Supplementary appendix to "Inference on the dimension of the nonstationary subspace in functional time series"

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Abstract

This supplementary appendix provides additional simulation results for our paper Nielsen, Seo, and Seong (2022). The statistics reported are CKP, which is the statistic of Chang, Kim, and Park (2016), and $\mathcal{T}$, which is our variance ratio statistic. For the latter, the subscripts $\mathcal{C}$ and $\mathcal{K}$ denote statistics based on $\widehat{\mathcal{C}}^{(1)}$ and $\widehat{\mathcal{K}}^{(1)}$, respectively. An asterisk means that our variance ratio statistic is implemented with $\ell = s_0$ and no asterisk means $\ell = s_0 + 2$.

References

Chang, Y., C. S. Kim, and J. Y. Park (2016). Nonstationarity in time series of state densities. *Journal of Econometrics 192*, 152–167.

Nielsen, M. Ø., W.-K. Seo, and D. Seong (2022). Inference on the dimension of the nonstationary subspace in functional time series. CREATES research paper 2022-04, Aarhus University.

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Table S.1: Additional simulation results for Experiment 1

		$T = 200$						$T = 500$					
Test	$s_0 \backslash q_1$	1.0	1.5	2.0	2.5	3.0	3.5	1.0	1.5	2.0	2.5	3.0	3.5
$\mathcal{T}_{\mathcal{C}}$	2	0.025	0.025	0.032	0.057	0.106	0.169	0.043	0.043	0.043	0.044	0.053	0.086
	3	0.591	0.568	0.474	0.287	0.163	0.153	0.936	0.936	0.932	0.925	0.853	0.521
	4	0.956	0.952	0.900	0.741	0.603	0.525	1.000	1.000	1.000	1.000	0.993	0.940
	5	0.997	0.996	0.985	0.942	0.868	0.779	1.000	1.000	1.000	1.000	1.000	0.997
$\mathcal{T}_{\mathcal{K}}^*$	2	0.043	0.030	0.027	0.032	0.040	0.053	0.052	0.049	0.046	0.045	0.045	0.049
	3	0.492	0.519	0.479	0.404	0.309	0.262	0.911	0.915	0.914	0.913	0.892	0.873
	4	0.912	0.897	0.841	0.749	0.663	0.597	1.000	0.999	1.000	0.999	0.998	0.995
	5	0.991	0.982	0.959	0.922	0.884	0.836	1.000	1.000	1.000	1.000	1.000	1.000
$\mathcal{T}_{\mathcal{C}}^*$	2	0.064	0.093	0.203	0.363	0.504	0.606	0.054	0.055	0.071	0.136	0.259	0.397
	3	0.718	0.362	0.160	0.128	0.119	0.121	0.980	0.976	0.822	0.335	0.199	0.151
	4	0.879	0.529	0.347	0.264	0.207	0.186	1.000	0.998	0.809	0.556	0.399	0.339
	5	0.991	0.860	0.577	0.417	0.240	0.170	1.000	1.000	0.988	0.920	0.758	0.607

Notes: Based on 10,000 Monte Carlo replications. The DGP true value is $s = 2$ and the H_0 value is s_0. The nominal size is 5%.

Table S.2: Additional simulation results for Experiment 2

		$T = 200$						$T = 500$					
Test	$s_0 \backslash q_2$	0.00	0.05	0.10	0.15	0.20	0.25	0.00	0.05	0.10	0.15	0.20	0.25
$\mathcal{T}_{\mathcal{C}}$	2	0.025	0.037	0.065	0.104	0.151	0.231	0.041	0.048	0.061	0.070	0.093	0.122
	3	0.583	0.597	0.669	0.755	0.756	0.686	0.934	0.951	0.970	0.990	0.996	0.995
	4	0.957	0.962	0.969	0.972	0.958	0.909	1.000	1.000	1.000	1.000	1.000	1.000
	5	0.996	0.998	0.999	0.998	0.996	0.990	1.000	1.000	1.000	1.000	1.000	1.000
$\mathcal{T}_{\mathcal{K}}^*$	2	0.043	0.048	0.066	0.101	0.146	0.211	0.049	0.057	0.065	0.076	0.103	0.134
	3	0.471	0.483	0.547	0.602	0.630	0.612	0.905	0.922	0.942	0.963	0.977	0.977
	4	0.897	0.908	0.928	0.932	0.929	0.892	0.999	1.000	1.000	1.000	1.000	1.000
	5	0.989	0.995	0.997	0.995	0.990	0.971	1.000	1.000	1.000	1.000	1.000	1.000
$\mathcal{T}_{\mathcal{C}}^*$	2	0.064	0.070	0.107	0.211	0.379	0.565	0.052	0.059	0.070	0.092	0.145	0.234
	3	0.741	0.794	0.896	0.651	0.334	0.210	0.981	0.989	0.999	1.000	0.996	0.851
	4	0.939	0.963	0.983	0.725	0.508	0.381	1.000	1.000	1.000	1.000	0.997	0.919
	5	0.999	0.999	0.994	0.898	0.820	0.750	1.000	1.000	1.000	1.000	1.000	0.998

Notes: Based on 10,000 Monte Carlo replications. The DGP true value is $s = 2$ and the H_0 value is s_0. The nominal size is 5%.

Table S.3: Additional simulation results for Experiment 3

			$T = 200$				$T = 500$			
θ	Test	s_0	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 0$	$s = 1$	$s = 2$	$s = 3$
0.0	$\mathcal{T}_{\mathcal{C}}$	s		0.057	0.075	0.109		0.056	0.051	0.061
		$s+1$	1.000	0.983	0.948	0.836	1.000	1.000	1.000	0.999
		$s+2$	1.000	1.000	0.998	0.975	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.066	0.098	0.168		0.057	0.055	0.071
		$s+1$	1.000	0.979	0.857	0.533	1.000	1.000	1.000	1.000
		$s+2$	1.000	0.994	0.920	0.777	1.000	1.000	1.000	1.000
		$s+3$	1.000	0.992	0.958	0.946	1.000	1.000	1.000	0.999
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.354	0.627	0.763		0.163	0.397	0.563
		$s+1$	0.223	0.114	0.114	0.126	0.679	0.222	0.134	0.119
		$s+2$	0.332	0.371	0.422	0.516	0.546	0.515	0.550	0.605
		$s+3$	0.622	0.747	0.859	0.933	0.768	0.850	0.926	0.964
0.5	$\mathcal{T}_{\mathcal{C}}$	s		0.059	0.077	0.099		0.052	0.052	0.066
		$s+1$	0.990	0.919	0.843	0.692	1.000	0.998	0.997	0.992
		$s+2$	1.000	0.999	0.993	0.984	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.068	0.109	0.187		0.053	0.057	0.082
		$s+1$	0.993	0.911	0.697	0.417	1.000	0.999	0.997	0.981
		$s+2$	1.000	0.976	0.885	0.814	1.000	1.000	1.000	0.997
		$s+3$	1.000	0.990	0.985	0.992	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.348	0.627	0.748		0.167	0.392	0.574
		$s+1$	0.259	0.132	0.110	0.152	0.653	0.242	0.156	0.144
		$s+2$	0.349	0.388	0.460	0.571	0.585	0.520	0.564	0.654
		$s+3$	0.673	0.818	0.911	0.986	0.808	0.880	0.959	0.995
0.8	$\mathcal{T}_{\mathcal{C}}$	s		0.047	0.061	0.067		0.049	0.056	0.067
		$s+1$	0.825	0.578	0.446	0.357	0.986	0.935	0.897	0.758
		$s+2$	0.969	0.934	0.909	0.878	1.000	1.000	0.998	0.993
		$s+3$	0.999	0.998	0.998	0.996	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.064	0.128	0.203		0.056	0.078	0.131
		$s+1$	0.836	0.507	0.333	0.217	0.988	0.931	0.822	0.530
		$s+2$	0.913	0.793	0.681	0.618	1.000	0.997	0.971	0.905
		$s+3$	0.976	0.954	0.948	0.968	1.000	1.000	0.999	0.999
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.353	0.595	0.658		0.207	0.462	0.612
		$s+1$	0.248	0.132	0.132	0.166	0.544	0.217	0.166	0.169
		$s+2$	0.352	0.379	0.432	0.500	0.528	0.515	0.574	0.641
		$s+3$	0.696	0.793	0.869	0.951	0.833	0.909	0.962	0.992

Notes: Based on 10,000 Monte Carlo replications. The DGP true value is s and the H_0 value is s_0. Nominal size is 5%.

Table S.4: Additional simulation results for Experiment 3 (without permutation)

			$T = 200$				$T = 500$			
θ	Test	s_0	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 0$	$s = 1$	$s = 2$	$s = 3$
0.0	$\mathcal{T}_{\mathcal{K}}$	s		0.046	0.040	0.037		0.052	0.048	0.043
		$s+1$	0.999	0.960	0.970	0.968	1.000	1.000	0.999	1.000
		$s+2$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	CKP	s		0.046	0.043	0.026		0.050	0.052	0.044
		$s+1$	1.000	0.998	0.843	0.406	1.000	1.000	1.000	1.000
		$s+2$	1.000	0.983	0.718	0.280	1.000	1.000	1.000	1.000
		$s+3$	0.999	0.894	0.486	0.240	1.000	1.000	1.000	0.997
0.5	$\mathcal{T}_{\mathcal{K}}$	s		0.044	0.030	0.028		0.047	0.044	0.040
		$s+1$	0.976	0.906	0.874	0.873	1.000	0.994	0.992	0.996
		$s+2$	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	CKP	s		0.048	0.057	0.054		0.051	0.064	0.061
		$s+1$	1.000	1.000	1.000	0.940	1.000	1.000	1.000	1.000
		$s+2$	1.000	1.000	0.960	0.858	1.000	1.000	1.000	1.000
		$s+3$	1.000	0.890	0.739	0.060	1.000	1.000	1.000	0.516
0.8	$\mathcal{T}_{\mathcal{K}}$	s		0.025	0.011	0.007		0.040	0.030	0.026
		$s+1$	0.756	0.632	0.520	0.457	0.968	0.906	0.876	0.866
		$s+2$	0.990	0.966	0.934	0.923	1.000	1.000	0.999	0.999
		$s+3$	1.000	0.999	0.999	0.997	1.000	1.000	1.000	1.000
	CKP	s		0.051	0.091	0.076		0.048	0.088	0.087
		$s+1$	1.000	0.992	0.767	0.413	1.000	1.000	1.000	1.000
		$s+2$	1.000	1.000	0.954	0.829	1.000	1.000	1.000	1.000
		$s+3$	1.000	0.996	0.975	0.828	1.000	1.000	1.000	0.999

Notes: Based on 10,000 Monte Carlo replications. The DGP true value is s and the H_0 value is s_0. Nominal size is 5%.

Table S.5: Additional simulation results for Experiment 3 (without permutation, continued)

			$T = 200$				$T = 500$			
θ	Test	s_0	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 0$	$s = 1$	$s = 2$	$s = 3$
0.0	$\mathcal{T}_{\mathcal{C}}$	s		0.050	0.045	0.042		0.053	0.049	0.044
		$s+1$	1.000	0.988	0.991	0.992	1.000	1.000	1.000	1.000
		$s+2$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.053	0.051	0.047		0.055	0.051	0.048
		$s+1$	1.000	0.981	0.987	0.984	1.000	1.000	1.000	1.000
		$s+2$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.053	0.054	0.054		0.055	0.052	0.049
		$s+1$	1.000	0.998	0.998	0.998	1.000	1.000	1.000	1.000
		$s+2$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	0.991	1.000	1.000	1.000	1.000
0.5	$\mathcal{T}_{\mathcal{C}}$	s		0.046	0.034	0.034		0.048	0.046	0.042
		$s+1$	0.982	0.947	0.911	0.913	1.000	0.998	0.995	0.998
		$s+2$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.057	0.046	0.048		0.051	0.054	0.048
		$s+1$	0.989	0.928	0.909	0.901	1.000	0.997	0.995	0.997
		$s+2$	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.058	0.052	0.060		0.051	0.055	0.050
		$s+1$	0.991	0.978	0.932	0.877	1.000	0.999	0.997	0.999
		$s+2$	1.000	1.000	0.999	0.998	1.000	1.000	1.000	1.000
		$s+3$	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
0.8	$\mathcal{T}_{\mathcal{C}}$	s		0.028	0.014	0.012		0.041	0.032	0.028
		$s+1$	0.786	0.685	0.536	0.451	0.969	0.926	0.889	0.883
		$s+2$	0.993	0.973	0.938	0.923	1.000	1.000	1.000	1.000
		$s+3$	1.000	0.999	0.999	0.996	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{K}}$	s		0.053	0.035	0.037		0.051	0.051	0.045
		$s+1$	0.799	0.591	0.496	0.397	0.979	0.909	0.891	0.874
		$s+2$	0.987	0.961	0.912	0.892	1.000	1.000	0.999	0.999
		$s+3$	1.000	0.999	0.997	0.992	1.000	1.000	1.000	1.000
	$\mathcal{T}^*_{\mathcal{C}}$	s		0.056	0.062	0.076		0.051	0.056	0.054
		$s+1$	0.820	0.694	0.433	0.271	0.981	0.959	0.902	0.860
		$s+2$	0.986	0.944	0.829	0.766	1.000	1.000	0.999	0.999
		$s+3$	1.000	0.996	0.989	0.974	1.000	1.000	1.000	1.000

Notes: Based on 10,000 Monte Carlo replications. The DGP true value is s and the H_0 value is s_0. Nominal size is 5%.