

Appendix A.

The length of l_i is listed as follow:

$$\begin{aligned} l_1 = \frac{1}{4} & (-4r_1 \cos \theta + 4x_1 \cos \theta - 2r_2 \cos(\theta - \theta y) + 2r_2 \cos(\theta + \theta y) + r_2 \cos(\theta - \theta y - \theta z) \\ & + r_2 \cos(\theta + \theta y - \theta z) + r_2 \cos(\theta - \theta y + \theta z) + r_2 \cos(\theta + \theta y + \theta z) \\ & - 4h \sin \theta + 4z_1 \sin \theta \\ & + ((4r_1 \cos \theta - 4x_1 \cos \theta + 2r_2 \cos(\theta - \theta y) - 2r_2 \cos(\theta + \theta y) \\ & - r_2 \cos(\theta - \theta y - \theta z) - r_2 \cos(\theta + \theta y - \theta z) - r_2 \cos(\theta - \theta y + \theta z) \\ & - r_2 \cos(\theta + \theta y + \theta z) + 4h \sin \theta - 4z_1 \sin \theta)^2 - 8(2h^2 - 21^2 + 2r_1^2 + 2r_2^2 \\ & - 4r_1 x_1 + 2x_1^2 + 2y_1^2 - 4hz_1 + 2z_1^2 - 2r_1 r_2 \cos(\theta y - \theta z) \\ & + 2r_2 x_1 \cos(\theta y - \theta z) - 2r_1 r_2 \cos(\theta y + \theta z) + 2r_2 x_1 \cos(\theta y + \theta z) \\ & + 4hr_2 \sin(\theta y) - 4r_2 z_1 \sin(\theta y) \\ & - 2r_2 y_1 \sin(\theta y - \theta z) + 2r_2 y_1 \sin(\theta y + \theta z)))^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
l_2 = & \frac{1}{32} (-32r_1 \cos \theta - 16x_1 \cos \theta + 16\sqrt{3}y_1 \cos \theta + 8r_2 \cos(\theta - \theta y)) \\
& + 4\sqrt{3}r_2 \cos(\theta - \theta x - \theta y) - 4\sqrt{3}r_2 \cos(\theta + \theta x - \theta y) - 8r_2 \cos(\theta + \theta y) \\
& + 4\sqrt{3}r_2 \cos(\theta - \theta x + \theta y) - 4\sqrt{3}r_2 \cos(\theta + \theta x + \theta y) \\
& + 6r_2 \cos(\theta - \theta x - \theta z) + 6r_2 \cos(\theta + \theta x - \theta z) + 2r_2 \cos(\theta - \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x - \theta y - \theta z) \\
& + 2r_2 \cos(\theta + \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y - \theta z) + 6r_2 \cos(\theta - \theta x + \theta z) \\
& + 6r_2 \cos(\theta + \theta x + \theta z) + 2r_2 \cos(\theta - \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x - \theta y + \theta z) \\
& + 2r_2 \cos(\theta + \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y + \theta z) - 32h \sin \theta + 32z_1 \sin \theta \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta x - \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta x - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta y - \theta z) + 3r_2 \sin(\theta - \theta x - \theta y - \theta z) \\
& - 3r_2 \sin(\theta + \theta x - \theta y - \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta y - \theta z) \\
& - 3r_2 \sin(\theta - \theta x + \theta y - \theta z) + 3r_2 \sin(\theta + \theta x + \theta y - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta x + \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta x + \theta z) \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta y + \theta z) - 3r_2 \sin(\theta - \theta x - \theta y + \theta z) \\
& + 3r_2 \sin(\theta + \theta x - \theta y + \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta y + \theta z) \\
& + 3r_2 \sin(\theta - \theta x + \theta y + \theta z) \\
& - 3r_2 \sin(\theta + \theta x + \theta y + \theta z) \\
& + (64(-16h^2 + 161^2 - 16r_1^2 - 16r_2^2 - 16r_1x_1 - 16x_1^2 + 16\sqrt{3}r_1y_1 - 16y_1^2 \\
& + 32hz_1 - 16z_1^2 + 12r_1r_2 \cos(\theta x - \theta z) - 8\sqrt{3}r_2y_1 \cos(\theta x - \theta z) \\
& - 2\sqrt{3}r_1r_2 \cos(\theta x - \theta y - \theta z) - 4\sqrt{3}r_2x_1 \cos(\theta x - \theta y - \theta z) \\
& + 4r_1r_2 \cos(\theta y - \theta z) + 8r_2x_1 \cos(\theta y - \theta z) + 2\sqrt{3}r_1r_2 \cos(\theta x + \theta y - \theta z) \\
& + 4\sqrt{3}r_2x_1 \cos(\theta x + \theta y - \theta z) + 12r_1r_2 \cos(\theta x + \theta z) \\
& - 8\sqrt{3}r_2y_1 \cos(\theta x + \theta z) - 2\sqrt{3}r_1r_2 \cos(\theta x - \theta y + \theta z) - 4\sqrt{3}r_2x_1 \cos(\theta x \\
& - \theta y + \theta z) + 4r_1r_2 \cos(\theta y + \theta z) + 8r_2x_1 \cos(\theta y + \theta z) \\
& + 2\sqrt{3}r_1r_2 \cos(\theta x + \theta y + \theta z) + 4\sqrt{3}r_2x_1 \cos(\theta x + \theta y + \theta z) \\
& + 8\sqrt{3}hr_2 \sin(\theta x - \theta y) - 8\sqrt{3}r_2z_1 \sin(\theta x - \theta y) + 16hr_2 \sin(\theta y) \\
& - 16r_2z_1 \sin(\theta y) + 8\sqrt{3}hr_2 \sin(\theta x + \theta y) - 8\sqrt{3}r_2z_1 \sin(\theta x + \theta y) \\
& - 4\sqrt{3}r_1r_2 \sin(\theta x - \theta z) - 8\sqrt{3}r_2x_1 \sin(\theta x - \theta z) - 6r_1r_2 \sin(\theta x - \theta y - \theta z) \\
& + 4\sqrt{3}r_2y_1 \sin(\theta x - \theta y - \theta z) + 4\sqrt{3}r_1r_2 \sin(\theta y - \theta z) - 8r_2y_1 \sin(\theta y - \theta z) \\
& + 6r_1r_2 \sin(\theta x + \theta y - \theta z) - 4\sqrt{3}r_2y_1 \sin(\theta x + \theta y - \theta z) \\
& + 4\sqrt{3}r_1r_2 \sin(\theta x + \theta z) + 8\sqrt{3}r_2x_1 \sin(\theta x + \theta z) + 6r_1r_2 \sin(\theta x - \theta y + \theta z) \\
& - 4\sqrt{3}r_2y_1 \sin(\theta x - \theta y + \theta z) - 4\sqrt{3}r_1r_2 \sin(\theta y + \theta z) + 8r_2y_1 \sin(\theta y + \theta z) \\
& - 6r_1r_2 \sin(\theta x + \theta y + \theta z) + 4\sqrt{3}r_2y_1 \sin(\theta x + \theta y + \theta z)) + (-32r_1 \cos \theta \\
& - 16x_1 \cos \theta + 16\sqrt{3}y_1 \cos \theta + 8r_2 \cos(\theta - \theta y) + 4\sqrt{3}r_2 \cos(\theta - \theta x - \theta y) \\
& - 4\sqrt{3}r_2 \cos(\theta + \theta x - \theta y) - 8r_2 \cos(\theta + \theta y) + 4\sqrt{3}r_2 \cos(\theta - \theta x + \theta y) \\
& - 4\sqrt{3}r_2 \cos(\theta + \theta x + \theta y) + 6r_2 \cos(\theta - \theta x - \theta z) + 6r_2 \cos(\theta + \theta x - \theta z) \\
& + 2r_2 \cos(\theta - \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x + \theta y - \theta z) \\
& + 2r_2 \cos(\theta + \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y - \theta z) + 6r_2 \cos(\theta - \theta x + \theta z)
\end{aligned}$$

$$\begin{aligned}
& + 6r_2 \cos(\theta + \theta x + \theta z) + 2r_2 \cos(\theta - \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x - \theta y + \theta z) \\
& + 2r_2 \cos(\theta + \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y + \theta z) - 32h \sin \theta + 32z_1 \sin \theta \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta x - \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta x - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta y - \theta z) + 3r_2 \sin(\theta - \theta x - \theta y - \theta z) \\
& - 3r_2 \sin(\theta + \theta x - \theta y - \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta y - \theta z) \\
& - 3r_2 \sin(\theta - \theta x + \theta y - \theta z) + 3r_2 \sin(\theta + \theta x + \theta y - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta x + \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta x + \theta z) \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta y + \theta z) - 3r_2 \sin(\theta - \theta x - \theta y + \theta z) \\
& + 3r_2 \sin(\theta + \theta x - \theta y + \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta y + \theta z) \\
& + 3r_2 \sin(\theta - \theta x + \theta y + \theta z) - 3r_2 \sin(\theta + \theta x + \theta y + \theta z))^{1/2}
\end{aligned}$$

$$\begin{aligned}
l_3 = \frac{1}{32} & \left(-32r_1 \cos \theta - 16x_1 \cos \theta - 16\sqrt{3}y_1 \cos \theta + 8r_2 \cos(\theta - \theta y) \right. \\
& - 4\sqrt{3}r_2 \cos(\theta - \theta x - \theta y) + 4\sqrt{3}r_2 \cos(\theta + \theta x - \theta y) - 8r_2 \cos(\theta + \theta y) \\
& - 4\sqrt{3}r_2 \cos(\theta - \theta x + \theta y) + 4\sqrt{3}r_2 \cos(\theta + \theta x + \theta y) \\
& + 6r_2 \cos(\theta - \theta x - \theta z) + 6r_2 \cos(\theta + \theta x - \theta z) + 2r_2 \cos(\theta - \theta y - \theta z) \\
& - \sqrt{3}r_2 \cos(\theta - \theta x - \theta y - \theta z) + \sqrt{3}r_2 \cos(\theta + \theta x - \theta y - \theta z) \\
& + 2r_2 \cos(\theta + \theta y - \theta z) + \sqrt{3}r_2 \cos(\theta - \theta x + \theta y - \theta z) \\
& - \sqrt{3}r_2 \cos(\theta + \theta x + \theta y - \theta z) + 6r_2 \cos(\theta - \theta x + \theta z) \\
& + 6r_2 \cos(\theta + \theta x + \theta z) + 2r_2 \cos(\theta - \theta y + \theta z) \\
& - \sqrt{3}r_2 \cos(\theta - \theta x - \theta y + \theta z) + \sqrt{3}r_2 \cos(\theta + \theta x - \theta y + \theta z) \\
& + 2r_2 \cos(\theta + \theta y + \theta z) + \sqrt{3}r_2 \cos(\theta - \theta x + \theta y + \theta z) \\
& - \sqrt{3}r_2 \cos(\theta + \theta x + \theta y + \theta z) - 32h \sin \theta + 32z_1 \sin \theta \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta x - \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta x - \theta z) \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta y - \theta z) + 3r_2 \sin(\theta - \theta x - \theta y - \theta z) \\
& - 3r_2 \sin(\theta + \theta x - \theta y - \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta y - \theta z) \\
& - 3r_2 \sin(\theta - \theta x + \theta y - \theta z) + 3r_2 \sin(\theta + \theta x + \theta y - \theta z) \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta x + \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta x + \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta y + \theta z) - 3r_2 \sin(\theta - \theta x - \theta y + \theta z) \\
& + 3r_2 \sin(\theta + \theta x - \theta y + \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta y + \theta z) \\
& + 3r_2 \sin(\theta - \theta x + \theta y + \theta z) \\
& - 3r_2 \sin(\theta + \theta x + \theta y + \theta z) \\
& + (-64(16h^2 - 16l^2 + 16r_1^2 + 16r_2^2 + 16r_1x_1 + 16x_1^2 + 16\sqrt{3}r_1y_1 + 16y_1^2 \\
& - 32hz_1 + 16z_1^2 - 12r_1r_2 \cos(\theta x - \theta z) - 8\sqrt{3}r_2y_1 \cos(\theta x - \theta z) \\
& - 2\sqrt{3}r_1r_2 \cos(\theta x - \theta y - \theta z) - 4\sqrt{3}r_2x_1 \cos(\theta x - \theta y - \theta z) \\
& - 4r_1r_2 \cos(\theta y - \theta z) - 8r_2x_1 \cos(\theta y - \theta z) + 2\sqrt{3}r_1r_2 \cos(\theta x + \theta y - \theta z) \\
& + 4\sqrt{3}r_2x_1 \cos(\theta x + \theta y - \theta z) - 12r_1r_2 \cos(\theta x + \theta z) \\
& - 8\sqrt{3}r_2y_1 \cos(\theta x + \theta z) - 2\sqrt{3}r_1r_2 \cos(\theta x - \theta y + \theta z) - 4\sqrt{3}r_2x_1 \cos(\theta x \\
& - \theta y + \theta z) - 4r_1r_2 \cos(\theta y + \theta z) - 8r_2x_1 \cos(\theta y + \theta z) \\
& + 2\sqrt{3}r_1r_2 \cos(\theta x + \theta y + \theta z) + 4\sqrt{3}r_2x_1 \cos(\theta x + \theta y + \theta z) \\
& + 8\sqrt{3}hr_2 \sin(\theta x - \theta y) - 8\sqrt{3}r_2z_1 \sin(\theta x - \theta y) - 16hr_2 \sin(\theta y) \\
& + 16r_2z_1 \sin(\theta y) + 8\sqrt{3}hr_2 \sin(\theta x + \theta y) - 8\sqrt{3}r_2z_1 \sin(\theta x + \theta y) \\
& - 4\sqrt{3}r_1r_2 \sin(\theta x - \theta z) - 8\sqrt{3}r_2x_1 \sin(\theta x - \theta z) + 6r_1r_2 \sin(\theta x - \theta y - \theta z) \\
& + 4\sqrt{3}r_2y_1 \sin(\theta x - \theta y - \theta z) + 4\sqrt{3}r_1r_2 \sin(\theta y - \theta z) + 8r_2y_1 \sin(\theta y - \theta z) \\
& - 6r_1r_2 \sin(\theta x + \theta y - \theta z) - 4\sqrt{3}r_2y_1 \sin(\theta x + \theta y - \theta z) \\
& + 4\sqrt{3}r_1r_2 \sin(\theta x + \theta z) + 8\sqrt{3}r_2x_1 \sin(\theta x + \theta z) - 6r_1r_2 \sin(\theta x - \theta y + \theta z) \\
& - 4\sqrt{3}r_2y_1 \sin(\theta x - \theta y + \theta z) - 4\sqrt{3}r_1r_2 \sin(\theta y + \theta z) - 8r_2y_1 \sin(\theta y + \theta z) \\
& + 6r_1r_2 \sin(\theta x + \theta y + \theta z) + 4\sqrt{3}r_2y_1 \sin(\theta x + \theta y + \theta z) \\
& + (32r_1 \cos \theta + 16x_1 \cos \theta + 16\sqrt{3}y_1 \cos \theta - 8r_2 \cos(\theta - \theta y) \\
& + 4\sqrt{3}r_2 \cos(\theta - \theta x - \theta y) - 4\sqrt{3}r_2 \cos(\theta + \theta x - \theta y) \\
& + 8r_2 \cos(\theta + \theta y) + 4\sqrt{3}r_2 \cos(\theta - \theta x + \theta y) - 4\sqrt{3}r_2 \cos(\theta + \theta x + \theta y) \\
& - 6r_2 \cos(\theta - \theta x - \theta z) - 6r_2 \cos(\theta + \theta x - \theta z) - 2r_2 \cos(\theta - \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x - \theta y - \theta z) \\
& - 2r_2 \cos(\theta + \theta y - \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y - \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y - \theta z) - 6r_2 \cos(\theta - \theta x + \theta z)
\end{aligned}$$

$$\begin{aligned}
& - 6r_2 \cos(\theta + \theta x + \theta z) - 2r_2 \cos(\theta - \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta - \theta x - \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta + \theta x - \theta y + \theta z) \\
& - 2r_2 \cos(\theta + \theta y + \theta z) - \sqrt{3}r_2 \cos(\theta - \theta x + \theta y + \theta z) \\
& + \sqrt{3}r_2 \cos(\theta + \theta x + \theta y + \theta z) + 32h \sin \theta - 32z_1 \sin \theta \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta x - \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta x - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta y - \theta z) - 3r_2 \sin(\theta - \theta x - \theta y - \theta z) \\
& + 3r_2 \sin(\theta + \theta x - \theta y - \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta y - \theta z) \\
& + 3r_2 \sin(\theta - \theta x + \theta y - \theta z) - 3r_2 \sin(\theta + \theta x + \theta y - \theta z) \\
& + 2\sqrt{3}r_2 \sin(\theta - \theta x + \theta z) + 2\sqrt{3}r_2 \sin(\theta + \theta x + \theta z) \\
& - 2\sqrt{3}r_2 \sin(\theta - \theta y + \theta z) + 3r_2 \sin(\theta - \theta x - \theta y + \theta z) \\
& - 3r_2 \sin(\theta + \theta x - \theta y + \theta z) - 2\sqrt{3}r_2 \sin(\theta + \theta y + \theta z) \\
& - 3r_2 \sin(\theta - \theta x + \theta y + \theta z) + 3r_2 \sin(\theta + \theta x + \theta y + \theta z) \Big)^{\frac{1}{2}}
\end{aligned}$$

Appendix B.

The Jacobian matrix is listed as follows:

$$J_x = \begin{pmatrix} J_{x11} & J_{x12} & J_{x13} \\ J_{x21} & J_{x22} & J_{x23} \\ J_{x31} & J_{x32} & J_{x33} \end{pmatrix}$$

$$J_{x11} = 2(-h + z_1 + l_1 \sin \theta - r_2 \sin \theta y)$$

$$J_{x12} = 0$$

$$J_{x13} = -2r_2 \cos \theta z (-r_1 + x_1 - l_1 \cos \theta + r_2 \cos \theta y \cos \theta z) \sin \theta y - 2r_2 \cos \theta y (-h + z_1 + l_2 \sin \theta - r_2 \sin \theta y) - 2r_2 \sin \theta y \sin \theta z (y_1 + r_2 \cos \theta y \sin \theta z)$$

$$J_{x21} = 2(-h + z_1 + l_2 \sin \theta + \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y)$$

$$J_{x22} = \sqrt{3} r_2 \cos \theta x \cos \theta y (-h + z_1 + l_2 \sin \theta + \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) +$$

$$\sqrt{3} r_2 (\cos \theta x \cos \theta z \sin \theta y + \sin \theta x \sin \theta z) (x_1 + \frac{1}{2} (r_1 + l_2 \cos \theta)) - \frac{1}{2} r_2 \cos \theta y \cos \theta z +$$

$$\frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z) + \sqrt{3} r_2 (-\cos \theta z \sin \theta x +$$

$$\cos \theta x \sin \theta y \sin \theta z) (y_1 - \frac{1}{2} \sqrt{3} (r_1 + l_2 \cos \theta)) - \frac{1}{2} r_2 \cos \theta y \sin \theta z + \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \sin \theta x \sin \theta y \sin \theta z)$$

$$J_{x23} = 2(-h + z_1 + l_2 \sin \theta + \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) (\frac{1}{2} r_2 \cos \theta y -$$

$$\frac{1}{2} \sqrt{3} r_2 \sin \theta x \sin \theta y) + 2 (\frac{1}{2} \sqrt{3} r_2 \cos \theta y \cos \theta z \sin \theta x + \frac{1}{2} r_2 \cos \theta z \sin \theta y) (x_1 + \frac{1}{2} (r_1 +$$

$$l_2 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \cos \theta z + \frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z) +$$

$$2 (\frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x \sin \theta z + \frac{1}{2} r_2 \sin \theta y \sin \theta z) (y_1 - \frac{1}{2} \sqrt{3} (r_1 + l_1 \cos \theta)) -$$

$$\frac{1}{2} r_2 \cos \theta y \sin \theta z + \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \sin \theta x \sin \theta y \sin \theta z))$$

$$J_{x31} = 2(-h + z_1 + l_3 \sin \theta - \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y)$$

$$J_{x32} = -\sqrt{3} r_2 \cos \theta x \cos \theta y (-h + z_1 + l_3 \sin \theta - \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) -$$

$$\sqrt{3} r_2 (\cos \theta x \cos \theta z \sin \theta y + \sin \theta x \sin \theta z) (x_1 + \frac{1}{2} (r_1 + l_3 \cos \theta)) - \frac{1}{2} r_2 \cos \theta y \cos \theta z -$$

$$\frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z) - \sqrt{3} r_2 (-\cos \theta z \sin \theta x +$$

$$\cos \theta x \sin \theta y \sin \theta z) (y_1 + \frac{1}{2} \sqrt{3} (r_1 + l_3 \cos \theta)) - \frac{1}{2} r_2 \cos \theta y \sin \theta z - \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \sin \theta x \sin \theta y \sin \theta z)$$

$$\begin{aligned}
J_{x_{33}} = & 2(-h + z_1 + l_3 \sin \theta - \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) (\frac{1}{2} r_2 \cos \theta y + \\
& \frac{1}{2} \sqrt{3} r_2 \sin \theta x \sin \theta y) + 2(-\frac{1}{2} \sqrt{3} r_2 \cos \theta y \cos \theta z \sin \theta x + \frac{1}{2} r_2 \cos \theta z \sin \theta y) (x_1 + \frac{1}{2} (r_1 + \\
& l_3 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \cos \theta z - \frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z)) + \\
& 2(-\frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x \sin \theta z + \frac{1}{2} r_2 \sin \theta y \sin \theta z) (y_1 + \frac{1}{2} \sqrt{3} (r_1 + l_3 \cos \theta) - \\
& \frac{1}{2} r_2 \cos \theta y \sin \theta z - \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \sin \theta x \sin \theta y \sin \theta z))
\end{aligned}$$

$$J_q = \begin{pmatrix} J_{q_{11}} & 0 & 0 \\ 0 & J_{q_{22}} & 0 \\ 0 & 0 & J_{q_{33}} \end{pmatrix}$$

$$J_{q_{11}} = -2 \cos \theta (-r_1 + x_1 - l_1 \cos \theta + r_2 \cos \theta y \cos \theta z) + 2 \sin \theta (-h + z_1 + l_1 \sin \theta - r_2 \sin \theta y)$$

$$\begin{aligned}
J_{q_{22}} = & 2 \sin \theta (-h + z_1 + l_2 \sin \theta + \frac{1}{2} \sqrt{3} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) + \cos \theta (x_1 + \frac{1}{2} (r_1 + \\
& l_2 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \cos \theta z + \frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z)) - \\
& \sqrt{3} \cos \theta (y_1 - \frac{1}{2} \sqrt{3} (r_1 + l_2 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \sin \theta z + \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \\
& \sin \theta x \sin \theta y \sin \theta z))
\end{aligned}$$

$$\begin{aligned}
J_{q_{33}} = & 2 \sin \theta (-h + z_1 + l_3 \sin \theta - \frac{1}{2} r_2 \cos \theta y \sin \theta x + \frac{1}{2} r_2 \sin \theta y) + \cos \theta (x_1 + \frac{1}{2} (r_1 + \\
& l_3 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \cos \theta z - \frac{1}{2} \sqrt{3} r_2 (\cos \theta z \sin \theta x \sin \theta y - \cos \theta x \sin \theta z)) + \\
& \sqrt{3} \cos \theta (y_1 + \frac{1}{2} \sqrt{3} (r_1 + l_3 \cos \theta) - \frac{1}{2} r_2 \cos \theta y \sin \theta z - \frac{1}{2} \sqrt{3} r_2 (\cos \theta x \cos \theta z + \\
& \sin \theta x \sin \theta y \sin \theta z))
\end{aligned}$$