Online Supplemental Material for "A Max-Correlation White Noise Test for Weakly Dependent Time Series"

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This Draft: October 21, 2019

Contents

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A Outline and Assumptions

In this supplemental material, we present theoretical and numerical details that are omitted in the main paper Hill and Motegi (2019). In the present appendix, we state our assumptions on the data generating process and plug-in estimator.

Appendix B contains a comparison between our imposed Near Epoch Dependence [NED] property with the moment contraction property invoked in Shao (2011), Guay, Guerre, and Lazarová (2013) and Xiao and Wu (2014).

Appendix C contains several examples of data generating processes, filters, and the resulting correlation expansion form.

Appendix D details local asymptotic power for the max-correlation test.

Appendix E presents a kernel variance estimator for self-standardization of the sample correlation as an option for weighting.

Appendix F presents omitted proofs for Lemma A.4 (expansion), Lemma A.5 (convergence in finite dimensional distributions) and Lemma A.6 (ULLN's).

Appendix G presents an environment in which an upper bound on the maximum lag rate of increase $\mathcal{L}_n \to \infty$ is available.

Appendix H contains the complete Monte Carlo study.

Throughout $|\cdot|$ is the l_1-matrix norm; $||\cdot||$ is the l_2-matrix norm; $||\cdot||_p$ is the L_p-norm. $I(\cdot)$ is the indicator function: $I(A) = 1$ if A is true, else $I(A) = 0$. $\mathcal{F}_t \equiv \sigma(y_\tau, x_\tau : \tau \leq t)$. All random variables lie in a complete probability measure space $(\Omega, \mathcal{P}, \mathcal{F})$, hence $\sigma(\cup_{t\in\mathbb{Z}}\mathcal{F}_t) \subseteq \mathcal{F}$. We drop the (pseudo) true value θ_0 from function arguments when there is no confusion. K is a positive constant the value of which may be different in different places.

Assumptions The class of time series models considered is:

$$y_t = f(x_{t-1}, \phi_0) + u_t \;\text{ and }\; u_t = \epsilon_t \sigma_t(\theta_0) \tag{A.1}$$

where $\phi \in \mathbb{R}^{k_\phi}$, $k_\phi \geq 0$, and $f(x,\phi)$ is a level response function. ϵ_t satisfies $E[\epsilon_t] = 0$, $E[\epsilon_t^2] < \infty$, and the regressors are $x_t \in \mathbb{R}^{k_x}$, $k_x \geq 0$. $\{x_t, y_t\}$ are strictly stationary in order to focus ideas. $\sigma_t^2(\theta_0)$ is a process measurable with respect to $\mathcal{F}_{t-1} \equiv \sigma(y_\tau, x_\tau : \tau \leq t-1)$, where θ_0 is decomposed as $[\phi_0', \delta_0']$ and $\delta_0 \in \mathbb{R}^{k_\delta}$ are volatility-specific parameters, $k_\delta \geq 0$.

Unless $y_t = \epsilon_t$ such that y_t is known to have a zero mean, let $\hat{\theta}_n = [\hat{\phi}_n', \hat{\delta}_n']$ estimate θ_0 where n is the sample size, and define the residual, and its sample serial covariance and correlation at lag $h \geq 1$:

$$\epsilon_t(\hat{\theta}_n) \equiv \frac{u_t(\hat{\phi}_n)}{\sigma_t(\hat{\theta}_n)} \equiv \frac{y_t - f(x_{t-1}, \hat{\phi}_n)}{\sigma_t(\hat{\theta}_n)} \;\text{ and }\; \hat{\gamma}_n(h) \equiv \frac{1}{n}\sum_{t=1+h}^{n} \epsilon_t(\hat{\theta}_n)\epsilon_{t-h}(\hat{\theta}_n) \;\text{ and }\; \hat{\rho}_n(h) \equiv \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)}.$$

In the pure volatility model set $f(x_{t-1}, \hat{\phi}_n) = 0$, and in the level model set $\sigma_t(\hat{\theta}_n) = 1$.

The general test statistic is based on the measurable function:

$$\vartheta : \mathbb{R}^{\mathcal{L}} \to [0, \infty) \tag{A.2}$$

that satisfies the following: $\vartheta(x)$ is continuous; lower bound $\vartheta(a) = 0$ *if and only if* $a = 0$; upper bound $\vartheta(a) \leq K\mathcal{L}\mathcal{M}$ for some $K > 0$ and any $a = [a_h]_{h=1}^{\mathcal{L}}$ such that $|a_h| \leq \mathcal{M}$ for each h; divergence $\vartheta(a) \to \infty$ as $||a|| \to \infty$; monotonicity $\vartheta(a_{\mathcal{L}_1}) \leq \vartheta([a'_{\mathcal{L}_1}, c'_{\mathcal{L}_2-\mathcal{L}_1}]')$ where $(a_{\mathcal{L}}, c_{\mathcal{L}}) \in \mathbb{R}^{\mathcal{L}}$, $\forall \mathcal{L}_2 \geq \mathcal{L}_1$ and any $c_{\mathcal{L}_2-\mathcal{L}_1} \in \mathbb{R}^{\mathcal{L}_2-\mathcal{L}_1}$; and the triangle inequality $\vartheta(a+b) \leq \vartheta(a) + \vartheta(b)$ $\forall a, b \in \mathbb{R}^{\mathcal{L}_n}$. Examples include the maximum $\vartheta(a) = \max_{1\leq h\leq\mathcal{L}} |a_h|$ and sum $\vartheta(a) = \sum_{h=1}^{\mathcal{L}} |a_h|$, where $a = [a_h]_{h=1}^{\mathcal{L}}$. The general test statistic is:

$$\hat{\mathcal{T}}_n \equiv \vartheta\left(\left[\sqrt{n}\hat{\omega}_n(h)\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \text{ with weights } \hat{\omega}_n(h) > 0 \text{ and } \hat{\omega}_n(h) \xrightarrow{p} \omega(h) \in (0, \infty).$$

Let $\{\upsilon_t\}$ be a stationary α-mixing process with σ-fields $\mathfrak{V}_s^t \equiv \sigma(\upsilon_\tau : s \leq \tau \leq t)$ and $\mathfrak{V}_t \equiv \mathfrak{V}_{-\infty}^t$ and coefficients

$$\alpha_h^{(\upsilon)} = \sup_{t\in\mathbb{Z}} \sup_{\mathcal{A}\subset\mathfrak{V}_t^\infty, \mathcal{B}\subset\mathfrak{V}_{-\infty}^{t-h}} |P(\mathcal{A}\cap\mathcal{B}) - P(\mathcal{A})P(\mathcal{B})|.$$

We say $\{\epsilon_t\}$ is stationary L_q-NED with size λ on a mixing *base* $\{\upsilon_t\}$ when ϵ_t is L_q-bounded and

$$\left\|\epsilon_t - E[\epsilon_t|\mathfrak{V}_{t-h}^{t+h}]\right\|_q \leq K\psi_h \text{ where } \psi_h = O\left(h^{-\lambda-\iota}\right) \text{ for tiny } \iota > 0.$$

If $\epsilon_t = \upsilon_t$ then $||\epsilon_t - E[\epsilon_t|\mathfrak{V}_{t-h}^{t+h}]||_q = 0$, hence NED includes mixing sequences, but it also includes non-mixing sequences since it covers infinite lag functions of mixing sequences that need not be mixing (see, e.g., Doukhan, 1994). See Davidson (1994) for historical references and deep results.

Assumption 1 (data generating process)**.**

a. $\{x_t, y_t\}$ *are stationary, ergodic, and* $L_{2+\delta}$*-bounded for tiny* $\delta > 0$.

b. ϵ_t *is stationary, ergodic,* $E[\epsilon_t] = 0$, L_r*-bounded,* $r > 4$*, and* L_4*-NED with size* $1/2$ *on stationary* α*-mixing* $\{\upsilon_t\}$ *with coefficients* $\alpha_h^{(\upsilon)} = O(h^{-r/(r-2)-\iota})$ *for tiny* $\iota > 0$.

c. The weights satisfy $\hat{\omega}_n(h) > 0$ *a.s. and* $\hat{\omega}_n(h) = \omega(h) + O_p(1/n^\kappa)$ *for some* $\kappa > 0$ *and non-random* $\omega(h) \in (0, \infty)$*, for each* h.

We require notation that makes use of estimating equations $m_t \in \mathbb{R}^{k_m}$ and a matrix $\mathcal{A} \in \mathbb{R}^{k_\theta \times k_m}$ defined under Assumption 2.c. Define

$$\mathcal{D}(h) \equiv E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\right] + E\left[\epsilon_t\left(\epsilon_{t-h}s_{t-h} + \frac{G_{t-h}}{\sigma_{t-h}}\right)\right] \in \mathbb{R}^{k_\theta}, \tag{A.3}$$

and

$$z_t(h) \equiv r_t(h) - \rho(h)r_t(0) \text{ where } r_t(h) \equiv \frac{\epsilon_t\epsilon_{t-h} - E\left[\epsilon_t\epsilon_{t-h}\right] - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]} \text{ and } \rho(h) \equiv \frac{E[\epsilon_t\epsilon_{t-h}]}{E[\epsilon_t^2]}.$$

The process that arises in the key approximation is:

$$\mathcal{Z}_n(h) \equiv \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} z_t(h). \tag{A.4}$$

Assumption 2 (plug-in: response and identification)**.**

a. Level response. *$f : \mathbb{R}^{k_x} \times \Phi \to \mathbb{R}$, where Φ is a compact subset of $\mathbb{R}^{k_\phi}$, $k_\phi \geq 0$; $f(x, \phi)$ is Borel measurable for each ϕ, and for each x three times continuously differentiable, where $(\partial/\partial\phi)^j f(x, \phi)$ is Borel measurable for each ϕ and $j = 1, 2, 3$; $E[\sup_{\phi \in \mathcal{N}_{\phi_0}} |(\partial/\partial\phi)^j f(x_t, \phi)|^6] < \infty$ for $j = 0, 1, 2, 3$ and some compact set with positive measure $\mathcal{N}_{\phi_0} \subseteq \Phi$ containing ϕ_0.*

b. Volatility. *$\sigma_t^2 : \Theta \to [0, \infty)$ where $\Theta = \Phi \times \Delta \in \mathbb{R}^{k_\theta}$, and Δ is a compact subset of $\mathbb{R}^{k_\delta}$, $k_\delta \geq 0$; $\sigma_t^2(\theta)$ is $\mathcal{F}_{t-1}$-measurable, continuous, and three times continuously differentiable, where $(\partial/\partial\theta)^j \ln \sigma_t^2(\theta)$ is Borel measurable for each θ and $j = 0, 1, 2, 3$; $\inf_{\theta \in \Theta} |\sigma_t^2(\theta)| \geq \iota > 0$ a.s. and $E[\sup_{\theta \in \mathcal{N}_{\theta_0}} |(\partial/\partial\theta)^j \ln \sigma_t^2(\theta)|^4] < \infty$ for $j = 0, 1, 2, 3$ and some compact subset $\mathcal{N}_{\theta_0} \subseteq \Theta$ containing θ_0.*

c. Estimator. *$\hat{\theta}_n \in \Theta$ for each n, and for a unique interior point $\theta_0 \in \Theta$ we have $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A} n^{-1/2} \sum_{t=1}^{n} m_t(\theta_0) + \mathcal{R}_m(n)$, where the $k_m \times 1$ stochastic remainder $\mathcal{R}_m(n) = O_p(n^{-\zeta})$ for some $\zeta > 0$, with $\mathcal{F}_t$-measurable estimating equations $m_t = [m_{i,t}]_{i=1}^{k_m} : \Theta \to \mathbb{R}^{k_m}$ for $k_m \geq k_\theta$, and non-stochastic $\mathcal{A} \in \mathbb{R}^{k_\theta \times k_m}$. Moreover, zero mean $m_t(\theta_0)$ is stationary, ergodic, $L_{r/2}$-bounded and L_2-NED with size $1/2$ on $\{\upsilon_t\}$, where $r > 4$ and $\{\upsilon_t\}$ appear in Assumption 1.b.*

d. Nondegenerate Finite Dimensional Variance. *Let $\mathcal{L} \in \mathbb{N}$ be arbitrary, and let $\lambda \equiv [\lambda_h]_{h=1}^{\mathcal{L}} \in \mathbb{R}^{\mathcal{L}}$. Then* $\liminf_{n\to\infty} \inf_{\lambda'\lambda=1} E[(\sum_{h=1}^{\mathcal{L}} \lambda_h \mathcal{Z}_n(h))^2] > 0$.

Assumption 2.c′. *$\hat{\theta}_n \in \Theta$ for each n, and for a unique interior point $\theta_0 \in \Theta$ we have $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A} n^{-1/2} \sum_{t=1}^{n} m_t(\theta_0) + \mathcal{R}_m(n)$ where the $k_m \times 1$ stochastic remainder $\mathcal{R}_m(n) = O_p(n^{-\zeta})$ for some $\zeta > 0$, with $\mathcal{F}_t$-measurable estimating equations $m_t = [m_{i,t}]_{i=1}^{k_m} : \Theta \to \mathbb{R}^{k_m}$ for $k_m \geq k_\theta$; and non-stochastic $\mathcal{A} \in \mathbb{R}^{k_\theta \times k_m}$. $m_t(\theta)$ is twice continuously differentiable, $(\partial/\partial\theta)^j m_t(\theta)$ is Borel measurable for each θ and $j = 1, 2$, and $E[\sup_{\theta \in \Theta} |(\partial/\partial\theta)^i m_{j,t}(\theta)|] < \infty$ for each $i = 0, 1, 2$ and $j = 1, ..., k_m$. Moreover, zero mean m_t is stationary, ergodic, $L_{r/2}$-bounded and L_2-NED with size $1/2$ on $\{\upsilon_t\}$, where $r > 4$ and $\{\upsilon_t\}$ appear in Assumption 1.b.*

We use the following variance bound for NED sequences repeatedly and without further citation. If w_t is zero mean, stationary, L_p-bounded for some $p > 2$, and L_2-NED with size $1/2$, on an α-mixing base with decay $O(h^{-p/(p-2)-\iota})$, then by Theorem 17.5 in Davidson (1994) and Theorem 1.6 in McLeish (1975):

$$E\left[\left(\frac{1}{\sqrt{n}} \sum_{t=1}^{n} w_t\right)^2\right] = O(1). \tag{A.5}$$

B Moment Contraction and NED

Consider Wu's (2005) physical dependence measure (see also Wu and Shao, 2004, Wu and Min, 2005) exploited in Shao (2011), Xiao and Wu (2014) and Guay, Guerre, and Lazarová (2013) for white noise tests. Assume there exists a measurable function g and random variables $\{v_t\}_{t\in\mathbb{Z}}$ such that

$$\epsilon_t = g(\nu_t, \nu_{t-1}, ...)$$

is a well-defined random variable. If v_t is iid then $\{\epsilon_t\}$ is a stationary and ergodic process. Let $\epsilon_t' = g(\nu_t, \nu_{t-1}, ..., v_0', v_{-1}', ...)$ where $\{v_t'\}_{t\in\mathbb{Z}}$ is an iid copy of $\{v_t\}_{t\in\mathbb{Z}}$, thus ϵ_t' is a coupled version of ϵ_t. The dependence quantity

$$d_t^{(\epsilon)}(q) \equiv \left\|\epsilon_t - \epsilon_t'\right\|_q$$

measures the impact of a distant past on ϵ_t as $t \to \infty$. We say that a random variable z_t is L_q*-moment contracting*, or MC_q, when z_t is L_q-bounded and

$$\sum_{k=0}^{\infty} d_k^{(z)}(q) < \infty.$$

Under stationary MC_2, $\{z_t\}$ is a short memory process since (see, e.g. Shao, 2011, Remark 2.1):

$$\sum_{h=0}^{\infty} |E[z_t z_{t-h}]| \leq \left(\sum_{k=0}^{\infty} d_k^{(z)}(2)\right)^2 < \infty.$$

We say z_t is geometrically MC_q, or GMC_q, if $E|\epsilon_t - \epsilon_t'|^q \leq K\rho^t$ for some $\rho \in (0,1)$.

It is easy to show

$$\left\|\epsilon_t\epsilon_{t-h} - \epsilon_t'\epsilon_{t-h}'\right\|_{q/2} \leq K\left\{d_t(q) + d_{t-h}(q)\right\},$$

hence $\epsilon_t\epsilon_{t-h}$ is MC_2 when ϵ_t is MC_4, and therefore $\{\epsilon_t\epsilon_{t-h}\}$ is a short memory process when ϵ_t is MC_4.

If $\epsilon_t\epsilon_{t-h}$ is L_2-NED with size $1/2$ then by Theorem 1.6 in McLeish (1975), cf. Davidson (1994, Theorem 17.2):

$$\mathcal{S} \equiv \lim_{n\to\infty} E\left[\left(\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\left\{\epsilon_t\epsilon_{t-h} - E[\epsilon_t\epsilon_{t-h}]\right\}\right)^2\right] < \infty.$$

Conversely, MC_4 does not generally imply $\mathcal{S} < \infty$, hence $\mathcal{S} < \infty$ would have to be assumed under an MC assumption. In the present context, $\mathcal{S} < \infty$ holds if: (i) a standard cumulant series bound holds which essentially reduces to $\mathcal{S} < \infty$ (Wu and Shao, 2004, Shao, 2011); or (ii) ϵ_t is GMC_4 (Wu and Shao, 2004, Proposition 2, cf. Wu, 2005); or (iii) $\epsilon_t = \sum_{i=0}^{\infty}\psi_i v_{t-i}$ with zero mean $v_t = g(\omega_t, \omega_{t-1}, ...)$, iid ω_t, $\sum_{i,j=0}^{\infty}|\psi_i\psi_j E[v_0 v_{i-j}]| < \infty$ and $\sum_{i=0}^{\infty} i^{1/2}|\psi_i| < \infty$, provided v_t and $\epsilon_t\epsilon_{t-h}$ are respectively L_2- and L_4-*weakly dependent* (Wu and Min, 2005, Theorem 3, Proposition 1). See Hannan (1973) and Wu and Min (2005) for definitions of L_p-*weak dependence*. If $\epsilon_t\epsilon_{t-h}$ is MC_2 then it is L_2-weakly dependent (Wu, 2005, Theorem 1), hence if $\mathcal{S} < \infty$ then central limit theorems in Hannan (1973) and Wu and Min (2005) apply.

Now define

$$\mathcal{P}_0\epsilon_t \equiv E[\epsilon_t|\mathfrak{V}_0] - E[\epsilon_t|\mathfrak{V}_{-1}] \text{ where } \mathfrak{V}_t \equiv \sigma(\upsilon_\tau : \tau \leq t).$$

By Theorem 1 in Wu (2005), $||\mathcal{P}_0\epsilon_t||_q \leq d_t^{(\epsilon)}(q)$, hence MC$_q$ implies $\sum_{k=0}^{\infty} ||\mathcal{P}_0\epsilon_t||_q < \infty$. Similarly, if ϵ_t is L_q-NED with size 1 on $\{\nu_t\}$ then ϵ_t is an $\mathfrak{V}_t$-adapted L_q-mixingale with size 1 and therefore $\sum_{k=0}^{\infty} ||E[\epsilon_k|\mathfrak{F}_0]||_q < 0$ hence $\sum_{k=0}^{\infty} ||\mathcal{P}_0\epsilon_t||_q < \infty$ (see, e.g., Meng and Lin, 2009). Thus, MC$_q$ and L_q-NED with size 1 equally imply a sufficient condition for linear processes of ϵ_t to satisfy a central limit theorem (Hannan, 1973, Wu and Min, 2005). However, in Assumption 1 we only need ϵ_t to be L_4-NED with size 1/2, which allows for slower memory decay than MC$_4$.

In addition to MC$_8$ and therefore a finite 8^{th} moment, Shao (2011) imposes an eighth order cumulant summability condition which ultimately ensures a cross product partial sum has a finite second moment. As discussed above, L_4-NED under Assumption 1.b implies $\mathcal{S} < \infty$, hence no additional cumulant condition is required.

Guay, Guerre, and Lazarová (2013) impose an MC$_{12}$ condition and therefore require at least a 12^{th} moment. Further, rather than impose sufficient conditions for weak convergence as in Lobato (2001), Shao (2011), and in the present paper under Assumption 1, they assume it under their Assumption M in the form of joint weak convergence of a covariance process and the plug-in estimator. They claim their Assumption M holds for linear processes and a least squares estimator, but no further processes or estimators are considered.

Xiao and Wu (2014) impose MC$_4$ and restrict the degree of dependence based on the rate of increase of the maximum lag. We do not tie the degree of dependence to the maximum lag, while L_4-NED with size 1/2 allows for slower memory decay.

C Examples

We give several examples of models under (A.1) in order to verify the assumptions. Recall

$$G_t(\phi) \equiv \left[\frac{\partial}{\partial \phi'} f(x_{t-1}, \phi), \mathbf{0}'_{k_\delta}\right]' \in \mathbb{R}^{k_\theta} \quad \text{and} \quad s_t(\theta) \equiv \frac{1}{2}\frac{\partial}{\partial \theta} \ln \sigma_t^2(\theta)$$

$$\mathcal{D}(h) \equiv E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\right] + E\left[\epsilon_t\left(\epsilon_{t-h}s_{t-h} + \frac{G_{t-h}}{\sigma_{t-h}}\right)\right] \in \mathbb{R}^{k_\theta}.$$

Under Assumption 2

$$\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A}\frac{1}{n^{1/2}}\sum_{t=1}^{n} m_t(\theta_0) + o_p(1),$$

with $\mathcal{F}_t$-measurable estimating equations $m_t = [m_{i,t}]_{i=1}^{k_m} : \Theta \to \mathbb{R}^{k_m}$ for $k_m \geq k_\theta$, and non-stochastic $\mathcal{A} \in \mathbb{R}^{k_\theta \times k_m}$. Define

$$z_t(h) \equiv r_t(h) - \rho(h)r_t(0) \text{ where } r_t(h) \equiv \frac{\epsilon_t\epsilon_{t-h} - E\left[\epsilon_t\epsilon_{t-h}\right] - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]} \text{ and } \rho(h) \equiv \frac{E[\epsilon_t\epsilon_{t-h}]}{E[\epsilon_t^2]},$$

and

$$\mathcal{Z}_n(h) \equiv \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} z_t(h).$$

Lemma 2.1 states that under Assumptions 1 and 2:

$$\left|\vartheta\left(\sqrt{n}\left[\hat{\rho}_n(h) - \rho(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\mathcal{Z}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right| \leq \vartheta\left(\left[\sqrt{n}\{\hat{\rho}_n(h) - \rho(h)\} - \mathcal{Z}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \xrightarrow{p} 0,$$

for some non-unique sequence $\{\mathcal{L}_n\}$ of positive integers, where $\mathcal{L}_n \to \infty$ and $\mathcal{L}_n = o(n)$.

We specifically characterize the form of $\mathcal{Z}_n(h)$ under the null in order to compute the bootstrapped p-value:

$$\mathcal{Z}_n(h) \equiv \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \frac{\{\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t\}}{E\left[\epsilon_t^2\right]}. \tag{C.1}$$

The exact form of $\mathcal{Z}_n(h)$ can be simplified depending on whether ϵ_t is assumed independent under the null, the regressors $\{x_t\}$ are independent of the sequence $\{\epsilon_t\}$, the stochastic volatility component σ_t is estimated, and the level response f is linear.

In the following we operate under the null.

C.1 Level Response

The level response model is

$$y_t = f(x_{t-1}, \phi_0) + u_t.$$

Assume $f(\cdot, \phi)$ is three times continuously differentiable in $\phi \in \mathbb{R}^{k_\phi}$, $E[u_t^2] < \infty$, and $E[G_t G_t']$ is finite and positive definite where $G_t = G_t(\phi_0) = [G_{t,i}(\phi_0)]_{i=1}^k \equiv (\partial/\partial\phi) f(x_{t-1}, \phi_0)$. Define nonlinear least squares estimating equations $m_t(\phi) = (y_t - f(x_{t-1}, \phi)) \times (\partial/\partial\phi) f(x_{t-1}, \phi)$. Assume $E[m_t(\phi)] = 0$ *if and only if* $\phi = \phi_0$, a unique interior point of compact Φ.

Assume $\{u_t, x_t\}$ are stationary L_r-bounded, $r > 4$, and α-mixing with coefficients $\alpha_h = O(h^{-r/(r-4)}/\ln(h))$. Assume $E[\sup_{\phi \in \mathcal{N}_{\phi_0}} |G_{t,i}(\phi)|^r] < \infty$ for each i, some $r > 4$, and some compact $\mathcal{N}_{\phi_0} \subseteq \Phi$ containing ϕ_0. Many nonlinear response functions satisfy this condition under L_r-boundedness of $\{u_t, x_t\}$, including linear, logistic, and trigonometric functions. Then m_t is stationary, L_r-bounded, and α-mixing. Sufficient conditions for stationary geometric ergodicity of nonlinear AR-GARCH with iid innovations are in Meitz and Saikkonen (2008), amongst others, cf. Doukhan (1994, Chapt. 2.4.2).

Define the nonlinear least squares estimator $\hat{\phi}_n = \arg\min_{\phi \in \Phi}\{1/n \sum_{t=1}^{n}(y_t - f(x_{t-1}, \phi))^2\}$. By construction $s_t = .5(\partial/\partial\theta)\ln\sigma_t^2 = 0$ since $\sigma_t^2 = 1$, and $\mathcal{D}(h) = E[u_{t-h}G_t] + E[u_t G_{t-h}] = E[u_{t-h}G_t]$. Then Assumptions 1 and 2 hold, with $m_t = u_t G_t$ and $\mathcal{A} = (E[G_t G_t'])^{-1}$, hence

$$\sqrt{n}\hat{\rho}_n(h) = \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \frac{u_t\left\{u_{t-h} - E[u_{t-h}G_t]'\left(E\left[G_t G_t'\right]\right)^{-1} G_t\right\}}{E\left[u_t^2\right]} + o_p(1). \tag{C.2}$$

If additionally u_t is independent of the sequence $\{x_t\}$, then $E[u_{t-h}G_t] = 0$, hence

$$\sqrt{n}\hat{\rho}_n(h) = \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} u_t u_{t-h}/E[u_t^2] + o_p(1),$$

the well-known result that $\hat{\phi}_n$ does not impact the limit distribution of $\sqrt{n}\hat{\rho}_n(h)$ (cf. Wooldridge, 1990).

C.2 Linear Response with Least Squares

The model is

$$y_t = \phi_0' x_{t-1} + u_t, \quad E[u_t] = 0.$$

Let $E[(y_t - \phi_0' x_{t-1})u_t] = 0$ for a unique interior $\phi_0 \in \Phi$, and assume $E[x_t x_t']$ is positive definite. Assume $\{x_t, u_t\}$ are stationary and ergodic, L_r-bounded, $p > 4$, and L_4-NED on an α-mixing base with coefficient decay $O(h^{-r/(r-4)-\iota})$. An AR process with an iid error that has a continuous bounded distribution is geometrically α-mixing and therefore geometrically NED. This extends to linear or nonlinear GARCH errors (see, e.g., Doukhan, 1994, Meitz and Saikkonen, 2008).

By construction $G_t = x_{t-1}$ and $s_t = 0$ hence $\mathcal{D}(h) \equiv E[u_{t-h}x_{t-1}]$. If $\hat{\phi}_n$ is the least squares estimator then Assumptions 1 and 2 are satisfied, with $\mathcal{A} = (E[x_t x_t'])^{-1}$ and $m_t = u_t x_{t-1}$. Therefore:

$$\sqrt{n}\hat{\rho}_n(h) = \frac{1}{E\left[u_t^2\right]} \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} u_t \left\{u_{t-h} - E[u_{t-h}x_{t-1}]'(E[x_t x_t'])^{-1} x_{t-1}\right\} + o_p(1).$$

If $u_t^2 - E[u_t^2]$ is an adapted mds then $E[m_{i,t}^2] = E[u_t^2]E[x_{i,t}^2] < \infty$ and $E[(u_t u_{t-h})^2] = (E[u_t^2])^2 < \infty$, hence we only need $\{x_t, u_t\}$ to be L_r-bounded, $r > 2$, and α-mixing.

C.3 Mean Model with Sample Mean

The mean model is $y_t = E[y_t] + u_t$, hence $f(x_{t-1}, \phi_0) = E[y_t]$, $\sigma_t = 1$ and $E[u_t] = 0$. Assume $\{y_t\}$ is stationary, ergodic, L_r-bounded for some $r > 4$, and L_4-NED on an α-mixing base with decay $O(h^{-r/(r-4)-\iota})$. Then $G_t = 1$ and $s_t = 0$, hence $\mathcal{D}(h) \equiv E[u_t u_{t-h}] + E[u_t] = E[(y_t - E[y_t])(y_{t-h} - E[y_{t-h}])]$ hence $\mathcal{D}(h) = 0$ under H_0 for $h \geq 1$. The plug-in estimator is the sample mean $\hat{\theta}_n = 1/n \sum_{t=1}^{n} y_t$, so that $m_t = u_t$ and $\mathcal{A} = 1$. Assumptions 1 and 2 are satisfied, hence from (C.2):

$$\sqrt{n}\hat{\rho}_n(h) = \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \frac{(y_t - E[y_t])(y_{t-h} - E[y_{t-h}])}{E\left[(y_t - E[y_t])^2\right]} + o_p(1).$$

C.4 GARCH(1,1) with QML

The model is GARCH(1,1) $y_t = \sigma_t \epsilon_t$ with $\sigma_t^2 = \omega_0 + \alpha_0 y_{t-1}^2 + \beta_0 \sigma_{t-1}^2$, $\omega_0, \alpha_0, \beta_0 > 0$, $E[\epsilon_t] = 0$ and $E[\epsilon_t^2] = 1$. We ignore boundary cases by assuming $\alpha_0, \beta_0 > 0$. The model includes weak, semi-strong or strong GARCH (see Drost and Nijman, 1993), in which case the model is correct in some sense since ϵ_t

is assumed to be serially uncorrelated. Conditions for strict stationarity in the case of iid or mds ϵ_t are given in Nelson (1990) and Lee and Hansen (1994), and Boussama (2006) proves geometric ergodicity.

Let $\theta \equiv [\omega, \alpha, \beta]'$, and $\Theta = [\iota_\omega, u_\omega] \times [0, 1-\iota] \times [0, 1-\iota]$, where $u_\omega > \iota_\omega > 0$ and $\iota \in (0,1)$. Define the unobserved volatility process $\sigma_t^2(\theta) = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2(\theta)$ on Θ, and define the iterated process used for estimation: $\tilde{\sigma}_0^2(\theta) = \omega$ and $\tilde{\sigma}_t^2(\theta) = \omega + \alpha y_{t-1}^2 + \beta \tilde{\sigma}_{t-1}^2(\theta)$ for $t \geq 1$. Let θ_0 be the unique interior point of Θ such that $\sigma_t^2(\theta_0) = \sigma_t^2$ and $E[(y_t^2/\sigma_t^2(\theta_0) - 1)(\partial/\partial\theta)\ln(\sigma_t^2(\theta_0))] = 0$, the QML first order moment condition. The feasible QML estimator is $\hat{\theta}_n \equiv \arg\inf_{\theta\in\Theta}\{\sum_{t=1}^n \{\ln \tilde{\sigma}_t^2(\theta) + \sum_{t=1}^n y_t^2/\tilde{\sigma}_t^2(\theta)\}\}$.[1] See Francq and Zakoïan (2004) for refined QML asymptotics when ϵ_t is iid, and see Lee and Hansen (1994) for the semi-strong case.

Since our assumptions must hold whether ϵ_t is white noise or not, under potentially much weaker conditions than weak-GARCH (Drost and Nijman, 1993), we assume $\{y_t, \epsilon_t\}$ are stationary and ergodic, $(E|y_t|^\iota, E|\sigma_t^2|^\iota) < \infty$ for some $\iota > 0$, $\inf_{\theta\in\Theta} |\sigma_t^2(\theta)| \geq \iota > 0$ $a.s.$, and $\{\epsilon_t, (\partial/\partial\theta)^i \ln(\sigma_t^2(\theta_0)) : i = 0,1,2,3\}$ are stationary geometrically α-mixing. Further, for each $\theta \in \Theta$ unique stationary and ergodic solutions exist for the iterated process and its derivatives $\{(\partial/\partial\theta)^j \tilde{\sigma}_i^2(\theta) : j = 0,1,2,3\}_{i=0}^t$ as $t \to \infty$ at a geometric rate. We also require for some compact subset $\mathcal{N}_{\theta_0} \subseteq \Theta$ containing θ_0:

$$E\left[s_t(\theta_0)s_t'(\theta_0)\right] - E\left[\left(\frac{y_t^2}{\sigma_t^2(\theta_0)} - 1\right)\frac{\partial^2}{\partial\theta\partial\theta'}\ln(\sigma_t^2(\theta_0))\right] \text{ is non-singular} \tag{C.3}$$

$$E\left[\sup_{\theta\in\mathcal{N}_{\theta_0}}\left|\left(\frac{\partial}{\partial\theta}\right)^j \ln \sigma_t^2(\theta)\right|^4\right] < \infty \text{ for each } j = 1,2,3. \tag{C.4}$$

If ϵ_t is iid, or $\{\epsilon_t, \epsilon_t^2 - 1\}$ are martingale differences adapted to some sequence of sigma fields $\{\mathcal{G}_t\}$, then stationary solutions exist respectively when $E[\ln(\omega_0 + \alpha_0\epsilon_t^2)] < 0$ and $E[\ln(\omega_0 + \alpha_0\epsilon_t^2)|\mathcal{G}_t] < 0$ $a.s.$ (Nelson, 1990, Lee and Hansen, 1994). Write $s_t(\theta) \equiv 0.5 \times (\partial/\partial\theta)\ln(\sigma_t^2(\theta))$. If ϵ_t is iid or $\{\epsilon_t, \epsilon_t^2 - 1\}$ are martingale differences then (C.3) holds, and (C.4) holds by arguments in Francq and Zakoïan (2004, Section 4.2). The latter assume an iid error, but their proofs of (C.4) do not make use of independence. See, e.g., their equation (4.28).

Under the above conditions, Assumptions 1 and 2 hold. Assuming θ_0 does not lie on the boundary of Θ, plug-in estimator Assumption 2.c holds with $m_t = (\epsilon_t^2 - 1)s_t$ and $\mathcal{A} = \{2E[\epsilon_t^2 s_t s_t'] - E[(\epsilon_t^2 - 1)(\partial/\partial\theta)s_t(\theta_0)]\} = \{2E[s_t s_t'] - E[(\epsilon_t^2 - 1)(\partial/\partial\theta)s_t(\theta_0)]\}^{-1}$. Finally, $G_t = 0$ hence $\mathcal{D}(h) \equiv E[\epsilon_t\epsilon_{t-h}(s_t + s_{t-h})]$, and expansion (C.1) holds.

D Local Asymptotic Power

Hong (1996) shows that his standardized periodogram yields non-trivial asymptotic power against a sequence of local alternatives applied to the spectrum, with a slower than $\sqrt{n}$ drift. Ultimately the

[1]Since we assume the start condition $\tilde{\sigma}_0^2(\theta) = \omega$ we avoid the case where $\alpha_0 = 0$, β_0 is not identified and is therefore a nuisance parameter, and there are no GARCH effects (see Andrews, 2001). We do not allow nuisance parameters for brevity, but their inclusion is straightforward, although beyond this paper's scope.

reduced rate arises from an increasing bandwidth parameter. Using a similar spectrum local alternative, but with $\sqrt{n}$ drift, Shao (2011) proves the Cramér-von Mises test achieves non-trivial local power. Delgado and Velasco (2011) impose local $\sqrt{n}$ drift to the correlations, and show that a weighted average portmanteau statistic yields non-trivial local power.

The proposed bootstrap p-value test operates on $\sqrt{n}\hat{\rho}_n(h)$ and $\sqrt{n}\hat{\rho}_n^{(dw)}(h)$. The test therefore achieves the parametric rate of local asymptotic power against the sequence of alternatives:

$$H_1^L : \rho(h) = \rho_n(h) = \frac{r(h)}{\sqrt{n}} \text{ for each } h, \text{ where } |r(h)| \leq \sqrt{n}.$$

Note that $r(h)$ is a fixed constant for each h, where $|r(h)| \leq \sqrt{n}$ ensures $|\rho_n(h)| \leq 1$.

Theorem D.1. *Let Assumptions 1 and 2, $\widehat{\mathcal{A}}_n \xrightarrow{p} \mathcal{A}$, and $M = M_n \to \infty$ hold. There exists a non-unique sequence of maximum lags $\{\mathcal{L}_n\}$, $\mathcal{L}_n \to \infty$ and $\mathcal{L}_n = o(n)$, such that under H_1^L, $\lim_{n\to\infty} P(\hat{p}_{n,M}^{(dw)} < \alpha)$ $> \alpha$ if $r(h) \neq 0$ for some $h \in \mathbb{N}$. Further $\lim_{n\to\infty} P(\hat{p}_{n,M}^{(dw)} < \alpha) \nearrow 1$ monotonically in $|r(h)| \nearrow \infty$.*

Proof.

Step 1. Recall $\bar{F}_n^{(0)}(c) \equiv P(\vartheta([\mathring{\mathcal{Z}}(h)]_{h=1}^{\mathcal{L}_n}) > c)$, where $\{\mathring{\mathcal{Z}}(h) : h \in \mathbb{N}\}$ is an independent copy of $\{\mathcal{Z}(h) : h \in \mathbb{N}\}$, the Lemma 2.2 zero mean Gaussian process with variance $E[\mathcal{Z}(h)^2] < \infty$. Recall $\rho_n(h)$ $= r(h)/\sqrt{n}$ under H_1^L. We prove in Step 2 that

$$\mathcal{P}_{n,1}^{(L)}(\alpha) + o_p(1) \leq \hat{p}_{n,M_n}^{(dw)} \leq \mathcal{P}_{n,2}^{(L)}(\alpha) + o_p(1), \tag{D.1}$$

where

$$\mathcal{P}_{n,1}^{(L)}(\alpha) = \bar{F}_n^{(0)}\left(\vartheta\left([\mathcal{Z}(h)]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left([r(h)]_{h=1}^{\mathcal{L}_n}\right)\right)$$
$$\mathcal{P}_{n,2}^{(L)}(\alpha) = \bar{F}_n^{(0)}\left(-\vartheta\left([\mathcal{Z}(h)]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left([r(h)]_{h=1}^{\mathcal{L}_n}\right)\right).$$

By construction

$$P\left(\bar{F}_n^{(0)}\left(\vartheta\left([\mathring{\mathcal{Z}}(h)]_{h=1}^{\mathcal{L}_n}\right)\right) < \alpha\right) = \alpha \quad \text{and} \quad P\left(-\bar{F}_n^{(0)}\left(\vartheta\left([\mathring{\mathcal{Z}}(h)]_{h=1}^{\mathcal{L}_n}\right)\right) < \alpha\right) = 1 - \alpha.$$

Therefore, if $r(h) \neq 0$ for some $h \in \mathbb{N}$:

$$\lim_{n\to\infty} P\left(\hat{p}_{n,M}^{(dw)} < \alpha\right) > \alpha$$

Moreover, monotonically $P(\mathcal{P}_{n,1}^{(L)}(\alpha) < \alpha) \searrow 0$ and $P(\mathcal{P}_{n,2}^{(L)}(\alpha) < \alpha) \searrow 0$ as $\vartheta([r(h)]_{h=1}^{\mathcal{L}_n}) \to \infty$ and therefore as $\max_{1\leq h\leq \mathcal{L}_n} |r(h)| \to \infty$ for each n. Therefore

$$P\left(\hat{p}_{n,M}^{(dw)} < \alpha\right) \leq P\left(o_p(1) < \alpha - \mathcal{P}_{n,2}^{(L)}(\alpha)\right) \nearrow 1 \text{ as } |r(h)| \nearrow \infty$$
$$P\left(\hat{p}_{n,M}^{(dw)} < \alpha\right) \geq P\left(o_p(1) < \alpha - \mathcal{P}_{n,1}^{(L)}(\alpha)\right) \nearrow 1 \text{ as } |r(h)| \nearrow \infty$$

hence $\lim_{n\to\infty} P(\hat{p}^{(dw)}_{n,M} < \alpha) \nearrow 1$ monotonically in $|r(h)| \nearrow \infty$.

Step 2. We now prove (D.1). Recall ϑ satisfies the triangle inequality. Thus

$$\begin{aligned}
\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) &\leq \vartheta\left(\left[\sqrt{n}\left\{\hat{\rho}_n(h) - \rho_n(h)\right\}\right]_{h=1}^{\mathcal{L}_n}\right) \\
\vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) &\leq \vartheta\left(\left[\sqrt{n}\left\{\hat{\rho}_n(h) - \rho_n(h)\right\}\right]_{h=1}^{\mathcal{L}_n}\right)
\end{aligned}$$

and therefore

$$\left|\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right| \leq \vartheta\left(\left[\sqrt{n}\left\{\hat{\rho}_n(h) - \rho_n(h)\right\}\right]_{h=1}^{\mathcal{L}_n}\right).$$

Now apply arguments used in the proof of Theorem 2.5 under H_0, and the triangle inequality, to yield:

$$\begin{aligned}
\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) &\leq \left|\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right| + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \\
&\leq \vartheta\left(\left[\sqrt{n}\left\{\hat{\rho}_n(h) - \rho_n(h)\right\}\right]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \\
&= \vartheta\left(\left[\mathcal{Z}(h)\right]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) + o_p(1)
\end{aligned}$$

hence

$$\begin{aligned}
\hat{p}^{(dw)}_{n,M_n} &= P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq \vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1) \\
&\geq P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq \left|\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right|\right. \\
&\qquad \left. + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1) \\
&\geq \bar{F}^{(0)}_n\left(\vartheta\left(\left[\mathcal{Z}(h)\right]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right) + o_p(1).
\end{aligned}$$

Similarly:

$$\begin{aligned}
\hat{p}^{(dw)}_{n,M_n} &= P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq \vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1) \\
&= P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq \vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right. \\
&\qquad \left. - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1) \\
&\leq P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq -\left|\vartheta\left(\left[\sqrt{n}\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right|\right. \\
&\qquad \left. + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1) \\
&\leq P\left(\vartheta\left(\left[\sqrt{n}\hat{\rho}^{(dw)}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) \geq -\left|\vartheta\left(\left[\sqrt{n}\left\{\hat{\rho}_n(h) - \rho_n(h)\right\}\right]_{h=1}^{\mathcal{L}_n}\right)\right|\right. \\
&\qquad \left. + \vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) |\mathfrak{X}_n\right) + o_p(1)
\end{aligned}$$

$$= \quad \bar{F}_n^{(0)}\left(-\vartheta\left(\left[\mathcal{Z}(h)\right]_{h=1}^{\mathcal{L}_n}\right)+\vartheta\left(\left[\sqrt{n}\rho_n(h)\right]_{h=1}^{\mathcal{L}_n}\right)\right)+o_p(1).$$

This completes the proof. $\mathcal{QED}$.

E Nonparametric Self-Standardization: Kernel Variance Estimation

In order to control for variable dispersion across lags, a natural choice for $\hat{\omega}_n(h)$ is an inverted standard deviation estimator. Based on Corollary 2.4 we need to estimate the asymptotic variance under the null, identically $\lim_{n\to\infty} E[(1/\sqrt{n}\sum_{t=1}^n r_t(h))^2]$. A now classic approach exploits a kernel estimator, cf. Newey and West (1987), Andrews (1991) and De Jong and Davidson (2000). See, e.g., Shao (2010) for a viable alternative to kernel estimators.

Recall $m_t(\theta)$ are the estimating equations for $\hat{\theta}_n$, let $\widehat{\mathcal{A}}_n$ be a consistent estimator of $\mathcal{A}$ in Assumption 2.c, and define

$$\hat{\mathcal{D}}_n(h) \equiv \frac{1}{n}\sum_{t=h+1}^{n}\left\{\left(\epsilon_t(\hat{\theta}_n)s_t(\hat{\theta}_n)+\frac{G_t(\hat{\theta}_n)}{\sigma_t(\hat{\theta}_n)}\right)\epsilon_{t-h}(\hat{\theta}_n)+\epsilon_t(\hat{\theta}_n)\left(\epsilon_{t-h}(\hat{\theta}_n)s_{t-h}(\hat{\theta}_n)+\frac{G_{t-h}(\hat{\theta}_n)}{\sigma_{t-h}(\hat{\theta}_n)}\right)\right\} \tag{E.1}$$

and

$$\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n) \equiv \epsilon_t(\hat{\theta}_n)\epsilon_{t-h}(\hat{\theta}_n) - \hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n m_t(\hat{\theta}_n).$$

The variance estimator and proposed weights are:

$$\hat{\nu}^2_{\mathcal{K},n}(h) \equiv \frac{1/n\sum_{s,t=1}^n \mathcal{K}\left((s-t)/b_n\right)\widehat{\mathcal{E}}_{n,s,h}(\hat{\theta}_n)\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n)}{1/n\sum_{s=1}^n \epsilon_t^2(\hat{\theta}_n)} \quad \text{and} \quad \hat{\omega}_n(h) \equiv \frac{1}{\hat{\nu}_{\mathcal{K},n}(h)},$$

where $\mathcal{K}$ is a kernel function and positive integer b_n is bandwidth satisfying $b_n \to \infty$ as $n \to \infty$. Recall

$$r_t(h) \equiv \frac{\epsilon_t\epsilon_{t-h} - E\left[\epsilon_t\epsilon_{t-h}\right] - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]}.$$

We need only show $\hat{\nu}^2_{\mathcal{K},n}(h) \xrightarrow{p} \lim_{n\to\infty} E[(1/\sqrt{n}\sum_{t=1}^n r_t(h))^2]$ since by Theorem 2.3 the latter is exactly the asymptotic variance of $\hat{\rho}_n(h)$ under the null. Let $i \equiv \sqrt{-1}$.

Assumption 3 (kernel variance)**.**

a. $\mathcal{K}: \mathbb{R} \to [-1,1]$ *satisfies* $\mathcal{K}(0) = 1, \mathcal{K}(x) = \mathcal{K}(-x)\ \forall x \in \mathbb{R}$, $\mathcal{K}(\cdot)$ *is continuous at* 0 *and at all but a finite number of points,* $\int_{-\infty}^{\infty}|\mathcal{K}(x)|dx < \infty$, *and* $\int_{-\infty}^{\infty}|\psi(\xi)|d\xi < \infty$ *where* $\psi(\xi) = (2\pi)^{-1}\int_{-\infty}^{\infty}\mathcal{K}(x)e^{i\xi x}dx$ $< \infty$.

b. $b_n/n + 1/b_n \to 0$.

Lemma E.1. *Under Assumptions 1, 2.a,b,c′,d and 3,* $\hat{\nu}^2_{\mathcal{K},n}(h) \xrightarrow{p} \lim_{n\to\infty} E[(1/\sqrt{n}\sum_{t=1}^n r_t(h))^2]$.

Proof. Define

$$\mathcal{E}_{t,h}(\theta) \equiv \epsilon_t(\theta)\epsilon_{t-h}(\theta) - \mathcal{D}(h)'\mathcal{A}m_t(\theta) \quad \text{and} \quad \mathcal{E}_{t,h} \equiv \mathcal{E}_{t,h}(\theta_0).$$

By the arguments leading to (A.27) in the main paper $1/n\sum_{s=1}^{n}\epsilon_t^2(\hat{\theta}_n)\overset{p}{\to}E[\epsilon_t^2]\in(0,\infty)$, hence we focus on $1/n\sum_{s,t=1}^{n}\mathcal{K}((s-t)/b_n)\widehat{\mathcal{E}}_{n,s,h}(\hat{\theta}_n)\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n)$. Add and subtract terms to yield:

$$\begin{aligned}\hat{\nu}^2_{\mathcal{K},n}(h) &= \frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)\widehat{\mathcal{E}}_{n,s,h}(\hat{\theta}_n)\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n)\\ &= \frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)\mathcal{E}_{s,h}(\hat{\theta}_n)\mathcal{E}_{t,h}(\hat{\theta}_n)\\ &\quad-\left\{\hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n-\mathcal{D}(h)'\mathcal{A}\right\}\frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)\mathcal{E}_{s,h}(\hat{\theta}_n)m_t(\hat{\theta}_n)\\ &\quad-\left\{\hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n-\mathcal{D}(h)'\mathcal{A}\right\}\frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)m_s(\hat{\theta}_n)\mathcal{E}_{t,h}(\hat{\theta}_n)\\ &\quad+\left\{\hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n-\mathcal{D}(h)'\mathcal{A}\right\}\frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)m_s(\hat{\theta}_n)m_t(\hat{\theta}_n)'\left\{\hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n-\mathcal{D}(h)'\mathcal{A}\right\}'.\end{aligned}$$

We will prove

$$\frac{1}{n}\sum_{s,t=1}^{n}\mathcal{K}\left(\frac{s-t}{b_n}\right)\mathcal{E}_{s,h}(\hat{\theta}_n)\mathcal{E}_{t,h}(\hat{\theta}_n)\overset{p}{\to}\lim_{n\to\infty}E\left[\left(\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\mathcal{E}_{t,h}\right)^2\right]<\infty. \qquad \text{(E.2)}$$

Proofs that the remaining summands converge in probability to finite constants is similar. Furthermore, $\hat{\mathcal{D}}_n(h)\overset{p}{\to}\mathcal{D}(h)$ by arguments in the proof of Lemma 2.1, and $\widehat{\mathcal{A}}_n\overset{p}{\to}\mathcal{A}$ by supposition. Therefore $\hat{\nu}^2_{\mathcal{K},n}(h)$ $\overset{p}{\to}\lim_{n\to\infty}E[(1/\sqrt{n}\sum_{t=1}^{n}\mathcal{E}_{t,h})^2]/E[\epsilon_t^2]=\lim_{n\to\infty}E[(1/\sqrt{n}\sum_{t=1}^{n}r_t(h))^2]$.

In order to prove (E.2) we will verify the conditions of Theorem 2.2 in De Jong and Davidson (2000), which requires demonstrating their Assumptions 1-4 hold, which we label A1-A4. A1 holds by our stated kernel properties. Under Assumptions 1 and 2, and Theorem 17.8 in Davidson (1994), $\{\mathcal{E}_{t,h}\}$ satisfies A2. A3 holds by our stated bandwidth properties.[2] A4.a holds since $\sqrt{n}(\hat{\theta}_n-\theta_0)=O_p(1)$ by our Assumption 2.c′ and (A.5). A4.b holds by the continuous differentiability properties under Assumption 2.a,b,c′. A4 holds sufficiently if for some open neighborhood $\mathcal{N}_0\subset\Theta$ of θ_0, and every $\xi\in\mathbb{R}$:

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}E\left[\sup_{\theta\in\mathcal{N}_0}\left\|\frac{\partial}{\partial\theta}\mathcal{E}_{t,h}(\theta)\right\|^2\right]<\infty \text{ and } \sup_{\theta\in\mathcal{N}_0}\left|\frac{1}{n}\sum_{t=1}^{n}e^{i\xi t/b_n}\left(\frac{\partial}{\partial\theta}\mathcal{E}_{t,h}(\theta)-E\left[\frac{\partial}{\partial\theta}\mathcal{E}_{t,h}(\theta)\right]\right)\right|\overset{p}{\to}0.$$

The first holds by the moment envelope bounds in Assumption 2.a,b. Consider the second, and note that the term inside $|\cdot|$ has real and imaginary components $\mathcal{R}_n(\theta)$ and $\mathcal{I}_n(\theta)$. Assumption 2a,b,c′ ensures $(\partial/\partial\theta)\mathcal{E}_{t,h}(\theta)$ is $\mathcal{F}_t$-measurable and therefore stationary and ergodic, while, $|e^{i\xi t/b_n}|\leq 1$ for any $t=1,...,n$. Therefore the same arguments used to prove the uniform laws in Step 2 of the proof of Lemma 2.1 carry over here to prove $\sup_{\theta\in\mathcal{N}_0}|\mathcal{R}_n(\theta)|\overset{p}{\to}0$ and $\sup_{\theta\in\mathcal{N}_0}|\mathcal{I}_n(\theta)|\overset{p}{\to}0$. $\mathcal{QED}$.

[2]Assumption 2 in De Jong and Davidson (2000) requires two sets of constants c_{nt} and d_{nt}. We have stationarity and standard asymptotics, hence $c_{nt}=1/\sqrt{n}$. Further, $d_{nt}=K$ since $\mathcal{E}_{t,h}$ is stationary and does not depend on n. Their Assumption 3 states $1/b_n+b_n\max_{1\leq t\leq n}c_{nt}\to 0$ which holds under our assumptions when $c_{nt}=1/\sqrt{n}$.

F Omitted Proofs: Lemmas A.4-A.6

We now present omitted proofs.

F.1 Lemma A.4 (Expansion)

Let $h \geq 0$. Recall $\rho(h) \equiv E[\epsilon_t \epsilon_{t-h}]/E[\epsilon_t^2]$ and

$$\begin{aligned}
&G_t(\phi) \equiv \left[\frac{\partial}{\partial \phi'} f(x_{t-1}, \phi), \mathbf{0}'_{k_\delta}\right]' \in \mathbb{R}^{k_\theta} \quad \text{and} \quad s_t(\theta) \equiv \frac{1}{2}\frac{\partial}{\partial \theta} \ln \sigma_t^2(\theta) \\
&\mathcal{D}(h) \equiv E\left[\left(\epsilon_t s_t + G_t/\sigma_t\right)\epsilon_{t-h}\right] + E\left[\epsilon_t\left(\epsilon_{t-h}s_{t-h} + G_{t-h}/\sigma_{t-h}\right)\right] \in \mathbb{R}^{k_\theta} \\
&z_t(h) \equiv r_t(h) - \rho(h) r_t(0) \text{ where } r_t(h) \equiv \frac{\epsilon_t\epsilon_{t-h} - E\left[\epsilon_t\epsilon_{t-h}\right] - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]},
\end{aligned}$$

where m_t and $\mathcal{A}$ appear in plug-in expansion Assumption 2.c: $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A}n^{-1/2}\sum_{t=1}^n m_t(\theta_0) + O_p(n^{-\zeta})$ for some $\zeta > 0$.

Lemma A.4. *Under Assumptions 1 and 2: for some $\zeta > 0$ that appears in Assumption 2.c, and each $h \geq 0$:*

$$\mathcal{X}_n(h) \equiv \left|\sqrt{n}\left\{\hat{\rho}_n(h) - \rho(h)\right\} - \frac{1}{n^{1/2}}\sum_{t=1+h}^n \left\{r_t(h) - \rho(h)r_t(0)\right\}\right| = O_p\left(\frac{1}{n^{\min\{\zeta, 1/2\}}}\right). \tag{F.1}$$

Proof. There exists θ_n^*, $||\theta_n^* - \theta_0|| \leq ||\hat{\theta}_n - \theta_0||$, that may be different in different places, such that

$$\sqrt{n}\hat{\gamma}_n(h) = \frac{1}{\sqrt{n}}\sum_{t=1+h}^n \epsilon_t\epsilon_{t-h} + \sqrt{n}\mathcal{A}_n(h) + \sqrt{n}\mathcal{A}_n(-h), \tag{F.2}$$

where

$$\begin{aligned}
&\mathcal{A}_n(h) \qquad\qquad (\text{F.3})\\
&= -\left(\hat{\theta}_n - \theta_0\right)'\frac{1}{n}\sum_{t=1+h}^n \left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} \\
&- \left(\hat{\theta}_n - \theta_0\right)'\frac{1}{n}\sum_{t=1+h}^n \epsilon_t s_t \frac{G_{t-h}}{\sigma_{t-h}}\left(\hat{\theta}_n - \theta_0\right) \\
&+ \left(\hat{\theta}_n - \theta_0\right)'\left\{\frac{1}{n}\sum_{t=1+h}^n \epsilon_{t-h}\frac{\partial}{\partial\theta'}\left(\frac{G_t(\phi_n^*)}{\sigma_t(\theta_n^*)}\right) - \frac{1}{n}\sum_{t=1+h}^n \frac{G_t(\phi_n^*)}{\sigma_t(\theta_n^*)}\frac{G_{t-h}(\phi_n^*)'}{\sigma_{t-h}(\theta_n^*)}\right\}\left(\hat{\theta}_n - \theta_0\right) \\
&- \left(\hat{\theta}_n - \theta_0\right)'\left\{\frac{1}{n}\sum_{t=1+h}^n \epsilon_{t-h}(\theta_n^*)\frac{G_t(\phi_n^*)}{\sigma_t(\theta_n^*)}s_{t-h}(\theta_n^*)' + \frac{1}{n}\sum_{t=1+h}^n \epsilon_t(\theta_n^*)s_t(\theta_n^*)\epsilon_{t-h}(\theta_n^*)s_{t-h}(\theta_n^*)'\right\}\left(\hat{\theta}_n - \theta_0\right) \\
&+ \left(\hat{\theta}_n - \theta_0\right)'\frac{1}{n}\sum_{t=1+h}^n \epsilon_t\epsilon_{t-h}\frac{\partial}{\partial\theta'}s_t(\theta_n^*)\left(\hat{\theta}_n - \theta_0\right)
\end{aligned}$$

$$+\left(\hat{\theta}_n-\theta_0\right)'\left\{\frac{1}{n}\sum_{t=1+h}^{n}\epsilon_{t-h}s_t(\theta_n^*)\frac{\partial}{\partial\theta'}\epsilon_t(\theta_n^*)-\frac{1}{n}\sum_{t=1+h}^{n}s_t(\theta_n^*)\frac{G_{t-h}(\phi_n^*)}{\sigma_{t-h}(\theta_n^*)}\frac{\partial}{\partial\theta'}\epsilon_t(\theta_n^*)\right\}\left(\hat{\theta}_n-\theta_0\right)$$

$$-\left(\hat{\theta}_n-\theta_0\right)'\left\{\frac{1}{n}\sum_{t=1+h}^{n}\epsilon_t s_t\frac{\partial}{\partial\theta'}\left(\frac{G_{t-h}(\phi_n^*)}{\sigma_{t-h}(\theta_n^*)}\right)+\frac{1}{n}\sum_{t=1+h}^{n}\epsilon_t\frac{G_{t-h}(\phi_n^*)}{\sigma_{t-h}(\theta_n^*)}\frac{\partial}{\partial\theta'}s_t(\theta_n^*)\right\}\left(\hat{\theta}_n-\theta_0\right)$$

$$=-\left(\hat{\theta}_n-\theta_0\right)'\mathcal{B}_n(h)+\left(\hat{\theta}_n-\theta_0\right)'\sum_{i=1}^{6}\mathcal{C}_{n,i}(h,\theta_n^*)\left(\hat{\theta}_n-\theta_0\right).$$

In the following we will show each

$$\sup_{\theta\in\Theta}\|\mathcal{C}_{n,i}(h,\theta)\|=O_p\left(1\right),$$

while Assumption 2.c implies $\sqrt{n}(\hat{\theta}_n-\theta_0)=O_p(1/\sqrt{n})$. This yields

$$\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)'\sum_{i=1}^{6}\mathcal{C}_{n,i}(h,\theta_n^*)\left(\hat{\theta}_n-\theta_0\right)=O_p\left(1/\sqrt{n}\right),$$

hence

$$\sqrt{n}\hat{\gamma}_n(h)=\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\epsilon_t\epsilon_{t-h}-\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)'\{\mathcal{B}_n(h)+\mathcal{B}_n(-h)\}+O_p\left(1/\sqrt{n}\right).$$

We set the convention that

$$\{\hat{\gamma}_n(h),\mathcal{A}_n(h),\mathcal{B}_n(h),\mathcal{C}_{n,i}(h,\theta)\}=0\ \forall h\geq n-1.$$

Stationarity, ergodicity, and the Assumptions 1 and 2 moment bounds imply $\mathcal{B}_n(h)\xrightarrow{p}E[(\epsilon_t s_t+G_t/\sigma_t)\epsilon_{t-l}]$ and $\mathcal{B}_n(-h)\xrightarrow{p}E[\epsilon_t(\epsilon_{t-h}s_{t-h}+G_{t-h}/\sigma_{t-h})]$ for each h, hence

$$\mathcal{B}_n(h)+\mathcal{B}_n(-h)\xrightarrow{p}\mathcal{D}(h)\equiv E\left[\left(\epsilon_t s_t+G_t/\sigma_t\right)\epsilon_{t-h}\right]+E\left[\epsilon_t\left(\epsilon_{t-h}s_{t-h}+G_{t-h}/\sigma_{t-h}\right)\right]\in\mathbb{R}^{k_\theta}. \tag{F.4}$$

Furthermore,

$$\mathcal{C}_{n,1}(h,\theta_n^*)=\mathcal{C}_{n,1}(h,\theta_0)\xrightarrow{p}E\left[\mathcal{C}_{n,i}(h,\theta_0)\right]\text{ and }E\left[|\mathcal{C}_{n,1}(h,\theta_0)|\right]=O(1).$$

Now let $i=2,...,6$. Stationarity, ergodicity, and the Assumptions 1 and 2 moment bounds imply pointwise $\mathcal{C}_{n,i}(h,\theta)\xrightarrow{p}E[\mathcal{C}_{n,i}(h,\theta)]$ and $E[\sup_{\theta\in\mathcal{N}_{\theta_0}}|\mathcal{C}_{n,i}(h,\theta)|]=O(1)$ for some compact set $\mathcal{N}_{\theta_0}\subseteq\Theta$ containing θ_0.

Moreover, $\mathcal{C}_{n,i}(\cdot,\theta)$ are stochastically equicontinuous on $\mathcal{N}_{\theta_0}$. Consider $i=2$, the remaining terms being similar. Use $\inf_{\theta\in\Theta}\sigma_t^2(\theta)\geq\iota>0$ $a.s.$ under Assumption 2, the mean value theorem and properties of the matrix norm to yield $|\mathcal{C}_{n,i}(h,\theta)-\mathcal{C}_{n,i}(h,\tilde{\theta})|\leq\mathcal{E}_t\times|\theta-\tilde{\theta}|$ for any $\{\theta,\tilde{\theta}\}\in\mathcal{N}_{\theta_0}$, where

$$\mathcal{E}_t\equiv K\left|\epsilon_{t-h}\right|\times\sup_{\theta\in\mathcal{N}_{\theta_0}}\left|\frac{\partial^2}{\partial\phi\partial\phi'}G_t(\phi)\right|\times\left|\theta-\tilde{\theta}\right|$$

$$+K\left|\epsilon_{t-h}\right| \times \sup_{\theta \in \mathcal{N}_{\theta_0}} |G_t(\phi)| \times \sup_{\theta \in \mathcal{N}_{\theta_0}} \left|\frac{\partial^2}{\partial\theta\partial\theta'} \ln\left(\sigma_t(\theta)\right)\right| \times \left|\theta - \tilde{\theta}\right|$$
$$+K\left|\epsilon_{t-h}\right| \times \sup_{\theta \in \mathcal{N}_{\theta_0}} \left\{ \left(K\left|\frac{\partial}{\partial\phi}G_t(\phi)\right| + |G_t(\phi)| \times \left|\frac{\partial}{\partial\theta}\ln\left(\sigma_t(\theta)\right)\right|\right) \times \sup_{\theta \in \mathcal{N}_{\theta_0}} \left|\frac{\partial}{\partial\theta}\ln\left(\sigma_t(\tilde{\theta})\right)\right| \right\} \times \left|\theta - \tilde{\theta}\right|$$
$$+2\sup_{\theta \in \mathcal{N}_{\theta_0}} \left\{ |G_t(\phi)| \times \left(K\left|\frac{\partial}{\partial\phi}G_t(\phi)\right| + |G_t(\phi)| \times \left|\frac{\partial}{\partial\theta}\ln\left(\sigma_t(\theta)\right)\right|\right) \right\} \times \left|\theta - \tilde{\theta}\right|.$$

Note that, e.g., $\sup_{\theta \in \mathcal{N}_{\theta_0}} |G_t(\phi)| = \sup_{\phi \in \mathcal{N}_{\phi_0}} |G_t(\phi)|$ for some compact $\mathcal{N}_{\phi_0} \subseteq \Delta$ containing ϕ_0. Under Assumptions 1 and 2, and multiple applications of the Cauchy-Schwartz inequality, $E[\mathcal{E}_t] < \infty$. Markov's inequality now yields for any $(\epsilon, \eta) > 0$ there exists $0 < \delta < \epsilon\eta/E[\mathcal{E}_t]$ sufficiently small such that

$$\begin{aligned}\lim_{n\to\infty} P\left(\sup_{\theta,\theta \in \mathcal{N}_{\theta_0}:|\theta-\tilde{\theta}|<\delta} \left|\mathcal{C}_{n,i}(h,\theta) - \mathcal{C}_{n,i}(h,\tilde{\theta})\right| > \eta\right) &\leq \lim_{n\to\infty} P\left(\sup_{\theta,\theta \in \mathcal{N}_{\theta_0}:|\theta-\tilde{\theta}|<\delta} \mathcal{E}_t \times \left|\theta - \tilde{\theta}\right| > \eta\right) \\ &\leq \lim_{n\to\infty} P\left(\delta\mathcal{E}_t > \eta\right) \leq \frac{\delta}{\eta}E\left[\mathcal{E}_t\right] < \epsilon.\end{aligned}$$

Therefore $\{\mathcal{C}_{n,i}(\cdot,\theta) : \theta \in \mathcal{N}_{\theta_0}\}$ is stochastically equicontinuous. Hence, in conjunction with pointwise probability convergence (see e.g. Newey, 1991, Corollary 3.1):

$$\sup_{\theta \in \mathcal{N}_{\theta_0}} \left|\mathcal{C}_{n,i}(h,\theta) - E\left[\mathcal{C}_{n,i}(h,\theta)\right]\right| \xrightarrow{p} 0. \tag{F.5}$$

Further, $||\theta_n^* - \theta_0|| \leq ||\hat{\theta}_n - \theta_0|| \xrightarrow{p} 0$ under Assumption 2.c. Therefore $\theta_n^* \in \mathcal{N}_{\theta_0}$ asymptotically with probability approaching one. Hence, by (F.5) and continuity of $\mathcal{C}_{n,i}(\cdot,\theta)$:

$$\left|\mathcal{C}_{n,i}(h,\theta_n^*) - E[\mathcal{C}_{n,i}(h,\theta_0)]\right| \xrightarrow{p} 0. \tag{F.6}$$

Finally, under Assumption 2.c

$$\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) = \mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1}^{n} m_t + O_p\left(\frac{1}{n^{\zeta}}\right),$$

where $1/\sqrt{n}\sum_{t=1}^{n} m_t = O_p(1)$ by square integrability and the NED property for m_t. Hence:

$$\hat{\theta}_n - \theta_0 = O_p\left(1/\sqrt{n}\right). \tag{F.7}$$

Combine (F.3)-(F.7) to yield:

$$\sqrt{n}\hat{\gamma}_n(h) = \frac{1}{\sqrt{n}}\sum_{t=1+h}^{n} \epsilon_t\epsilon_{t-h} - \sqrt{n}\left(\hat{\theta}_n - \theta_0\right)'\mathcal{D}(h) + O_p\left(1/\sqrt{n}\right). \tag{F.8}$$

Now use $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A}n^{-1/2}\sum_{t=1}^n m_t + O_p(n^{-\zeta})$, $\zeta > 0$, to yield

$$\begin{aligned}\sqrt{n}\left\{\hat{\gamma}_n(h) - \gamma(h)\right\} &= \frac{1}{\sqrt{n}}\sum_{t=1+h}^n \epsilon_t\epsilon_{t-h} - \sqrt{n}E\left[\epsilon_t\epsilon_{t-h}\right] \\ &\quad -\mathcal{D}(h)'\left\{\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1}^n m_t + O_p\left(1/n^{\zeta}\right)\right\} + O_p\left(1/\sqrt{n}\right) \\ &= \frac{1}{\sqrt{n}}\sum_{t=1+h}^n \left(\epsilon_t\epsilon_{t-h} - E\left[\epsilon_t\epsilon_{t-h}\right] - \mathcal{D}(h)'\mathcal{A}m_t\right) + O_p\left(1/n^{\min\{\zeta,1/2\}}\right).\end{aligned}$$

If we set $h = 0$ then similarly:

$$\sqrt{n}\left\{\hat{\gamma}_n(0) - \gamma(0)\right\} = \frac{1}{\sqrt{n}}\sum_{t=1+h}^n \left(\epsilon_t^2 - E\left[\epsilon_t^2\right] - \mathcal{D}(0)'\mathcal{A}m_t\right) + O_p\left(1/n^{\min\{\zeta,1/2\}}\right). \quad \text{(F.9)}$$

By the assumed NED properties under Assumption 1 and 2.c, (A.5) applies to $\epsilon_t^2 - E[\epsilon_t^2] - \mathcal{D}(0)'\mathcal{A}m_t$ (see Davidson, 1994, Chap. 17), hence:

$$\hat{\gamma}_n(0) - \gamma(0) = \frac{1}{n}\sum_{t=1+h}^n \left(\epsilon_t^2 - E\left[\epsilon_t^2\right] - \mathcal{D}(0)'\mathcal{A}m_t\right) + O_p\left(\frac{1}{n^{1/2+\min\{\zeta,1/2\}}}\right) = O_p\left(\frac{1}{\sqrt{n}}\right).$$

Next we tackle the key term

$$r_t(h) \equiv \frac{\{\epsilon_t\epsilon_{t-h} - E[\epsilon_t\epsilon_{t-h}] - \mathcal{D}(h)'\mathcal{A}m_t\}}{E\left[\epsilon_t^2\right]}.$$

Under Assumptions 1 and 2.c, $r_t(h)$ is zero mean, stationary, L_p-bounded for some $p > 2$, and L_2-NED with size 1/2, on an α-mixing base with decay $O(h^{-p/(p-2)-\iota})$ (Davidson, 1994, Theorems 17.8 and 17.9). Therefore $E[(1/\sqrt{n}\sum_{t=1}^n r_t(h))^2] = O(1)$ by (A.5). This implies

$$\frac{1}{\sqrt{n}}\sum_{t=1+h}^n r_t(h)(1 + o_p(1)) = \frac{1}{\sqrt{n}}\sum_{t=1+h}^n r_t(h) + o_p(1).$$

Additionally, $E[r_t^2(0)] < \infty$ implies

$$\frac{1}{\sqrt{n}}\sum_{t=1}^n r_t(0) = \frac{1}{\sqrt{n}}\sum_{t=1+h}^n r_t(0) + O_p\left(1/\sqrt{n}\right).$$

Combine the above derivations to yield:

$$\begin{aligned}\sqrt{n}\left\{\hat{\rho}_n(h) - \rho(h)\right\} &= \sqrt{n}\left\{\frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)} - \frac{\gamma(h)}{\gamma(0)}\right\} \\ &= \frac{1}{\hat{\gamma}_n(0)}\sqrt{n}\left\{\hat{\gamma}_n(h) - \gamma(h)\right\} - \frac{\gamma(h)}{\hat{\gamma}_n(0)\gamma(0)}\sqrt{n}\left\{\hat{\gamma}_n(0) - \gamma(0)\right\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\gamma(0)+O_p\left(1/\sqrt{n}\right)}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left(\epsilon_t\epsilon_{t-h}-E\left[\epsilon_t\epsilon_{t-h}\right]-\mathcal{D}(h)'\mathcal{A}m_t\right) \\
&\quad -\frac{\gamma(h)}{\left(\gamma(0)+O_p\left(1/\sqrt{n}\right)\right)\gamma(0)}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left(\epsilon_t^2-E\left[\epsilon_t^2\right]-\mathcal{D}(0)'\mathcal{A}m_t\right)+O_p\left(\frac{1}{n^{\min\{\zeta,1/2\}}}\right) \\
&= \frac{1}{\gamma(0)}\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\left(\epsilon_t\epsilon_{t-h}-E\left[\epsilon_t\epsilon_{t-h}\right]-\mathcal{D}(h)'\mathcal{A}m_t\right) \\
&\quad -\frac{\rho(h)}{\gamma(0)}\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\left(\epsilon_t^2-E\left[\epsilon_t^2\right]-\mathcal{D}(0)'\mathcal{A}m_t\right)+O_p\left(\frac{1}{n^{\min\{\zeta,1/2\}}}\right) \\
&= \frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left\{r_t(h)-\rho(h)r_t(0)\right\}+O_p\left(\frac{1}{n^{\min\{\zeta,1/2\}}}\right).
\end{aligned}
$$

This proves (F.1) as required. $\mathcal{QED}$.

F.2 Lemma A.5 (Convergence in Finite Dimensional Distributions)

Define

$$
z_t(h) \equiv r_t(h)-\rho(h)r_t(0) \text{ where } r_t(h) \equiv \frac{\epsilon_t\epsilon_{t-h}-E\left[\epsilon_t\epsilon_{t-h}\right]-\mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]}.
$$

Lemma A.5. *Let Assumptions 1 and 2 hold, and write $\mathcal{Z}_n(h) \equiv 1/\sqrt{n}\sum_{t=1+h}^{n} z_t(h)$. For each $\mathcal{L} \in \mathbb{N}$:*

$$
\left\{\mathcal{Z}_n(h) : 1 \le h \le \mathcal{L}\right\} \overset{d}{\to} \left\{\mathcal{Z}(h) : 1 \le h \le \mathcal{L}\right\}, \tag{F.10}
$$

where $\{\mathcal{Z}(h) : 1 \le h \le \mathcal{L}\}$ is a zero mean Gaussian process with variance $\lim_{n\to\infty} n^{-1}\sum_{s,t=1}^{n} E[z_s(h)z_t(h)]$ $\in (0,\infty)$, and covariance function $\lim_{n\to\infty} n^{-1}\sum_{s,t=1}^{n} E[z_s(h)z_t(\tilde{h})]$.

Proof. For arbitrary $\mathcal{L} \in \mathbb{N}$, and $\lambda \in \mathbb{R}^{\mathcal{L}}$, $\lambda'\lambda = 1$:

$$
\begin{aligned}
\sum_{h=1}^{\mathcal{L}}\lambda_h\mathcal{Z}_n(h) &= \frac{1}{\sqrt{n}}\sum_{h=1}^{\mathcal{L}}\lambda_h\sum_{t=1+h}^{n} z_t(h) = \frac{1}{\sqrt{n}}\sum_{h=1}^{\mathcal{L}}\lambda_h\sum_{t=1}^{n} z_t(h)I\left(t\in\{1,...,n-h\}\right) \\
&= \frac{1}{\sqrt{n}}\sum_{t=1}^{n}\sum_{h=1}^{\mathcal{L}}\lambda_h z_t(h)I\left(t\in\{1,...,n-h\}\right) = \frac{1}{\sqrt{n}}\sum_{t=1}^{n}\lambda'\boldsymbol{z}_t(\mathcal{L}),
\end{aligned}
$$

where $\boldsymbol{z}_t(\mathcal{L}) \equiv [z_t(h)I(t\in\{1,...,n-h\})]_{h=1}^{\mathcal{L}} \in \mathbb{R}^{\mathcal{L}}$. Define $\sigma_n^2(\lambda) \equiv E[(\sum_{h=1}^{\mathcal{L}}\lambda_h\mathcal{Z}_n(h))^2]$. We need:

$$
\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\lambda'\boldsymbol{z}_t(\mathcal{L}) \overset{d}{\to} N\left(0, \lim_{n\to\infty}\sigma_n^2(\lambda)\right) \text{ where } \lim_{n\to\infty}\sigma_n^2(\lambda) \in (0,\infty). \tag{F.11}
$$

The claim (F.10) then follows by the Cramér-Wold theorem.

Under Assumptions 1 and 2.c, $\lambda'\boldsymbol{z}_t(\mathcal{L})$ is zero mean, L_p-bounded for some $p > 2$, and L_2-NED with size 1/2, on an α-mixing base with decay $O(h^{-p/(p-2)-\iota})$ (Davidson, 1994, Theorems 17.8, 17.9).

Therefore $\sigma_n^2(\lambda) = O(1)$ by (A.5). By Assumption 2.d, $\liminf_{n\to\infty} \inf_{\lambda'\lambda} \sigma_n^2(\lambda) > 0$. Now (F.11) follows by Theorem 2 in de Jong (1997). $\mathcal{QED}$

F.3 Lemma A.6 (ULLN's)

Set a block size b_n such that $1 \leq b_n < n$, $b_n \to \infty$ and $b_n/n \to 0$. Denote the blocks by $\mathcal{B}_s = \{(s-1)b_n + 1, \ldots, sb_n\}$ with $s = 1, \ldots, n/b_n$. Assume for simplicity that the number of blocks n/b_n is an integer. Generate iid random numbers $\{\xi_1, \ldots, \xi_{n/b_n}\}$ with $E[\xi_i] = 0$, $E[\xi_i^2] = 1$, and $E[\xi_i^4] < \infty$. Define an auxiliary variable $\omega_t = \xi_s$ if $t \in \mathcal{B}_s$.

Lemma A.6. *Under Assumptions 1 and 2.a,b,c′,d:*

$$\sup_{\theta\in\Theta} \left\| \frac{1}{n} \sum_{t=1}^{n} \omega_t \frac{\partial}{\partial\theta} m_t(\theta) \right\| \xrightarrow{p} 0 \tag{F.12}$$

$$\sup_{\theta\in\Theta} \left\| \frac{1}{n} \sum_{t=1}^{n} \frac{\partial}{\partial\theta} m_t(\theta) - E\left[\frac{\partial}{\partial\theta} m_t(\theta)\right] \right\| \xrightarrow{p} 0 \tag{F.13}$$

$$\frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \omega_t m_t = O_p(1). \tag{F.14}$$

Proof.

Step 1. Consider (F.12). The proof for (F.13) is similar, although simpler. For iid ξ_s that is distributed $N(0,1)$ and independent of the data:

$$\frac{1}{n} \sum_{t=1}^{n} \omega_t \frac{\partial}{\partial\theta} m_t(\theta) = \frac{1}{n/b_n} \sum_{s=1}^{n/b_n} \xi_s \frac{1}{b_n} \sum_{t=(s-1)b_n+1}^{sb_n} \frac{\partial}{\partial\theta} m_t(\theta),$$

where $(\partial/\partial\theta)m_t(\theta)$ is integrable uniformly on Θ under Assumption 2.c′. Therefore, for each $i = 1, ..., k_\theta$ and $j = 1, ..., k_m$, stationarity and Minkowski's inequality yield:

$$\begin{aligned} E\left[\left(\frac{1}{n}\sum_{t=1}^{n} \omega_t \frac{\partial}{\partial\theta_i} m_{j,t}(\theta)\right)^2\right] &= \frac{1}{n/b_n} E\left[\left(\frac{1}{b_n} \sum_{t=(s-1)b_n+1}^{sb_n} \frac{\partial}{\partial\theta_i} m_{j,t}(\theta)\right)^2\right] \\ &\leq \frac{b_n}{n} \sup_{\theta\in\Theta} E\left[\left(\frac{\partial}{\partial\theta_i} m_{j,t}(\theta)\right)^2\right] \to 0. \end{aligned} \tag{F.15}$$

Therefore pointwise $1/n \sum_{t=1}^{n} \omega_t (\partial/\partial\theta) m_t(\theta) \xrightarrow{p} 0$. Moreover, $1/n \sum_{t=1}^{n} \omega_t (\partial/\partial\theta) m_t(\theta)$ is stochastically equicontinuous. This follows by using a first order expansion of $(\partial/\partial\theta)m_t(\theta)$ and integrability of $\sup_{\theta\in\Theta} |1/n \sum_{t=1}^{n} \omega_t (\partial^2/\partial\theta\partial\theta) m_{i,t}(\theta)|$ under Assumption 2.c′. For each i, by the mean-value-theorem:

$$\sup_{|\theta-\tilde{\theta}|<\delta} \left| \frac{1}{n} \sum_{t=1}^{n} \omega_t \left(\frac{\partial}{\partial\theta} m_{i,t}(\theta) - \frac{\partial}{\partial\theta} m_{i,t}(\tilde{\theta}) \right) \right| \leq \sup_{|\theta-\tilde{\theta}|<\delta} \left\{ \left| \frac{1}{n} \sum_{t=1}^{n} \omega_t \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| \times \left| \theta - \tilde{\theta} \right| \right\}$$

$$\leq \sup_{\theta \in \Theta} \left| \frac{1}{n} \sum_{t=1}^{n} \omega_t \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| \times \delta.$$

Hence, for any $(\epsilon, \eta) > 0$ there exists

$$0 < \delta < \begin{cases} \infty & \text{if } E\left[\sup_{\theta\in\Theta}\left|\frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta)\right|\right] = 0 \\ \frac{\eta\epsilon}{E\left[\sup_{\theta\in\Theta}\left|\frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta)\right|\right]} & \text{if } 0 < E\left[\sup_{\theta\in\Theta}\left|\frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta)\right|\right] < \infty \end{cases}$$

such that:

$$\begin{aligned} &\lim_{n\to\infty} P\left(\sup_{\theta,\theta\in\mathcal{N}_{\theta_0}:|\theta-\tilde{\theta}|<\delta} \left| \frac{1}{n}\sum_{t=1}^{n} \omega_t \left(\frac{\partial}{\partial\theta} m_t(\theta) - \frac{\partial}{\partial\theta} m_t(\theta) \right) \right| > \eta \right) \\ &\quad \leq \lim_{n\to\infty} P\left(\sup_{\theta\in\Theta} \left| \frac{1}{n}\sum_{t=1}^{n} \omega_t \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| > \frac{\eta}{\delta} \right) \leq \frac{\delta}{\eta} \lim_{n\to\infty} E\left[\sup_{\theta\in\Theta} \left| \frac{1}{n}\sum_{t=1}^{n} \omega_t \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| \right] \\ &\quad = \frac{\delta}{\eta} \lim_{n\to\infty} E\left[\sup_{\theta\in\Theta} \left| \frac{1}{n/b_n} \sum_{s=1}^{n/b_n} \xi_s \frac{1}{b_n} \sum_{t=(s-1)b_n+1}^{sb_n} \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| \right] \leq \frac{\delta}{\eta} E\left[\sup_{\theta\in\Theta} \left| \frac{\partial^2}{\partial\theta\partial\theta} m_{i,t}(\theta) \right| \right] < \epsilon. \end{aligned}$$

This proves stochastic equicontinuity (see, e.g., Newey, 1991, Andrews, 1992). Therefore (F.12) holds by Newey (1991, Corollary 3.1).

Step 2. Next we show (F.14). Under Assumption 2.c′, m_t is zero mean, stationary, L_p-bounded for some $p > 2$, and L_2-NED with size 1/2, on an α-mixing base with decay $O(h^{-p/(p-2)-\iota})$ (Davidson, 1994, Theorems 17.8 and 17.9). Therefore $E[(1/\sqrt{b_n}\sum_{t=1}^{b_n} m_{i,t})^2] = O(1)$ by Theorem 17.5 in Davidson (1994) and Theorem 1.6 in McLeish (1975). By construction of ω_t this yields:

$$\begin{aligned} E\left[\left(\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\omega_t m_{i,t}\right)^2\right] &= E\left[\left(\frac{1}{\sqrt{n/b_n}}\sum_{s=1}^{n/b_n}\xi_s \frac{1}{\sqrt{b_n}} \sum_{t=(s-1)b_n+1}^{sb_n} m_{i,t}\right)^2\right] \\ &= E\left[\left(\frac{1}{\sqrt{b_n}} \sum_{t=(s-1)b_n+1}^{sb_n} m_{i,t}\right)^2\right] \leq K, \end{aligned}$$

hence $1/\sqrt{n}\sum_{t=1}^{n}\omega_t m_{i,t} = O_p(1)$. $\mathcal{QED}$.

G Maximum Lag Upper Bound

In this appendix we present conditions that lead to an upper bound on the maximum lag rate of increase $\mathcal{L}_n \to \infty$, cf. Lemma 2.1.

Recall the Lemma 2.1 expansion result. This requires $\kappa > 0$ in $\hat{\omega}_n(h) = \omega(h) + O_p(1/n^{\kappa})$ and $\zeta > 0$ in $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A}n^{-1/2}\sum_{t=1}^{n} m_t(\theta_0) + \mathcal{R}_m(n)$ where $\mathcal{R}_m(n) = O_p(n^{-\zeta})$.

Lemma 2.1. *Let Assumptions 1 and 2 hold. Then*

$$\tilde{\mathcal{X}}_n(h) \equiv \left| \sqrt{n}\hat{\omega}_n(h)\left\{\hat{\rho}_n(h) - \rho(h)\right\} - \omega(h)\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left\{r_t(h) - \rho(h)r_t(0)\right\}\right| = O_p\left(1/n^{\min\{\zeta,\kappa,1/2\}}\right). \quad \text{(G.1)}$$

Moreover, for some non-unique monotonic sequence of positive integers $\{\mathcal{L}_n\}$, $\mathcal{L}_n \to \infty$ and $\mathcal{L}_n = o(n)$, we have: $|\vartheta(\sqrt{n}[\hat{\omega}_n(h)\{\hat{\rho}_n(h) - \rho(h)\}]_{h=1}^{\mathcal{L}_n}) - \vartheta([\omega(h)\mathcal{Z}_n(h)]_{h=1}^{\mathcal{L}_n})| \leq \vartheta([\sqrt{n}\hat{\omega}_n(h)\{\hat{\rho}_n(h) - \rho(h)\} - \omega(h)\mathcal{Z}_n(h)]_{h=1}^{\mathcal{L}_n}) \xrightarrow{p} 0$. Therefore, under the null hypothesis:

$$\left|\vartheta\left(\left[\sqrt{n}\hat{\omega}_n(h)\hat{\rho}_n(h)\right]_{h=1}^{\mathcal{L}_n}\right) - \vartheta\left(\left[\omega(h)\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left\{\frac{\epsilon_t\epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]}\right\}\right]_{h=1}^{\mathcal{L}_n}\right)\right| \xrightarrow{p} 0. \quad \text{(G.2)}$$

Finally, if $\vartheta(\cdot)$ is the maximum transform, and $(n^{\min\{\zeta,\kappa,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$ for all h is uniformly integrable, then $\mathcal{L}_n = O(n^{\min\{\zeta,\kappa,1/2\}}/\ln(n))$ must be satisfied.

In this appendix we seek conditions such that $(n^{\min\{\zeta,\kappa,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$ is uniformly integrable. In order to simplify notation we assume $\hat{\omega}_n(h) = \omega(h) = 1$, but the main results carry over under straightforward modifications (see also Remark 4 below). Hence, from Assumption 1.c $\hat{\omega}_n(h) = \omega(h) + O_p(1/n^{\kappa})$ for some $\kappa > 0$, we may take $\kappa = \infty$. Thus:

$$\tilde{\mathcal{X}}_n(h) \equiv \left|\sqrt{n}\left\{\hat{\rho}_n(h) - \rho(h)\right\} - \frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left\{r_t(h) - \rho(h)r_t(0)\right\}\right| = O_p\left(1/n^{\min\{\zeta,1/2\}}\right) \quad \text{(G.3)}$$

and

$$\left|\max_{1\leq i\leq\mathcal{L}_n}\sqrt{n}\left|\hat{\rho}_n(h) - \rho(h)\right| \xrightarrow{p} 0 - \max_{1\leq i\leq\mathcal{L}_n}\left|\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}\left\{\frac{\epsilon_t\epsilon_{t-h} - \rho(h)\epsilon_t^2 - \left(\mathcal{D}(h) - \rho(h)\mathcal{D}(0)\right)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]}\right\}\right|\right| \xrightarrow{p} 0. \quad \text{(G.4)}$$

Demonstrating $(n^{\min\{\zeta,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$ is uniformly integrable requires several additional technical conditions. In the following $\iota > 0$ is a tiny number that may be different in different places.

Assumption 4 (maximum lag conditions)**.**

a. ϵ_t is L_r-bounded for some $r > 6$.

b. $||1/\hat{\gamma}_n(0)||_p = O(1)$ for some $p > 1$, $\sqrt{n}||\hat{\theta}_n - \theta_0||_4 = O(1)$, and $||n^{\lambda}\mathcal{R}_m(n)||_q = O(1)$ for some $\lambda > 0$ and $q > 2$.

c. $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h},\ (\epsilon_{t-h}s_{t-h} + G_{t-h}/\sigma_{t-h})\epsilon_t\}$ are L_p-bounded, $p > 2$, L_r-NED, $2 < r \leq p$, with size 1, on an α-mixing base $\{\upsilon_t\}$ with σ-fields $\mathfrak{V}_t \equiv \sigma(\upsilon_\tau : \tau \leq t)$ and decay $O(h^{-\eta p/(p-2)-\iota})$ where $\eta = (1/(2+\iota) - 1/p)^{-1}$.

Remark 1. The higher error moment (a) extends $||\epsilon_t||_r < \infty$ for some $r > 4$ under Assumption 1.b. This is required since we demonstrate uniform integrability by establishing $L_{1+\iota}$-boundedness for $(n^{\min\{\zeta,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$.

Remark 2. All conditions save $||1/\hat{\gamma}_n(0)||_p = O(1)$ are easily verified for most of the simulation study processes.[3]

Remark 3. NED condition (c) is imposed because we require a moment bound similar to (A.5) for $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}, (\epsilon_{t-h}s_{t-h} + G_{t-h}/\sigma_{t-h})\epsilon_t\}$. In view of the higher moment conditions in Assumption 2.a,b, and the NED definition, it always suffices to assume $\{\epsilon_t, x_t\}$ are L_p-bounded, $p > 6$, stationary α-mixing with mixing coefficients $\alpha_h = O(h^{-\eta p/(p-2)-\iota})$.

Remark 4. In the general case for $\hat{\omega}_n(h) = \omega(h) + O_p(1/n^\kappa)$ we additionally need $n^{\min\{\zeta,\kappa,1/2\}}||\hat{\omega}_n(h) - \omega(h)||_q = O(1)$ for some $q > 2$.

We now have the main result in this appendix. Under the appropriate conditions $(n^{\min\{\zeta,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$ is uniformly integrable, hence the Lemma 2.1 upper bound on $\mathcal{L}_n$ applies.

Lemma G.1. *Let Assumptions 1, 2 and 4 hold, and assume H_0 is true. Then $(n^{\min\{\zeta,1/2\}}/\ln(n))\tilde{\mathcal{X}}_n(h)$ is uniformly integrable for each h.*

Before we prove the claim, we verify Assumption 4 for a stationary AR(1) process.

Example 1. Let $y_t = \theta_0 y_{t-1} + \epsilon_t$, $|\theta_0| < 1$, where $E[\epsilon_t] = 0$, $E\,|\epsilon_t|^p < \infty$ for some $p > 12$, and $E[\epsilon_t y_{t-1}] = 0$. Assume $\{\epsilon_t, y_t\}$ are stationary geometrically α-mixing. Let $||(1/n\sum_{t=1}^n y_t^2)^{-1}||_{12+\iota} = O(1)$. The plug-in estimator $\hat{\theta}_n$ is least squares. It is easily verified that $\mathcal{D}(h) = E[y_{t-1}\epsilon_{t-h}]$ and $s_t(\theta) = 0$ for any θ.

If ϵ_t is iid with a bounded density function, then the mixing condition follows. See Gorodetskii (1977) and Doukhan (1994, p. 77). If ϵ_t follows a stationary GARCH process, then the mixing condition holds under mild regularity conditions (see Meitz and Saikkonen, 2008). Conversely, the stationary solution $y_t = \sum_{i=0}^\infty \theta_0^i \epsilon_{t-i}$ *a.s.* is an infinite order lag function of a stationary α-mixing process $\{\epsilon_t\}$. See Doukhan (1994, Chapt. 2.3.2) for deep theory and conditions under which such a process is also α-mixing.

Assumption 4.a. $E|\epsilon_t|^r < \infty$ for some $r > 6$ holds by construction.

Assumption 4.b. The first condition $||1/\hat{\gamma}_n(0)||_p = O(1)$ for some $p > 1$ holds by supposition. For the second condition $\sqrt{n}||\hat{\theta}_n - \theta_0||_4 = O(1)$, by Hölder's inequality and $||1/\hat{\gamma}_n(0)||_{12+\iota} = O(1)$:

$$\left\|\sqrt{n}\left(\hat{\theta}_n - \theta_0\right)\right\|_4 \leq \left\|\frac{1}{1/n\sum_{t=2}^n y_{t-1}^2}\right\|_{12} \left\|\frac{1}{\sqrt{n}}\sum_{t=2}^n \epsilon_t y_{t-1}\right\|_6 \leq K\left(E\left[\left(\frac{1}{\sqrt{n}}\sum_{t=2}^n \epsilon_t y_{t-1}\right)^6\right]\right)^{1/6}.$$

Under the stated conditions and Cauchy-Schwartz inequality, $\epsilon_t y_{t-1}$ is stationary L_p-bounded, $p > 6$, and geometrically α-mixing. Hence $E|1/\sqrt{n}\sum_{t=2}^n \epsilon_t y_{t-1}|^6 \leq K$ by Corollary 3 in Hansen (1991). Therefore $||\sqrt{n}(\hat{\theta}_n - \theta_0)||_4 = O(1)$.

[3] Some processes in the simulation study likely fail to have the required higher moments, but are used to demonstrate the sensitivity of the proposed test to moment condition failure. See Section 4 in the main paper, and see Appendix H.

Next, for the third condition $||n^{\lambda}\mathcal{R}_m(n)||_q = O(1)$ for some $\lambda > 0$ and $q > 2$, the least squares expansion satisfies:

$$\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) = \frac{1}{E\left[y_{t-1}^2\right]}\frac{1}{\sqrt{n}}\sum_{t=2}^{n}\epsilon_t y_{t-1} + \mathcal{R}_m(n)$$
$$\text{where } \mathcal{R}_m(n) = -\frac{\left(1/n\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)}{E\left[y_{t-1}^2\right]n^{-1}\sum_{t=2}^{n} y_{t-1}^2}\frac{1}{\sqrt{n}}\sum_{t=2}^{n}\epsilon_t y_{t-1}.$$

The geometric α-mixing property and $E[\epsilon_t y_{t-1}] = 0$ yields $1/\sqrt{n}\sum_{t=2}^{n}\epsilon_t y_{t-1} = O_p(1)$ by Theorem 1.6 in McLeish (1975). Similarly, y_t^2 is L_2-bounded, geometrically α-mixing hence:

$$\frac{1}{n}\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right] = \frac{1}{n}\sum_{t=2}^{n}\left(y_{t-1}^2 - E\left[y_{t-1}^2\right]\right) - \frac{1}{n}E\left[y_{t-1}^2\right] = O_p(1/\sqrt{n}).$$

Therefore

$$\sqrt{n}\mathcal{R}_m(n) = -\frac{\sqrt{n}\left(1/n\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)}{E\left[y_{t-1}^2\right]n^{-1}\sum_{t=2}^{n} y_{t-1}^2}\frac{1}{\sqrt{n}}\sum_{t=2}^{n}\epsilon_t y_{t-1} = O_p(1)$$

so that $\mathcal{R}_m(n) = O_p(n^{-\lambda})$ with $\lambda = 1/2$.

Hölder's inequality, $||1/\sqrt{n}\sum_{t=2}^{n}\epsilon_t y_{t-1}||_6 \leq K$ from above, and $||1/\hat{\gamma}_n(0)||_{12+\iota} = O(1)$ yield:

$$\begin{aligned}
\left\|\sqrt{n}\mathcal{R}_m(n)\right\|_{2+\iota} &\leq \left\|\frac{\sqrt{n}\left(1/n\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)}{E\left[y_{t-1}^2\right]n^{-1}\sum_{t=2}^{n} y_{t-1}^2}\right\|_{3+\iota}\left\|\frac{1}{\sqrt{n}}\sum_{t=2}^{n}\epsilon_t y_{t-1}\right\|_6 \\
&\leq K\left\|\frac{\sqrt{n}\left(1/n\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)}{1/n\sum_{t=2}^{n} y_{t-1}^2}\right\|_{3+\iota} \\
&\leq K\left\|\frac{1}{1/n\sum_{t=2}^{n} y_{t-1}^2}\right\|_{12+\iota}\left\|\sqrt{n}\left(\frac{1}{n}\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)\right\|_4 \\
&\leq K\left\|\sqrt{n}\left(\frac{1}{n}\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right]\right)\right\|_4.
\end{aligned}$$

Coupled with L_p-boundedness for ϵ_t for some $p > 12$, and the α-mixing property, y_{t-1}^2 is stationary $L_{p/2}$-bounded geometrically α-mixing. Hence $||\sqrt{n}(1/n\sum_{t=2}^{n} y_{t-1}^2 - E\left[y_{t-1}^2\right])||_4 = O(1)$ by Corollary 3 in Hansen (1991). Therefore $||\sqrt{n}\mathcal{R}_m(n)||_{2+\iota} = O(1)$ which verifies the third condition.

Assumption 4.c. By construction $G_t(\phi) = y_{t-1}$, $\sigma_t = 1$ and $s_t(\theta) = 0$, hence $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}, (\epsilon_{t-h}s_{t-h} + G_{t-h}/\sigma_{t-h})\epsilon_t\} = \{y_{t-1t}\epsilon_{t-h}, y_{t-1-h}\epsilon_t\}$. From the above arguments, $\{y_{t-1t}\epsilon_{t-h}, y_{t-1-h}\epsilon_t\}$ is L_p-bounded, $p > 6$, geometrically α-mixing. Therefore they are L_r-NED, $2 < r \leq p$, with arbitrary size, on a geometrically α-mixing base which therefore has arbitrarily fast decay, i.e. $\alpha_h = O(h^{-\lambda})$ for any $\lambda > 0$. Hence all components of Assumption 4.c hold (see, e.g., Davidson, 1994, Chapt. 7).

Proof of Lemma G.1. In order to reduce notation, let $H_0 : \rho(h) = 0 \ \forall h \geq 1$ be true. Then:

$$\tilde{\mathcal{X}}_n(h) \equiv \left| \sqrt{n}\hat{\rho}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} r_t(h) \right| = \left| \sqrt{n}\hat{\rho}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\frac{\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]} \right) \right|.$$

Hence:

$$\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \tilde{\mathcal{X}}_n(h) = \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\hat{\rho}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\frac{\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]} \right) \right|.$$

By the triangle inequality:

$$\begin{aligned}
\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \tilde{\mathcal{X}}_n(h) &= \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\hat{\rho}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\frac{\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t}{E\left[\epsilon_t^2\right]} \right) \right| \\
&\leq \frac{1}{\gamma(0)} \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\hat{\gamma}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t \right) \right| \\
&\quad + \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \sqrt{n}\hat{\gamma}_n(h) \left| \frac{1}{\hat{\gamma}_n(0)} - \frac{1}{\gamma(0)} \right| \\
&\leq \frac{1}{\gamma(0)} \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\hat{\gamma}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t \right) \right| \\
&\quad + \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)\gamma(0)} \sqrt{n} \left| \hat{\gamma}_n(0) - \gamma(0) \right| \\
&\equiv \mathcal{M}_{n,1}(h) + \mathcal{M}_{n,2}(h),
\end{aligned}$$

say. It suffices to prove each $\mathcal{M}_{n,i}(h)$ is L_p-bounded for some $p > 1$ (Billingsley, 1999, eq. (3.18)). In the following $p > 1$ and tiny $\iota > 0$ may be different in different places.

Step 1 ($M_{n,1}(h)$): Using notation from the proof of Lemma A.3, by (F.2) and (F.3):

$$\begin{aligned}
\mathcal{M}_{n,1}(h) &= \frac{1}{\gamma(0)} \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\hat{\gamma}_n(h) - \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} \left(\epsilon_t \epsilon_{t-h} - \mathcal{D}(h)'\mathcal{A}m_t \right) \right| \\
&= \frac{1}{\gamma(0)} \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| \sqrt{n}\mathcal{A}_n(h) + \sqrt{n}\mathcal{A}_n(-h) + \mathcal{D}(h)'\mathcal{A}m_t \right| \\
&\leq \frac{1}{\gamma(0)} \frac{n^{\min\{\zeta,1/2\}}}{\ln(n)} \left| -\sqrt{n} \left(\hat{\theta}_n - \theta_0 \right)' \{\mathcal{B}_n(h) + \mathcal{B}_n(-h)\} + \mathcal{D}(h)'\mathcal{A} \frac{1}{\sqrt{n}} \sum_{t=1+h}^{n} m_t \right| \qquad \text{(G.5)} \\
&\quad + \frac{1}{\gamma(0)} \left| \sqrt{n} \left(\hat{\theta}_n - \theta_0 \right)' \sum_{i=1}^{6} \{\mathcal{C}_{n,i}(h, \theta_n^*) + \mathcal{C}_{n,i}(-h, \theta_n^*)\} \frac{\sqrt{n}}{\ln(n)} \left(\hat{\theta}_n - \theta_0 \right) \right|.
\end{aligned}$$

By multiple applications of Hölder and Minkowsky inequalities, for some $\iota > 0$ the second term in (G.5) satisfies:

$$\begin{aligned}
&\left\|\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)'\sum_{i=1}^{6}\left\{\mathcal{C}_{n,i}(h,\theta_n^*)+\mathcal{C}_{n,i}(-h,\theta_n^*)\right\}\frac{\sqrt{n}}{\ln(n)}\left(\hat{\theta}_n-\theta_0\right)\right\|_{1+\iota} \\
&\qquad\leq\left\|\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)\right\|_{3+\iota}\left\|\sum_{i=1}^{6}\left\{\mathcal{C}_{n,i}(h,\theta_n^*)+\mathcal{C}_{n,i}(-h,\theta_n^*)\right\}\right\|_{3/2+\iota}\left\|\frac{\sqrt{n}}{\ln(n)}\left(\hat{\theta}_n-\theta_0\right)\right\|_{3+\iota} \\
&\qquad\leq\frac{1}{\ln(n)}\left\|\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)\right\|_{3+\iota}^2\sup_{\theta\in\Theta}\sum_{i=1}^{6}\left\|\mathcal{C}_{n,i}(h,\theta)+\mathcal{C}_{n,i}(-h,\theta)\right\|_{3/2+\iota} \\
&\qquad=O\left(1/\ln(n)\right)=o(1).
\end{aligned}$$

The last line follows from Assumptions 1 and 2: each $\sup_{\theta\in\Theta}||\mathcal{C}_{n,i}(h,\theta)||$ satisfies $E|\sup_{\theta\in\Theta}||\mathcal{C}_{n,i}(h,\theta)|||^{4+\iota}$ $= O(1)$, and by Assumption 4.b $||\sqrt{n}(\hat{\theta}_n-\theta_0)||_4 = O(1)$.

Use $\sqrt{n}(\hat{\theta}_n-\theta_0)=\mathcal{A}n^{-1/2}\sum_{t=1}^{n}m_t+\mathcal{R}_m(n)$ under Assumption 2, and the definitions of $\mathcal{D}(h)$ and $\mathcal{B}_n(h)$ in (A.3) and (F.3) to deduce for the first term in (G.5):

$$\begin{aligned}
&\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)}\left|\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)'\left\{\mathcal{B}_n(h)+\mathcal{B}_n(-h)\right\}-\mathcal{D}(h)'\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}m_t\right| \\
&\quad=\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)}\left|\left\{\mathcal{B}_n(h)+\mathcal{B}_n(-h)\right\}'\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)-\mathcal{D}(h)'\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}m_t\right| \\
&\quad=\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)}\left|\left\{\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t+\frac{G_t}{\sigma_t}\right)\epsilon_{t-h}+\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h}+\frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right\}'\right. \\
&\qquad\qquad\left.\times\left\{\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1}^{n}m_t+\mathcal{R}_m(n)\right\}-\mathcal{D}(h)'\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}m_t\right| \\
&\quad\leq\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)}\left|\left\{\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t+\frac{G_t}{\sigma_t}\right)\epsilon_{t-h}+\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h}+\frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right.\right. \\
&\qquad\qquad\left.\left.-E\left[\left(\epsilon_t s_t+\frac{G_t}{\sigma_t}\right)\epsilon_{t-h}+\left(\epsilon_{t-h}s_{t-h}+\frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right]\right\}'\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}m_t\right| \\
&\qquad+\frac{1}{\ln(n)}\left|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t+\frac{G_t}{\sigma_t}\right)\epsilon_{t-h}+\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h}+\frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right|\times\left|n^{\min\zeta}\mathcal{R}_m(n)\right|.
\end{aligned}$$

Now apply Hölder and Minkowsky inequalities to yield for some $\iota>0$:

$$\begin{aligned}
&\frac{n^{\min\{\zeta,1/2\}}}{\ln(n)}\left\|\sqrt{n}\left(\hat{\theta}_n-\theta_0\right)'\left\{\mathcal{B}_n(h)+\mathcal{B}_n(-h)\right\}-\mathcal{D}(h)'\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n}m_t\right\|_{1+\iota} \\
&\quad\leq\frac{1}{\ln(n)}\sqrt{n}\left\|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t+\frac{G_t}{\sigma_t}\right)\epsilon_{t-h}+\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h}+\frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right.
\end{aligned}$$

$$-E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} + \left(\epsilon_{t-h}s_{t-h} + \frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right]\Bigg\|_{2+i} \times \left\|\mathcal{A}\frac{1}{\sqrt{n}}\sum_{t=1+h}^{n} m_t\right\|_2$$

$$+\frac{1}{\ln(n)}\left\|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} + \frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h} + \frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right\|_2 \times \left\|n^{\min\zeta}\mathcal{R}_m(n)\right\|_{2+i}.$$

By variance property (A.5) $||\mathcal{A}n^{-1/2}\sum_{t=1+h}^{n} m_t||_2 = O(1)$, and by supposition $||n^{\min\zeta}\mathcal{R}_m(n)||_{2+i} = O(1)$. Under Assumptions 1, 2 and 4, applications of Hölder and Minkowsky inequalities yield:

$$\left\|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} + \frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_{t-h}s_{t-h} + \frac{G_{t-h}}{\sigma_{t-h}}\right)\epsilon_t\right\|_2 \leq K.$$

Finally, we show

$$\sqrt{n}\left\|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} - E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\epsilon_t\right]\right\|_{2+\iota} = O(1). \tag{G.6}$$

The same argument extends to $(\epsilon_{t-h}s_{t-h} + G_{t-h}/\sigma_{t-h})\epsilon_t$. Observe that:

$$\begin{aligned}\sqrt{n}&\left\|\frac{1}{n}\sum_{t=1+h}^{n}\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} - E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\epsilon_t\right]\right\|_{2+\iota} \\ &\leq \sqrt{n}\left\|\frac{1}{n}\sum_{t=1}^{n}\left\{\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} - E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\epsilon_t\right]\right\}\right\|_{2+\iota} \\ &\quad + \frac{h}{\sqrt{n}}\left\|\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\right\|_{2+\iota}.\end{aligned} \tag{G.7}$$

Hölder and Minkowsky inequalities combined with Assumptions 2 and 4.a imply $||(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}||_{2+\iota} < \infty$.

By Assumption 4.c $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}\}$ is L_p-bounded, $p > 2$, L_r-NED, $2 < r \leq p$, with size 1, on an α-mixing base $\{\upsilon_t\}$ with σ-fields $\mathfrak{V}_t \equiv \sigma(\upsilon_\tau : \tau \leq t)$ and decay $O(h^{-\eta p/(p-2)-\iota})$ where $\eta = (\frac{1}{2+\iota} - \frac{1}{p})^{-1}$. Since $\iota > 0$ in (G.7) is an arbitrary tiny number, we can always choose it small enough such that $2 + \iota < r \leq p$. By Hölder's inequality $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}\}$ is therefore also $L_{2+\iota}$-NED. If $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}, \mathfrak{V}_t\}$ subsequently forms an $L_{2+\iota}$-mixingale sequence with size 1, then by Corollary 1 in Hansen (1991):

$$\sqrt{n}\left\|\frac{1}{n}\sum_{t=1}^{n}\left\{\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h} - E\left[\left(\epsilon_t s_t + \frac{G_t}{\sigma_t}\right)\epsilon_{t-h}\epsilon_t\right]\right\}\right\|_{2+\iota} = O(1),$$

and the proof is complete.

Apply Theorem 17.6 of Davidson (1994) to deduce that $\{(\epsilon_t s_t + G_t/\sigma_t)\epsilon_{t-h}, \mathfrak{V}_t\}$ forms an $L_{2+\iota}$-

mixingale sequence with size

$$\min\left\{1, \frac{1}{\frac{1}{2+\iota}-\frac{1}{p}}\frac{p}{p-2}\left(\frac{1}{2+\iota}-\frac{1}{p}\right)\right\} = \min\left\{1, \frac{1}{\frac{1}{2+\iota}-\frac{1}{p}}\left(\frac{1}{2+\iota}-\frac{1}{p}\right)\right\} = \min\{1,1\} = 1.$$

This completes the proof. $\mathcal{QED}$.

H Complete Monte Carlo Study

In this appendix we perform a large scale Monte Carlo study by presenting all tests noted in the main paper Hill and Motegi (2019, Section 4).

H.1 Simulation Design

We first construct an error term e_t that drives an observed variable y_t. Let ν_t be iid $N(0,1)$. We consider iid $e_t = \nu_t$; GARCH(1,1) $e_t = \nu_t w_t$ with random volatility process $w_1^2 = 1$ and $w_t^2 = 1 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$ for $t \geq 2$; MA(2) $e_t = \nu_t + 0.5\nu_{t-1} + 0.25\nu_{t-2}$ for $t \geq 3$, with initial values $e_1 = 0$ and $e_2 = \nu_2 + 0.5\nu_1$; and AR(1) $e_t = 0.7e_{t-1} + \nu_t$ for $t \geq 2$ with initial $e_1 = 0$. Each error process is strictly stationary and ergodic.[4]

Now recall Scenarios #1–#9 from the main paper:

Scenario #1: Simple $y_t = e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$.

Scenario #2: Bilinear $y_t = 0.5e_{t-1}y_{t-2} + e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$.

Scenario #3: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(2) filter $\epsilon_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2}$; least squares.

Scenario #4: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(1) filter $\epsilon_t = y_t - \phi_1 y_{t-1}$; least squares.

Scenario #5: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; no filter.

Scenario #6: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; GARCH(1,1) filter $\epsilon_t = y_t/\sigma_t$ with $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$; quasi-maximum likelihood.[5]

Scenario #7: Remote MA(6) $y_t = e_t + 0.25e_{t-6}$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$.

Scenario #8: Remote MA(12) $y_t = e_t + 0.25e_{t-12}$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$.

Scenario #9: Remote MA(24) $y_t = e_t + 0.25e_{t-24}$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$.

[4] Ergodicity follows since each error process is stationary α-mixing. See, e.g., Kolmogorov and Rozanov (1960) for processes with continuous bounded spectral densities (e.g. stationary Gaussian AR, Gaussian MA(2)); Nelson (1990) for GARCH process stationarity; and Carrasco and Chen (2002) for mixing properties of stationary GARCH processes.

[5] QML is performed using the iterated process $\tilde{\sigma}_1^2(\theta) = \omega$ and $\tilde{\sigma}_t^2(\theta) = \omega + \alpha y_{t-1}^2 + \beta\tilde{\sigma}_{t-1}^2(\theta)$ for $t = 2, \ldots, n$. We impose $(\omega, \alpha, \beta) > 0$ and $\alpha + \beta \leq 1$ during estimation.

H.1.1 White Noise Tests

In the main paper, we study the max-correlation test with the dependent wild bootstrap and automatic lag selection, and Shao's (2011) Cramér-von Mises test with the dependent wild bootstrap and $\mathcal{L}_n = n-1$. In this appendix, we consider several additional white noise tests and a non-random lag length.

We consider fourteen total tests: eight tests with a non-random $\mathcal{L}_n = o(n)$ and five tests with $\mathcal{L}_n = n-1$, including CvM^{dw}; and the max-correlation test with automatic lag $\mathcal{L}_n^*$.

Tests with $\mathcal{L}_n = o(n)$ require a specific selection of lag length. We use a fixed length at $\mathcal{L}_n = 5$ and sample-size dependent length $\mathcal{L}_n = [\delta n/\ln(n)]$ with $\delta \in \{.5, 1\}$, where $[\cdot]$ truncates to an integer value. In this set-up, we have $\mathcal{L}_n \in \{5, 10, 21\}$ for $n = 100$; $\mathcal{L}_n \in \{5, 22, 45\}$ for $n = 250$; $\mathcal{L}_n \in \{5, 40, 80\}$ for $n = 500$; and $\mathcal{L}_n \in \{5, 72, 144\}$ for $n = 1000$. We summarize the choices in Table 1.

Table 1: Non-Stochastic Lag Values: $\mathcal{L}_n \in \{5, [\delta n/\ln(n)]\}$, $\delta \in \{.5, 1\}$

n	100	250	500	1000
$\mathcal{L}_n$	$\{5, 10, 21\}$	$\{5, 22, 45\}$	$\{5, 40, 80\}$	$\{5, 72, 144\}$

The following tests are summarized in Table 2.

Max-Correlation with Fixed $\mathcal{L}_n$ or Automatic Lag $\mathcal{L}_n^*$ We perform both a max-correlation test with automatic lag $\mathcal{L}_n^*$ and dependent wild bootstrap, and a max-correlation test with a non-random lag $\mathcal{L}_n$ and wild bootstrap or dependent wild bootstrap.

The dependent wild bootstrap is always valid asymptotically under our assumptions, but the wild bootstrap requires independence or a martingale difference assumption under the null (see Wu, 1986, Liu, 1988, Hansen, 1996). We omit results for the max-correlation test with $\mathcal{L}_n^*$ and wild bootstrap because the max-correlation test with $\mathcal{L}_n^*$ and dependent wild bootstrap is both valid under the null and our assumptions, and overall dominates all other tests in this study.

The test with non-random $\mathcal{L}_n$ from Table 1 is based on $\hat{\mathcal{T}}_n(\mathcal{L}_n) \equiv \sqrt{n} \max_{1 \leq h \leq \mathcal{L}_n} |\hat{\omega}_n(h)\hat{\rho}_n(h)|$ with weight $\hat{\omega}_n(h) = 1$.

The test statistic with the automatic lag is similarly $\hat{\mathcal{T}}_n(\mathcal{L}_n^*) \equiv \sqrt{n} \max_{1 \leq h \leq \mathcal{L}_n^*} |\hat{\rho}_n(h)|$. The automatic lag selection $\mathcal{L}_n^*$ is chosen from a set $\{1, ..., \bar{\mathcal{L}}_n\}$ for some pre-chosen upper-bound $\bar{\mathcal{L}}_n \to \infty$. In the case of the maximum and $\hat{\omega}_n(h) = \omega(h) = 1$, we have from expansion Lemma 2.1 and dependent wild bootstrap Theorem 2.5 that

$$\bar{\mathcal{L}}_n = o\left(\frac{n^{\min\{\zeta, 1/2\}}}{\ln(n)}\right)$$

must hold, where $\zeta > 0$ appears in the Assumption 2.c or 2.c$'$ plug-in expansion $\sqrt{n}(\hat{\theta}_n - \theta_0) = \mathcal{A}n^{-1/2}\sum_{t=1}^n m_t(\theta_0) + O_p(n^{-\zeta})$. Under standard regularity conditions many plug-in estimators will satisfy $\zeta = 1/2$, hence $\bar{\mathcal{L}}_n = O(\sqrt{n}/\ln(n))$. We use

$$\bar{\mathcal{L}}_n = \left[\delta \times \frac{\sqrt{n}}{\ln n}\right] \text{ with } \delta = 10,$$

where $[\cdot]$ denotes the integer part, hence

$$\bar{\mathcal{L}}_n \in \{21, 28, 35, 45\} \text{ for } n \in \{100, 250, 500, 1000\}.$$

We also require the tuning parameter q (cf. (12) and (13) in the main paper). In order to choose a plausible value of q, in the main paper we performed a preliminary simulation study that computes empirical size and size-adjusted power for the max-correlation test with $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$ across $q \in \{1.50, 1.75, \ldots, 4.50\}$. We considered two cases. In Case 1, size is computed under Scenario #1 with an iid error; and size-adjusted power is computed under #4 with an iid error. In Case 2, size is computed under #5 with an iid error; and size-adjusted power is computed under #5 with MA(2) error. For each case, sample size is $\{100, 500\}$; nominal size is 0.05; and 1000 Monte Carlo samples and 500 bootstrap samples are generated.

See Figure 1 for results. Variation of empirical size and size-adjusted power for the test based on $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ across the values of q is fairly small in each experiment, implying that a choice of q should not have a critical impact on the test performance. For each case and sample size, we obtain relatively accurate size and high power around $q = 3$. We therefore use $q = 3$ throughout.

Andrews and Ploberger's Sup-LM Test Andrews and Ploberger's (1996) sup-LM test is based on the representation (see Nankervis and Savin, 2010):

$$\mathcal{AP}_n \equiv \sup_{\lambda \in \Lambda} \text{LM}_n(\lambda, n-1) \quad \text{where} \quad \text{LM}_n(\lambda, n-1) \equiv n(1-\lambda^2)\left\{\sum_{h=1}^{n-1} \lambda^{h-1}\hat{\rho}_n(h)\right\}^2.$$

Andrews and Ploberger (1996) compute $\text{LM}_n(\lambda, n-1)$ but use a truncated maximum lag $\mathcal{L} = 50$ for simulating critical values. Nankervis and Savin (2010) compute $\text{LM}_n(\lambda, \mathcal{L})$ for $\mathcal{L} = 20$, and use the same $\mathcal{L}$ for simulating critical values. Thus, the test proposed in Nankervis and Savin (2010) is, strictly speaking, not the same test as in Andrews and Ploberger (1996), but a truncated variant that is therefore inconsistent against some deviations from the white noise null hypothesis.

We use either $\mathcal{L}_n = o(n)$ or $\mathcal{L}_n = n - 1$ in order to accomplish test consistency and fair comparison. Hence we compute $\text{LM}_n(\lambda, \mathcal{L}_n) = n(1-\lambda^2)\{\sum_{h=1}^{\mathcal{L}_n} \lambda^{h-1}\hat{\rho}_n(h)\}^2$. Since the lags vary with n, we do not simulate critical values. We only bootstrap a p-value via the wild bootstrap or the dependent wild bootstrap.

The parameter space Λ is discretized to $\{-.800, -.795, \ldots, .795, .800\}$. We use the same end-points $\pm.8$ as in Andrews and Ploberger (1996) and Nankervis and Savin (2010), but a twice finer grid (their increment is .010). Nankervis and Savin (2012) use endpoints $\pm.95$ with increment .010. The distance from the maximum endpoints ± 1 is due to their placing the test in a stationary ARMA framework. We perform the sup-LM test without an ARMA interpretation and therefore also relax the endpoints to $\pm.95$ for the sake of comparison. We only report results based on $\pm.8$ because using $\pm.95$ yields similar rejection frequencies.

We apply the wild bootstrap and dependent wild bootstrap procedures for the sup-LM test.

Hong's Test Hong's (1996) test is based on a standardized periodogram. If the periodogram is computed with a truncated kernel, then the statistic is just a standardized Box-Pierce statistic. We use a standardized Ljung-Box statistic $\mathcal{N}_n \equiv (2\mathcal{L}_n)^{-1/2} \sum_{h=1}^{\mathcal{L}_n} \hat{\omega}_n(h)\{n\hat{\rho}_n^2(h) - 1\}$ with $\hat{\omega}_n(h) = (n+2)/(n - h)$, cf Hong (1996, eq. (3)). If the null hypothesis of white noise is true *and* $\{\sqrt{n}\hat{\rho}_n^2(h)\}_{h=1}^{\mathcal{L}_n}$ are asymptotically independent, then under Hong's Assumptions 1.a, 2 and 3 we have $\mathcal{N}_n \xrightarrow{d} N(0,1)$, else $\mathcal{N}_n \xrightarrow{p} \infty$. This is a one-sided test where the rejection region exists only at the upper tail of $N(0,1)$. The asymptotic independence of $\{\sqrt{n}\hat{\rho}_n^2(h)\}_{h=1}^{\mathcal{L}_n}$ holds if tested variable ϵ_t is iid, but may not hold if ϵ_t is only mds or white noise. Hence, the asymptotic $N(0,1)$ test is not a white noise test in a strict sense.

We compute p-values via the asymptotic standard normal distribution, wild bootstrap, and dependent wild bootstrap. Since $\mathcal{N}_n$ is just an affine transformation of the Ljung-Box statistic, a bootstrapped test based on $\mathcal{N}_n$ is identical to a bootstrapped Ljung-Box Q-test when they are performed at the same lag $\mathcal{L}_n$. Bootstrapped p-values are computed as $\hat{p}_{n,M} = 1/M \sum_{i=1}^{M} I(\mathcal{N}_{n,i}^{(boot)} \geq \mathcal{N}_n)$.

Cramér-von Mises Test In the main paper, we run the Cramér-von Mises test with the dependent wild bootstrap. In this study we use three bootstrap procedures: the wild bootstrap, dependent wild bootstrap, and Zhu and Li's (2015) blockwise random weighting bootstrap [BRWB].

The BRWB algorithm is performed as follows (we ignore the first-order correlation expansion for notational clarity). Suppose that the objective function to be minimized is written as $1/n\sum_{t=1}^{n} l_t(\theta)$. Set a block size b_n, $1 \leq b_n < n$, and denote the blocks by $\mathcal{B}_s = \{(s-1)b_n + 1, \ldots, sb_n\}$ with $s = 1, \ldots, n/b_n$. Assume n/b_n is an integer for simplicity. Generate positive iid random numbers $\{\delta_1, \ldots, \delta_{n/b_n}\}$ from a common distribution with mean 1 and variance 1. Following Zhu and Li (2015), we use the Bernoulli distribution with $P[\delta_t = 0.5 \times (3 - \sqrt{5})] = (2\sqrt{5})^{-1} \times (1 + \sqrt{5})$ and $P[\delta_t = 0.5 \times (3 + \sqrt{5})] = 1 - (2\sqrt{5})^{-1} \times (1 + \sqrt{5})$. Define an auxiliary variable $\omega_t^* = \delta_s$ if $t \in \mathcal{B}_s$. Calculate $\hat{\theta}_n^* = \text{argmin}_{\theta\in\Theta} 1/n\sum_{t=1}^{n} \omega_t^* l_t(\theta)$. Compute $\hat{\gamma}_n^*(h) = 1/n\sum_{t=1+h}^{n} \omega_t^* \epsilon_t(\hat{\theta}_n^*)\epsilon_{t-h}(\hat{\theta}_n^*)$ and $S_n^*(\lambda) = \sum_{h=1}^{n-1} \sqrt{n}\hat{\gamma}_n^*(h)\psi_h(\lambda)$, where $\psi_h(\lambda) = (h\pi)^{-1}\sin(h\lambda)$. Define the bootstrapped process $\Delta_n(\lambda) = S_n^*(\lambda) - S_n(\lambda) - Z_n(\lambda)$, where $Z_n(\lambda) = n\sum_{h=1}^{n-1}\{\sum_{t=1+h}^{n} \omega_t^* - n + h\}\hat{\gamma}_n(h)\psi_h(\lambda)$. Then compute the bootstrapped CvM test statistic $\mathcal{C}_n^* = \int_0^\pi \{\Delta_n(\lambda)\}^2 d\lambda$. Repeat M times, resulting in the sequence $\{\mathcal{C}_{n,i}^*\}_{i=1}^{M}$ and approximate p-value $1/M\sum_{i=1}^{M} I(\mathcal{C}_{n,i}^* \geq \mathcal{C}_n)$.

Ljung-Box Q-test We perform the Ljung-Box Q-test with $\mathcal{L}_n$ from Table 1. P-values are computed from the $\chi^2(\mathcal{L}_n - k_\theta)$ distribution. We do not report bootstrap test because the bootstrapped Q-test is arithmetically identical to Hong's (1996) bootstrapped standardized Q-test, as noted above.

Wild Bootstrap and Dependent Wild Bootstrap P-Value Computation The following details how sample autocorrelations are bootstrapped by the wild bootstrap (Wu, 1986, Liu, 1988) and dependent wild bootstrap (Shao, 2011). Recall $m_t(\theta)$ are the estimating equations for $\hat{\theta}_n$, let $\widehat{\mathcal{A}}_n$ be a consistent estimator of $\mathcal{A}$ in Assumption 2.c, and define

$$\hat{\mathcal{D}}_n(h) \equiv \frac{1}{n}\sum_{t=h+1}^{n}\left\{\left(\epsilon_t(\hat{\theta}_n)s_t(\hat{\theta}_n)+\frac{G_t(\hat{\theta}_n)}{\sigma_t(\hat{\theta}_n)}\right)\epsilon_{t-h}(\hat{\theta}_n)+\epsilon_t(\hat{\theta}_n)\left(\epsilon_{t-h}(\hat{\theta}_n)s_{t-h}(\hat{\theta}_n)+\frac{G_{t-h}(\hat{\theta}_n)}{\sigma_{t-h}(\hat{\theta}_n)}\right)\right\} \quad \text{(H.1)}$$

and

$$\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n) \equiv \epsilon_t(\hat{\theta}_n)\epsilon_{t-h}(\hat{\theta}_n) - \hat{\mathcal{D}}_n(h)'\widehat{\mathcal{A}}_n m_t(\hat{\theta}_n).$$

The dependent wild bootstrapped sample correlation $\hat{\rho}_{n,i}^{(dw)}(h)$ is computed as:

$$\hat{\rho}_n^{(dw)}(h) \equiv \frac{1}{1/n\sum_{t=1}^{n}\epsilon_t^2(\hat{\theta}_n)}\frac{1}{n}\sum_{t=1+h}^{n}\varphi_t\left\{\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n)-\frac{1}{n}\sum_{s=1+h}^{n}\widehat{\mathcal{E}}_{n,s,h}(\hat{\theta}_n)\right\}.$$

Notice $\hat{\rho}_n^{(dw)}(h)$ exploits the Lemma 2.1 correlation expansion via $\widehat{\mathcal{E}}_{n,t,h}(\hat{\theta}_n)$, which correctly accounts for the first order (asymptotic) impact of the i^{th} sample's plug-in $\hat{\theta}_{n,i}$.

In each case we generate $M = 500$ bootstrap samples. The dependent wild bootstrap requires a block size b_n, while Shao (2011) uses $b_n = b\sqrt{n}$ with $b \in \{.5, 1, 2\}$, leading to qualitatively similar results. We therefore simply use the middle value $b_n = \sqrt{n}$.[6] The wild bootstrap has block size $b_n = 1$ and no re-centering with $1/n\sum_{s=1+h}^{n}\widehat{\mathcal{E}}_{n,s,h}(\hat{\theta}_n)$.

We perform the max-correlation test with fixed $\mathcal{L}_n$ and wild bootstrap or dependent wild bootstrap, denoted $\hat{\mathcal{T}}_n^{w}(\mathcal{L}_n)$ and $\hat{\mathcal{T}}_n^{dw}(\mathcal{L}_n)$. We perform the max-correlation test with automatic lag by dependent wild bootstrap, $\hat{\mathcal{T}}_n^{dw}(\mathcal{L}_n^*)$.

Consider $\hat{\mathcal{T}}_n^{dw}(\mathcal{L}_n^*)$. We compute the bootstrapped statistic $\hat{\mathcal{T}}_{n,i}^{(dw)}(\mathcal{L}_{n,i}^*) \equiv \sqrt{n}\max_{1\leq h\leq \mathcal{L}_{n,i}^*}|\hat{\rho}_{n,i}^{(dw)}(h)|$ for each bootstrap sample $i \in \{1,\ldots,M\}$ with $M = 500$. Note that $\mathcal{L}_{n,i}^*$ is the automatic lag for the i^{th} bootstrap sample specifically. The approximate p-value is computed as

$$\hat{p}_{n,M}^{(dw)} = \frac{1}{M}\sum_{i=1}^{M} I\left(\hat{\mathcal{T}}_{n,i}^{(dw)}(\mathcal{L}_{n,i}^*) \geq \hat{\mathcal{T}}_n(\mathcal{L}_n^*)\right).$$

The test proposed rejects the null at nominal size α when $\hat{p}_{n,M}^{(dw)} < \alpha$.

The Q-statistic, Hong's normalized Q-statistic, the sup-LM statistic and the CvM statistic are linear combinations of (squared) residual correlations. Therefore, exactly as with the max-correlation test, both bootstrap procedures are applied based on the Lemma 2.1 correlation expansion in order to account for the impact of the estimator $\hat{\theta}_n$ on asymptotics.

[6] In simulations not reported here, we compared $b_n = b\sqrt{n}$ across $b \in \{.5, 1, 2\}$ and found there is little difference in test performance.

H.2 Simulation Results

As in the main paper, sample size is $n \in \{100, 250, 500, 1000\}$, and $J = 1000$ independent Monte Carlo samples and $M = 500$ independent bootstrap samples are generated. Rejection frequencies with respect to nominal size $\alpha \in \{.010, .050, .100\}$ are reported. See Tables 3-29 for rejection frequencies from Sup-LM, Hong, Q, and $\hat{\mathcal{T}}_n(\mathcal{L}_n)$ each based on $\mathcal{L}_n = o(n)$ from Table 1. Tables 30-34 contain results for sup-LM with $\mathcal{L}_n = n - 1$ and CvM with $\mathcal{L}_n = n - 1$ and $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$. The groupings follow from the fact that the former group uses various $\mathcal{L}_n$, while the latter group uses exactly one maximum lag.

H.2.1 Empirical Size

Max-Correlation test We focus on the max-correlation test with the dependent wild bootstrap and fixed $\mathcal{L}_n$ (denoted $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$). We then compare results with $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ in order to see how our proposed lag selection approach improves inference. We comment below on the wild bootstrap results as compared to the dependent wild bootstrap.

Consider Scenario #1, $n = 100$, and iid error e_t (see Table 3). The empirical size with respect to nominal sizes $\alpha \in \{.010, .050, .100\}$ is $\{.009, .060, .135\}$ at $\mathcal{L}_n = 5$, $\{.009, .047, .113\}$ at $\mathcal{L}_n = 10$, and $\{.000, .026, .077\}$ at $\mathcal{L}_n = 21$. The empirical size of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$, in contrast, is $\{.017, .068, .128\}$ (see Table 30). Those results suggest that $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ with $\mathcal{L}_n = o(n)$ has a tendency of under-rejections when lag length is large.

The same implication holds under more complex scenarios which involve filters. See, for example, Scenario #6, $n = 1000$, and iid error (Table 26). The empirical size of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ is $\{.011, .052, .106\}$ at $\mathcal{L}_n = 5$, $\{.002, .028, .083\}$ at $\mathcal{L}_n = 72$, and $\{.004, .022, .066\}$ at $\mathcal{L}_n = 144$. The empirical size of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$, in contrast, is $\{.011, .047, .101\}$ (Table 33). Those results highlight that the automatic lag selection significantly stabilizes the empirical size of the max-correlation test by trimming redundant lags.

Hong's Test with Dependent Wild Bootstrap Hong's test with the dependent wild bootstrap ($\mathcal{N}^{dw}$) is even more conservative than $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ at large lags. In Scenario #1, $n = 100$, and iid error e_t (Table 3), the empirical size of $\mathcal{N}^{dw}$ is $\{.007, .054, .122\}$ at $\mathcal{L}_n = 5$, $\{.002, .023, .073\}$ at $\mathcal{L}_n = 10$, and $\{.000, .026, .075\}$ at $\mathcal{L}_n = 21$. In Scenario #6, $n = 1000$, and iid error (Table 26), the empirical size of $\mathcal{N}^{dw}$ is $\{.018, .061, .137\}$ at $\mathcal{L}_n = 5$, $\{.001, .006, .028\}$ at $\mathcal{L}_n = 72$, and $\{.000, .001, .014\}$ at $\mathcal{L}_n = 144$. Given $\mathcal{L}_n = o(n)$, the max-correlation test statistic leads to a more accurate size than Hong's statistic arguably because the former has a simpler structure than the latter. The former picks only the largest sample correlation, while the latter sums up all $\mathcal{L}_n$ sample correlations into one test statistic.

Bootstrapped Sup-LM Test and CvM Test We now discuss the sup-LM tests assisted by the dependent wild bootstrap ($\mathcal{AP}^{dw}$) with $\mathcal{L}_n = o(n)$ from Table 1 and $\mathcal{L}_n = n - 1$; and the CvM test assisted by the dependent wild bootstrap (CvM^{dw}) or BRWB (CvM^{brw}). Each has similar empirical size. $\mathcal{AP}^{dw}$ with $\mathcal{L}_n = o(n)$ produces stable empirical size across various lag lengths, which is a considerable advantage over $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ and $\mathcal{N}^{dw}$. See, for example, Scenario #1, $n = 100$, and iid error e_t (Table

3). The empirical size of $\mathcal{AP}^{dw}$ is $\{.014, .071, .135\}$ at $\mathcal{L}_n = 5$, $\{.020, .082, .154\}$ at $\mathcal{L}_n = 10$, and $\{.020, .064, .134\}$ at $\mathcal{L}_n = 21$.

A disadvantage of the sup-LM and CvM tests is that they sometimes lead to over-rejections. Such a tendency is indeed observed from the rejection frequencies of $\mathcal{AP}^{dw}$ reported above. Another example is $\mathcal{L}_n = n-1$, Scenario #2, $n = 100$, and iid error (Table 30). The empirical size is $\{.022, .083, .151\}$ for $\mathcal{AP}^{dw}$, $\{.018, .076, .149\}$ for CvM^{dw}, and $\{.045, .115, .186\}$ for CvM^{brw}. The empirical size of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is $\{.008, .047, .090\}$, which is much sharper than the size of sup-LM and CvM tests.

Asymptotic Q-Test and Asymptotic Hong Test The asymptotic Ljung-Box test (Q_{χ^2}) and the asymptotic Hong test ($\mathcal{N}^{N_{0,1}}$) produce fatal size distortions when the true process is non-iid. See, for example, Scenario #2, $n = 1000$, and iid error e_t (Table 10), where the tested variable is bilinear. The empirical size at $\mathcal{L}_n = 5$ is $\{.064, .169, .254\}$ for Q_{χ^2} and $\{.135, .207, .264\}$ for $\mathcal{N}^{N_{0,1}}$. Similar results are observed in Scenario #5, $n = 1000$, and iid error e_t (Table 22), where the tested variable is GARCH. Those results confirm that the asymptotic convergence of the Ljung-Box test statistic to χ^2 and that of the Hong test statistic to $N(0,1)$ require more than serial uncorrelatedness. Hence, those tests are not white noise tests in a strict sense.

Wild Bootstrap Versus Dependent Wild Bootstrap Given our simulation design, the wild bootstrap often results in more accurate size than the dependent wild bootstrap for each test under study. Revisit Scenario #6, $n = 1000$, and iid error (Table 26), where $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ becomes more and more conservative as lag length increases as stated above. The max-correlation test with the wild bootstrap ($\hat{\mathcal{T}}^{w}(\mathcal{L}_n)$), in contrast, leads to stable empirical size of $\{.011, .048, .102\}$ at $\mathcal{L}_n = 5$, $\{.006, .049, .090\}$ at $\mathcal{L}_n = 72$, and $\{.005, .041, .103\}$ at $\mathcal{L}_n = 144$. Similar results arise for Hong's test, the sup-LM test, and the CvM test as well. Interestingly, the wild bootstrap still operates well under non-mds white noise cases like bilinear. Those results, however, call for some caution since there is not a theoretical guarantee that the wild bootstrap is valid under non-mds white noise.[7]

Summary The empirical size of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is overall more accurate than the size of any other test in this study. $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is free of an ad-hoc selection of lag length, and has sharp size even in small samples with relatively challenging scenarios like bilinear or GARCH.

H.2.2 Empirical Power

We now discuss empirical power of each test. We do not discuss Q_{χ^2} or $\mathcal{N}^{N_{0,1}}$ since they produce fatal size distortions under non-iid scenarios. We do not discuss $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$ or $\mathcal{N}^{dw}$ with $\mathcal{L}_n = o(n)$ since they tend to be conservative at large lags. Since $\mathcal{AP}^{dw}$ with $\mathcal{L}_n = o(n)$ and $\mathcal{L}_n = n-1$ produce virtually identical results, we discuss the latter only. Thus, we compare $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ with $\mathcal{AP}^{dw}$, CvM^{dw}, and CvM^{brw} with $\mathcal{L}_n = n-1$.

[7] It is left as a future task to find a numerical example of non-mds white noise where the wild bootstrap fails and the dependent wild bootstrap operates well.

In Scenarios #1–#6, there is not an obvious ranking among $\{\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*), \mathcal{AP}^{dw}, CvM^{dw}, CvM^{brw}\}$. $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is more powerful than the other tests in some cases, but not in other cases. Overall we do not observe a drastic difference in power. See, for example, Scenario #2, $n = 1000$, and AR(1) error (Table 33). The empirical power with respect to nominal sizes $\alpha \in \{.010, .050, .100\}$ is $\{.723, .823, .864\}$ for $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$, $\{.408, .661, .774\}$ for $\mathcal{AP}^{dw}$, $\{.474, .697, .810\}$ for CvM^{dw}, and $\{.615, .761, .833\}$ for CvM^{brw}. In this specific case, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is most powerful; CvM^{brw} is the second; CvM^{dw} is the third; $\mathcal{AP}^{dw}$ is least powerful.

Another example is Scenario #3, $n = 1000$, and AR(1) error (Table 33). The empirical power is $\{.599, .847, .922\}$ for $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$, $\{.459, .681, .787\}$ for $\mathcal{AP}^{dw}$, $\{.688, .876, .923\}$ for CvM^{dw}, and $\{.700, .878, .913\}$ for CvM^{brw}. In this specific case, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is more powerful than $\mathcal{AP}^{dw}$, but slightly less powerful than $\{CvM^{dw}, CvM^{brw}\}$ when the nominal size is 1%.

In Scenarios #7–#9, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ dominates $\{\mathcal{AP}^{dw}, CvM^{dw}, CvM^{brw}\}$ completely. The former can detect remote autocorrelations given a large enough sample size n, while the latter cannot. The power of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ under Scenario #8 (Remote MA(12)), for instance, is $\{.013, .067, .117\}$ for $n = 100$, $\{.024, .134, .244\}$ for $n = 250$, $\{.371, .673, .770\}$ for $n = 500$, and $\{.983, .997, .997\}$ for $n = 1000$ (Table 34). We observe that $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ captures remote autocorrelation with increasing probability as n grows. The reason for this desired result is that, as confirmed in Hill and Motegi (2019, Table 2), $\mathcal{L}_n^*$ converges to $h^* = 12$ sufficiently fast under Remote MA(12). See Section 4.2.3 in the main paper for further discussion.

The power of CvM^{brw}, by contrast, is $\{.056, .125, .189\}$ for $n = 100$, $\{.020, .094, .156\}$ for $n = 250$, $\{.020, .068, .128\}$ for $n = 500$, and $\{.026, .081, .152\}$ for $n = 1000$ (Table 34). CvM^{brw} has no power against the remote autocorrelation even when sample size is as large as $n = 1000$. The power of $\mathcal{AP}^{dw}$ and CvM^{dw} is virtually identical to the power of CvM^{brw}. Hence, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is the only test under study that has power against remote autocorrelations.

The reason why the sup-LM and CvM tests fail to capture remote autocorrelations is that those test statistics assign the largest weights to small lags. That feature delivers sharp size and high power against adjacent correlations like those in Scenarios #1–#6, but critically low power against remote correlations like those in Scenarios #7–#9. The present max-correlation test statistic, in contrast, assigns equal weights to all lags. That feature itself delivers under-rejections and low power against adjacent correlations when $\mathcal{L}_n$ is large, but such a shortcoming is addressed by our proposed automatic lag selection mechanism.[8]

Summary Taking both size and power into account, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ generally dominates each white noise test studied here. First, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ achieves the most accurate size. Second, under H_1 with adjacent correlations, the relative performance of $\{\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*), \mathcal{AP}^{dw}, CvM^{dw}, CvM^{brw}\}$ varies across cases, but in any case there is not a large difference in power. Third, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ has much higher power against remote

[8]As mentioned in the main paper, a weighted max-correlation test where the weights are inverted standard errors may provide necessary improvements when the data are extremely volatile (e.g. bilinear with GARCH error). The difficulty there, however, is the need for a non-parametric variance estimator that will be sensitive to choice of tuning parameter. These issues are left for a future study.

autocorrelations than the strongest competitors $\{\mathcal{AP}^{dw}, CvM^{dw}, CvM^{brw}\}$. The latter tests cannot detect remote autocorrelations even when sample size is as large as $n = 1000$. Thus, $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ is indeed the *only* white noise test in this study that achieves *both* accurate size under each null *and* high power under each alternative.

Table 2: List of All White Noise Tests

Tests with non-random $\mathcal{L}_n = o(n)$ (cf. Table 1)

Symbol	Test Statistic	P-Value Computation	Reference
$\hat{\mathcal{T}}^{w}(\mathcal{L}_n)$	max-corr with fixed $\mathcal{L}_n$	wild bootstrap	Hill and Motegi (2019)
$\hat{\mathcal{T}}^{dw}(\mathcal{L}_n)$	max-corr with fixed $\mathcal{L}_n$	dependent wild bootstrap	Hill and Motegi (2019)
$\mathcal{AP}^{w}$	sup-LM	wild bootstrap	Andrews and Ploberger (1996)
$\mathcal{AP}^{dw}$	sup-LM	dependent wild bootstrap	Andrews and Ploberger (1996)
Q_{χ^2}	Ljung-Box Q-statistic	asymptotic χ^2	Ljung and Box (1978)
$\mathcal{N}^{N_{0,1}}$	std. Ljung-Box Q-statistic	asymptotic $N(0,1)$	Hong (1996)
$\mathcal{N}^{w}$	std. Ljung-Box Q-statistic	wild bootstrap	Hong (1996)
$\mathcal{N}^{dw}$	std. Ljung-Box Q-statistic	dependent wild bootstrap	Hong (1996)

Tests with $\mathcal{L}_n = n - 1$

Symbol	Test Statistic	P-Value Computation	Reference
$\mathcal{AP}^{w}$	sup-LM	wild bootstrap	Andrews and Ploberger (1996)
$\mathcal{AP}^{dw}$	sup-LM	dependent wild bootstrap	Andrews and Ploberger (1996)
CvM^{w}	Cramér-von Mises	wild bootstrap	-
CvM^{dw}	Cramér-von Mises	dependent wild bootstrap	Shao (2011)
CvM^{brw}	Cramér-von Mises	blockwise random weighting bootstrap	Zhu and Li (2015)

Test with automatic lag $\mathcal{L}_n^*$

Symbol	Test Statistic	P-Value Computation	Reference
$\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$	max-corr with automatic $\mathcal{L}_n^*$	dependent wild bootstrap	Hill and Motegi (2019)

std. = standardized. For Q_{χ^2}, the degrees of freedom of the asymptotic χ^2-distribution are $\mathcal{L}_n - 2$ in Scenario #3 since an AR(2) filter is used; $\mathcal{L}_n - 1$ in Scenario #4 since an AR(1) filter is used; and $\mathcal{L}_n$ in the other scenarios. The automatic $\mathcal{L}_n^*$ is computed such that $\mathcal{L}_n^*/\bar{\mathcal{L}}_n \to [0,1]$ for a non-random upper bound $\bar{\mathcal{L}}_n = [10 \times \sqrt{n}/(\ln n)] = o(\sqrt{n}/\ln(n))$.

Table 3: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #1 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.006, .039, .089	.004, .038, .096	.005, .028, .066	.005, .041, .090	.003, .043, .082	.003, .044, .089
$\hat{\mathcal{T}}^{dw}$	.009, .060, .135	.009, .047, .113	.000, .026, .077	.002, .048, .093	.005, .044, .099	.001, .021, .057
$\mathcal{AP}^w$	.002, .047, .097	.008, .026, .087	.001, .033, .071	.007, .050, .087	.003, .038, .090	.007, .035, .073
$\mathcal{AP}^{dw}$	.014, .071, .135	.020, .082, .154	.020, .064, .134	.014, .062, .134	.013, .063, .145	.012, .058, .125
Q_{χ^2}	.010, .045, .094	.020, .057, .113	.029, .095, .157	.033, .098, .172	.036, .091, .150	.049, .103, .152
$\mathcal{N}^{N_{0,1}}$	.021, .041, .087	.019, .055, .096	.023, .050, .070	.087, .139, .191	.042, .094, .144	.032, .058, .092
$\mathcal{N}^w$	.002, .032, .077	.005, .036, .092	.003, .040, .086	.008, .042, .086	.008, .031, .088	.005, .044, .081
$\mathcal{N}^{dw}$	.007, .054, .122	.002, .023, .073	.000, .026, .075	.003, .040, .103	.000, .027, .082	.000, .012, .047

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.918, .984, .991	.907, .974, .984	.902, .970, .985	1.00, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.788, .956, .990	.706, .950, .984	.623, .922, .974	.955, .998, 1.00	.941, 1.00, 1.00	.936, .995, .999
$\mathcal{AP}^w$	.922, .992, 1.00	.900, .986, .996	.904, .982, .995	1.00, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	.743, .956, .991	.697, .934, .979	.677, .935, .985	.831, .981, .998	.756, .965, .992	.758, .956, .998
Q_{χ^2}	.937, .977, .991	.870, .960, .970	.762, .880, .929	.998, 1.00, 1.00	1.00, 1.00, 1.00	.996, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	.971, .991, .994	.913, .963, .978	.774, .856, .899	1.00, 1.00, 1.00	.998, .998, .998	.996, 1.00, 1.00
$\mathcal{N}^w$	.844, .975, .986	.644, .894, .950	.434, .771, .884	.996, 1.00, 1.00	.987, 1.00, 1.00	.964, .998, 1.00
$\mathcal{N}^{dw}$	.492, .839, .950	.168, .608, .841	.048, .253, .529	.703, .961, .996	.482, .897, .978	.172, .594, .853

Scenario #1: Simple $y_t = e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 4: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #1 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.008, .043, .086	.007, .039, .090	.007, .039, .072	.005, .039, .084	.008, .050, .086	.004, .041, .082
$\hat{\mathcal{T}}^{dw}$	.013, .065, .135	.003, .046, .094	.001, .032, .080	.007, .041, .108	.001, .033, .080	.001, .019, .052
$\mathcal{AP}^w$	.006, .040, .093	.001, .037, .092	.008, .047, .093	.004, .038, .077	.007, .042, .082	.006, .039, .073
$\mathcal{AP}^{dw}$	.008, .060, .123	.013, .062, .134	.010, .064, .109	.018, .064, .108	.007, .055, .118	.011, .050, .109
Q_{χ^2}	.015, .056, .109	.022, .057, .097	.026, .086, .133	.049, .114, .188	.039, .108, .164	.035, .091, .151
$\mathcal{N}^{N_{0,1}}$	.024, .049, .085	.028, .057, .092	.020, .053, .083	.080, .149, .201	.045, .096, .132	.016, .054, .082
$\mathcal{N}^w$	.010, .051, .095	.005, .039, .088	.006, .031, .073	.006, .044, .097	.008, .040, .084	.002, .030, .080
$\mathcal{N}^{dw}$	.004, .046, .116	.000, .015, .050	.001, .007, .033	.003, .044, .103	.000, .005, .036	.000, .009, .031

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	.995, 1.00, 1.00	.974, .999, 1.00	.971, .999, 1.00	.991, .999, 1.00	.973, 1.00, 1.00	.980, 1.00, 1.00
Q_{χ^2}	1.00, 1.00, 1.00	.999, 1.00, 1.00	.990, .998, .998	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	.999, 1.00, 1.00	.991, .998, .999	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	1.00, 1.00, 1.00	.998, 1.00, 1.00	.949, .996, .998	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{dw}$	.979, 1.00, 1.00	.435, .917, .987	.091, .534, .824	.987, 1.00, 1.00	.823, .986, .998	.365, .904, .990

Scenario #1: Simple $y_t = e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 5: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #1 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.006, .048, .109	.010, .043, .098	.004, .033, .081	.005, .056, .103	.006, .048, .081	.005, .039, .095
$\hat{\mathcal{T}}^{dw}$	.012, .055, .111	.001, .029, .062	.000, .021, .060	.007, .043, .102	.001, .031, .070	.000, .014, .052
$\mathcal{AP}^w$	.005, .038, .082	.009, .050, .087	.006, .050, .103	.010, .051, .102	.014, .053, .094	.008, .046, .095
$\mathcal{AP}^{dw}$	.014, .065, .121	.008, .050, .120	.007, .061, .115	.008, .044, .100	.011, .045, .089	.009, .065, .122
Q_{χ^2}	.012, .044, .104	.014, .057, .099	.026, .073, .110	.042, .135, .221	.028, .081, .158	.028, .098, .145
$\mathcal{N}^{N_{0,1}}$	.023, .059, .122	.019, .049, .079	.014, .028, .058	.089, .157, .213	.040, .084, .135	.019, .053, .096
$\mathcal{N}^w$	.008, .053, .108	.003, .045, .094	.005, .035, .085	.010, .048, .096	.005, .037, .105	.007, .028, .075
$\mathcal{N}^{dw}$	.004, .051, .121	.001, .010, .030	.000, .001, .018	.007, .043, .104	.000, .004, .030	.000, .002, .015

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.998, 1.00, 1.00	.999, 1.00, 1.00
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{dw}$	1.00, 1.00, 1.00	.876, .997, 1.00	.269, .898, .991	1.00, 1.00, 1.00	.974, 1.00, 1.00	.770, .994, 1.00

Scenario #1: Simple $y_t = e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 6: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #1 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.006, .043, .090	.003, .053, .104	.015, .059, .110	.005, .055, .098	.009, .047, .089	.008, .039, .094
$\hat{\mathcal{T}}^{dw}$	.013, .068, .128	.004, .033, .072	.003, .019, .069	.006, .054, .122	.003, .025, .068	.002, .025, .069
$\mathcal{AP}^w$	.008, .044, .094	.010, .044, .096	.011, .051, .094	.001, .035, .085	.005, .042, .094	.007, .037, .086
$\mathcal{AP}^{dw}$	.015, .063, .120	.013, .061, .106	.007, .056, .111	.012, .059, .100	.006, .040, .101	.005, .044, .101
Q_{χ^2}	.013, .050, .105	.014, .050, .122	.021, .067, .127	.048, .137, .213	.024, .089, .148	.024, .075, .128
$\mathcal{N}^{N_{0,1}}$	.020, .053, .097	.017, .064, .102	.008, .022, .057	.093, .162, .208	.020, .056, .112	.007, .028, .044
$\mathcal{N}^w$	.015, .050, .114	.013, .064, .115	.005, .037, .078	.009, .050, .096	.010, .048, .095	.008, .041, .082
$\mathcal{N}^{dw}$	.007, .041, .103	.000, .003, .021	.000, .004, .014	.005, .051, .099	.000, .006, .026	.000, .000, .011

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.842, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.987, 1.00, 1.00

Scenario #1: Simple $y_t = e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^{n} y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 7: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #2 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.007, .047, .108	.006, .040, .090	.006, .041, .080	.005, .035, .098	.004, .032, .078	.000, .020, .069
$\hat{\mathcal{T}}^{dw}$	.016, .066, .147	.002, .035, .105	.003, .032, .080	.000, .022, .076	.001, .015, .052	.000, .008, .035
$\mathcal{AP}^w$	.010, .062, .125	.011, .055, .126	.008, .040, .103	.004, .026, .066	.006, .040, .080	.007, .036, .091
$\mathcal{AP}^{dw}$	.028, .093, .167	.018, .080, .167	.024, .084, .156	.008, .038, .095	.002, .040, .084	.006, .038, .092
Q_{χ^2}	.045, .138, .229	.037, .104, .165	.057, .114, .181	.295, .440, .531	.238, .376, .462	.145, .234, .298
$\mathcal{N}^{N_{0,1}}$	.084, .151, .211	.058, .097, .153	.046, .080, .107	.366, .471, .522	.275, .367, .428	.113, .173, .213
$\mathcal{N}^w$	.007, .058, .120	.005, .040, .110	.007, .050, .105	.005, .030, .069	.002, .027, .078	.004, .025, .062
$\mathcal{N}^{dw}$	.003, .045, .128	.004, .028, .090	.002, .015, .051	.002, .024, .060	.000, .007, .039	.000, .005, .020

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.614, .819, .885	.548, .765, .867	.576, .805, .869	.685, .891, .942	.701, .891, .947	.689, .893, .945
$\hat{\mathcal{T}}^{dw}$	.328, .657, .803	.319, .628, .794	.235, .577, .762	.262, .538, .695	.258, .498, .682	.234, .489, .677
$\mathcal{AP}^w$	.471, .809, .901	.440, .817, .900	.451, .793, .900	.612, .874, .944	.596, .874, .944	.619, .870, .939
$\mathcal{AP}^{dw}$	.313, .629, .783	.319, .608, .770	.338, .613, .770	.247, .497, .674	.218, .476, .661	.221, .511, .694
Q_{χ^2}	.811, .929, .967	.726, .846, .905	.621, .742, .809	.965, .986, .996	.939, .971, .987	.912, .947, .966
$\mathcal{N}^{N_{0,1}}$	.863, .910, .934	.769, .834, .877	.586, .707, .765	.985, .993, .996	.957, .977, .988	.896, .925, .938
$\mathcal{N}^w$	.365, .718, .842	.320, .644, .794	.224, .532, .702	.565, .831, .919	.546, .839, .921	.481, .783, .875
$\mathcal{N}^{dw}$	.151, .453, .666	.062, .284, .546	.012, .121, .297	.166, .416, .619	.078, .342, .561	.042, .209, .453

Scenario #2: Bilinear $y_t = 0.5e_{t-1}y_{t-2} + e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 8: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #2 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.015, .073, .140	.008, .056, .108	.007, .044, .100	.000, .029, .084	.006, .039, .082	.002, .026, .071
$\hat{\mathcal{T}}^{dw}$	.006, .046, .099	.002, .034, .091	.002, .026, .071	.008, .030, .064	.008, .018, .052	.007, .018, .038
$\mathcal{AP}^w$	.014, .073, .128	.015, .063, .117	.024, .075, .128	.005, .030, .091	.005, .032, .084	.005, .026, .073
$\mathcal{AP}^{dw}$	.019, .081, .152	.014, .068, .141	.012, .058, .126	.010, .039, .084	.007, .024, .071	.008, .037, .069
Q_{χ^2}	.065, .155, .226	.031, .099, .164	.051, .114, .168	.564, .698, .755	.432, .522, .584	.244, .325, .386
$\mathcal{N}^{N_{0,1}}$	.113, .191, .249	.037, .094, .129	.039, .066, .091	.611, .672, .715	.439, .530, .581	.247, .307, .342
$\mathcal{N}^w$	.014, .072, .138	.011, .044, .100	.011, .042, .090	.005, .028, .072	.001, .020, .064	.000, .011, .060
$\mathcal{N}^{dw}$	.006, .050, .096	.001, .012, .041	.000, .004, .032	.005, .013, .049	.003, .011, .031	.001, .005, .010

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.946, .980, .991	.945, .985, .994	.951, .981, .989	.856, .956, .980	.863, .951, .977	.855, .952, .971
$\hat{\mathcal{T}}^{dw}$	.743, .915, .964	.717, .907, .964	.711, .898, .957	.344, .601, .744	.343, .584, .738	.383, .626, .753
$\mathcal{AP}^w$	.917, .986, .995	.911, .990, .997	.900, .977, .995	.846, .946, .969	.841, .955, .982	.813, .947, .975
$\mathcal{AP}^{dw}$	.582, .839, .920	.601, .810, .901	.572, .811, .909	.290, .562, .717	.281, .538, .698	.298, .563, .720
Q_{χ^2}	.997, .998, 1.00	.968, .987, .992	.925, .969, .983	.999, .999, .999	1.00, 1.00, 1.00	.998, .999, .999
$\mathcal{N}^{N_{0,1}}$	.999, .999, .999	.979, .992, .995	.923, .956, .973	1.00, 1.00, 1.00	.999, 1.00, 1.00	.995, .997, .998
$\mathcal{N}^w$	.847, .962, .979	.746, .924, .957	.623, .866, .924	.799, .956, .983	.817, .947, .974	.777, .939, .967
$\mathcal{N}^{dw}$	.489, .797, .907	.123, .523, .778	.025, .227, .508	.260, .525, .691	.139, .442, .642	.074, .337, .586

Scenario #2: Bilinear $y_t = 0.5e_{t-1}y_{t-2} + e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 9: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #2 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.010, .062, .118	.015, .053, .116	.015, .055, .114	.003, .015, .073	.002, .027, .061	.002, .024, .065
$\hat{\mathcal{T}}^{dw}$	.015, .054, .112	.001, .028, .069	.001, .019, .056	.007, .012, .052	.005, .023, .047	.006, .014, .036
$\mathcal{AP}^w$	.012, .069, .121	.019, .084, .147	.013, .077, .132	.002, .025, .080	.004, .028, .078	.002, .028, .068
$\mathcal{AP}^{dw}$	.020, .063, .132	.012, .074, .126	.013, .069, .130	.002, .023, .063	.008, .017, .047	.003, .016, .055
Q_{χ^2}	.052, .136, .215	.034, .089, .151	.039, .085, .147	.742, .817, .862	.534, .613, .663	.362, .438, .480
$\mathcal{N}^{N_{0,1}}$	.120, .187, .240	.041, .086, .134	.022, .055, .085	.781, .838, .863	.550, .619, .665	.321, .398, .433
$\mathcal{N}^w$	.016, .073, .145	.010, .047, .110	.004, .037, .082	.001, .021, .065	.001, .018, .060	.001, .011, .052
$\mathcal{N}^{dw}$	.006, .040, .095	.000, .006, .039	.000, .003, .018	.007, .023, .043	.002, .008, .017	.002, .003, .006

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.992, .996, .998	.993, .998, .999	.991, .993, .996	.907, .979, .989	.891, .967, .987	.889, .969, .984
$\hat{\mathcal{T}}^{dw}$	.907, .971, .991	.894, .976, .990	.897, .977, .987	.402, .648, .764	.372, .629, .761	.411, .621, .770
$\mathcal{AP}^w$	.977, .994, .996	.984, 1.00, 1.00	.976, .993, .999	.913, .975, .988	.892, .971, .985	.891, .974, .986
$\mathcal{AP}^{dw}$	.767, .923, .967	.767, .921, .966	.767, .936, .970	.378, .628, .784	.347, .623, .751	.343, .590, .740
Q_{χ^2}	1.00, 1.00, 1.00	.999, .999, 1.00	.999, .999, .999	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.994, .998, .998	1.00, 1.00, 1.00	.998, .998, .999	.999, .999, .999
$\mathcal{N}^w$	.969, .993, .999	.959, .991, .996	.924, .984, .996	.872, .967, .988	.856, .962, .984	.875, .966, .982
$\mathcal{N}^{dw}$	.744, .917, .965	.315, .788, .924	.055, .460, .777	.317, .570, .721	.193, .483, .681	.094, .411, .667

Scenario #2: Bilinear $y_t = 0.5e_{t-1}y_{t-2} + e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^{n} y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 10: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #2 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.017, .087, .137	.013, .066, .125	.009, .052, .107	.002, .026, .083	.001, .028, .076	.002, .026, .064
$\hat{\mathcal{T}}^{dw}$	.012, .062, .112	.008, .026, .072	.004, .016, .057	.002, .012, .040	.003, .007, .030	.000, .005, .018
$\mathcal{AP}^w$	.015, .075, .138	.015, .079, .149	.016, .080, .147	.001, .030, .082	.003, .024, .073	.003, .036, .065
$\mathcal{AP}^{dw}$	.014, .063, .109	.015, .076, .137	.012, .066, .119	.001, .014, .039	.004, .019, .047	.000, .010, .042
Q_{χ^2}	.064, .169, .254	.030, .087, .153	.020, .081, .145	.855, .907, .931	.632, .715, .765	.436, .509, .550
$\mathcal{N}^{N_{0,1}}$	.135, .207, .264	.033, .087, .127	.009, .024, .044	.877, .901, .917	.632, .698, .718	.411, .469, .518
$\mathcal{N}^w$	.029, .085, .147	.009, .053, .107	.007, .037, .076	.001, .031, .069	.001, .016, .053	.000, .007, .047
$\mathcal{N}^{dw}$	.007, .045, .102	.000, .003, .036	.000, .000, .005	.001, .010, .024	.001, .001, .006	.000, .000, .001

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.999, 1.00, 1.00	.997, .997, .997	.995, 1.00, 1.00	.929, .974, .987	.924, .977, .986	.901, .968, .980
$\hat{\mathcal{T}}^{dw}$	.966, .994, .995	.966, .992, .999	.961, .989, .997	.486, .678, .789	.482, .705, .811	.480, .680, .791
$\mathcal{AP}^w$	.996, .998, .998	.996, 1.00, 1.00	.996, 1.00, 1.00	.924, .972, .988	.933, .977, .989	.924, .984, .989
$\mathcal{AP}^{dw}$	.891, .973, .991	.902, .973, .991	.895, .968, .981	.396, .659, .789	.375, .627, .786	.415, .677, .807
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.998, .998, .999
$\mathcal{N}^w$	.997, .999, 1.00	.990, .998, 1.00	.990, .996, .997	.926, .977, .984	.918, .977, .992	.917, .973, .986
$\mathcal{N}^{dw}$	.911, .971, .990	.674, .933, .972	.218, .793, .953	.392, .643, .775	.254, .548, .706	.145, .492, .669

Scenario #2: Bilinear $y_t = 0.5e_{t-1}y_{t-2} + e_t$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^n y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 11: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #3 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.008, .048, .099	.008, .049, .090	.005, .034, .093	.007, .037, .088	.010, .036, .072	.006, .038, .080
$\hat{\mathcal{T}}^{dw}$	.025, .106, .181	.015, .068, .142	.006, .047, .118	.019, .075, .164	.005, .053, .133	.002, .034, .088
$\mathcal{AP}^w$	.010, .045, .102	.005, .036, .107	.006, .047, .095	.005, .045, .100	.006, .042, .106	.004, .029, .075
$\mathcal{AP}^{dw}$	.036, .110, .185	.037, .120, .198	.028, .103, .182	.033, .109, .177	.034, .112, .194	.037, .121, .204
Q_{χ^2}	.008, .056, .107	.009, .050, .098	.019, .069, .128	.023, .062, .118	.019, .050, .111	.014, .049, .085
$\mathcal{N}^{N_{0,1}}$	.005, .013, .021	.010, .020, .041	.008, .023, .042	.010, .020, .035	.008, .024, .037	.006, .014, .024
$\mathcal{N}^w$	.007, .050, .119	.006, .031, .063	.005, .035, .070	.007, .043, .084	.003, .037, .087	.002, .025, .073
$\mathcal{N}^{dw}$	.025, .102, .191	.006, .044, .116	.000, .023, .084	.022, .082, .155	.002, .044, .122	.000, .018, .078

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.008, .068, .139	.006, .049, .114	.007, .045, .099	.028, .101, .192	.005, .058, .129	.010, .040, .092
$\hat{\mathcal{T}}^{dw}$	.041, .143, .235	.012, .078, .174	.007, .063, .141	.039, .143, .239	.011, .070, .170	.004, .061, .141
$\mathcal{AP}^w$	.015, .068, .115	.022, .061, .111	.014, .050, .108	.023, .115, .191	.028, .113, .203	.030, .112, .188
$\mathcal{AP}^{dw}$	.030, .108, .194	.031, .108, .192	.032, .118, .196	.091, .218, .308	.098, .211, .299	.109, .227, .298
Q_{χ^2}	.020, .084, .153	.010, .063, .131	.024, .069, .116	.030, .113, .185	.027, .100, .168	.035, .084, .142
$\mathcal{N}^{N_{0,1}}$	.021, .036, .057	.017, .037, .062	.019, .035, .058	.028, .054, .078	.017, .047, .079	.015, .034, .055
$\mathcal{N}^w$	.013, .085, .170	.015, .070, .134	.009, .041, .091	.026, .101, .184	.013, .069, .151	.009, .052, .114
$\mathcal{N}^{dw}$	.033, .129, .219	.007, .064, .142	.007, .045, .109	.046, .148, .234	.013, .086, .171	.003, .040, .111

Scenario #3: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(2) filter $\epsilon_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 12: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #3 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.011, .054, .104	.005, .039, .083	.007, .045, .099	.011, .050, .088	.010, .044, .092	.007, .035, .092
$\hat{\mathcal{T}}^{dw}$	.020, .063, .125	.005, .033, .082	.004, .033, .082	.015, .065, .139	.005, .029, .084	.006, .032, .080
$\mathcal{AP}^w$	.014, .053, .118	.007, .049, .096	.010, .044, .096	.003, .044, .085	.006, .035, .087	.007, .034, .092
$\mathcal{AP}^{dw}$	.019, .092, .149	.015, .079, .144	.022, .080, .139	.018, .078, .152	.014, .075, .146	.023, .082, .169
Q_{χ^2}	.006, .045, .103	.014, .058, .089	.016, .069, .113	.023, .079, .138	.017, .081, .134	.026, .068, .110
$\mathcal{N}^{N_{0,1}}$	.003, .006, .012	.011, .032, .056	.010, .024, .041	.016, .031, .049	.014, .034, .064	.012, .024, .043
$\mathcal{N}^w$	.005, .049, .111	.008, .037, .080	.006, .039, .080	.011, .045, .104	.004, .039, .081	.007, .036, .076
$\mathcal{N}^{dw}$	.007, .062, .135	.002, .022, .072	.001, .007, .038	.012, .065, .136	.001, .014, .060	.000, .007, .034

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.035, .137, .227	.013, .065, .132	.008, .046, .098	.052, .187, .309	.014, .063, .142	.007, .054, .107
$\hat{\mathcal{T}}^{dw}$	.038, .143, .246	.005, .045, .110	.004, .038, .104	.059, .190, .309	.007, .053, .148	.006, .039, .109
$\mathcal{AP}^w$	.028, .098, .184	.032, .085, .156	.021, .097, .164	.082, .220, .341	.058, .195, .303	.085, .208, .306
$\mathcal{AP}^{dw}$	.030, .114, .184	.022, .110, .196	.024, .089, .162	.136, .298, .424	.169, .310, .393	.154, .307, .397
Q_{χ^2}	.040, .149, .239	.025, .081, .139	.042, .107, .172	.108, .255, .373	.047, .137, .207	.049, .127, .198
$\mathcal{N}^{N_{0,1}}$	.033, .077, .106	.020, .050, .078	.019, .039, .054	.087, .126, .172	.034, .075, .123	.023, .058, .093
$\mathcal{N}^w$	.048, .157, .258	.011, .066, .128	.014, .066, .117	.074, .210, .331	.032, .105, .192	.014, .062, .136
$\mathcal{N}^{dw}$	.032, .134, .253	.001, .029, .089	.001, .012, .054	.084, .256, .377	.007, .050, .143	.002, .016, .075

Scenario #3: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(2) filter $\epsilon_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 13: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #3 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.011, .055, .116	.012, .041, .084	.012, .054, .106	.005, .043, .094	.010, .054, .101	.005, .035, .088
$\hat{\mathcal{T}}^{dw}$	.009, .067, .120	.002, .027, .095	.003, .026, .081	.020, .076, .131	.002, .030, .078	.001, .021, .055
$\mathcal{AP}^w$	.007, .059, .110	.005, .043, .090	.011, .051, .098	.014, .049, .102	.007, .048, .106	.006, .044, .087
$\mathcal{AP}^{dw}$	.015, .063, .119	.019, .076, .149	.024, .074, .132	.015, .060, .115	.012, .058, .125	.012, .062, .128
Q_{χ^2}	.011, .054, .111	.009, .049, .091	.019, .070, .121	.018, .076, .136	.023, .069, .125	.023, .071, .122
$\mathcal{N}^{N_{0,1}}$	.004, .015, .030	.009, .034, .062	.005, .016, .029	.010, .023, .048	.009, .038, .069	.007, .023, .041
$\mathcal{N}^w$	.012, .051, .091	.004, .046, .103	.007, .032, .085	.006, .050, .104	.009, .038, .085	.004, .036, .079
$\mathcal{N}^{dw}$	.018, .053, .114	.001, .008, .046	.000, .002, .023	.016, .066, .126	.001, .006, .036	.000, .001, .012

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.087, .251, .379	.014, .080, .150	.015, .069, .116	.149, .372, .539	.026, .091, .171	.008, .084, .165
$\hat{\mathcal{T}}^{dw}$	.098, .284, .390	.005, .054, .117	.005, .041, .099	.133, .357, .505	.006, .061, .137	.004, .038, .107
$\mathcal{AP}^w$	.042, .158, .232	.034, .126, .205	.037, .115, .199	.177, .407, .539	.128, .342, .492	.143, .339, .488
$\mathcal{AP}^{dw}$	.048, .139, .220	.026, .118, .211	.018, .085, .161	.276, .489, .613	.263, .441, .550	.257, .442, .572
Q_{χ^2}	.119, .294, .413	.029, .102, .180	.037, .114, .184	.267, .459, .596	.069, .175, .264	.064, .163, .252
$\mathcal{N}^{N_{0,1}}$	.085, .155, .212	.035, .073, .117	.019, .037, .065	.191, .287, .364	.058, .139, .198	.033, .079, .109
$\mathcal{N}^w$	.108, .293, .398	.031, .090, .168	.025, .077, .144	.217, .415, .548	.052, .144, .232	.025, .097, .192
$\mathcal{N}^{dw}$	.097, .258, .410	.003, .032, .092	.001, .008, .048	.178, .425, .552	.001, .042, .130	.000, .010, .054

Scenario #3: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(2) filter $\epsilon_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 14: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #3 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.011, .049, .104	.009, .048, .091	.011, .045, .089	.013, .056, .113	.010, .056, .093	.009, .040, .085
$\hat{\mathcal{T}}^{dw}$	.013, .064, .110	.003, .034, .091	.003, .025, .052	.008, .060, .132	.002, .026, .071	.004, .022, .059
$\mathcal{AP}^w$	.010, .044, .104	.012, .053, .100	.008, .057, .097	.010, .051, .109	.004, .028, .084	.005, .045, .094
$\mathcal{AP}^{dw}$	.012, .065, .121	.020, .073, .138	.018, .065, .106	.020, .059, .114	.011, .067, .119	.016, .064, .133
Q_{χ^2}	.008, .049, .098	.012, .055, .085	.021, .081, .133	.033, .100, .164	.019, .066, .123	.020, .072, .130
$\mathcal{N}^{N_{0,1}}$	.009, .028, .040	.007, .027, .058	.007, .018, .036	.017, .032, .050	.011, .037, .068	.006, .021, .033
$\mathcal{N}^w$	.010, .047, .093	.013, .044, .094	.009, .036, .088	.007, .056, .103	.006, .033, .081	.009, .036, .092
$\mathcal{N}^{dw}$	.010, .054, .111	.000, .012, .034	.000, .003, .014	.006, .046, .114	.000, .006, .028	.000, .001, .010

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.236, .496, .630	.038, .117, .186	.042, .105, .171	.405, .685, .798	.060, .205, .316	.053, .159, .250
$\hat{\mathcal{T}}^{dw}$	.223, .485, .656	.016, .070, .156	.009, .055, .131	.390, .673, .800	.029, .125, .255	.015, .088, .184
$\mathcal{AP}^w$	.079, .240, .366	.065, .208, .322	.056, .190, .288	.426, .684, .797	.351, .627, .756	.355, .632, .745
$\mathcal{AP}^{dw}$	.071, .231, .363	.047, .179, .281	.042, .171, .298	.502, .751, .854	.493, .707, .801	.506, .735, .819
Q_{χ^2}	.304, .543, .677	.065, .163, .254	.058, .159, .234	.547, .757, .832	.119, .265, .364	.106, .261, .366
$\mathcal{N}^{N_{0,1}}$	.270, .390, .477	.040, .098, .159	.019, .062, .092	.492, .613, .685	.072, .189, .272	.037, .097, .144
$\mathcal{N}^w$	.294, .550, .680	.036, .118, .209	.026, .113, .189	.541, .759, .859	.067, .212, .316	.057, .170, .267
$\mathcal{N}^{dw}$	.256, .527, .674	.000, .022, .063	.000, .002, .018	.473, .730, .845	.004, .043, .130	.001, .005, .034

Scenario #3: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(2) filter $\epsilon_t = y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 15: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #4 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.048, .204, .340	.021, .111, .213	.021, .118, .198	.047, .193, .327	.031, .138, .224	.031, .130, .210
$\hat{\mathcal{T}}^{dw}$	.082, .227, .343	.023, .131, .265	.019, .098, .204	.055, .203, .329	.026, .137, .259	.005, .080, .181
$\mathcal{AP}^w$	.051, .214, .347	.053, .190, .313	.040, .179, .311	.043, .175, .302	.031, .163, .298	.041, .150, .277
$\mathcal{AP}^{dw}$	.199, .349, .465	.185, .377, .494	.195, .364, .483	.131, .302, .417	.121, .297, .430	.131, .328, .472
Q_{χ^2}	.070, .200, .281	.068, .172, .245	.076, .177, .263	.106, .252, .361	.077, .184, .279	.068, .147, .212
$\mathcal{N}^{N_{0,1}}$	.090, .164, .207	.072, .122, .161	.063, .095, .125	.114, .194, .246	.081, .138, .196	.058, .100, .143
$\mathcal{N}^w$	.060, .195, .335	.043, .149, .251	.029, .094, .169	.050, .183, .310	.033, .139, .237	.019, .090, .160
$\mathcal{N}^{dw}$	.045, .195, .323	.015, .098, .220	.006, .048, .148	.048, .211, .338	.010, .100, .222	.005, .043, .104

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.259, .580, .728	.177, .424, .576	.124, .341, .473	.147, .436, .618	.079, .286, .447	.041, .198, .315
$\hat{\mathcal{T}}^{dw}$	.229, .533, .705	.089, .350, .546	.032, .182, .360	.135, .404, .605	.035, .239, .443	.009, .102, .265
$\mathcal{AP}^w$	.455, .703, .818	.455, .702, .814	.437, .706, .803	.441, .753, .851	.440, .709, .832	.413, .722, .821
$\mathcal{AP}^{dw}$	.547, .757, .850	.516, .734, .834	.529, .758, .857	.558, .782, .860	.564, .787, .875	.556, .786, .868
Q_{χ^2}	.540, .750, .829	.397, .609, .714	.345, .519, .626	.296, .558, .681	.227, .455, .569	.194, .367, .483
$\mathcal{N}^{N_{0,1}}$	.575, .691, .755	.442, .554, .633	.316, .437, .500	.340, .489, .574	.258, .373, .455	.143, .233, .289
$\mathcal{N}^w$	.426, .694, .817	.256, .547, .695	.157, .404, .563	.214, .517, .669	.129, .380, .557	.067, .249, .406
$\mathcal{N}^{dw}$	.288, .640, .797	.086, .350, .568	.022, .165, .340	.202, .496, .684	.051, .237, .439	.014, .097, .229

Scenario #4: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(1) filter $\epsilon_t = y_t - \phi_1 y_{t-1}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 16: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #4 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.221, .443, .574	.101, .266, .377	.086, .228, .350	.174, .385, .512	.107, .282, .396	.085, .208, .312
$\hat{\mathcal{T}}^{dw}$	.190, .455, .611	.047, .200, .345	.026, .153, .280	.153, .365, .522	.043, .192, .308	.033, .135, .253
$\mathcal{AP}^w$	.202, .451, .597	.171, .395, .564	.172, .406, .556	.151, .386, .523	.141, .356, .522	.131, .329, .476
$\mathcal{AP}^{dw}$	.314, .564, .672	.280, .523, .656	.293, .535, .652	.242, .512, .649	.263, .484, .618	.243, .466, .598
Q_{χ^2}	.257, .449, .578	.103, .248, .358	.082, .221, .299	.302, .521, .631	.141, .293, .382	.124, .260, .346
$\mathcal{N}^{N_{0,1}}$	.251, .373, .445	.108, .204, .268	.069, .141, .207	.308, .438, .507	.157, .250, .319	.087, .161, .211
$\mathcal{N}^w$	.223, .462, .600	.064, .205, .321	.039, .152, .259	.172, .384, .531	.054, .172, .288	.037, .150, .250
$\mathcal{N}^{dw}$	.171, .428, .590	.007, .078, .201	.000, .039, .104	.127, .369, .525	.001, .067, .172	.000, .028, .098

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.784, .944, .985	.577, .810, .889	.505, .759, .843	.699, .910, .962	.390, .685, .811	.307, .601, .736
$\hat{\mathcal{T}}^{dw}$	.665, .932, .980	.284, .650, .805	.193, .514, .724	.567, .875, .963	.163, .521, .734	.073, .366, .590
$\mathcal{AP}^w$	.938, .986, .996	.912, .978, .988	.910, .987, .995	.953, .988, .996	.958, .992, .997	.938, .985, .995
$\mathcal{AP}^{dw}$	.933, .983, .993	.901, .975, .992	.899, .975, .988	.945, .988, .999	.944, .993, 1.00	.941, .987, .994
Q_{χ^2}	.961, .991, .994	.794, .908, .955	.661, .823, .875	.853, .957, .975	.504, .729, .825	.430, .637, .740
$\mathcal{N}^{N_{0,1}}$	.970, .987, .994	.823, .898, .929	.581, .726, .788	.874, .932, .962	.523, .678, .761	.323, .459, .548
$\mathcal{N}^w$	.952, .987, .993	.643, .879, .929	.426, .718, .831	.778, .956, .982	.365, .693, .828	.237, .526, .659
$\mathcal{N}^{dw}$	.851, .974, .990	.133, .550, .802	.029, .215, .466	.639, .916, .975	.041, .325, .576	.009, .100, .280

Scenario #4: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(1) filter $\epsilon_t = y_t - \phi_1 y_{t-1}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 17: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #4 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.570, .782, .871	.336, .542, .637	.268, .455, .560	.431, .657, .756	.289, .492, .602	.289, .466, .575
$\hat{\mathcal{T}}^{dw}$	.538, .783, .866	.184, .442, .570	.143, .376, .516	.412, .679, .787	.180, .386, .534	.135, .333, .475
$\mathcal{AP}^w$	.479, .776, .865	.446, .728, .838	.444, .721, .850	.363, .656, .780	.345, .614, .757	.307, .593, .718
$\mathcal{AP}^{dw}$	.566, .794, .881	.553, .751, .850	.567, .781, .869	.461, .701, .806	.441, .673, .788	.439, .675, .792
Q_{χ^2}	.579, .776, .854	.191, .372, .507	.168, .322, .422	.587, .762, .839	.248, .423, .527	.166, .301, .411
$\mathcal{N}^{N_{0,1}}$	.614, .730, .781	.226, .352, .440	.098, .178, .269	.631, .755, .804	.197, .332, .424	.122, .213, .292
$\mathcal{N}^w$	.567, .795, .874	.132, .345, .463	.071, .229, .376	.439, .688, .779	.108, .276, .409	.076, .223, .348
$\mathcal{N}^{dw}$	.488, .768, .867	.010, .076, .230	.001, .027, .100	.311, .625, .763	.006, .071, .211	.000, .022, .077

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.998, 1.00, 1.00	.954, .994, .998	.941, .983, .992	.992, 1.00, 1.00	.913, .981, .994	.872, .967, .988
$\hat{\mathcal{T}}^{dw}$	.979, 1.00, 1.00	.834, .979, .992	.751, .962, .982	.977, 1.00, 1.00	.713, .958, .989	.643, .919, .972
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.999, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.997, 1.00, 1.00	.995, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
Q_{χ^2}	1.00, 1.00, 1.00	.970, .991, .998	.921, .970, .986	.997, 1.00, 1.00	.836, .944, .968	.718, .870, .924
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	.968, .993, .997	.876, .926, .951	1.00, 1.00, 1.00	.840, .933, .960	.625, .761, .826
$\mathcal{N}^w$	1.00, 1.00, 1.00	.945, .991, .996	.798, .946, .978	.997, .998, 1.00	.765, .920, .969	.519, .794, .892
$\mathcal{N}^{dw}$	.998, 1.00, 1.00	.294, .817, .949	.036, .345, .663	.981, 1.00, 1.00	.105, .524, .789	.017, .175, .419

Scenario #4: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(1) filter $\epsilon_t = y_t - \phi_1 y_{t-1}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 18: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #4 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.921, .979, .991	.778, .904, .938	.724, .878, .918	.833, .955, .980	.705, .828, .874	.655, .801, .856
$\hat{\mathcal{T}}^{dw}$	.914, .975, .994	.658, .845, .914	.573, .788, .869	.787, .921, .958	.595, .795, .868	.502, .733, .827
$\mathcal{AP}^w$	.854, .969, .990	.825, .953, .980	.788, .958, .983	.733, .914, .955	.678, .875, .939	.659, .889, .941
$\mathcal{AP}^{dw}$	.893, .965, .985	.881, .976, .988	.864, .968, .995	.771, .925, .958	.726, .892, .940	.724, .916, .951
Q_{χ^2}	.937, .983, .996	.338, .573, .705	.265, .467, .599	.901, .956, .976	.405, .615, .708	.303, .495, .594
$\mathcal{N}^{N_{0,1}}$	.942, .969, .980	.382, .558, .643	.175, .319, .415	.941, .967, .979	.387, .549, .633	.175, .307, .387
$\mathcal{N}^w$	.934, .991, .997	.279, .524, .662	.140, .362, .502	.818, .935, .964	.270, .525, .664	.162, .358, .509
$\mathcal{N}^{dw}$	.900, .977, .991	.016, .166, .372	.000, .029, .113	.716, .921, .970	.003, .110, .291	.001, .022, .099

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	.999, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
Q_{χ^2}	1.00, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.995, .999, 1.00	.955, .991, .995
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	.999, 1.00, 1.00	.986, .996, .998	1.00, 1.00, 1.00	.988, .997, .998	.899, .958, .975
$\mathcal{N}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.989, 1.00, 1.00	1.00, 1.00, 1.00	.972, .995, .998	.876, .978, .991
$\mathcal{N}^{dw}$	1.00, 1.00, 1.00	.761, .989, .999	.129, .708, .919	1.00, 1.00, 1.00	.380, .892, .981	.028, .320, .638

Scenario #4: AR(2) $y_t = 0.3y_{t-1} - 0.15y_{t-2} + e_t$; AR(1) filter $\epsilon_t = y_t - \phi_1 y_{t-1}$; least squares estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 19: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #5 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.001, .037, .089	.004, .036, .081	.005, .031, .065	.001, .037, .096	.003, .032, .074	.003, .037, .079
$\hat{\mathcal{T}}^{dw}$	.005, .052, .108	.004, .036, .093	.004, .029, .071	.003, .034, .099	.002, .024, .073	.000, .016, .057
$\mathcal{AP}^w$	.006, .036, .087	.006, .032, .072	.001, .042, .080	.002, .033, .077	.007, .038, .082	.003, .034, .077
$\mathcal{AP}^{dw}$	.009, .060, .127	.006, .060, .136	.012, .059, .129	.004, .044, .095	.008, .058, .111	.010, .049, .122
Q_{χ^2}	.037, .108, .162	.032, .088, .140	.021, .082, .134	.476, .620, .704	.462, .617, .684	.334, .476, .548
$\mathcal{N}^{N_{0,1}}$	.062, .128, .165	.044, .082, .124	.033, .063, .088	.566, .654, .710	.515, .624, .685	.317, .409, .453
$\mathcal{N}^w$	.005, .033, .072	.009, .033, .076	.004, .030, .069	.003, .022, .075	.003, .034, .082	.000, .014, .061
$\mathcal{N}^{dw}$	.003, .041, .112	.000, .019, .068	.001, .012, .049	.002, .018, .066	.000, .011, .045	.000, .005, .029

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.893, .974, .991	.848, .966, .981	.823, .962, .985	.967, .998, 1.00	.965, .998, .999	.972, .998, .999
$\hat{\mathcal{T}}^{dw}$	.612, .889, .962	.536, .857, .941	.478, .849, .941	.594, .843, .933	.579, .832, .933	.580, .836, .929
$\mathcal{AP}^w$	.770, .965, .990	.774, .967, .988	.776, .957, .988	.948, .994, .998	.936, .989, 1.00	.950, .994, .999
$\mathcal{AP}^{dw}$	.550, .844, .945	.517, .830, .942	.485, .799, .925	.489, .785, .904	.465, .793, .923	.419, .768, .902
Q_{χ^2}	.967, .988, .994	.934, .979, .989	.877, .956, .973	.996, .997, .999	.997, .998, 1.00	.995, .997, .999
$\mathcal{N}^{N_{0,1}}$	.987, .996, .998	.958, .985, .988	.865, .920, .936	.999, .999, .999	.998, .998, .998	.986, .991, .994
$\mathcal{N}^w$	.702, .946, .980	.543, .871, .952	.446, .790, .907	.929, .993, .999	.923, .993, .999	.893, .988, .998
$\mathcal{N}^{dw}$	.339, .704, .885	.112, .496, .742	.025, .229, .489	.422, .775, .902	.270, .678, .851	.082, .422, .710

Scenario #5: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; no filter. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 20: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #5 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.009, .057, .119	.006, .047, .091	.008, .037, .084	.007, .040, .100	.005, .027, .073	.002, .029, .075
$\hat{\mathcal{T}}^{dw}$	.007, .054, .119	.005, .032, .075	.001, .021, .075	.005, .037, .083	.007, .025, .054	.005, .015, .048
$\mathcal{AP}^w$	.011, .048, .102	.007, .042, .084	.009, .036, .093	.004, .032, .080	.007, .025, .075	.007, .041, .097
$\mathcal{AP}^{dw}$	.009, .063, .121	.012, .065, .136	.008, .063, .135	.012, .054, .104	.012, .036, .088	.013, .042, .094
Q_{χ^2}	.043, .122, .196	.027, .080, .143	.031, .090, .142	.808, .883, .916	.801, .876, .903	.652, .741, .781
$\mathcal{N}^{N_{0,1}}$	.089, .143, .208	.029, .076, .117	.024, .051, .086	.874, .909, .923	.844, .894, .904	.611, .693, .736
$\mathcal{N}^w$	.003, .036, .085	.006, .040, .084	.007, .045, .083	.005, .031, .069	.003, .025, .086	.001, .020, .061
$\mathcal{N}^{dw}$	.004, .041, .106	.001, .008, .044	.000, .007, .031	.003, .021, .059	.000, .006, .020	.000, .002, .015

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.992, .999, .999	.993, 1.00, 1.00	.997, .999, .999	.981, .996, 1.00	.985, .997, 1.00	.982, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.902, .974, .994	.905, .975, .993	.913, .976, .995	.696, .861, .931	.667, .848, .930	.672, .850, .937
$\mathcal{AP}^w$	.982, .999, 1.00	.985, .998, 1.00	.983, .997, 1.00	.983, .997, .999	.979, .997, .999	.978, .997, 1.00
$\mathcal{AP}^{dw}$	.829, .964, .987	.801, .942, .980	.800, .945, .979	.631, .833, .914	.608, .832, .912	.563, .825, .904
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.997, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.998, .998, .998
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	.999, 1.00, 1.00	.999, .999, .999	.998, .998, .999
$\mathcal{N}^w$	.981, .998, .999	.973, .997, 1.00	.935, .993, .998	.975, 1.00, 1.00	.970, .999, 1.00	.971, 1.00, 1.00
$\mathcal{N}^{dw}$	.780, .945, .980	.289, .781, .931	.049, .450, .785	.584, .826, .912	.396, .733, .857	.170, .645, .825

Scenario #5: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; no filter. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 21: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #5 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.006, .040, .097	.005, .047, .089	.009, .042, .083	.000, .028, .080	.001, .027, .077	.001, .033, .083
$\hat{\mathcal{T}}^{dw}$	.006, .046, .087	.002, .019, .067	.003, .019, .058	.006, .027, .058	.003, .020, .049	.006, .020, .041
$\mathcal{AP}^w$	.010, .042, .090	.005, .054, .098	.006, .044, .093	.003, .035, .079	.003, .047, .081	.002, .031, .085
$\mathcal{AP}^{dw}$	.012, .053, .105	.008, .053, .101	.007, .065, .129	.007, .031, .091	.008, .029, .070	.008, .031, .073
Q_{χ^2}	.058, .155, .239	.025, .088, .147	.034, .085, .140	.933, .955, .972	.940, .960, .972	.801, .847, .878
$\mathcal{N}^{N_{0,1}}$	.083, .149, .209	.032, .062, .105	.014, .043, .067	.948, .958, .971	.946, .966, .976	.818, .855, .872
$\mathcal{N}^w$	.006, .039, .086	.006, .039, .089	.005, .041, .084	.004, .041, .079	.004, .028, .060	.001, .021, .043
$\mathcal{N}^{dw}$	.005, .041, .100	.000, .001, .026	.000, .004, .013	.003, .024, .052	.003, .007, .017	.000, .001, .006

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	.998, .999, 1.00	.999, 1.00, 1.00	.983, 1.00, 1.00	.995, 1.00, 1.00	.986, .998, 1.00
$\hat{\mathcal{T}}^{dw}$	.970, .995, .998	.955, .991, .997	.956, .991, .995	.716, .868, .926	.725, .874, .930	.700, .850, .918
$\mathcal{AP}^w$	.998, 1.00, 1.00	.996, 1.00, 1.00	.994, .999, 1.00	.984, .998, .999	.982, .998, 1.00	.979, .995, 1.00
$\mathcal{AP}^{dw}$	.930, .983, .995	.910, .979, .994	.905, .979, .992	.672, .835, .901	.623, .818, .895	.658, .844, .916
Q_{χ^2}	.999, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, .999, .999
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	.999, 1.00, 1.00
$\mathcal{N}^w$	.997, 1.00, 1.00	.993, .999, 1.00	.993, 1.00, 1.00	.986, .997, 1.00	.974, .999, 1.00	.981, .998, 1.00
$\mathcal{N}^{dw}$	.908, .974, .991	.624, .923, .970	.197, .772, .955	.645, .824, .896	.505, .753, .879	.293, .694, .852

Scenario #5: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; no filter. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 22: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #5 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.013, .056, .107	.004, .048, .082	.011, .033, .068	.002, .041, .099	.006, .029, .080	.004, .032, .069
$\hat{\mathcal{T}}^{dw}$	.004, .044, .092	.004, .038, .078	.003, .017, .056	.006, .020, .050	.000, .011, .044	.006, .011, .028
$\mathcal{AP}^w$	.006, .059, .097	.008, .050, .107	.010, .045, .086	.006, .045, .095	.003, .032, .075	.006, .038, .083
$\mathcal{AP}^{dw}$	.009, .045, .095	.010, .043, .093	.007, .056, .111	.003, .025, .062	.004, .017, .047	.008, .019, .052
Q_{χ^2}	.045, .138, .215	.032, .085, .153	.031, .094, .153	.988, .996, .996	.974, .983, .986	.921, .940, .946
$\mathcal{N}^{N_{0,1}}$	.103, .168, .221	.025, .063, .121	.016, .038, .059	.990, .993, .993	.977, .982, .986	.894, .917, .924
$\mathcal{N}^w$	.011, .044, .091	.009, .043, .090	.004, .037, .087	.005, .031, .077	.003, .027, .065	.000, .019, .047
$\mathcal{N}^{dw}$	.005, .045, .109	.000, .002, .023	.000, .000, .007	.001, .017, .044	.000, .001, .004	.000, .001, .004

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.988, .999, .999	.987, .998, 1.00	.986, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.985, .997, .999	.987, .996, .996	.991, 1.00, 1.00	.751, .870, .917	.768, .884, .935	.762, .883, .933
$\mathcal{AP}^w$	.998, 1.00, 1.00	.997, 1.00, 1.00	.997, 1.00, 1.00	.981, 1.00, 1.00	.985, 1.00, 1.00	.982, .998, 1.00
$\mathcal{AP}^{dw}$	.975, .996, .999	.967, .987, .992	.973, .993, .999	.728, .864, .923	.704, .841, .904	.687, .848, .915
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	.998, 1.00, 1.00	.999, 1.00, 1.00	.998, 1.00, 1.00	.981, .999, 1.00	.981, .998, 1.00	.977, .995, .998
$\mathcal{N}^{dw}$	.961, .988, .997	.892, .983, .992	.603, .958, .987	.680, .838, .900	.570, .780, .871	.471, .747, .870

Scenario #5: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; no filter. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 23: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #6 ($n = 100$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.006, .047, .089	.009, .047, .084	.005, .033, .072	.007, .047, .097	.004, .040, .083	.003, .035, .083
$\hat{\mathcal{T}}^{dw}$	.015, .095, .163	.010, .051, .114	.005, .031, .070	.016, .080, .157	.010, .046, .128	.003, .027, .093
$\mathcal{AP}^w$	.009, .050, .095	.008, .036, .075	.009, .041, .090	.012, .054, .102	.013, .044, .096	.010, .044, .094
$\mathcal{AP}^{dw}$	.020, .075, .149	.021, .084, .152	.017, .065, .130	.028, .097, .175	.038, .100, .184	.033, .091, .173
Q_{χ^2}	011, .055, .107	.016, .051, .093	.030, .082, .127	.033, .092, .151	.040, .103, .169	.035, .087, .146
$\mathcal{N}^{N_{0,1}}$	.025, .060, .088	.021, .058, .092	.015, .042, .063	.050, .116, .147	.046, .090, .136	.044, .077, .116
$\mathcal{N}^w$	.007, .050, .107	.011, .044, .084	.005, .030, .064	.008, .059, .114	.008, .043, .089	.013, .052, .093
$\mathcal{N}^{dw}$	.009, .063, .141	.004, .039, .105	.001, .016, .057	.018, .085, .168	.002, .045, .107	.001, .029, .084

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 10	Lag = 21
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.865, .946, .960	.794, .923, .946	.737, .850, .885	.970, .977, .983	.949, .958, .962	.927, .946, .954
$\hat{\mathcal{T}}^{dw}$	.799, .924, .950	.655, .882, .926	.471, .800, .882	.953, .975, .976	.946, .971, .976	.894, .948, .957
$\mathcal{AP}^w$	.902, .965, .974	.873, .955, .968	.873, .949, .962	.964, .972, .980	.967, .973, .975	.964, .976, .979
$\mathcal{AP}^{dw}$	.821, .943, .960	.782, .926, .947	.801, .943, .966	.959, .974, .979	.923, .966, .970	.939, .978, .981
Q_{χ^2}	.903, .976, .992	.794, .909, .948	.670, .825, .888	.998, 1.00, 1.00	.997, 1.00, 1.00	.989, .997, .998
$\mathcal{N}^{N_{0,1}}$	.941, .976, .984	.861, .925, .955	.688, .798, .838	1.00, 1.00, 1.00	.997, 1.00, 1.00	.990, .994, .998
$\mathcal{N}^w$	.760, .912, .942	.572, .826, .895	.355, .652, .778	.949, .965, .972	.927, .959, .970	.821, .916, .937
$\mathcal{N}^{dw}$	.546, .840, .931	.199, .569, .770	.056, .267, .494	.909, .970, .978	.589, .910, .961	.212, .586, .816

Scenario #6: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; GARCH(1,1) filter $\epsilon_t = y_t/\sigma_t$ with $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$; QML estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 24: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #6 ($n = 250$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^{w}$	.014, .055, .115	.008, .045, .102	.008, .048, .083	.011, .055, .108	.008, .046, .091	.009, .052, .104
$\hat{\mathcal{T}}^{dw}$	.010, .073, .139	.002, .034, .096	.002, .026, .068	.010, .058, .127	.005, .036, .094	.003, .024, .083
$\mathcal{AP}^{w}$	.011, .045, .084	.011, .042, .086	.009, .048, .107	.017, .057, .114	.018, .052, .103	.015, .056, .098
$\mathcal{AP}^{dw}$	.019, .075, .143	.015, .064, .140	.017, .072, .128	.019, .074, .133	.014, .072, .150	.014, .065, .137
Q_{χ^2}	.015, .053, .099	.019, .073, .114	.032, .077, .120	.030, .096, .156	.027, .074, .139	.035, .099, .155
$\mathcal{N}^{N_{0,1}}$	.031, .089, .124	.031, .061, .095	.022, .043, .072	.069, .132, .179	.038, .078, .123	.025, .057, .083
$\mathcal{N}^{w}$	.010, .045, .095	.009, .049, .102	.007, .031, .069	.014, .066, .127	.009, .049, .093	.010, .052, .113
$\mathcal{N}^{dw}$	.007, .056, .118	.000, .026, .062	.000, .015, .041	.009, .061, .126	.000, .023, .065	.000, .011, .045

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 22	Lag = 45	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^{w}$	.992, .992, .993	.988, .989, .992	.982, .988, .989	.995, .999, .999	.991, .993, .993	.991, .991, .992
$\hat{\mathcal{T}}^{dw}$	.994, .997, .997	.989, .991, .993	.967, .977, .978	.997, .997, .998	.991, .993, .994	.995, .997, .997
$\mathcal{AP}^{w}$	.993, .994, .997	.993, .994, .995	.994, .994, .997	.999, 1.00, 1.00	.999, .999, .999	.993, .994, .994
$\mathcal{AP}^{dw}$	.990, .993, .994	.988, .995, .995	.992, .995, .996	.994, .994, .996	.994, .994, .996	.998, .999, .999
Q_{χ^2}	1.00, 1.00, 1.00	.997, 1.00, 1.00	.979, .991, .998	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	.999, 1.00, 1.00	.975, .990, .998	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{w}$	.991, .992, .993	.969, .986, .988	.838, .948, .967	.993, .994, .994	.995, .995, .996	.991, .992, .993
$\mathcal{N}^{dw}$	.979, .992, .993	.410, .894, .963	.089, .470, .774	.997, .997, .997	.866, .986, .988	.383, .903, .982

Scenario #6: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; GARCH(1,1) filter $\epsilon_t = y_t/\sigma_t$ with $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$; QML estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 25: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #6 ($n = 500$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.012, .057, .106	.011, .051, .104	.002, .040, .086	.012, .048, .089	.010, .040, .093	.009, .043, .089
$\hat{\mathcal{T}}^{dw}$	.007, .058, .113	.006, .036, .107	.001, .029, .073	.006, .050, .116	.003, .029, .079	.003, .021, .064
$\mathcal{AP}^w$	.006, .047, .096	.008, .046, .087	.017, .057, .104	.010, .055, .110	.015, .049, .102	.009, .040, .094
$\mathcal{AP}^{dw}$	.013, .072, .125	.014, .075, .145	.016, .060, .111	.016, .071, .126	.021, .075, .130	.016, .060, .133
Q_{χ^2}	.013, .046, .092	.018, .067, .105	.029, .084, .135	.033, .104, .169	.022, .078, .148	.020, .083, .137
$\mathcal{N}^{N_{0,1}}$	.021, .064, .101	.019, .063, .096	.016, .039, .069	.065, .141, .192	.032, .076, .120	.026, .055, .080
$\mathcal{N}^w$	.009, .054, .105	.004, .040, .098	.010, .037, .071	.016, .059, .125	.011, .043, .093	.005, .033, .080
$\mathcal{N}^{dw}$	.014, .053, .109	.000, .011, .047	.000, .003, .025	.011, .048, .113	.001, .015, .048	.000, .006, .035

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 40	Lag = 80
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.999, 1.00, 1.00	.998, .998, .998	.998, .998, .998	.998, 1.00, 1.00	.995, .995, .995	.997, .997, .997
$\hat{\mathcal{T}}^{dw}$	.996, .996, .997	.997, .998, .998	.996, .996, .996	.996, .996, .996	.999, .999, .999	.998, .998, .999
$\mathcal{AP}^w$	.999, .999, .999	.995, .995, .995	1.00, 1.00, 1.00	.998, .998, .998	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.998, .998, .998	1.00, 1.00, 1.00	.999, .999, 1.00	.999, 1.00, 1.00	.999, .999, .999
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	.999, .999, .999	.997, .999, .999	.991, .994, .994	.999, 1.00, 1.00	.999, .999, .999	.996, .996, .996
$\mathcal{N}^{dw}$	.999, 1.00, 1.00	.821, .995, .996	.210, .819, .962	.997, .998, .998	.990, .997, .998	.774, .994, .995

Scenario #6: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; GARCH(1,1) filter $\epsilon_t = y_t/\sigma_t$ with $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$; QML estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 26: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #6 ($n = 1000$)

	e_t is iid: $e_t = \nu_t$			e_t is GARCH(1,1): $w_t^2 = 1.0 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.011, .048, .102	.006, .049, .090	.005, .041, .103	.011, .049, .097	.010, .065, .109	.009, .056, .098
$\hat{\mathcal{T}}^{dw}$	.011, .052, .106	.002, .028, .083	.004, .022, .066	.012, .068, .121	.001, .038, .088	.001, .034, .080
$\mathcal{AP}^w$	.011, .046, .096	.010, .043, .093	.011, .047, .106	.013, .046, .091	.009, .059, .102	.007, .052, .101
$\mathcal{AP}^{dw}$	.021, .071, .123	.013, .051, .108	.020, .079, .129	.011, .057, .137	.012, .063, .121	.008, .051, .114
Q_{χ^2}	.013, .060, .100	.023, .073, .123	.019, .078, .148	.042, .115, .183	.020, .075, .123	.027, .088, .141
$\mathcal{N}^{N_{0,1}}$	.032, .073, .100	.022, .053, .092	.011, .030, .047	.069, .137, .190	.030, .078, .121	.013, .042, .068
$\mathcal{N}^w$	.011, .068, .110	.008, .045, .098	.004, .045, .094	.009, .046, .096	.008, .047, .094	.004, .036, .091
$\mathcal{N}^{dw}$	.018, .061, .137	.001, .006, .028	.000, .001, .014	.006, .051, .118	.000, .002, .020	.000, .004, .017

	e_t is MA(2): $e_t = \nu_t + 0.50\nu_{t-1} + 0.25\nu_{t-2}$			e_t is AR(1): $e_t = 0.7e_{t-1} + \nu_t$		
	Lag = 5	Lag = 72	Lag = 144	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.998, .998, .999
$\hat{\mathcal{T}}^{dw}$	1.00, 1.00, 1.00	.999, .999, 1.00	.999, .999, .999	.998, .998, .998	.999, .999, 1.00	.999, .999, .999
$\mathcal{AP}^w$	.999, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, .999, .999	.999, .999, .999	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.998, .998, .998	1.00, 1.00, 1.00	.999, .999, .999	1.00, 1.00, 1.00	.999, .999, .999
Q_{χ^2}	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^{N_{0,1}}$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{N}^w$	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.999, .999, .999	.997, .998, .998
$\mathcal{N}^{dw}$	.999, .999, .999	.998, 1.00, 1.00	.670, .992, 1.00	1.00, 1.00, 1.00	.999, 1.00, 1.00	.994, 1.00, 1.00

Scenario #6: GARCH(1,1) $y_t = \sigma_t e_t$, $\sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.5\sigma_{t-1}^2$; GARCH(1,1) filter $\epsilon_t = y_t/\sigma_t$ with $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$; QML estimation. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 27: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #7 (Remote MA(6))

	$n = 100$			$n = 250$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.014, .080, .146	.068, .231, .381	.057, .201, .336	.024, .077, .131	.519, .732, .814	.464, .666, .750
$\hat{\mathcal{T}}^{dw}$	.011, .063, .143	.048, .226, .388	.025, .147, .293	.005, .040, .098	.319, .649, .789	.269, .543, .678
$\mathcal{AP}^w$	.012, .066, .123	.023, .110, .182	.032, .104, .175	.027, .086, .159	.052, .147, .226	.034, .147, .232
$\mathcal{AP}^{dw}$	.019, .075, .151	.016, .086, .158	.018, .084, .153	.018, .069, .136	.018, .093, .180	.017, .072, .164
Q_{χ^2}	.033, .107, .178	.138, .290, .400	.141, .283, .381	.044, .111, .188	.342, .570, .703	.302, .488, .599
$\mathcal{N}^{N_{0,1}}$	.071, .125, .175	.194, .319, .399	.132, .228, .286	.067, .124, .164	.384, .545, .634	.256, .381, .467
$\mathcal{N}^w$	.017, .065, .120	.060, .229, .357	.042, .154, .265	.019, .089, .157	.219, .469, .634	.131, .338, .482
$\mathcal{N}^{dw}$	.016, .074, .155	.018, .149, .303	.006, .056, .159	.013, .050, .103	.021, .173, .362	.001, .051, .171

	$n = 500$			$n = 1000$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.019, .092, .161	.942, .988, .993	.952, .986, .991	.034, .092, .169	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.010, .064, .122	.878, .968, .985	.823, .944, .974	.007, .059, .121	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^w$	.020, .098, .171	.102, .244, .343	.092, .245, .363	.025, .091, .162	.165, .341, .455	.178, .359, .481
$\mathcal{AP}^{dw}$	.008, .048, .115	.026, .129, .236	.027, .112, .211	.009, .058, .124	.063, .223, .347	.064, .221, .357
Q_{χ^2}	.037, .106, .186	.661, .846, .910	.495, .706, .801	.029, .097, .167	.941, .983, .988	.817, .926, .964
$\mathcal{N}^{N_{0,1}}$	.070, .137, .186	.704, .833, .889	.441, .589, .680	.083, .143, .189	.935, .977, .987	.692, .825, .893
$\mathcal{N}^w$	.034, .102, .171	.488, .763, .869	.295, .585, .720	.025, .108, .183	.851, .960, .988	.633, .869, .941
$\mathcal{N}^{dw}$	.007, .049, .115	.037, .297, .571	.004, .080, .243	.009, .058, .113	.177, .676, .877	.010, .170, .433

Scenario #7: Remote MA(6) $y_t = e_t + 0.25e_{t-6}$ with $e_t \overset{i.i.d.}{\sim} N(0,1)$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^{n} y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 28: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #8 (Remote MA(12))

	$n = 100$			$n = 250$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.010, .062, .126	.008, .067, .132	.053, .180, .283	.012, .060, .122	.488, .715, .799	.491, .705, .800
$\hat{\mathcal{T}}^{dw}$	.019, .067, .148	.010, .067, .159	.034, .163, .280	.019, .077, .155	.383, .659, .779	.320, .596, .729
$\mathcal{AP}^w$	.011, .048, .097	.012, .054, .109	.016, .062, .132	.013, .055, .123	.021, .087, .162	.020, .079, .143
$\mathcal{AP}^{dw}$	.029, .115, .184	.015, .085, .180	.024, .097, .176	.023, .083, .139	.022, .098, .175	.015, .084, .165
Q_{χ^2}	.024, .075, .125	.034, .100, .168	.130, .252, .351	.029, .080, .156	.327, .563, .692	.261, .457, .570
$\mathcal{N}^{N_{0,1}}$	.046, .099, .139	.062, .122, .167	.120, .193, .253	.050, .098, .136	.410, .572, .646	.200, .309, .402
$\mathcal{N}^w$	.007, .061, .121	.021, .072, .135	.037, .159, .280	.015, .064, .119	.224, .476, .641	.117, .331, .483
$\mathcal{N}^{dw}$	.012, .073, .147	.006, .060, .140	.011, .082, .215	.009, .067, .154	.048, .287, .500	.017, .128, .305

	$n = 500$			$n = 1000$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.014, .061, .128	.939, .977, .989	.901, .959, .981	.019, .082, .140	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.009, .061, .121	.883, .964, .976	.854, .958, .979	.014, .051, .114	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^w$	.012, .065, .135	.023, .094, .161	.027, .098, .170	.007, .054, .110	.027, .109, .168	.029, .118, .198
$\mathcal{AP}^{dw}$	.010, .070, .131	.005, .049, .119	.013, .063, .145	.016, .066, .125	.011, .071, .149	.014, .077, .137
Q_{χ^2}	.010, .066, .118	.621, .804, .897	.500, .698, .785	.016, .075, .137	.936, .978, .992	.809, .924, .955
$\mathcal{N}^{N_{0,1}}$	.059, .108, .151	.667, .814, .877	.417, .574, .674	.042, .089, .130	.930, .981, .992	.710, .845, .890
$\mathcal{N}^w$	.014, .058, .132	.497, .768, .855	.296, .584, .733	.019, .086, .143	.862, .966, .991	.618, .864, .926
$\mathcal{N}^{dw}$	.006, .071, .139	.094, .440, .697	.015, .145, .372	.011, .054, .115	.229, .737, .926	.020, .232, .527

Scenario #8: Remote MA(12) $y_t = e_t + 0.25e_{t-12}$ with $e_t \overset{i.i.d.}{\sim} N(0,1)$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n \sum_{t=1}^{n} y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 29: Rejection Frequencies of Tests with Various Fixed $\mathcal{L}_n = o(n)$ in Scenario #9 (Remote MA(24))

	$n = 100$			$n = 250$		
	Lag = 5	Lag = 10	Lag = 21	Lag = 5	Lag = 22	Lag = 45
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.014, .058, .115	.008, .042, .092	.012, .056, .109	.016, .060, .117	.019, .077, .142	.362, .586, .688
$\hat{\mathcal{T}}^{dw}$	.017, .094, .168	.011, .073, .150	.010, .068, .143	.014, .083, .160	.011, .066, .167	.258, .534, .670
$\mathcal{AP}^w$	.005, .044, .093	.009, .044, .089	.008, .045, .097	.010, .064, .124	.014, .078, .129	.010, .056, .112
$\mathcal{AP}^{dw}$	.026, .084, .164	.022, .083, .154	.021, .094, .164	.014, .073, .142	.015, .075, .158	.018, .069, .142
Q_{χ^2}	.019, .074, .122	.032, .097, .152	.066, .135, .194	.016, .067, .123	.048, .140, .213	.229, .426, .533
$\mathcal{N}^{N_{0,1}}$	.034, .074, .105	.053, .088, .124	.055, .094, .128	.041, .088, .127	.075, .141, .183	.224, .337, .415
$\mathcal{N}^w$	.008, .063, .131	.012, .059, .117	.012, .059, .136	.017, .071, .142	.028, .094, .160	.106, .324, .457
$\mathcal{N}^{dw}$	.010, .065, .135	.006, .056, .135	.005, .050, .118	.004, .053, .144	.005, .052, .108	.020, .130, .287

	$n = 500$			$n = 1000$		
	Lag = 5	Lag = 40	Lag = 80	Lag = 5	Lag = 72	Lag = 144
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}^w$	.018, .069, .129	.914, .974, .983	.891, .965, .977	.008, .061, .119	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\hat{\mathcal{T}}^{dw}$	.012, .059, .133	.867, .959, .979	.851, .949, .979	.012, .060, .128	.999, 1.00, 1.00	.999, 1.00, 1.00
$\mathcal{AP}^w$	.013, .051, .105	.013, .063, .119	.015, .064, .129	.015, .067, .119	.002, .049, .113	.015, .065, .113
$\mathcal{AP}^{dw}$	.019, .079, .153	.021, .075, .156	.017, .064, .138	.018, .073, .134	.012, .072, .133	.016, .070, .150
Q_{χ^2}	.016, .083, .140	.626, .818, .877	.487, .696, .791	.016, .075, .126	.936, .981, .989	.821, .932, .955
$\mathcal{N}^{N_{0,1}}$	.058, .106, .162	.673, .800, .860	.404, .567, .652	.057, .101, .166	.943, .979, .988	.712, .842, .888
$\mathcal{N}^w$	.012, .073, .133	.514, .772, .869	.293, .559, .717	.009, .069, .147	.867, .975, .989	.646, .864, .928
$\mathcal{N}^{dw}$	.008, .069, .144	.134, .516, .756	.029, .247, .512	.016, .076, .126	.316, .787, .929	.044, .354, .663

Scenario #9: Remote MA(24) $y_t = e_t + 0.25e_{t-24}$ with $e_t \overset{i.i.d.}{\sim} N(0,1)$; mean filter $\epsilon_t = y_t - E[y_t]$; $\hat{\phi}_n = 1/n\sum_{t=1}^{n} y_t$. In this table we report rejection frequencies with the nominal sizes 1%, 5%, and 10%. $\hat{\mathcal{T}}$ is the proposed max-correlation test. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. Q_{χ^2} is the asymptotic Ljung-Box test based on the χ^2 distribution. $\mathcal{N}^{N_{0,1}}$ is Hong's (1996) asymptotic test based on $N(0,1)$. Superscript "w" means the wild bootstrap while "dw" means Shao's (2011) dependent wild bootstrap. Lag length is $\mathcal{L}_n \in \{5, [.5n/\ln(n)], [n/\ln(n)]\}$.

Table 30: Rejection frequencies of $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$, $\mathcal{AP}$, and CvM (Scenarios #1–#6, $n = 100$)

IID error: $e_t = \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.017, .068, .128	.008, .047, .090	.002, .061, .129	.034, .197, .327	.006, .031, .068	.026, .091, .140
$\mathcal{AP}^w$	.007, .038, .086	.011, .072, .128	.007, .042, .098	.042, .195, .329	.004, .028, .068	.006, .039, .086
$\mathcal{AP}^{dw}$	.017, .079, .144	.022, .083, .151	.022, .102, .190	.182, .376, .504	.020, .072, .136	.019, .078, .135
CvM^w	.012, .063, .132	.013, .075, .135	.009, .060, .124	.106, .287, .413	.005, .035, .079	.012, .043, .076
CvM^{dw}	.023, .081, .138	.018, .076, .149	.020, .086, .167	.133, .338, .483	.021, .077, .141	.034, .087, .144
CvM^{brw}	.028, .082, .142	.045, .115, .186	.009, .041, .076	.123, .276, .390	.034, .076, .139	.001, .024, .075

GARCH(1,1) error: $e_t = \nu_t w_t$ with $w_t^2 = 1 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.005, .026, .075	.007, .021, .040	.004, .054, .109	.027, .150, .248	.001, .003, .012	.026, .090, .162
$\mathcal{AP}^w$	.003, .034, .083	.004, .032, .078	.003, .040, .091	.049, .170, .287	.006, .028, .075	.013, .055, .102
$\mathcal{AP}^{dw}$	.010, .071, .141	.006, .034, .077	.034, .117, .218	.148, .343, .480	.005, .038, .109	.028, .102, .175
CvM^w	.004, .048, .105	.007, .045, .102	.012, .053, .108	.075, .245, .364	.005, .038, .099	.018, .072, .117
CvM^{dw}	.017, .081, .149	.002, .030, .070	.026, .086, .168	.118, .287, .430	.006, .049, .103	.036, .100, .168
CvM^{brw}	.028, .079, .140	.013, .056, .113	.009, .032, .061	.098, .209, .300	.015, .060, .113	.006, .044, .091

MA(2) error: $e_t = \nu_t + 0.5\nu_{t-1} + 0.25\nu_{t-2}$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.693, .901, .951	.582, .769, .825	.012, .068, .135	.242, .601, .762	.461, .707, .788	.908, .966, .979
$\mathcal{AP}^w$	.896, .989, .996	.484, .816, .913	.013, .052, .112	.438, .693, .792	.789, .965, .985	.860, .932, .953
$\mathcal{AP}^{dw}$	.673, .916, .976	.331, .627, .784	.029, .115, .181	.516, .751, .836	.508, .828, .931	.813, .922, .963
CvM^w	.963, .994, 1.00	.681, .898, .943	.015, .071, .156	.604, .829, .896	.900, .989, .997	.882, .917, .927
CvM^{dw}	.898, .984, .995	.450, .743, .866	.029, .113, .182	.570, .805, .898	.681, .908, .969	.878, .927, .940
CvM^{brw}	.929, .991, .995	.712, .841, .899	.018, .058, .110	.587, .802, .884	.791, .932, .967	.247, .739, .904

AR(1) error: $e_t = 0.7e_{t-1} + \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.531, .752, .847	.477, .637, .685	.021, .128, .227	.263, .636, .788	.179, .345, .432	.987, .991, .991
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.588, .872, .946	.017, .078, .162	.405, .735, .820	.936, .996, .999	.972, .987, .989
$\mathcal{AP}^{dw}$	.774, .972, .996	.232, .500, .665	.115, .243, .315	.530, .756, .865	.470, .758, .898	.953, .987, .990
CvM^w	.998, 1.00, 1.00	.743, .938, .975	.035, .152, .250	.434, .727, .838	.961, .997, .999	.938, .948, .954
CvM^{dw}	.925, .996, 1.00	.282, .567, .741	.064, .193, .299	.472, .741, .849	.564, .818, .923	.958, .970, .973
CvM^{brw}	.981, 1.00, 1.00	.564, .744, .830	.044, .118, .192	.520, .748, .845	.687, .862, .921	.481, .863, .932

Table 31: Rejection frequencies of $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$, $\mathcal{AP}$, and CvM (Scenarios #1–#6, $n = 250$)

IID error: $e_t = \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.011, .045, .087	.012, .042, .087	.005, .048, .093	.178, .479, .634	.005, .029, .063	.019, .064, .125
$\mathcal{AP}^w$	.008, .052, .105	.011, .064, .134	.011, .049, .098	.162, .397, .537	.012, .047, .114	.007, .040, .085
$\mathcal{AP}^{dw}$	.018, .056, .114	.016, .079, .145	.023, .071, .126	.298, .537, .666	.012, .062, .114	.013, .062, .136
CvM^w	.008, .047, .093	.019, .076, .148	.007, .042, .100	.376, .621, .730	.011, .058, .104	.005, .044, .091
CvM^{dw}	.016, .072, .144	.030, .085, .154	.011, .065, .127	.370, .615, .735	.011, .058, .118	.019, .065, .112
CvM^{brw}	.029, .075, .120	.020, .064, .119	.012, .054, .097	.388, .630, .728	.016, .054, .107	.002, .019, .061

GARCH(1,1) error: $e_t = \nu_t w_t$ with $w_t^2 = 1 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.004, .031, .069	.008, .023, .038	.007, .040, .084	.116, .312, .451	.004, .010, .015	.011, .063, .107
$\mathcal{AP}^w$	.003, .039, .081	.007, .033, .088	.006, .030, .076	.130, .355, .524	.008, .037, .081	.009, .043, .082
$\mathcal{AP}^{dw}$	.008, .045, .100	.014, .036, .079	.014, .066, .133	.250, .461, .587	.010, .030, .080	.029, .098, .171
CvM^w	.007, .058, .118	.007, .035, .095	.004, .045, .097	.291, .528, .642	.003, .047, .096	.007, .047, .095
CvM^{dw}	.013, .059, .108	.029, .048, .083	.012, .058, .127	.242, .501, .648	.009, .037, .080	.020, .075, .132
CvM^{brw}	.013, .062, .108	.006, .019, .052	.009, .045, .086	.299, .509, .634	.010, .039, .094	.006, .055, .111

MA(2) error: $e_t = \nu_t + 0.5\nu_{t-1} + 0.25\nu_{t-2}$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.993, .998, 1.00	.841, .935, .962	.006, .060, .114	.677, .927, .982	.707, .834, .868	.992, .993, .993
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.919, .983, .997	.021, .095, .166	.928, .977, .991	.982, .999, .999	.992, .994, .995
$\mathcal{AP}^{dw}$	.969, .999, 1.00	.592, .818, .921	.016, .073, .144	.911, .978, .989	.801, .952, .982	.982, .986, .989
CvM^w	1.00, 1.00, 1.00	.974, .994, .995	.026, .103, .189	.980, .993, .999	.998, 1.00, 1.00	.975, .979, .984
CvM^{dw}	.999, 1.00, 1.00	.769, .924, .968	.019, .086, .189	.951, .996, .999	.903, .979, .994	.983, .989, .991
CvM^{brw}	1.00, 1.00, 1.00	.901, .946, .975	.024, .088, .144	.973, .997, .999	.939, .978, .990	.616, .952, .981

AR(1) error: $e_t = 0.7e_{t-1} + \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.903, .979, .990	.676, .774, .812	.041, .217, .355	.714, .954, .984	.156, .271, .334	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.830, .958, .989	.072, .217, .320	.936, .984, .996	.970, .995, 1.00	.997, .998, .999
$\mathcal{AP}^{dw}$	.971, .996, 1.00	.285, .535, .709	.150, .296, .386	.940, .989, .994	.566, .788, .899	.997, .999, .999
CvM^w	1.00, 1.00, 1.00	.888, .979, .988	.136, .328, .464	.953, .996, .998	.974, .999, 1.00	.987, .989, .991
CvM^{dw}	.999, 1.00, 1.00	.341, .572, .718	.136, .341, .465	.935, .991, .999	.680, .849, .912	.984, .987, .988
CvM^{brw}	1.00, 1.00, 1.00	.575, .706, .792	.127, .289, .419	.948, .991, .996	.725, .860, .903	.816, .983, .995

Table 32: Rejection frequencies of $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$, $\mathcal{AP}$, and CvM (Scenarios #1–#6, $n = 500$)

IID error: $e_t = \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.007, .047, .096	.010, .036, .077	.004, .045, .094	.462, .803, .894	.005, .033, .083	.016, .053, .096
$\mathcal{AP}^w$	.006, .043, .088	.008, .059, .117	.008, .045, .095	.440, .729, .828	.008, .039, .079	.014, .048, .096
$\mathcal{AP}^{dw}$	.012, .055, .118	.016, .057, .111	.010, .069, .139	.561, .780, .870	.007, .055, .120	.025, .069, .128
CvM^w	.011, .048, .098	.014, .081, .150	.013, .052, .097	.759, .910, .944	.009, .050, .108	.014, .052, .102
CvM^{dw}	.010, .051, .102	.014, .072, .124	.012, .059, .132	.710, .882, .939	.009, .053, .103	.016, .072, .141
CvM^{brw}	.023, .064, .115	.027, .070, .134	.014, .057, .099	.712, .879, .934	.009, .052, .103	.005, .037, .087

GARCH(1,1) error: $e_t = \nu_t w_t$ with $w_t^2 = 1 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.001, .031, .063	.017, .029, .042	.008, .032, .078	.283, .588, .733	.003, .005, .006	.012, .046, .089
$\mathcal{AP}^w$	.014, .053, .094	.004, .037, .090	.011, .048, .106	.347, .609, .745	.002, .030, .090	.008, .052, .118
$\mathcal{AP}^{dw}$	.004, .049, .107	.008, .021, .064	.016, .066, .142	.444, .687, .793	.006, .030, .066	.021, .080, .137
CvM^w	.009, .056, .107	.006, .026, .084	.010, .048, .092	.579, .813, .887	.002, .045, .098	.007, .044, .092
CvM^{dw}	.015, .066, .115	.026, .038, .075	.011, .051, .104	.550, .802, .881	.013, .052, .111	.026, .072, .143
CvM^{brw}	.013, .052, .105	.003, .021, .056	.011, .063, .116	.583, .802, .882	.008, .029, .065	.010, .055, .099

MA(2) error: $e_t = \nu_t + 0.5\nu_{t-1} + 0.25\nu_{t-2}$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	1.00, 1.00, 1.00	.932, .965, .976	.019, .086, .151	.972, .999, .999	.798, .874, .908	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.982, .996, .999	.040, .141, .234	.999, 1.00, 1.00	.996, .998, 1.00	.998, .998, .999
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.767, .921, .970	.025, .100, .189	1.00, 1.00, 1.00	.920, .972, .990	.997, .998, .998
CvM^w	1.00, 1.00, 1.00	.997, 1.00, 1.00	.028, .150, .275	1.00, 1.00, 1.00	.998, 1.00, 1.00	.995, .996, .996
CvM^{dw}	1.00, 1.00, 1.00	.884, .966, .990	.032, .144, .250	1.00, 1.00, 1.00	.959, .994, .995	.995, .998, .998
CvM^{brw}	1.00, 1.00, 1.00	.955, .980, .989	.031, .131, .232	1.00, 1.00, 1.00	.971, .988, .992	.901, .993, .997

AR(1) error: $e_t = 0.7e_{t-1} + \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.998, 1.00, 1.00	.715, .819, .860	.181, .512, .636	.991, 1.00, 1.00	.111, .176, .230	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.891, .964, .984	.162, .371, .501	1.00, 1.00, 1.00	.986, 1.00, 1.00	.997, .998, .998
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.338, .585, .743	.270, .467, .559	1.00, 1.00, 1.00	.626, .805, .896	.998, .998, .999
CvM^w	1.00, 1.00, 1.00	.919, .985, .993	.333, .597, .709	1.00, 1.00, 1.00	.986, .999, 1.00	.995, .995, .997
CvM^{dw}	1.00, 1.00, 1.00	.393, .630, .781	.325, .592, .700	.999, 1.00, 1.00	.700, .852, .918	.999, 1.00, 1.00
CvM^{brw}	1.00, 1.00, 1.00	.584, .730, .803	.365, .603, .713	1.00, 1.00, 1.00	.794, .876, .911	.945, .999, 1.00

Table 33: Rejection frequencies of $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$, $\mathcal{AP}$, and CvM (Scenarios #1–#6, $n = 1000$)

IID error: $e_t = \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.012, .050, .109	.008, .056, .103	.006, .057, .097	.929, .996, .998	.008, .038, .077	.011, .047, .101
$\mathcal{AP}^w$	.008, .043, .085	.015, .067, .134	.007, .041, .089	.822, .946, .977	.010, .056, .106	.012, .050, .110
$\mathcal{AP}^{dw}$	.012, .047, .095	.011, .057, .123	.016, .068, .124	.870, .964, .984	.013, .061, .123	.006, .052, .115
CvM^w	.008, .053, .104	.022, .074, .138	.011, .050, .093	.974, .996, .998	.012, .055, .111	.014, .057, .102
CvM^{dw}	.008, .060, .108	.016, .063, .106	.010, .049, .102	.974, .991, .993	.015, .058, .107	.013, .057, .103
CvM^{brw}	.016, .057, .105	.023, .072, .128	.016, .051, .102	.972, .997, .999	.017, .063, .104	.003, .044, .081

GARCH(1,1) error: $e_t = \nu_t w_t$ with $w_t^2 = 1 + 0.2e_{t-1}^2 + 0.5w_{t-1}^2$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.006, .032, .071	.014, .026, .031	.008, .033, .085	.741, .925, .961	.002, .002, .002	.006, .051, .102
$\mathcal{AP}^w$	.007, .045, .098	.001, .028, .079	.006, .057, .091	.712, .907, .959	.009, .034, .074	.014, .052, .097
$\mathcal{AP}^{dw}$	.010, .058, .117	.003, .008, .028	.010, .054, .110	.728, .900, .943	.002, .032, .078	.010, .058, .118
CvM^w	.007, .054, .101	.002, .030, .092	.009, .061, .102	.901, .980, .991	.007, .043, .096	.011, .018, .018
CvM^{dw}	.010, .060, .116	.004, .014, .028	.008, .056, .105	.880, .973, .993	.006, .032, .065	.049, .065, .073
CvM^{brw}	.015, .054, .104	.002, .014, .051	.014, .065, .109	.911, .967, .985	.006, .035, .078	.096, .122, .116

MA(2) error: $e_t = \nu_t + 0.5\nu_{t-1} + 0.25\nu_{t-2}$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	1.00, 1.00, 1.00	.968, .982, .986	.063, .184, .257	1.00, 1.00, 1.00	.900, .934, .952	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.995, .996, .998	.083, .226, .350	1.00, 1.00, 1.00	1.00, 1.00, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.898, .967, .989	.053, .177, .296	1.00, 1.00, 1.00	.971, .994, .997	.999, 1.00, 1.00
CvM^w	1.00, 1.00, 1.00	.999, .999, .999	.095, .287, .442	1.00, 1.00, 1.00	1.00, 1.00, 1.00	.998, .999, .999
CvM^{dw}	1.00, 1.00, 1.00	.974, .994, .997	.068, .295, .471	1.00, 1.00, 1.00	.986, .997, 1.00	.998, .998, .998
CvM^{brw}	1.00, 1.00, 1.00	.991, .994, .995	.092, .270, .438	1.00, 1.00, 1.00	.998, .998, .999	.988, 1.00, 1.00

AR(1) error: $e_t = 0.7e_{t-1} + \nu_t$

	#1. Simple	#2. Bilin	#3. AR2/AR2	#4. AR2/AR1	#5. GARCH/wo	#6. GARCH/w
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	1.00, 1.00, 1.00	.723, .823, .864	.599, .847, .922	1.00, 1.00, 1.00	.066, .126, .158	1.00, 1.00, 1.00
$\mathcal{AP}^w$	1.00, 1.00, 1.00	.923, .981, .992	.369, .667, .792	1.00, 1.00, 1.00	.976, .998, 1.00	1.00, 1.00, 1.00
$\mathcal{AP}^{dw}$	1.00, 1.00, 1.00	.408, .661, .774	.459, .681, .787	1.00, 1.00, 1.00	.677, .832, .901	.998, .999, .999
CvM^w	1.00, 1.00, 1.00	.941, .984, .991	.690, .886, .935	1.00, 1.00, 1.00	.982, .998, 1.00	.999, 1.00, 1.00
CvM^{dw}	1.00, 1.00, 1.00	.474, .697, .810	.688, .876, .923	1.00, 1.00, 1.00	.750, .877, .929	.998, .999, .999
CvM^{brw}	1.00, 1.00, 1.00	.615, .761, .833	.700, .878, .913	1.00, 1.00, 1.00	.819, .882, .927	.989, 1.00, 1.00

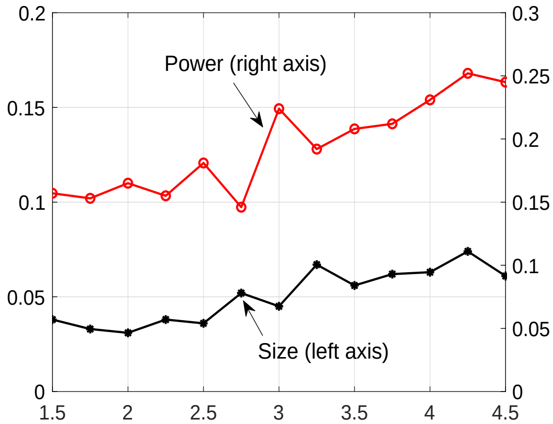
Table 34: Rejection frequencies of $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$, $\mathcal{AP}$, and CvM (Scenarios #7–#9)

	$n = 100$			$n = 250$		
	#7. MA(6)	#8. MA(12)	#9. MA(24)	#7. MA(6)	#8. MA(12)	#9. MA(24)
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.016, .084, .139	.013, .067, .117	.017, .065, .118	.155, .289, .352	.024, .134, .244	.012, .042, .088
$\mathcal{AP}^w$	.036, .104, .163	.013, .062, .133	.014, .054, .107	.050, .149, .233	.020, .087, .148	.006, .050, .110
$\mathcal{AP}^{dw}$	.009, .063, .139	.026, .090, .171	.022, .082, .171	.020, .089, .162	.017, .077, .139	.015, .085, .149
CvM^w	.006, .065, .123	.008, .048, .110	.010, .057, .124	.024, .096, .163	.010, .079, .144	.010, .065, .121
CvM^{dw}	.040, .098, .171	.034, .110, .179	.029, .098, .186	.026, .080, .142	.025, .087, .155	.022, .088, .143
CvM^{brw}	.049, .112, .193	.056, .125, .189	.038, .091, .143	.025, .080, .161	.020, .094, .156	.017, .083, .137

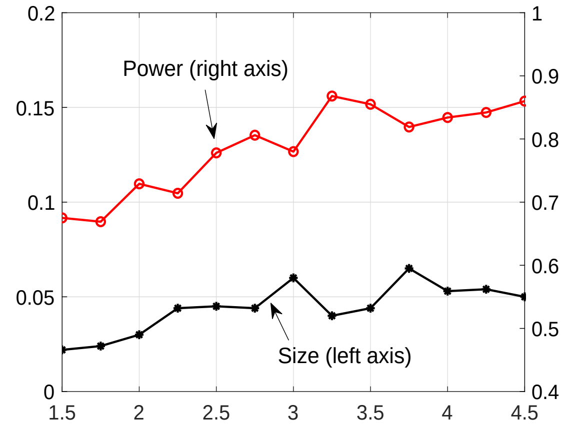
	$n = 500$			$n = 1000$		
	#7. MA(6)	#8. MA(12)	#9. MA(24)	#7. MA(6)	#8. MA(12)	#9. MA(24)
	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%	1%, 5%, 10%
$\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$	.710, .812, .826	.371, .673, .770	.024, .097, .192	.999, 1.00, 1.00	.983, .997, .997	.578, .833, .918
$\mathcal{AP}^w$	.096, .226, .311	.025, .095, .168	.015, .063, .129	.162, .356, .485	.037, .120, .195	.012, .067, .119
$\mathcal{AP}^{dw}$	.022, .130, .226	.015, .069, .142	.018, .079, .158	.060, .199, .332	.011, .066, .152	.008, .049, .115
CvM^w	.027, .112, .207	.020, .086, .133	.011, .060, .135	.034, .158, .344	.021, .078, .151	.020, .075, .146
CvM^{dw}	.014, .087, .175	.026, .092, .161	.024, .071, .133	.038, .160, .320	.017, .083, .166	.028, .079, .144
CvM^{brw}	.033, .101, .188	.020, .068, .128	.019, .078, .142	.037, .150, .321	.026, .081, .152	.027, .078, .138

#7: Remote MA(6) $y_t = e_t + 0.25e_{t-6}$ with a mean filter. #8: Remote MA(12) $y_t = e_t + 0.25e_{t-12}$ with a mean filter. #9: Remote MA(24) $y_t = e_t + 0.25e_{t-24}$ with a mean filter. For each scenario, $e_t \overset{i.i.d.}{\sim} N(0,1)$. $\hat{\mathcal{T}}_n(\mathcal{L}_n^*)$ is the max-correlation test based on Shao's (2011) dependent wild bootstrap, with automatic lag $\mathcal{L}_n^*$. $\mathcal{AP}$ is Andrews and Ploberger's (1996) sup-LM test. CvM is the Cramér-von Mises test. The $\mathcal{AP}$ and CvM tests use $\mathcal{L}_n = n - 1$ lags. Superscript "w" means the wild bootstrap; "dw" means the dependent wild bootstrap; "brw"means Zhu and Li's (2015) block-wise random weighting bootstrap.

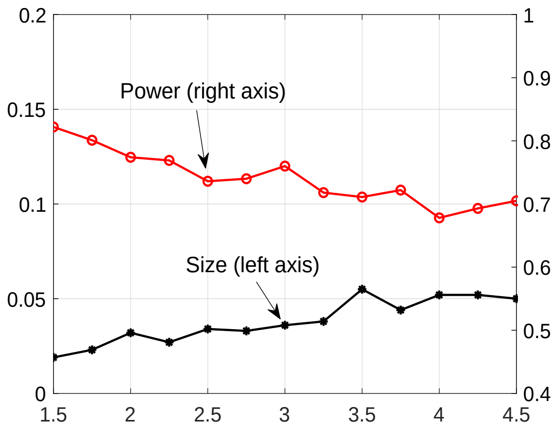
Figure 1: Empirical Size and Size-Adjusted Power of $\hat{\mathcal{T}}^{dw}(\mathcal{L}_n^*)$ with $\alpha = 0.05$



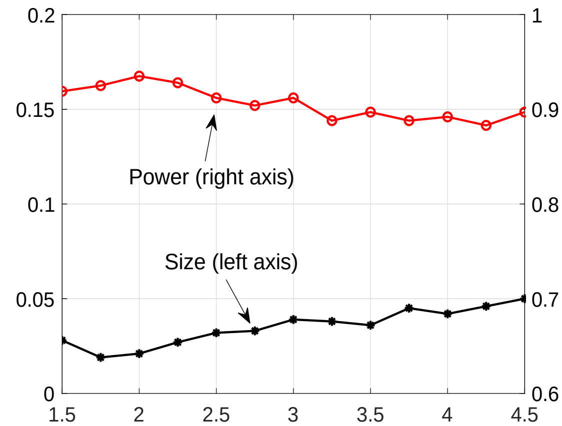
Case 1, $n = 100$



Case 1, $n = 500$



Case 2, $n = 100$



Case 2, $n = 500$

We plot empirical size and size-adjusted power of the bootstrapped max-correlation test with automatic lag selection given nominal size 5%. In Case 1, the empirical size and empirical quantiles for size adjustment are computed under Scenario #1 (iid y_t and mean filter) with i.i.d. error; then the size-adjusted power is computed under Scenario #4 (AR(2) y_t and AR(1) filter) with i.i.d. error. In Case 2, the empirical size and empirical quantiles for size adjustment are computed under Scenario #5 (GARCH y_t and no filter) with i.i.d. error; then the size-adjusted power is computed under Scenario #5 with MA(2) error. The tuning parameter that affects the penalty term $\mathcal{P}_n(\mathcal{L})$ is $q \in \{1.50, 1.75, \ldots, 4.50\}$. The largest possible lag length is $\bar{\mathcal{L}}_n = [10\sqrt{n}/(\ln n)]$, which implies that $\bar{\mathcal{L}}_{100} = 21$ and $\bar{\mathcal{L}}_{500} = 35$. $J = 1000$ Monte Carlo samples and $M = 500$ bootstrap samples are generated.

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