Supplementary Material to 'Estimation of spatial autoregressions with stochastic weight matrices'[*]

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A simulation study when spatial weights are generated by dependent random variables

Section 6.1 of 'Estimation of spatial autoregressions with stochastic weight matrices' took the spatial weights to be iid but in some cases, especially for asymmetric spatial weight matrices, this may not be reasonable. For instance, if the distance from unit r to s is small the distance from s to r may also be expected to be small. To capture such behaviour we use the same designs as described in Section 6.1, but with the following alteration: after generating the V_j, replace $v_{rs,j} = \left(v_{sr,j}^2 + 5\right)^{\frac{1}{2}}$ for each $r = 1, \ldots, m$ and $s \leq r$, where $v_{rs,j}$ denotes the (r, s)-th element of V_j. Thus we replace the part of V_j below the diagonal with a transformation of the part above the diagonal. The choice of transformation is uniformly continuous, in keeping with the idea of 'preserving' the distance between units discussed earlier in the paragraph. Similar operations are carried out with the sparse and circulant specifications of W. We then proceed with the experiment design as in the corresponding parts of Section 6.1.

The results are in Tables 1(a)-(c), where we report the stochastic case. They indicate that the procedure of generating dependent weights in this way does little to alter the character and behaviour of the estimates. The same features that we saw in Tables 6.1, 6.3 and 6.5 are evident. We may also compare the dense, sparse and circulant cases to see if stochastic dependent spatial weights yield any difference in performance as opposed to stochastic iid ones. Out of 72 comparisons for each type of weight matrix, the dependent setting exhibits a smaller bias in 54 (dense), 61 (sparse) and 41 (circulant) cases, while the MSE is smaller in all 72 (dense), 54 (sparse) and 32 (circulant) cases. Thus in our experiment designs dependent spatial weights do not contaminate the performance of estimates.

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[*] This appendix should be read in conjunction with Section 6 of 'Estimation of spatial autoregressions with stochastic weight matrices'.

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		(a) Dense W_1, W_2						(b) Sparse W						(c) Circulant W					
	m/n	48		72		144		96		144		288		96		144		288	
Bias	v	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β
$\hat{\theta}$	1	0.0292	0.0025	0.0423	0.0046	0.0459	0.0009	0.0280	0.0061	0.0585	0.0059	0.0586	0.0038	0.0058	0.0309	0.0024	0.0151	0.0027	0.0152
	10	0.0126	0.0143	0.0059	0.0069	0.0051	0.0046	0.0017	0.0063	0.0062	0.0062	0.0005	0.0009	0.0047	0.0367	0.0017	0.0139	0.0015	0.0128
	20	0.0086	0.0104	0.0067	0.0104	0.0016	0.0050	0.0033	0.0151	0.0027	0.0046	0.0001	0.0031	0.0066	0.0422	0.0024	0.0099	0.0001	0.0074
	100	0.0036	0.0030	0.0023	0.0084	0.0012	0.0013	0.0100	0.0109	0.0046	0.0076	0.0021	0.0027	0.0055	0.0369	0.0047	0.0346	0.0003	0.0044
$\tilde{\theta}$	1	0.0698	0.0151	0.0622	0.0084	0.0793	0.0066	0.0008	0.0063	0.0396	0.0055	0.0417	0.0039	0.0697	0.4591	0.0769	0.4970	0.0874	0.5451
	10	0.0007	0.0021	0.0016	0.0020	0.0012	0.0009	0.0091	0.0052	0.0116	0.0107	0.0031	0.0049	0.0810	0.5066	0.0908	0.5429	0.1008	0.5947
	20	0.0036	0.0038	0.0017	0.0032	0.0031	0.0005	0.0125	0.0154	0.0090	0.0116	0.0027	0.0044	0.0819	0.5032	0.0889	0.5336	0.1000	0.5902
	100	0.0104	0.0131	0.0052	0.0084	0.0058	0.0050	0.0207	0.0226	0.0120	0.0117	0.0016	0.0026	0.0811	0.5006	0.0896	0.5452	0.0994	0.5836
$\check{\theta}$	1	0.0516	0.0073	0.0625	0.0078	0.0531	0.0029	0.1230	0.0081	0.1311	0.0076	0.1335	0.0063	0.0008	0.0102	0.0034	0.0204	0.0090	0.0548
	10	0.0155	0.0187	0.0116	0.0137	0.0039	0.0062	0.0306	0.0280	0.0263	0.0265	0.0107	0.0132	0.0048	0.0315	0.0059	0.0339	0.0118	0.0715
	20	0.0176	0.0202	0.0113	0.0106	0.0080	0.0063	0.0336	0.0292	0.0231	0.0268	0.0100	0.0124	0.0040	0.0222	0.0065	0.0363	0.0122	0.0749
	100	0.0240	0.0287	0.0144	0.0177	0.0105	0.0104	0.0402	0.0430	0.0258	0.0264	0.0087	0.0094	0.0037	0.0225	0.0080	0.0505	0.0135	0.0823
MSE	v	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β
$\hat{\theta}$	1	0.1665	0.0398	0.1529	0.0245	0.3150	0.0131	1.9196	0.0822	0.9467	0.0518	1.3855	0.0242	0.0048	0.2282	0.0028	0.1434	0.0016	0.0746
	10	0.0119	0.0530	0.0083	0.0350	0.0039	0.0171	0.0257	0.0998	0.0158	0.0665	0.0077	0.0339	0.0039	0.2033	0.0033	0.1548	0.0018	0.0797
	20	0.0120	0.0530	0.0085	0.0358	0.0043	0.0174	0.0269	0.1028	0.0167	0.0650	0.0080	0.0346	0.0043	0.2106	0.0038	0.1583	0.0018	0.0782
	100	0.0120	0.0534	0.0076	0.0335	0.0040	0.0177	0.0235	0.1031	0.0171	0.0646	0.0078	0.0335	0.0042	0.2075	0.0033	0.1530	0.0017	0.0783
$\tilde{\theta}$	1	0.0461	0.0395	0.0408	0.0241	0.0515	0.0130	0.5264	0.0784	0.6161	0.0502	0.5338	0.0233	0.0069	0.3462	0.0081	0.3464	0.0099	0.3710
	10	0.0117	0.0525	0.0084	0.0350	0.0039	0.0171	0.0266	0.1002	0.0156	0.0662	0.0077	0.0338	0.0084	0.3664	0.0102	0.3831	0.0119	0.4084
	20	0.0118	0.0525	0.0084	0.0356	0.0042	0.0172	0.0261	0.1024	0.0166	0.0649	0.0080	0.0346	0.0086	0.3675	0.0098	0.3736	0.0116	0.4051
	100	0.0120	0.0532	0.0076	0.0336	0.0040	0.0177	0.0229	0.1024	0.0169	0.0643	0.0078	0.0335	0.0085	0.3699	0.0098	0.3818	0.0115	0.3975
$\check{\theta}$	1	0.0287	0.0387	0.0301	0.0240	0.0302	0.0128	0.1552	0.0769	0.1628	0.0495	0.1677	0.0231	0.0016	0.1481	0.0017	0.1136	0.0020	0.1014
	10	0.0109	0.0512	0.0081	0.0344	0.0038	0.0169	0.0243	0.0970	0.0150	0.0651	0.0075	0.0335	0.0023	0.1610	0.0023	0.1392	0.0027	0.1235
	20	0.0111	0.0514	0.0081	0.0349	0.0041	0.0170	0.0242	0.0996	0.0158	0.0639	0.0078	0.0343	0.0024	0.1635	0.0024	0.1372	0.0027	0.1279
	100	0.0115	0.0524	0.0073	0.0331	0.0040	0.0176	0.0217	0.1008	0.0162	0.0634	0.0076	0.0332	0.0022	0.1606	0.0025	0.1415	0.0029	0.1312
Size	v	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β
$\hat{\theta}$	1	0.0430	0.0575	0.0330	0.0475	0.0480	0.0580	0.0540	0.0555	0.0530	0.0565	0.0500	0.0415	0.0530	0.0540	0.0380	0.0415	0.0560	0.0535
	10	0.0610	0.0565	0.0640	0.0470	0.0440	0.0430	0.0490	0.0445	0.0410	0.0480	0.0420	0.0535	0.0520	0.0440	0.0470	0.0580	0.0490	0.0605
	20	0.0510	0.0550	0.0500	0.0555	0.0660	0.0500	0.0560	0.0565	0.0500	0.0415	0.0460	0.0535	0.0470	0.0520	0.0460	0.0510	0.0430	0.0445
	100	0.0510	0.0570	0.0450	0.0435	0.0490	0.0520	0.0470	0.0530	0.0550	0.0505	0.0490	0.0535	0.0470	0.0540	0.0450	0.0505	0.0450	0.0455
$\tilde{\theta}$	1	0.1040	0.0585	0.0760	0.0455	0.1070	0.0615	0.0640	0.0535	0.0740	0.0610	0.0710	0.0430	0.8110	0.4335	0.8890	0.6105	0.9390	0.8100
	10	0.0560	0.0570	0.0640	0.0480	0.0440	0.0425	0.0580	0.0485	0.0390	0.0485	0.0490	0.0525	0.9070	0.4670	0.9460	0.6540	0.9840	0.8845
	20	0.0560	0.0540	0.0550	0.0535	0.0590	0.0510	0.0620	0.0570	0.0500	0.0425	0.0530	0.0555	0.8930	0.4820	0.9380	0.6420	0.9760	0.8800
	100	0.0590	0.0570	0.0490	0.0440	0.0480	0.0520	0.0440	0.0520	0.0550	0.0505	0.0520	0.0515	0.8920	0.4785	0.9430	0.6730	0.9790	0.8735
$\check{\theta}$	1	0.0210	0.0570	0.0150	0.0465	0.0160	0.0585	0.0240	0.0545	0.0130	0.0630	0.0120	0.0435	0.1120	0.1040	0.1260	0.1000	0.1510	0.1210
	10	0.0430	0.0560	0.0600	0.0455	0.0390	0.0445	0.0520	0.0510	0.0360	0.0510	0.0420	0.0520	0.1310	0.1095	0.1560	0.1330	0.1730	0.1475
	20	0.0510	0.0570	0.0520	0.0545	0.0540	0.0490	0.0610	0.0575	0.0520	0.0435	0.0470	0.0550	0.1440	0.1190	0.1470	0.1245	0.1800	0.1630
	100	0.0530	0.0575	0.0470	0.0420	0.0490	0.0515	0.0430	0.0555	0.0580	0.0515	0.0480	0.0520	0.1290	0.1165	0.1550	0.1280	0.1730	0.1470

Table 1: Monte Carlo absolute bias, mean squared error and size, nominal size 5%, dependent weight matrices regenerated in each trial.