A simulation study when spatial weights are generated by dependent random variables

Section 6.1 of ‘Estimation of spatial autoregressions with stochastic weight matrices’ took the spatial weights to be iid but in some cases, especially for asymmetric spatial weight matrices, this may not be reasonable. For instance, if the distance from unit $r$ to $s$ is small the distance from $s$ to $r$ may also be expected to be small. To capture such behaviour we use the same designs as described in Section 6.1, but with the following alteration: after generating the $V_j$, replace $v_{rs,j} = \left( v_{sr,j}^2 + 5 \right)^{\frac{1}{2}}$ for each $r = 1, \ldots, m$ and $s \leq r$, where $v_{rs,j}$ denotes the $(r, s)$-th element of $V_j$. Thus we replace the part of $V_j$ below the diagonal with a transformation of the part above the diagonal. The choice of transformation is uniformly continuous, in keeping with the idea of ‘preserving’ the distance between units discussed earlier in the paragraph. Similar operations are carried out with the sparse and circulant specifications of $W$. We then proceed with the experiment design as in the corresponding parts of Section 6.1.

The results are in Tables 1(a)-(c), where we report the stochastic case. They indicate that the procedure of generating dependent weights in this way does little to alter the character and behaviour of the estimates. The same features that we saw in Tables 6.1, 6.3 and 6.5 are evident. We may also compare the dense, sparse and circulant cases to see if stochastic dependent spatial weights yield any difference in performance as opposed to stochastic iid ones. Out of 72 comparisons for each type of weight matrix, the dependent setting exhibits a smaller bias in 54 (dense), 61 (sparse) and 41 (circulant) cases, while the MSE is smaller in all 72 (dense), 54 (sparse) and 32 (circulant) cases. Thus in our experiment designs dependent spatial weights do not contaminate the performance of estimates.

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*This appendix should be read in conjunction with Section 6 of ‘Estimation of spatial autoregressions with stochastic weight matrices’.

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Table 1: Monte Carlo absolute bias, mean squared error and size, nominal size 5%, dependent weight matrices regenerated in each trial.