

Supplemental Appendix - Link of moments before and
after transformations, with an application to
resampling from fat-tailed distributions

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Table 1: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.1)$, $k = 2$, $E_*(x) = 48.113837381$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	36.02031518	79.52389775	0.748647731	31.85063757	112.8084388	0.661984978
10	22.75139093	739.1814387	0.472865857	59.81391499	232.8180046	1.243174900
50	50.59838790	3.901968792	1.051639001	49.48870901	2.079265927	1.028575389
100	48.72227366	0.882972177	1.012645765	48.45340538	0.488301544	1.007057595
500	48.13800244	0.034209684	1.000502248	48.12736420	0.019144562	1.000281142
1000	48.11987728	0.008543904	1.000125534	48.11721864	0.004783143	1.000070276

Table 2: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.1)$, $k = 3$, $E_*(x) = 48.113837381$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	686.7873982	4271.121409	14.27421789	1295.665613	9251.947117	26.92916806
10	62.26001429	66.30685719	1.29401473	53.4084961	18.29835898	1.110044407
50	48.13614219	0.183205259	1.000463584	48.12306332	0.087691534	1.000191752
100	48.11525298	0.022820961	1.000029422	48.11441994	0.010941749	1.000012108
500	48.11383966	0.000182483	1.000000047	48.11383832	8.74994E-05	1.000000019

Table 3: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.1)$, $k = 4$, $E_*(x) = 48.113837381$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	656.6211587	17041.23391	13.64724151	1280.201915	34238.65768	26.60776992
10	45.93426141	5.088503652	0.954699602	47.10456151	2.057393128	0.979023168
50	48.10824978	0.008049875	0.999883867	48.11153053	0.003306909	0.999952054
100	48.11348322	0.000503168	0.999992639	48.11369172	0.000206678	0.999996973

Table 4: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.3)$, $k = 2$, $E_*(x) = 16.2588462282$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	10.90027129	39.49266418	0.67042096	12.04682277	28.36346446	0.740939585
10	20.05461030	42.98010744	1.233458391	19.51817111	14.75765738	1.200464709
50	16.56024987	0.450947284	1.018537825	16.41722038	0.232551018	1.009740799
100	16.33345748	0.106994115	1.004588963	16.29821230	0.056215388	1.002421210
500	16.26182072	0.004208903	1.000182946	16.26041783	0.002224563	1.000096661
1000	16.25958977	0.001051677	1.000045732	16.25923910	0.000555951	1.000024164

Table 5: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.3)$, $k = 3$, $E_*(x) = 16.2588462282$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	357.3868597	2437.040583	21.98107139	179.4132451	1207.692768	11.03480792
10	17.27127102	3.435483172	1.062269166	16.65696979	1.263222364	1.024486582
50	16.26061542	0.017805411	1.000108814	16.25954838	0.008141218	1.000043186
100	16.25895783	0.002222322	1.000006864	16.25889039	0.001016834	1.000002716
500	16.25884641	1.77665E-05	1.000000011	16.2588463	8.12314E-06	1.000000004

Table 6: Accuracy of expansion and bound, $y \sim \text{Gam}(2, 1.3)$, $k = 4$, $E_*(x) = 16.2588462282$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	347.0041084	9138.275446	21.34248049	175.6001415	4476.162379	10.80028306
10	16.05949307	0.393932431	0.987738788	16.17271909	0.155668905	0.994702752
50	16.25840397	0.000633313	0.999972799	16.25867084	0.000250341	0.999989213
100	16.25881833	3.95755E-05	0.999998284	16.25883519	1.56374E-05	0.999999321

Table 7: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.1)$, $k = 2$, $E_*(x) = 7.64132294$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	5.348890454	18.11267799	0.69999534	5.547758808	13.9716725	0.726020723
10	9.070487448	21.76358109	1.18703103	9.250433075	11.70554432	1.210580045
50	7.779876421	0.208442261	1.018132132	7.738604908	0.144474011	1.012731037
100	7.675580602	0.049193328	1.00448321	7.665445087	0.034536134	1.003156802
500	7.642688123	0.001931828	1.000178657	7.642285167	0.001361787	1.000125924
1000	7.641664198	0.000482679	1.000044659	7.641563478	0.000340293	1.000031478

Table 8: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.1)$, $k = 3$, $E_*(x) = 7.64132294$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	132.5229086	1132.385351	17.34292734	120.6061654	847.8742229	15.78341424
10	8.141174023	1.725595451	1.065414207	7.936898676	0.954853041	1.038681225
50	7.642193478	0.008490408	1.000113925	7.641844525	0.005513082	1.000068258
100	7.641377893	0.001059513	1.000007191	7.641355802	0.000688324	1.0000043
500	7.641323032	8.47005E-06	1.000000012	7.641322996	5.50162E-06	1.000000007

Table 9: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.1)$, $k = 4$, $E_*(x) = 7.64132294$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	137.4851024	3506.24111	17.99231669	116.763729	2896.920893	15.28056462
10	7.544974283	0.194066642	0.987391102	7.580423302	0.115830347	0.992030223
50	7.641105291	0.000311884	0.999971516	7.641192612	0.00018637	0.999982944
100	7.641309205	1.94899E-05	0.999998202	7.64131473	1.1645E-05	0.999998925

Table 10: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.3)$, $k = 2$, $E_*(x) = 4.1276252974$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	3.222377874	7.159466827	0.780685659	3.459858499	5.498965123	0.838220102
10	4.610126678	1.932521013	1.116895635	4.478563404	1.101408133	1.085021794
50	4.150268872	0.033086837	1.005485860	4.142876889	0.022165928	1.003695004
100	4.133258332	0.008031035	1.001364716	4.131424273	0.005412930	1.000920378
500	4.127850246	0.000318227	1.000054498	4.127777070	0.000214898	1.000036770
1000	4.127681532	7.95323E-05	1.000013624	4.127663239	5.37101E-05	1.000009192

Table 11: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.3)$, $k = 3$, $E_*(x) = 4.1276252974$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	41.10209787	298.1583280	9.957807433	35.42614118	246.8631630	8.582693104
10	4.179104175	0.166211312	1.012471790	4.157808888	0.098258707	1.007312580
50	4.127715779	0.001092510	1.000021921	4.127678059	0.000686532	1.000012783
100	4.127630986	0.000136464	1.000001378	4.127628611	8.57695E-05	1.000000803
500	4.127625307	1.08802E-06	1.000000002	4.127625303	6.83128E-07	1.000000001

Table 12: Accuracy of expansion and bound, $y \sim \text{Gam}(1, 1.3)$, $k = 4$, $E_*(x) = 4.1276252974$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	41.16945050	1034.516608	9.974124958	35.33930121	867.5052936	8.561654382
10	4.116393816	0.020057047	0.997278948	4.120839821	0.011688021	0.998356082
50	4.127602697	3.22434E-05	0.999994525	4.127612121	1.87732E-05	0.999996808
100	4.127623875	2.01233E-06	0.999999655	4.127624469	1.17092E-06	0.999999799

Table 13: Accuracy of expansion and bound, $y \sim N(1, 1)$, $k = 2$, $E_*(x) = 4.4703660027$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	2.98772013	11.18380332	0.668339042	4.098873968	3.711456770	0.916898967
10	4.573313241	0.160009011	1.023028816	4.499708029	0.045612240	1.006563674
50	4.474510726	0.005887077	1.000927155	4.471546512	0.001749706	1.000264074
100	4.471402220	0.001467883	1.000231797	4.470661165	0.000436851	1.000066026
500	4.470407452	5.86654E-05	1.000009272	4.470377810	1.74653E-05	1.000002641
1000	4.470376365	1.46655E-05	1.000002318	4.470368954	4.36445E-06	1.000000660

Table 14: Accuracy of expansion and bound, $y \sim N(1, 1)$, $k = 3$, $E_*(x) = 4.4703660027$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	49.77001850	409.4277092	11.13332073	11.14246140	66.45933835	2.492516585
10	4.472402982	0.011294884	1.000455663	4.470738257	0.002604166	1.000083272
50	4.470369340	8.97957E-05	1.000000747	4.470366608	2.07783E-05	1.000000135
100	4.470366211	1.12231E-05	1.000000047	4.470366041	2.59425E-06	1.000000008
500	4.470366003	8.13632E-08	1.000000000	4.470366003	1.52491E-08	1.000000000

Table 15: Accuracy of expansion and bound, $y \sim N(1, 1)$, $k = 4$, $E_*(x) = 4.4703660027$.

m	Raw expansion			Centered expansion		
	E_k	Bound	$E_k / E_*(x)$	E_k	Bound	$E_k / E_*(x)$
1	51.04836227	1304.891883	11.41928026	11.12050304	190.6877168	2.487604601
10	4.469867668	0.000738275	0.999888525	4.470274316	0.000136150	0.999979490
50	4.470365169	1.17793E-06	0.999999813	4.470365852	2.13008E-07	0.999999966
100	4.470365951	6.65426E-08	0.999999988	4.470365993	9.62026E-09	0.999999998