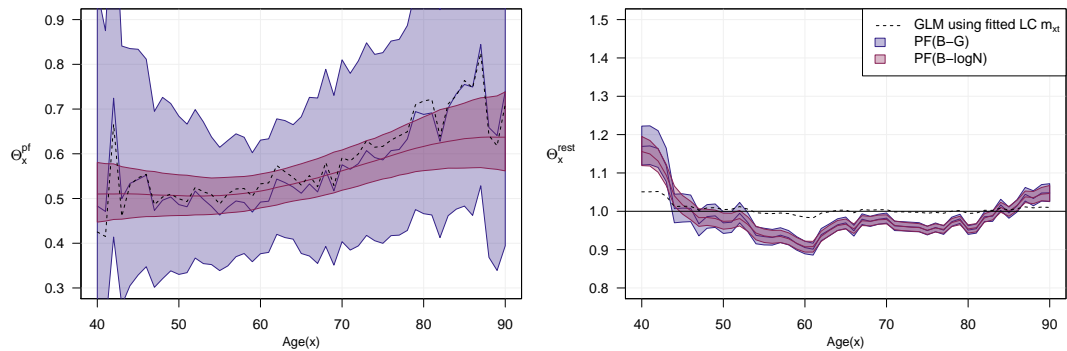
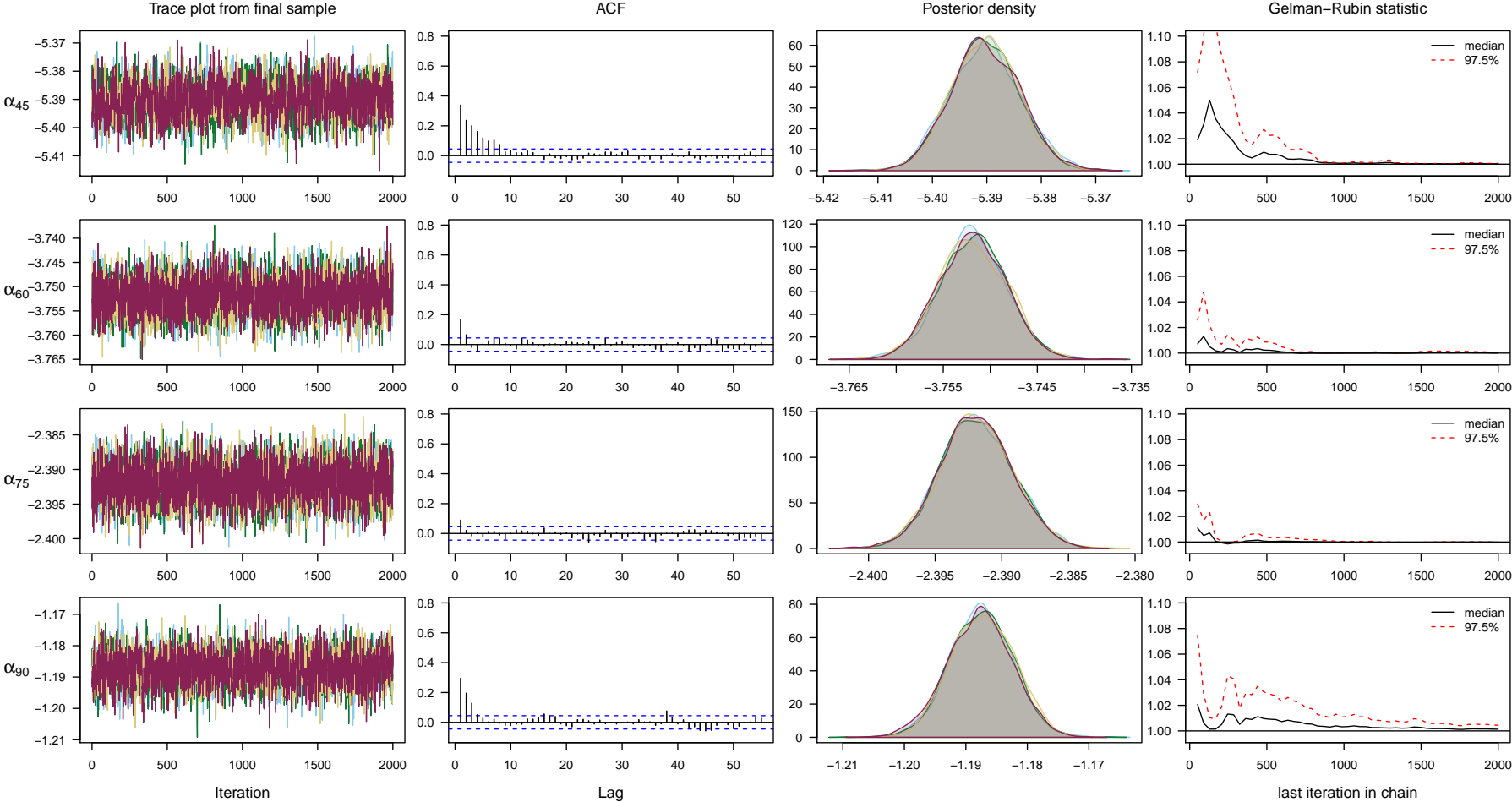


Figure 25: Parameter estimates for α_x , β_x , κ_t , δ and σ_ϵ^2 using the reduced CMI portfolio.

Convergence diagnostics for α_x in PF(B-G) (reduced portfolio size).



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Figure 26: For comments: see Figure 2.

Convergence diagnostics for β_x in PF(B-G) (reduced portfolio size).

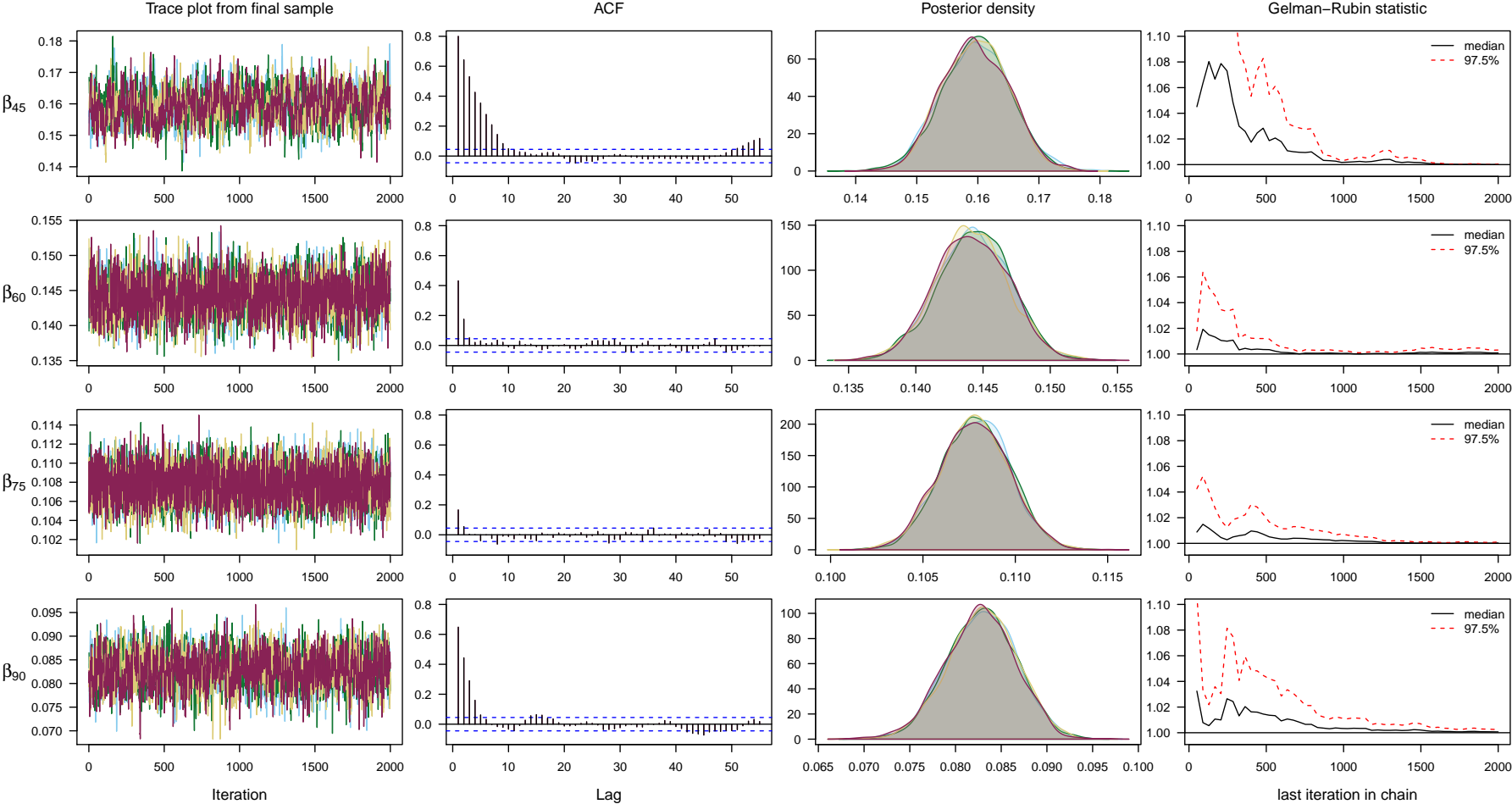


Figure 27: For comments: see Figure 2.

Convergence diagnostics for κ_t in PF(B-G) (reduced portfolio size).

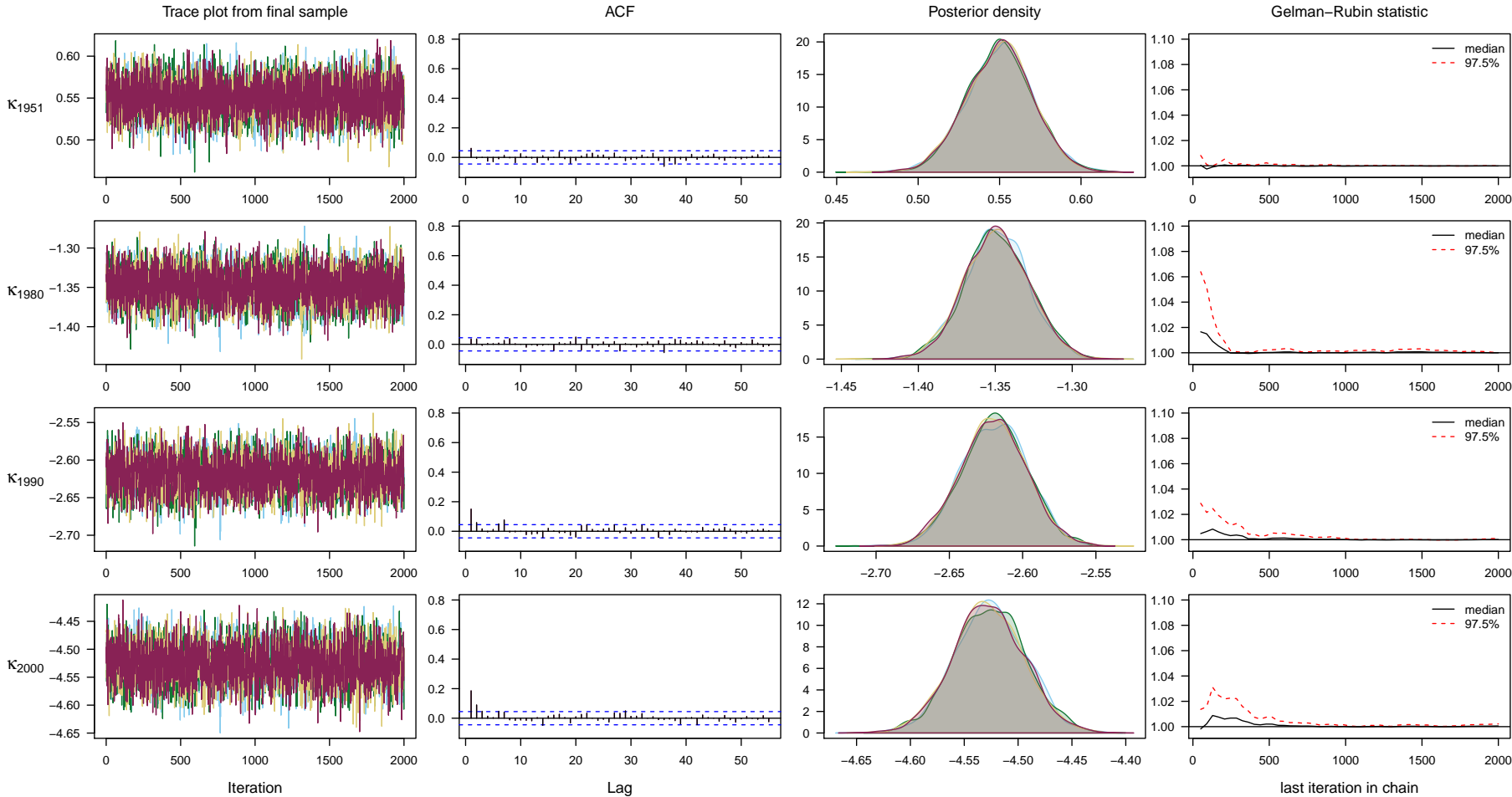


Figure 28: For comments: see Figure 2.

Convergence diagnostics for δ and σ_ε^2 in PF(B-G) (reduced portfolio size).

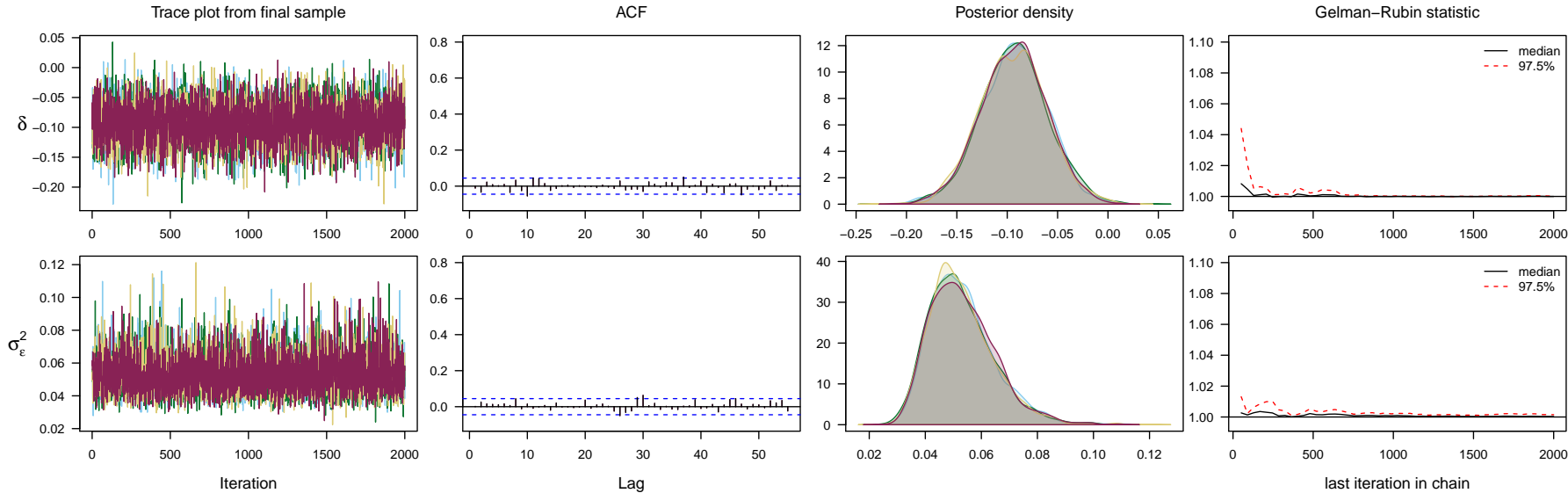
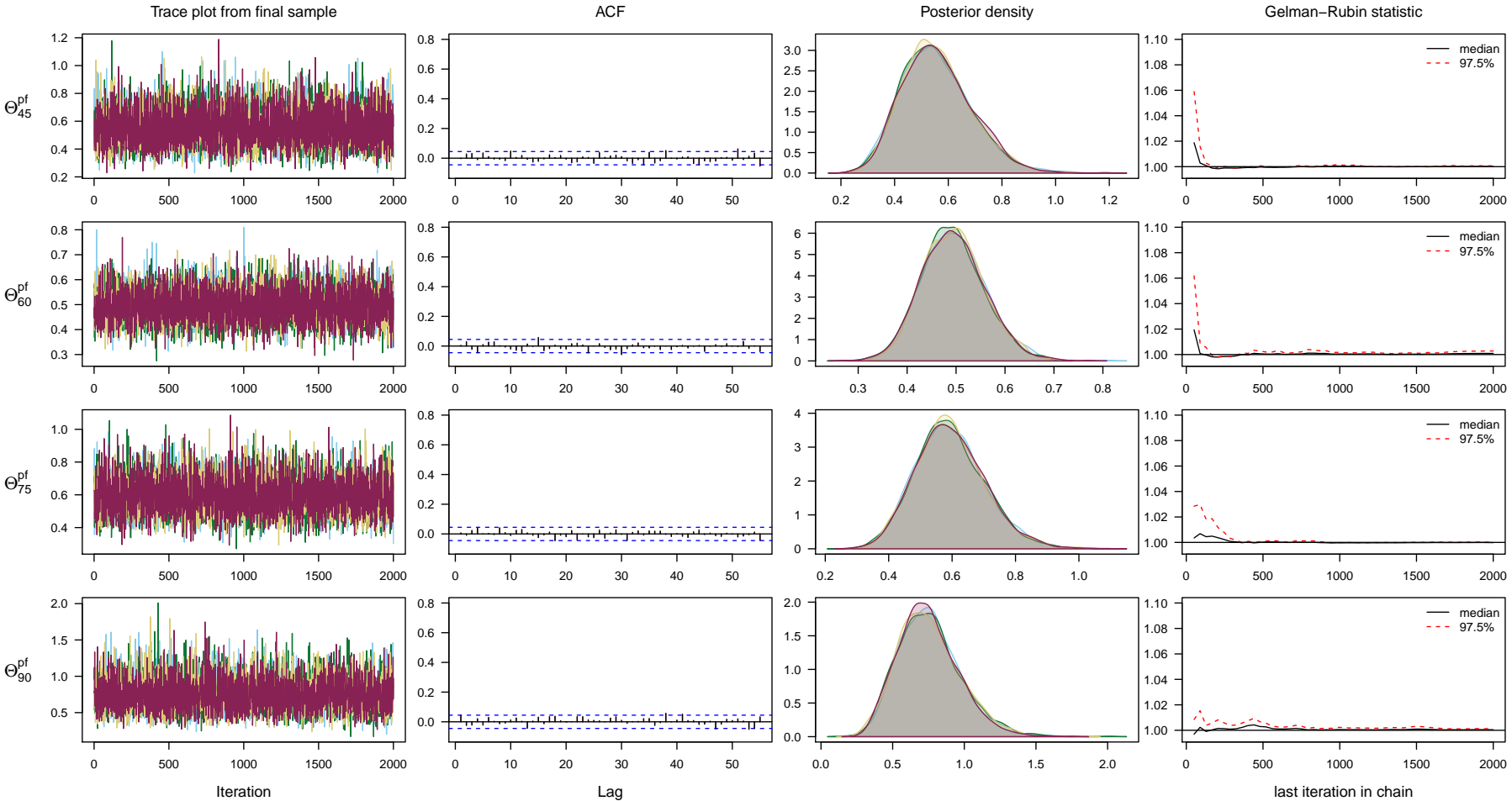


Figure 29: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{pf} in PF(B-G) (reduced portfolio size).



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Figure 30: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{rest} in PF(B-G) (reduced portfolio size).

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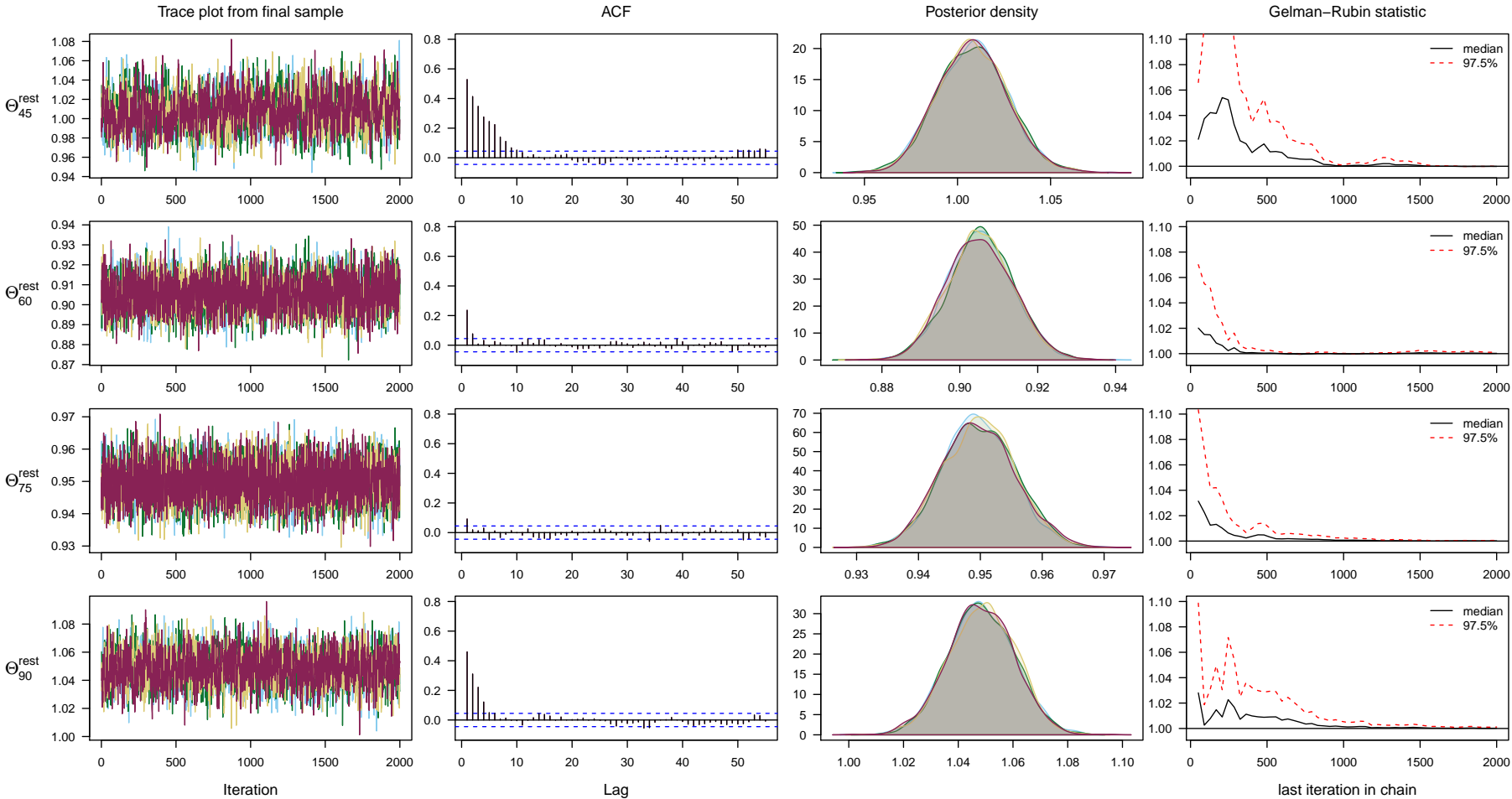


Figure 31: For comments: see Figure 2.

MH-sampling variances and acceptance probabilities in PF(B-G) (reduced portfolio size).

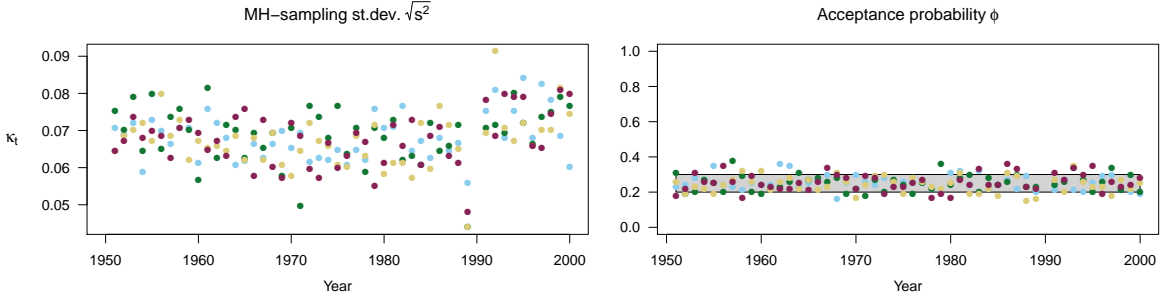
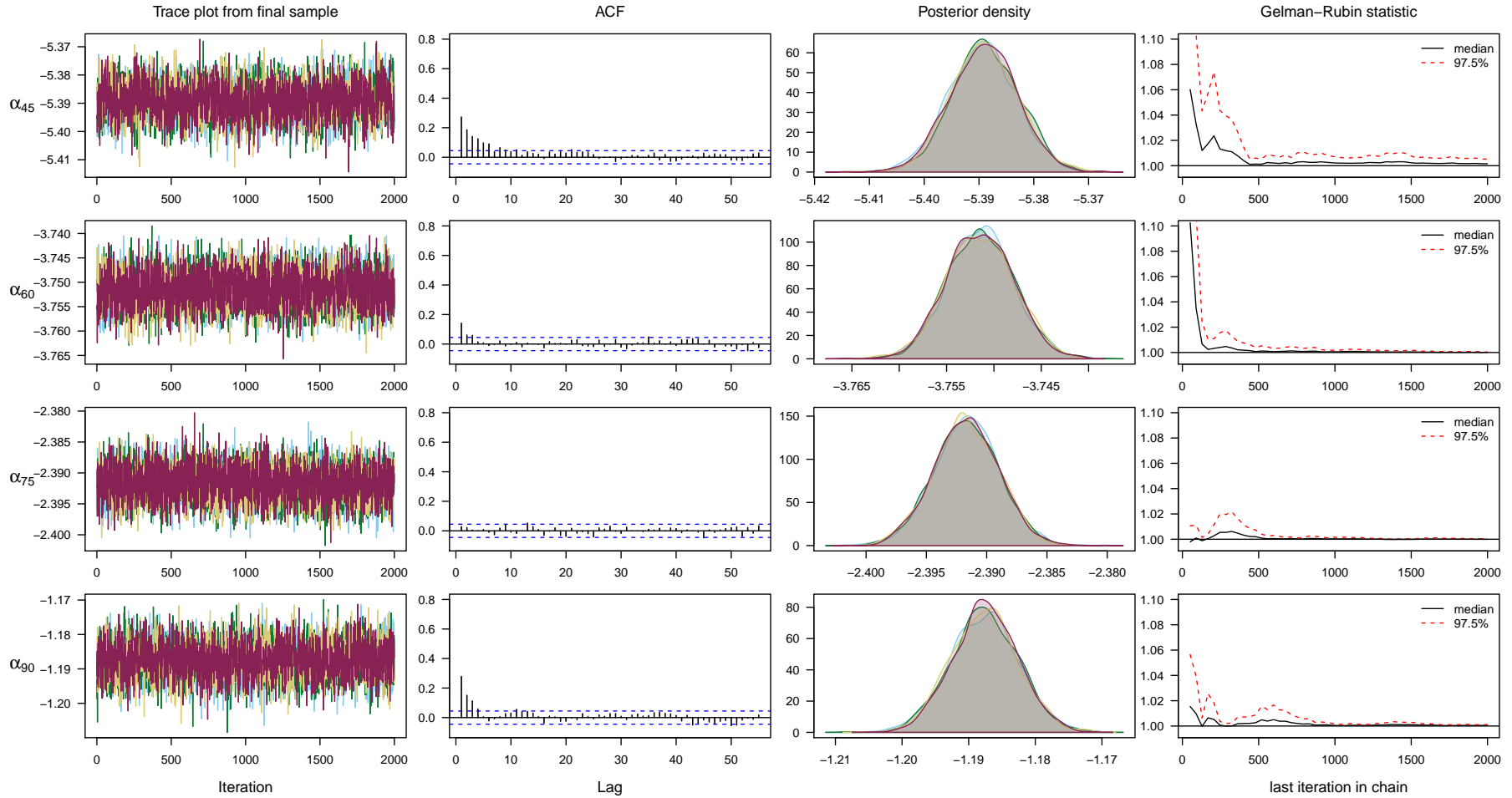


Figure 32: Metropolis(-Hastings) sampling variances used during the final sample phase and the acceptance probabilities from the last sample.

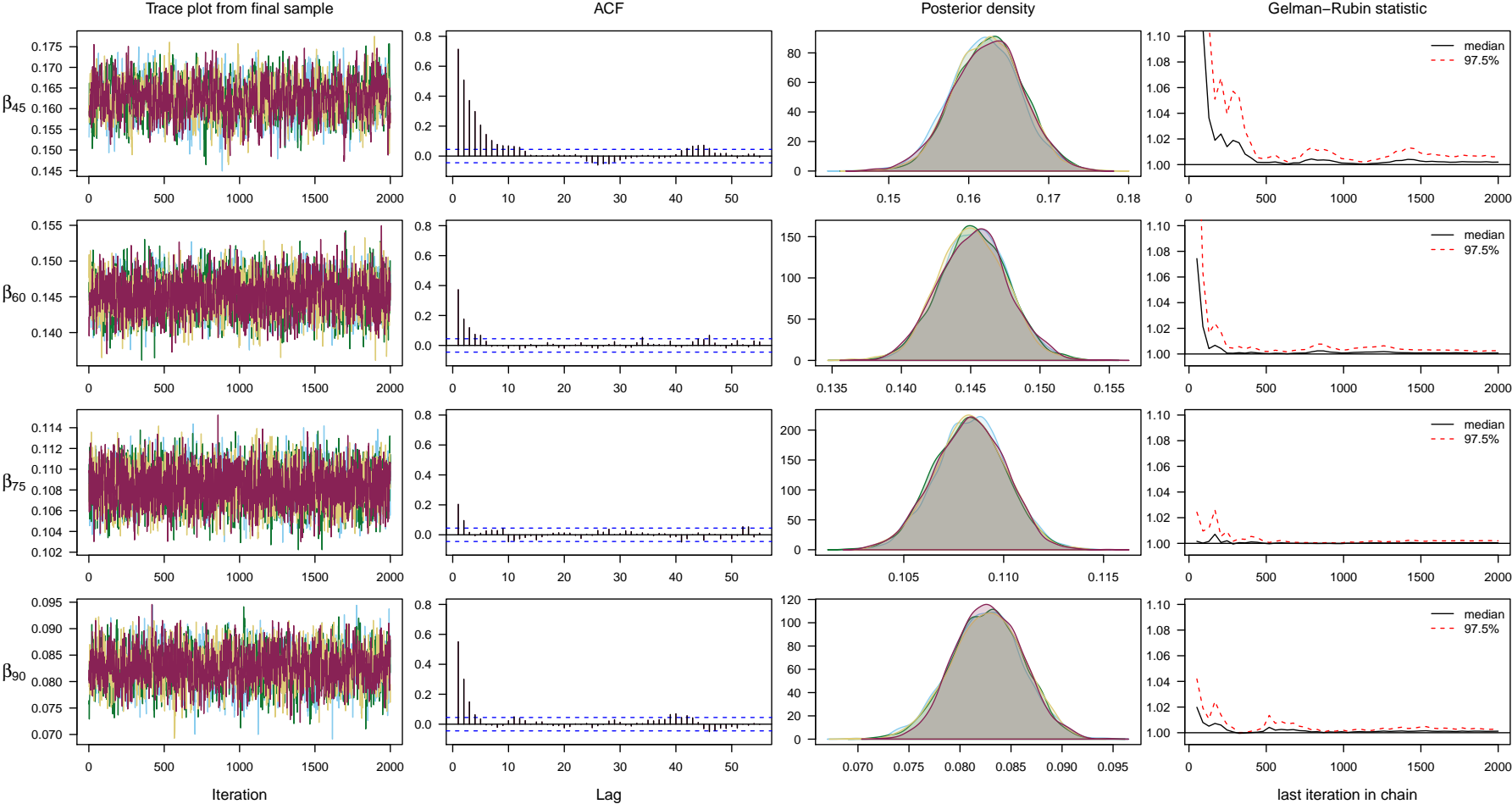
Convergence diagnostics for α_x in PF(B-logN) (reduced portfolio size).



32

Figure 33: For comments: see Figure 2.

Convergence diagnostics for β_x in PF(B-logN) (reduced portfolio size).



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Figure 34: For comments: see Figure 2.

Convergence diagnostics for κ_t in PF(B-logN) (reduced portfolio size).

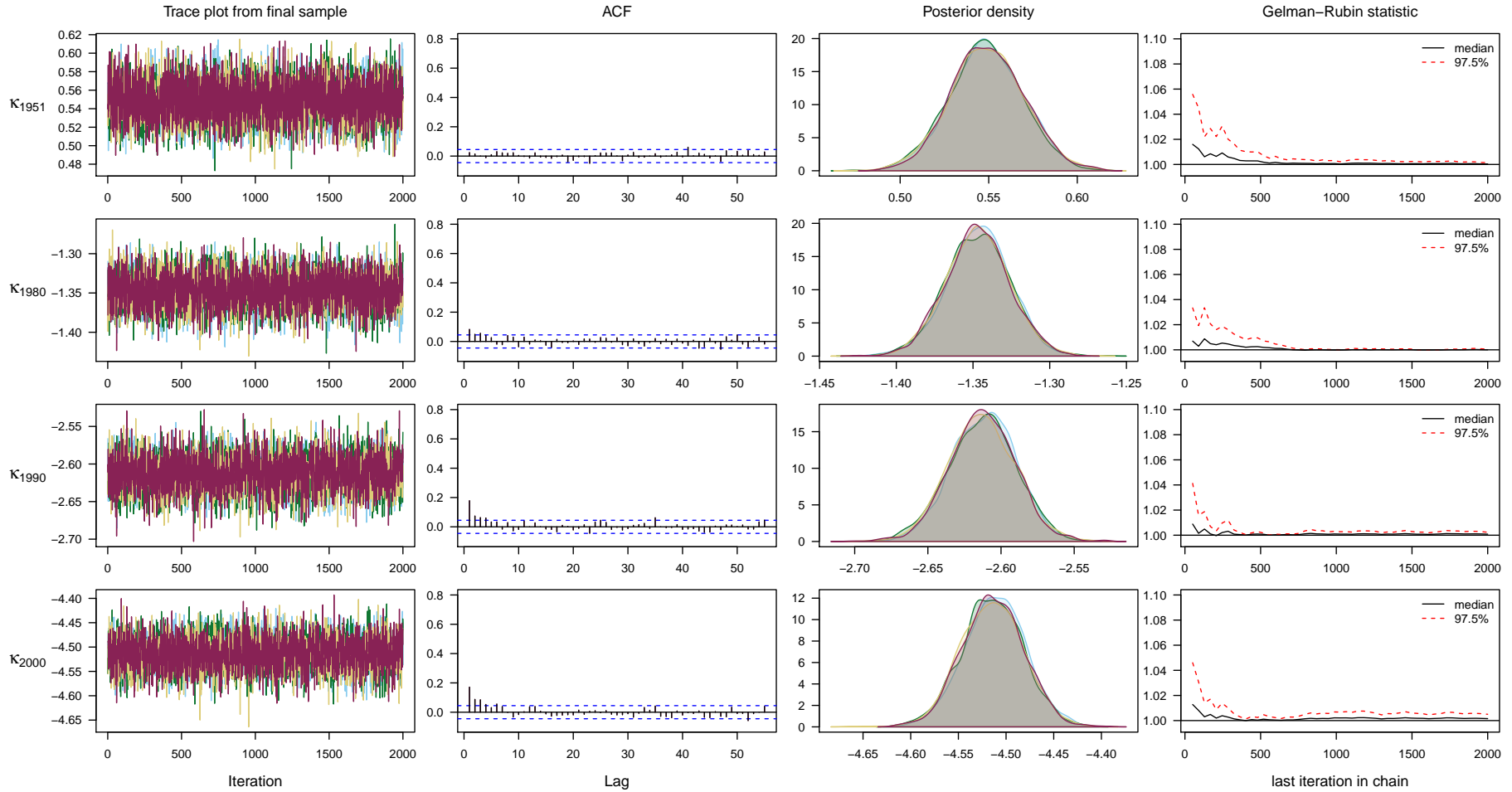
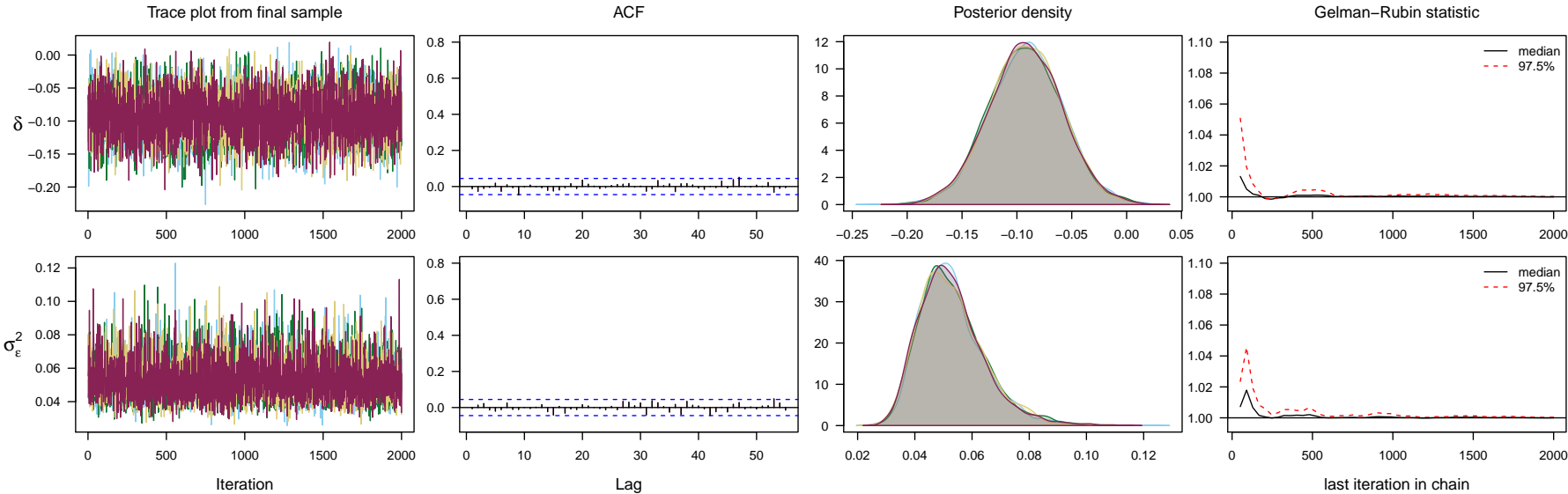


Figure 35: For comments: see Figure 2.

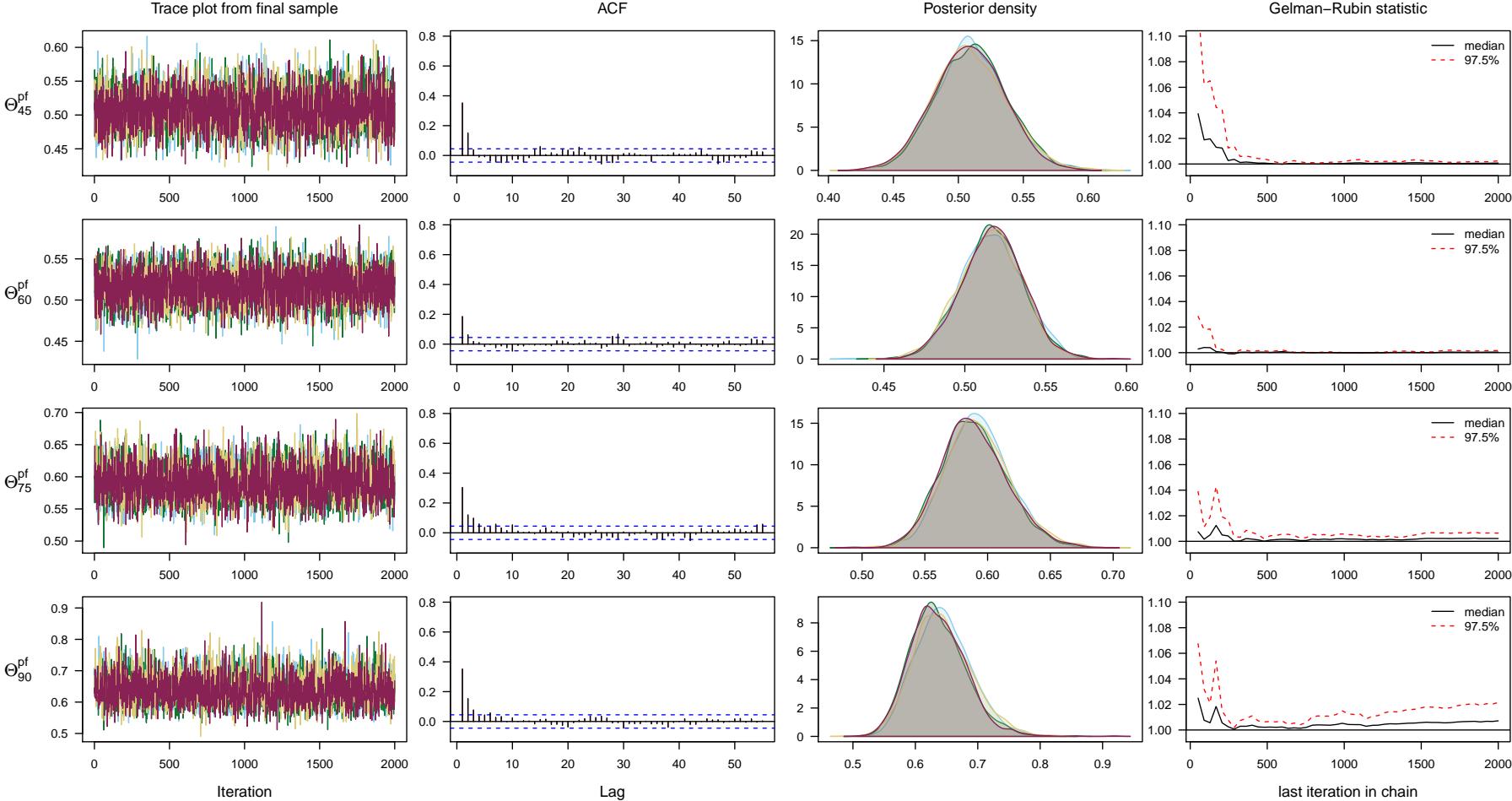
Convergence diagnostics for δ and σ_ε^2 in PF(B-logN) (reduced portfolio size).



35

Figure 36: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{pf} in PF(B-logN) (reduced portfolio size).



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Figure 37: For comments: see Figure 2.

Convergence diagnostics for Θ_x^{rest} in PF(B-logN) (reduced portfolio size).

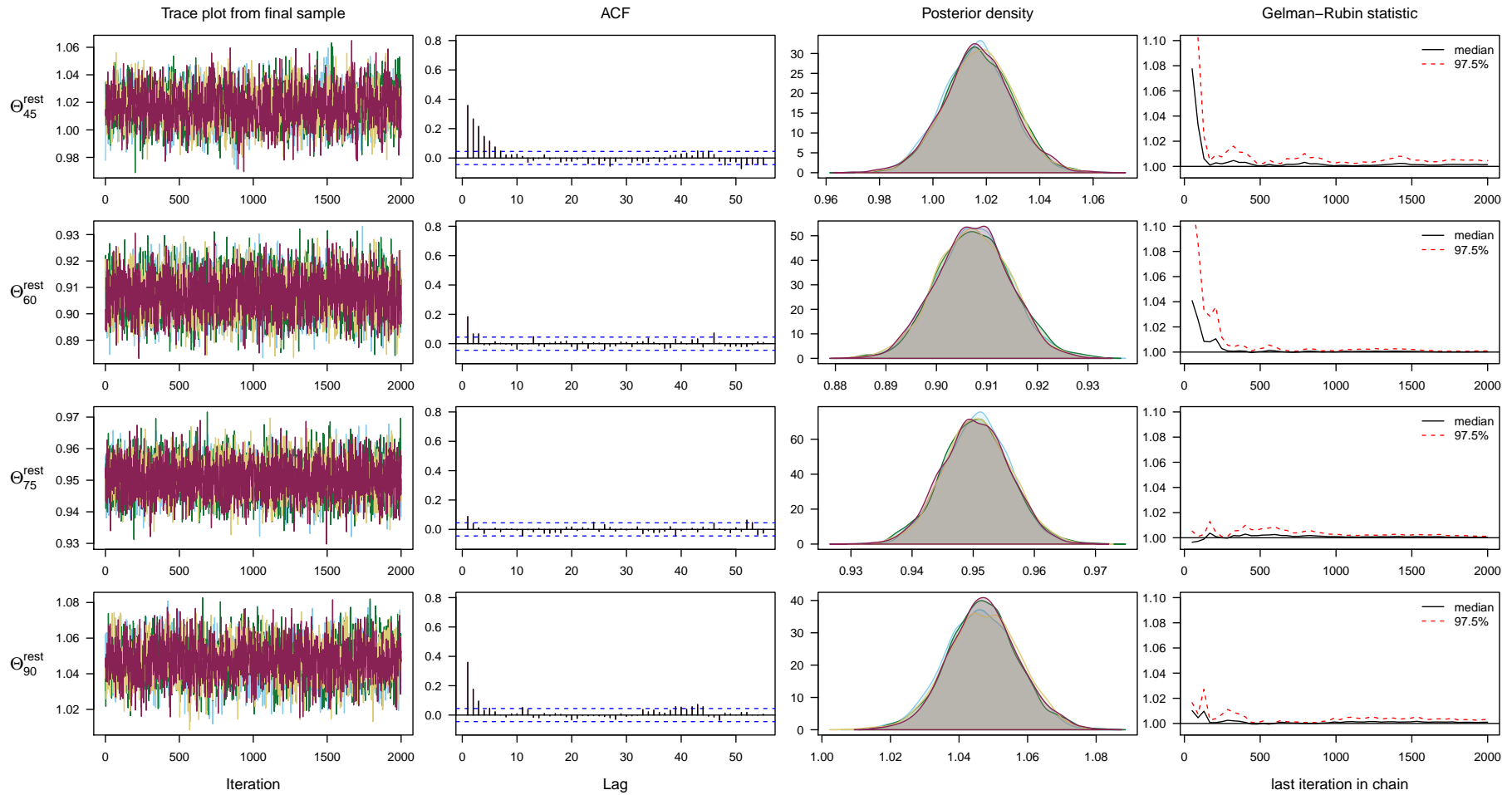


Figure 38: For comments: see Figure 2.

Convergence diagnostics for $\sigma_{\Theta_i}^2$ and ρ_{Θ_i} in PF(B-logN) (reduced portfolio size).

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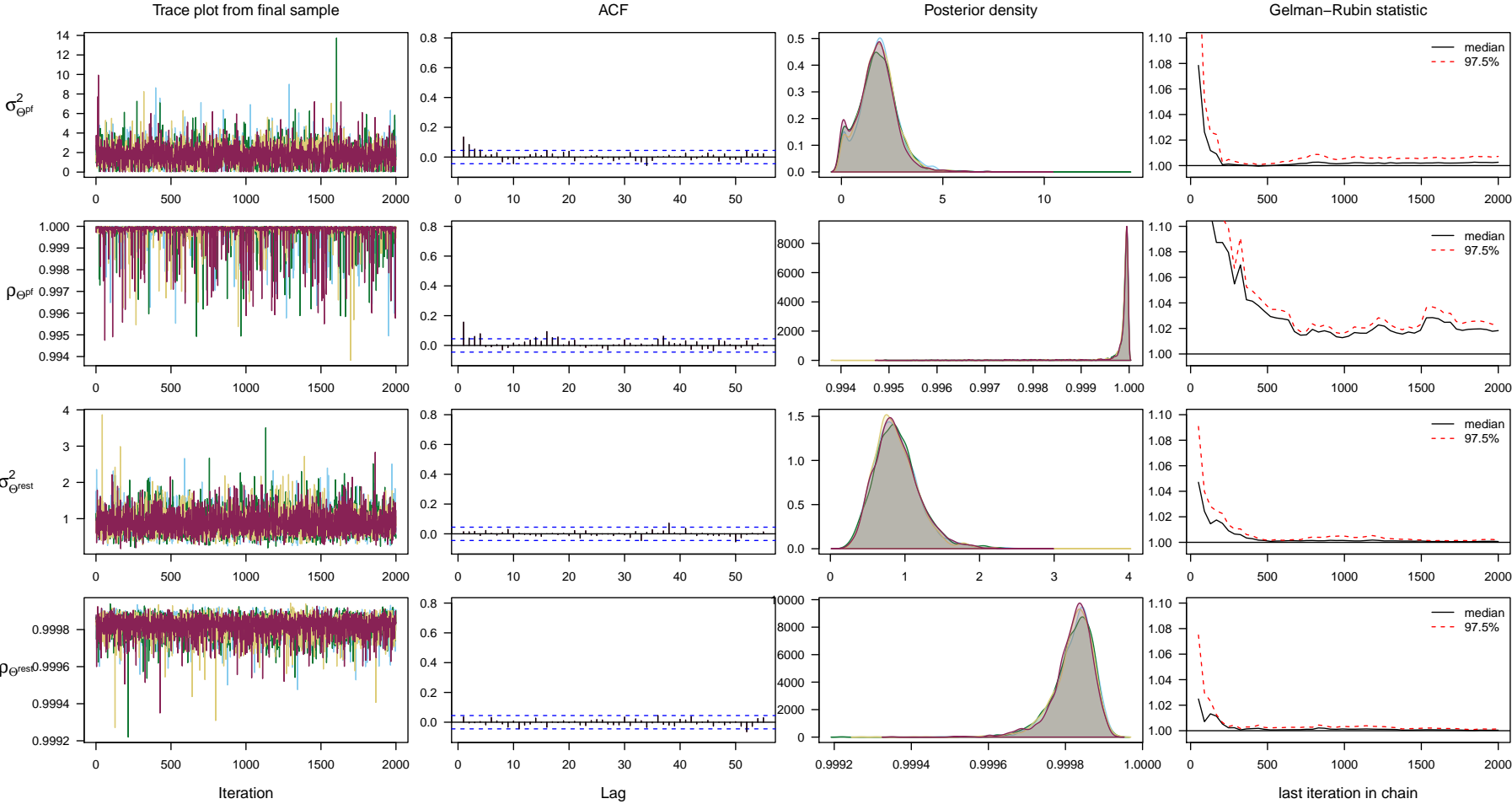


Figure 39: For comments: see Figure 2.

MH-sampling variances and acceptance probabilities in PF(B-logN) (reduced portfolio size).

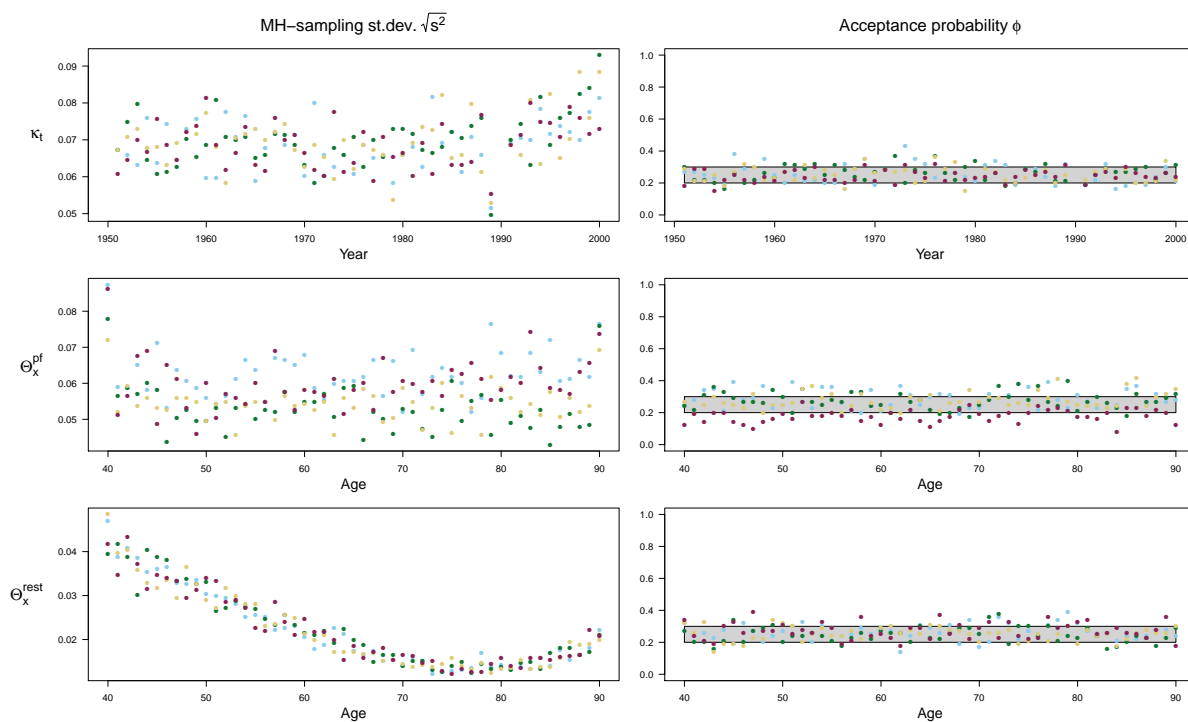


Figure 40: Metropolis(-Hastings) sampling variances used during the final sample phase and the acceptance probabilities from the last sample.

References

- Gelman, A. and Rubin, D. (1992), ‘Inference from iterative simulation using multiple sequences’, *Statistical Science* **7**(4), 457 – 472.
- van Berkum, F., Antonio, K. and Vellekoop, M. (2017), ‘A Bayesian joint model for population and portfolio-specific mortality’.