**Supplementary Material of the paper**

**Accurate model for ultra relativistic interactions between laser beams and electrons.**

**Alexandru Popa**

**PROGRAMS FOR NUMERICAL CALCULATIONS WITH MATHEMATICA 7**

We have the following equivalences between the notations from the paper and the symbols used in the scripts: $I_L \equiv IL$, $\lambda_L \equiv \lambda L$, $\tau_L \equiv \tau L$, $P \equiv P$, $\eta_i \equiv \eta i$, $\beta_{xi} \equiv \beta xi$, $\beta_{yi} \equiv \beta yi$, $\beta_{zi} \equiv \beta zi$, $\gamma_i \equiv \gamma i$, $\theta \equiv \theta$, $\phi \equiv \phi$, $c \equiv c$, $m \equiv m$, $\varepsilon_0 \equiv \varepsilon 0$, $e \equiv q$, $E_{M1} \equiv EM1$, $E_{M2} \equiv EM2$, $\omega_L \equiv \omega L$, $a_1 \equiv a1$, $a_2 \equiv a2$, $f_0 \equiv f0$, $f_1 \equiv f1$, $f_2 \equiv f2$, $f_2 \equiv f3$, $\gamma \equiv \gamma$, $ft \equiv t$, $\Delta\eta \equiv \Delta\eta$, $f\Delta\eta \equiv$ $a\ function\ equal\ to\ 1\ for\ correct\ value\ of\ \Delta\eta$, $\beta_x \equiv \beta x$, $\beta_y \equiv \beta y$, $\beta_z \equiv \beta z$, $n_x \equiv nx$, $n_y \equiv ny$, $n_z \equiv nz$, $g_1 \equiv g1$,$g_2 \equiv g2$, $g_3 \equiv g3$, $F_1 \equiv F1$, $F_2 \equiv F2$, $h_1 \equiv h1$, $h_2 \equiv h2$, $h_3 \equiv h3$, $E_x \equiv Ex$, $E_y \equiv Ey$, $E_z \equiv Ez$ $I_{av} \equiv Iav$, $I \equiv Ii$, $\underline{I}_{av} \equiv Iavn$, $\Delta t_1 \equiv \Delta t1$, $E \equiv G$, $\eta_{1i} \equiv \eta 1i$, $\eta_{1f} \equiv \eta 1f$, $\eta_{2i} \equiv \eta 2i$, $\eta_{2f} \equiv \eta 2f$, $\eta_{3i} \equiv \eta 3i$, $\eta_{3f} \equiv \eta 3f$

## 1. Calculation of $\Delta\eta$

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$ and $\beta_{zi}$ . The output date is $\Delta\eta$.

Input data.

```
IL = 5 * 10^(24);
λL = 1.054 * 10^(−6);
τL = 600 * 10^(−15);
P = 1;
ηi = 0;
βxi = 0;
βyi = 0;
```

$\beta\text{zi} = 0.01;$

$\gamma\text{i} = \frac{1}{\left(1-\beta\text{xi}^2-\beta\text{yi}^2-\beta\text{zi}^2\right)\text{^}(1/2)};$

$\phi = 0;$

$c = 299792458;$

$m = 9.10938291 * 10\text{^}(-31);$

$\varepsilon 0 = 8.8541878176 * 10\text{^}(-12);$

$q = 1.60217657 * 10\text{^}(-19);$

$\text{EM1} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 + P)};$

$\text{EM2} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 - P)};$

$\omega\text{L} = 2 * \pi * c/\lambda\text{L};$

$\text{a1} = q * \text{EM1}/(m * c * \omega\text{L});$

$\text{a2} = q * \text{EM2}/(m * c * \omega\text{L});$

Calculation of $\Delta\eta$

$\text{f0} = \gamma\text{i} * (1 - \beta\text{zi});$

$\text{f1} = -\text{a1} * (\text{Sin}[\eta] - \text{Sin}[\eta\text{i}]) + \gamma\text{i} * \beta\text{xi};$

$\text{f2} = -\text{a2} * (\text{Cos}[\eta\text{i}] - \text{Cos}[\eta]) + \gamma\text{i} * \beta\text{yi};$

$\gamma = \frac{1}{2*\text{f0}} * \left(1 + \text{f0}^2 + \text{f1}^2 + \text{f2}^2\right);$

$\Delta\eta = 10.61;$

$\text{f}\Delta\eta = \frac{1}{\omega\text{L}*\tau\text{L}*\text{f0}} * (\text{NIntegrate}[\gamma, \{\eta, \eta\text{i}, \eta\text{i} + \Delta\eta\}, \text{AccuracyGoal} \to 12])$

1.00092

## 2. Time as function of EM phase

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$ and $\beta_{zi}$ . The output data is the variation of time versus EM phase from Fig. 1.

Input data.

```
IL = 5 * 10^(24);
λL = 1.054 * 10^(-6);
τL = 600 * 10^(-15);
P = 1;
ηi = 0;
βxi = 0;
βyi = 0;
βzi = 0.01;
γi = 1/((1 - βxi^2 - βyi^2 - βzi^2)^(1/2));
c = 299792458;
m = 9.10938291 * 10^(-31);
ε0 = 8.8541878176 * 10^(-12);
q = 1.60217657 * 10^(-19);
EM1 = Sqrt[IL/(ε0 * c) * (1 + P)];
EM2 = Sqrt[IL/(ε0 * c) * (1 - P)];
ωL = 2 * π * c/λL;
a1 = q * EM1/(m * c * ωL);
a2 = q * EM2/(m * c * ωL);
```

Calculation of variation of time versus phase

```
f0 = γi * (1 - βzi);
f1 = -a1 * (Sin[η] - Sin[ηi]) + γi * βxi;
f2 = -a2 * (Cos[ηi] - Cos[η]) + γi * βyi;
γ = 1/(2 * f0) * (1 + f0^2 + f1^2 + f2^2);
f3 = γ - f0;
ft = 1/(ωL * f0) * (NIntegrate[γ, {η, ηi, ηmax}, AccuracyGoal → 12]);
Plot[ft, {ηmax, 0, 10}, PlotRange → {{0, 12}}]
```

## 3. Calculations for Fig. 2(a)

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$ and $\beta_{zi}$ . The output is Fig. 2(a).

Input data.

$\mathrm{IL} = 1 * 10\hat{}(22);$

$\lambda\mathrm{L} = 1.054 * 10\hat{}(-6);$

$\tau\mathrm{L} = 600 * 10\hat{}(-15);$

$P = 1;$

$\eta\mathrm{i} = 0;$

$\beta\mathrm{xi} = 0;$

$\beta\mathrm{yi} = 0;$

$\beta\mathrm{zi} = 0.01;$

$\gamma\mathrm{i} = \frac{1}{(1-\beta\mathrm{xi}^2-\beta\mathrm{yi}^2-\beta\mathrm{zi}^2)\hat{}(1/2)};$

$c = 299792458;$

$m = 9.10938291 * 10\hat{}(-31);$

$\varepsilon 0 = 8.8541878176 * 10\hat{}(-12);$

$q = 1.60217657 * 10\hat{}(-19);$

$\mathrm{EM1} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1+P)};$

$\mathrm{EM2} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1-P)};$

$\omega\mathrm{L} = 2 * \pi * c/\lambda\mathrm{L};$

$\mathrm{a1} = q * \mathrm{EM1}/(m * c * \omega\mathrm{L});$

$\mathrm{a2} = q * \mathrm{EM2}/(m * c * \omega\mathrm{L});$

Calculation of $\Delta\eta$

$\mathrm{f0} = \gamma\mathrm{i} * (1 - \beta\mathrm{zi});$

$\mathrm{f1} = -\mathrm{a1} * (\mathrm{Sin}[\eta] - \mathrm{Sin}[\eta\mathrm{i}]) + \gamma\mathrm{i} * \beta\mathrm{xi};$

$\mathrm{f2} = -\mathrm{a2} * (\mathrm{Cos}[\eta\mathrm{i}] - \mathrm{Cos}[\eta]) + \gamma\mathrm{i} * \beta\mathrm{yi};$

$\gamma = \frac{1}{2 * \mathrm{f0}} * \left(1 + \mathrm{f0}^2 + \mathrm{f1}^2 + \mathrm{f2}^2\right);$

$\Delta\eta = 881.1;$

$\mathrm{f}\Delta\eta = \frac{1}{\omega\mathrm{L} * \tau\mathrm{L} * \mathrm{f0}} * (\mathrm{NIntegrate}[\gamma, \{\eta, \eta\mathrm{i}, \eta\mathrm{i} + \Delta\eta\}, \mathrm{AccuracyGoal} \to 12])$

1.00015

Calculation of $\beta$z

$\mathrm{f3} = \gamma - \mathrm{f0};$

$\beta\mathrm{x} = \mathrm{f1}/\gamma;$

$\beta\mathrm{y} = \mathrm{f2}/\gamma;$

$\beta\mathrm{z} = \mathrm{f3}/\gamma;$

$\mathrm{Plot}[\beta\mathrm{z}, \{\eta, \eta\mathrm{i}, \eta\mathrm{i} + 10 * \pi\}, \mathrm{PlotRange} \to \{\{0, 32\}, \{0, 0.3\}\}]$

## 4. Calculations for Fig. 2(b)

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$ and $\beta_{zi}$ . The output is Fig. 2(b).

Input data.

$\mathrm{IL} = 5 * 10\hat{}(24);$

$\lambda\mathrm{L} = 1.054 * 10\hat{}(-6);$

$\tau\mathrm{L} = 600 * 10\hat{}(-15);$

$P = 1;$

$\eta\mathrm{i} = 0;$

$\beta\mathrm{xi} = 0;$

$\beta\mathrm{yi} = 0;$

$\beta\mathrm{zi} = 0.01;$

$\gamma\mathrm{i} = \frac{1}{\left(1-\beta\mathrm{xi}^2-\beta\mathrm{yi}^2-\beta\mathrm{zi}^2\right)\hat{}(1/2)};$

$c = 299792458;$

$m = 9.10938291 * 10\hat{}(-31);$

$\varepsilon0 = 8.8541878176 * 10\hat{}(-12);$

$q = 1.60217657 * 10\hat{}(-19);$

$\mathrm{EM1} = \sqrt{\frac{\mathrm{IL}}{\varepsilon0 * c} * (1 + P)};$

$\mathrm{EM2} = \sqrt{\frac{\mathrm{IL}}{\varepsilon0 * c} * (1 - P)};$

$\omega\mathrm{L} = 2 * \pi * c/\lambda\mathrm{L};$

$\mathrm{a1} = q * \mathrm{EM1}/(m * c * \omega\mathrm{L});$

$\mathrm{a2} = q * \mathrm{EM2}/(m * c * \omega\mathrm{L});$

Calculation of $\Delta\eta$

$\mathrm{f0} = \gamma\mathrm{i} * (1 - \beta\mathrm{zi});$

$\mathrm{f1} = -\mathrm{a1} * (\mathrm{Sin}[\eta] - \mathrm{Sin}[\eta\mathrm{i}]) + \gamma\mathrm{i} * \beta\mathrm{xi};$

$\mathbf{f2 = -a2 * (Cos[\eta i] - Cos[\eta]) + \gamma i * \beta yi;}$

$\gamma = \frac{1}{2*f0} * \left(1 + f0^2 + f1^2 + f2^2\right);$

$\Delta\eta = 10.61;$

$f\Delta\eta = \frac{1}{\omega L * \tau L * f0} * (\text{NIntegrate}[\gamma, \{\eta, \eta i, \eta i + \Delta\eta\}, \text{AccuracyGoal} \to 12])$

1.00092

Calculation of $\beta$z

$f3 = \gamma - f0;$

$\beta x = f1/\gamma;$

$\beta y = f2/\gamma;$

$\beta z = f3/\gamma;$

$\text{Plot}[\beta z, \{\eta, \eta i, \eta i + \Delta\eta\}, \text{PlotRange} \to \{\{\eta i, \eta i + 1.2 * \Delta\eta\}, \{0, 1.1\}\}]$

## 5. Calculations for Fig. 3

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$, $\beta_{zi}$ and $\phi$ . The output is Fig. 3.

Input data.

$IL = 5 * 10\text{^}(24);$

$\lambda L = 1.054 * 10\text{^}(-6);$

$\tau L = 600 * 10\text{^}(-15);$

$P = 1;$

$\eta i = 0;$

$\beta xi = 0;$

$\beta yi = 0;$

$\beta zi = 0.01;$

$\gamma i = \frac{1}{\left(1 - \beta xi^2 - \beta yi^2 - \beta zi^2\right)\text{^}(1/2)};$

$\phi = 0;$

$c = 299792458;$

$m = 9.10938291 * 10\text{^}(-31);$

$\varepsilon 0 = 8.8541878176 * 10\text{^}(-12);$

$q = 1.60217657 * 10\text{^}(-19);$

$\mathrm{EM1} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1 + P)};$

$\mathrm{EM2} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1 - P)};$

$\omega\mathrm{L} = 2 * \pi * c/\lambda\mathrm{L};$

$\mathrm{a1} = q * \mathrm{EM1}/(m * c * \omega\mathrm{L})$

$\mathrm{a2} = q * \mathrm{EM2}/(m * c * \omega\mathrm{L});$

20.1491

Calculation of $\Delta\eta$

$\mathrm{f0} = \gamma\mathrm{i} * (1 - \beta\mathrm{zi});$

$\mathrm{f1} = -\mathrm{a1} * (\mathrm{Sin}[\eta] - \mathrm{Sin}[\eta\mathrm{i}]) + \gamma\mathrm{i} * \beta\mathrm{xi};$

$\mathrm{f2} = -\mathrm{a2} * (\mathrm{Cos}[\eta\mathrm{i}] - \mathrm{Cos}[\eta]) + \gamma\mathrm{i} * \beta\mathrm{yi};$

$\gamma = \frac{1}{2*\mathrm{f0}} * \left(1 + \mathrm{f0}^2 + \mathrm{f1}^2 + \mathrm{f2}^2\right);$

$\Delta\eta = 10.61;$

$\mathrm{f}\Delta\eta = \frac{1}{\omega\mathrm{L} * \tau\mathrm{L} * \mathrm{f0}} * (\mathrm{NIntegrate}[\gamma, \{\eta, \eta\mathrm{i}, \eta\mathrm{i} + \Delta\eta\}, \mathrm{AccuracyGoal} \to 12])$

1.00092

Calculation of kinematic quantities

$\mathrm{f0} = \gamma\mathrm{i} * (1 - \beta\mathrm{zi});$

$\mathrm{f1} = -\mathrm{a1} * (\mathrm{Sin}[\eta] - \mathrm{Sin}[\eta\mathrm{i}]) + \gamma\mathrm{i} * \beta\mathrm{xi};$

$\mathrm{f2} = -\mathrm{a2} * (\mathrm{Cos}[\eta\mathrm{i}] - \mathrm{Cos}[\eta]) + \gamma\mathrm{i} * \beta\mathrm{yi};$

$\gamma = \frac{1}{2*\mathrm{f0}} * \left(1 + \mathrm{f0}^2 + \mathrm{f1}^2 + \mathrm{f2}^2\right);$

$\mathrm{f3} = \gamma - \mathrm{f0};$

$\beta\mathrm{x} = \mathrm{f1}/\gamma;$

$\beta\mathrm{y} = \mathrm{f2}/\gamma;$

$\beta\mathrm{z} = \mathrm{f3}/\gamma;$

Calculation of scattered field components

$\mathrm{nx} = \mathrm{Sin}[\theta] * \mathrm{Cos}[\phi];$

$\mathrm{ny} = \mathrm{Sin}[\theta] * \mathrm{Sin}[\phi];$

$\mathrm{nz} = \mathrm{Cos}[\theta];$

$\mathrm{g1} = -\frac{\mathrm{a1}*\mathrm{f0}}{\gamma^2} * \mathrm{Cos}[\eta] + \frac{\mathrm{f1}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g2} = -\frac{\mathrm{a2}*\mathrm{f0}}{\gamma^2} * \mathrm{Sin}[\eta] + \frac{\mathrm{f2}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g3} = -\frac{\mathrm{f0}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathbf{F1 = 1 - \frac{nx*f1}{\gamma} - \frac{ny*f2}{\gamma} - \frac{nz*f3}{\gamma};}$

$\mathbf{F2 = nx * g1 + ny * g2 + nz * g3;}$

$\mathbf{h1 = -F1 * g1 + F2 * \left(nx - \frac{f1}{\gamma}\right);}$

$\mathbf{h2 = -F1 * g2 + F2 * \left(ny - \frac{f2}{\gamma}\right);}$

$\mathbf{h3 = -F1 * g3 + F2 * \left(nz - \frac{f3}{\gamma}\right);}$

Calculation of Iav, normalized to maximum value, as function of $\theta$

$\boldsymbol{F = \frac{1}{1.94*10^{10}*F1^6} * \left(h1^2 + h2^2 + h3^2\right);}$

$\mathbf{Iavn = NIntegrate[\boldsymbol{F}/\Delta\eta, \{\eta, \eta i, \eta i + \Delta\eta\}];}$

$\mathbf{Plot[Iavn, \{\boldsymbol{\theta}, 0, 0.3\}, PlotRange \rightarrow \{\{0, 0.3\}, \{0, 1\}\}]}$

## 6. Calculations for Fig. 4(a)

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$, $\beta_{zi}$, $\theta$ and $\phi$. The output is Fig. 4(a).

Input data.

$\mathbf{IL = 10\hat{}(22);}$

$\mathbf{\lambda L = 1.054 * 10\hat{}(-6);}$

$\mathbf{\tau L = 600 * 10\hat{}(-15);}$

$\boldsymbol{P = 1;}$

$\mathbf{\eta i = 0;}$

$\mathbf{\beta xi = 0;}$

$\mathbf{\beta yi = 0;}$

$\mathbf{\beta zi = 0.01;}$

$\mathbf{\gamma i = \frac{1}{\left(1 - \beta xi^2 - \beta yi^2 - \beta zi^2\right)\hat{}(1/2)};}$

$\boldsymbol{\theta = 0.108;}$

$\boldsymbol{\phi = 0;}$

$\boldsymbol{c = 299792458;}$

$\boldsymbol{m = 9.10938291 * 10\hat{}(-31);}$

$\boldsymbol{\varepsilon 0 = 8.8541878176 * 10\hat{}(-12);}$

$\boldsymbol{q = 1.60217657 * 10\hat{}(-19);}$

$\boldsymbol{R = 0.01;}$

$\mathrm{EM1} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1 + P)};$

$\mathrm{EM2} = \sqrt{\frac{\mathrm{IL}}{\varepsilon 0 * c} * (1 - P)};$

$\omega\mathrm{L} = 2 * \pi * c/\lambda\mathrm{L};$

$\mathrm{a1} = q * \mathrm{EM1}/(m * c * \omega\mathrm{L});$

$\mathrm{a2} = q * \mathrm{EM2}/(m * c * \omega\mathrm{L});$

$K = -\frac{q * \omega\mathrm{L}}{\varepsilon 0 * c * 4 * \pi * R};$

$\mathrm{K1} = \varepsilon 0 * c * K\hat{}2;$

Calculation of $\Delta\eta$

$\mathrm{f0} = \gamma\mathrm{i} * (1 - \beta\mathrm{zi});$

$\mathrm{f1} = -\mathrm{a1} * (\mathrm{Sin}[\eta] - \mathrm{Sin}[\eta\mathrm{i}]) + \gamma\mathrm{i} * \beta\mathrm{xi};$

$\mathrm{f2} = -\mathrm{a2} * (\mathrm{Cos}[\eta\mathrm{i}] - \mathrm{Cos}[\eta]) + \gamma\mathrm{i} * \beta\mathrm{yi};$

$\gamma = \frac{1}{2 * \mathrm{f0}} * \left(1 + \mathrm{f0}^2 + \mathrm{f1}^2 + \mathrm{f2}^2\right);$

$\Delta\eta = 881.1;$

$\mathrm{f}\Delta\eta = \frac{1}{\omega\mathrm{L} * \tau\mathrm{L} * \mathrm{f0}} * (\mathrm{NIntegrate}[\gamma, \{\eta, \eta\mathrm{i}, \eta\mathrm{i} + \Delta\eta\}, \mathrm{AccuracyGoal} \to 12])$

1.00015

Calculation of kinematic quantities

$\mathrm{f3} = \gamma - \mathrm{f0};$

$\beta\mathrm{x} = \mathrm{f1}/\gamma;$

$\beta\mathrm{y} = \mathrm{f2}/\gamma;$

$\beta\mathrm{z} = \mathrm{f3}/\gamma;$

Calculation of scattered field components

$\mathrm{nx} = \mathrm{Sin}[\theta] * \mathrm{Cos}[\phi];$

$\mathrm{ny} = \mathrm{Sin}[\theta] * \mathrm{Sin}[\phi];$

$\mathrm{nz} = \mathrm{Cos}[\theta];$

$\mathrm{g1} = -\frac{\mathrm{a1} * \mathrm{f0}}{\gamma^2} * \mathrm{Cos}[\eta] + \frac{\mathrm{f1}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g2} = -\frac{\mathrm{a2} * \mathrm{f0}}{\gamma^2} * \mathrm{Sin}[\eta] + \frac{\mathrm{f2}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g3} = -\frac{\mathrm{f0}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{Plot}[\mathrm{g3}, \{\eta, \eta\mathrm{i}, \eta\mathrm{i} + \Delta\eta\}, \mathrm{PlotRange} \to \{\{0, 1.5\Delta\eta\}\}];$

$\mathrm{F1} = 1 - \frac{\mathrm{nx} * \mathrm{f1}}{\gamma} - \frac{\mathrm{ny} * \mathrm{f2}}{\gamma} - \frac{\mathrm{nz} * \mathrm{f3}}{\gamma};$

$\mathrm{F2} = \mathrm{nx} * \mathrm{g1} + \mathrm{ny} * \mathrm{g2} + \mathrm{nz} * \mathrm{g3};$

$\mathbf{h1 = -F1 * g1 + F2 * \left(nx - \frac{f1}{\gamma}\right);}$

$\mathbf{h2 = -F1 * g2 + F2 * \left(ny - \frac{f2}{\gamma}\right);}$

$\mathbf{h3 = -F1 * g3 + F2 * \left(nz - \frac{f3}{\gamma}\right);}$

$\mathbf{Ex = \mathit{K} * \frac{h1}{F1^3};}$

$\mathbf{Ey = \mathit{K} * \frac{h2}{F1^3};}$

$\mathbf{Ez = \mathit{K} * \frac{h3}{F1^3};}$

$\mathbf{Ii = \varepsilon 0 * \mathit{c} * \left(Ex^2 + Ey^2 + Ez^2\right);}$

$\mathbf{Plot[Ii, \{\eta, 0, \eta i + 20 * \pi\}, PlotRange \to \{\{0, 20 * \pi\}, \{0, 0.002\}\}]}$

## 7. Calculations for Figs. 4(b), 5 and 6

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$, $\beta_{zi}$, $\theta$ and $\phi$. The output are Figs. 4(b) and 5.

Input data.

$\mathbf{IL = 5 * 10\hat{}(24);}$

$\mathbf{\lambda L = 1.054 * 10\hat{}(-6);}$

$\mathbf{\tau L = 600 * 10\hat{}(-15);}$

$\mathbf{\mathit{P} = 1;}$

$\mathbf{\eta i = 0;}$

$\mathbf{\beta xi = 0;}$

$\mathbf{\beta yi = 0;}$

$\mathbf{\beta zi = 0.01;}$

$\mathbf{\gamma i = \frac{1}{\left(1 - \beta xi^2 - \beta yi^2 - \beta zi^2\right)\hat{}(1/2)};}$

$\mathbf{\theta = 0.108;}$

$\mathbf{\phi = 0;}$

$\mathbf{\mathit{c} = 299792458;}$

$\mathbf{\mathit{m} = 9.10938291 * 10\hat{}(-31);}$

$\mathbf{\varepsilon 0 = 8.8541878176 * 10\hat{}(-12);}$

$\mathbf{\mathit{q} = 1.60217657 * 10\hat{}(-19);}$

$\mathbf{\mathit{R} = 0.01;}$

$\text{EM1} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 + P)};$

$\text{EM2} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 - P)};$

$\omega\text{L} = 2 * \pi * c/\lambda\text{L};$

$\text{a1} = q * \text{EM1}/(m * c * \omega\text{L});$

$\text{a2} = q * \text{EM2}/(m * c * \omega\text{L});$

$K = -\frac{q * \omega\text{L}}{\varepsilon 0 * c * 4 * \pi * R};$

Calculation of $\Delta\eta$

$\text{f0} = \gamma\text{i} * (1 - \beta\text{zi});$

$\text{f1} = -\text{a1} * (\text{Sin}[\eta] - \text{Sin}[\eta\text{i}]) + \gamma\text{i} * \beta\text{xi};$

$\text{f2} = -\text{a2} * (\text{Cos}[\eta\text{i}] - \text{Cos}[\eta]) + \gamma\text{i} * \beta\text{yi};$

$\gamma = \frac{1}{2 * \text{f0}} * \left(1 + \text{f0}^2 + \text{f1}^2 + \text{f2}^2\right);$

$\Delta\eta = 10.61;$

$\text{f}\Delta\eta = \frac{1}{\omega\text{L} * \tau\text{L} * \text{f0}} * (\text{NIntegrate}[\gamma, \{\eta, \eta\text{i}, \eta\text{i} + \Delta\eta\}, \text{AccuracyGoal} \to 12])$

1.00092

Calculation of kinematic quantities

$\text{f3} = \gamma - \text{f0};$

$\beta\text{x} = \text{f1}/\gamma;$

$\beta\text{y} = \text{f2}/\gamma;$

$\beta\text{z} = \text{f3}/\gamma;$

Calculation of scattered field components

$\text{nx} = \text{Sin}[\theta] * \text{Cos}[\phi];$

$\text{ny} = \text{Sin}[\theta] * \text{Sin}[\phi];$

$\text{nz} = \text{Cos}[\theta];$

$\text{g1} = -\frac{\text{a1} * \text{f0}}{\gamma^2} * \text{Cos}[\eta] + \frac{\text{f1}}{\gamma^3} * (\text{a1} * \text{f1} * \text{Cos}[\eta] + \text{a2} * \text{f2} * \text{Sin}[\eta]);$

$\text{g2} = -\frac{\text{a2} * \text{f0}}{\gamma^2} * \text{Sin}[\eta] + \frac{\text{f2}}{\gamma^3} * (\text{a1} * \text{f1} * \text{Cos}[\eta] + \text{a2} * \text{f2} * \text{Sin}[\eta]);$

$\text{g3} = -\frac{\text{f0}}{\gamma^3} * (\text{a1} * \text{f1} * \text{Cos}[\eta] + \text{a2} * \text{f2} * \text{Sin}[\eta]);$

$\text{Plot}[\text{g3}, \{\eta, \eta\text{i}, \eta\text{i} + \Delta\eta\}, \text{PlotRange} \to \{\{0, 1.5\Delta\eta\}\}];$

$\text{F1} = 1 - \frac{\text{nx} * \text{f1}}{\gamma} - \frac{\text{ny} * \text{f2}}{\gamma} - \frac{\text{nz} * \text{f3}}{\gamma};$

$\text{F2} = \text{nx} * \text{g1} + \text{ny} * \text{g2} + \text{nz} * \text{g3};$

$\text{h1} = -\text{F1} * \text{g1} + \text{F2} * \left(\text{nx} - \frac{\text{f1}}{\gamma}\right);$

$\mathbf{h2 = -F1 * g2 + F2 * \left(ny - \frac{f2}{\gamma}\right);}$

$\mathbf{h3 = -F1 * g3 + F2 * \left(nz - \frac{f3}{\gamma}\right);}$

$\mathbf{Ex = \mathit{K}\frac{h1}{F1^3};}$

$\mathbf{Ey = \mathit{K}\frac{h2}{F1^3};}$

$\mathbf{Ez = \mathit{K}\frac{h3}{F1^3};}$

$\mathbf{Ii = \varepsilon 0 * \mathit{c} * \left(Ex^2 + Ey^2 + Ez^2\right);}$

Calculation for Fig. 4(b)

$\mathbf{Plot[Ii, \{\eta, 0, \eta i + 1.05 * \Delta\eta\}, PlotRange \to \{\{0, 1.2 * \Delta\eta\}, \{0, 2000000000\}\}]}$

Calculation for Fig. 5.

$\mathbf{Plot[Ii, \{\eta, 4.1, 4.5\}, PlotRange \to \{\{4.1, 4.5\}, \{0, 2000000000\}\}]}$

$\mathbf{\eta 1i = 4.18;}$

$\mathbf{\eta 1f = 4.38;}$

$\mathbf{\Delta t1 = NIntegrate\left[\frac{\gamma}{\omega L * f0}, \{\eta, \eta 1i, \eta 1f\}\right]}$

1.9169409783726637*^-14

Variation of E (denoted by G) versus $\eta$. Calculation for Fig. 6.

$\mathbf{\mathit{G} = \frac{Abs[\mathit{K}]}{F1^3}(h1\text{^}2 + h2\text{^}2 + h3\text{^}2)\text{^}\{1/2\};}$

$\mathbf{Plot[\mathit{G}, \{\eta, \eta i, \eta i + 1.05 * \Delta\eta\}, PlotRange \to \{\{\eta i, \eta i + 1.2 * \Delta\eta\}, \{0, 900000\}\}]}$

## 8. Calculations of harmonic intensities

The input data are $I_L$, $\lambda_L$, $\tau_L$, $P$, $\eta_i$, $\beta_{xi}$, $\beta_{yi}$, $\beta_{zi}$, $\theta$ and $\phi$. The output is the intensity spectrum.

Input data.

$\mathbf{IL = 5 * 10\text{^}(24);}$

$\mathbf{\lambda L = 1.054 * 10\text{^}(-6);}$

$\mathbf{\tau L = 600 * 10\text{^}(-15);}$

$\mathbf{\mathit{P} = 1;}$

$\mathbf{\eta i = 0;}$

$\mathbf{\beta xi = 0;}$

$\mathbf{\beta yi = 0;}$

$\beta\text{zi} = 0.01;$

$\gamma\text{i} = \frac{1}{\left(1-\beta\text{xi}^2-\beta\text{yi}^2-\beta\text{zi}^2\right)\text{^}(1/2)};$

$\theta = 0.108;$

$\phi = 0;$

$c = 299792458;$

$m = 9.10938291 * 10\text{^}(-31);$

$\varepsilon 0 = 8.8541878176 * 10\text{^}(-12);$

$q = 1.60217657 * 10\text{^}(-19);$

$R = 0.01;$

$\text{EM1} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 + P)};$

$\text{EM2} = \sqrt{\frac{\text{IL}}{\varepsilon 0 * c} * (1 - P)};$

$\omega\text{L} = 2 * \pi * c/\lambda\text{L};$

$\text{a1} = q * \text{EM1}/(m * c * \omega\text{L});$

$\text{a2} = q * \text{EM2}/(m * c * \omega\text{L});$

$K = -\frac{q*\omega\text{L}}{\varepsilon 0*c*4*\pi*R};$

$\text{K1} = \frac{\varepsilon 0*c*K\text{^}2}{2*\pi};$

Calculation of $\Delta\eta$

$\text{f0} = \gamma\text{i} * (1 - \beta\text{zi});$

$\text{f1} = -\text{a1} * (\text{Sin}[\eta] - \text{Sin}[\eta\text{i}]) + \gamma\text{i} * \beta\text{xi};$

$\text{f2} = -\text{a2} * (\text{Cos}[\eta\text{i}] - \text{Cos}[\eta]) + \gamma\text{i} * \beta\text{yi};$

$\gamma = \frac{1}{2*\text{f0}} * \left(1 + \text{f0}^2 + \text{f1}^2 + \text{f2}^2\right);$

$\Delta\eta = 10.61;$

$\text{f}\Delta\eta = \frac{1}{\omega\text{L}*\tau\text{L}*\text{f0}} * (\text{NIntegrate}[\gamma, \{\eta, \eta\text{i}, \eta\text{i} + \Delta\eta\}, \text{AccuracyGoal} \to 12])$

1.00092

Calculation of kinematic quantities

$\text{f3} = \gamma - \text{f0};$

$\beta\text{x} = \text{f1}/\gamma;$

$\beta\text{y} = \text{f2}/\gamma;$

$\beta\text{z} = \text{f3}/\gamma;$

Calculation of scattered field components

$\mathrm{nx} = \mathrm{Sin}[\theta] * \mathrm{Cos}[\phi];$

$\mathrm{ny} = \mathrm{Sin}[\theta] * \mathrm{Sin}[\phi];$

$\mathrm{nz} = \mathrm{Cos}[\theta];$

$\mathrm{g1} = -\frac{\mathrm{a1*f0}}{\gamma^2} * \mathrm{Cos}[\eta] + \frac{\mathrm{f1}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g2} = -\frac{\mathrm{a2*f0}}{\gamma^2} * \mathrm{Sin}[\eta] + \frac{\mathrm{f2}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{g3} = -\frac{\mathrm{f0}}{\gamma^3} * (\mathrm{a1} * \mathrm{f1} * \mathrm{Cos}[\eta] + \mathrm{a2} * \mathrm{f2} * \mathrm{Sin}[\eta]);$

$\mathrm{F1} = 1 - \frac{\mathrm{nx*f1}}{\gamma} - \frac{\mathrm{ny*f2}}{\gamma} - \frac{\mathrm{nz*f3}}{\gamma};$

$\mathrm{F2} = \mathrm{nx} * \mathrm{g1} + \mathrm{ny} * \mathrm{g2} + \mathrm{nz} * \mathrm{g3};$

$\mathrm{h1} = -\mathrm{F1} * \mathrm{g1} + \mathrm{F2} * \left(\mathrm{nx} - \frac{\mathrm{f1}}{\gamma}\right);$

$\mathrm{h2} = -\mathrm{F1} * \mathrm{g2} + \mathrm{F2} * \left(\mathrm{ny} - \frac{\mathrm{f2}}{\gamma}\right);$

$\mathrm{h3} = -\mathrm{F1} * \mathrm{g3} + \mathrm{F2} * \left(\mathrm{nz} - \frac{\mathrm{f3}}{\gamma}\right);$

Calculation of the spectrum

$j = 800;$

$\eta\mathrm{1i} = 4.18;$

$\eta\mathrm{1f} = 4.38;$

$\eta\mathrm{2i} = 5.04;$

$\eta\mathrm{2f} = 5.24;$

$\eta\mathrm{3i} = 10.4;$

$\eta\mathrm{3f} = 10.61;$

$\mathrm{I1cj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{1i}, \eta\mathrm{1f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{2i}, \eta\mathrm{2f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{3i}, \eta\mathrm{3f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{I2cj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{1i}, \eta\mathrm{1f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{2i}, \eta\mathrm{2f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{3i}, \eta\mathrm{3f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{I3cj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{1i}, \eta\mathrm{1f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{2i}, \eta\mathrm{2f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Cos}[j * \eta], \{\eta, \eta\mathrm{3i}, \eta\mathrm{3f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{I1sj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta\mathrm{1i}, \eta\mathrm{1f}\}, \mathrm{AccuracyGoal} \to 100\right] +$
$\mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta\mathrm{2i}, \eta\mathrm{2f}\}, \mathrm{AccuracyGoal} \to 100\right] +$

$\mathrm{NIntegrate}\left[\frac{\mathrm{h1}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 3\mathrm{i}, \eta 3\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{I2sj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 1\mathrm{i}, \eta 1\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right] +$

$\mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 2\mathrm{i}, \eta 2\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right] +$

$\mathrm{NIntegrate}\left[\frac{\mathrm{h2}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 3\mathrm{i}, \eta 3\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{I3sj} = \mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 1\mathrm{i}, \eta 1\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right] +$

$\mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 2\mathrm{i}, \eta 2\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right] +$

$\mathrm{NIntegrate}\left[\frac{\mathrm{h3}}{\mathrm{F1}^3} * \mathrm{Sin}[j * \eta], \{\eta, \eta 3\mathrm{i}, \eta 3\mathrm{f}\}, \mathrm{AccuracyGoal} \to 100\right];$

$\mathrm{Ij} = \frac{\mathrm{K1}}{3493.33} * (\mathrm{I1cj}\hat{}2 + \mathrm{I2cj}\hat{}2 + \mathrm{I3cj}\hat{}2 + \mathrm{I1sj}\hat{}2 + \mathrm{I2sj}\hat{}2 + \mathrm{I3sj}\hat{}2)$

0.0511397

ListLogLogPlot[{{60, 23.90}, {70, 6.756}, {80, 2.534}, {90, 1.786}, {100, 2.178}, {120, 3.075}, {140, 1.684}, {160, 0.8221}, {180, 0.3328}, {200, 0.4955}, {238, 1}, {250, 0.4004}, {300, 0.3217}, {350, 0.2479}, {400, 0.06457}, {450, 0.2223}, {500, 0.1230}, {600, 0.1181}, {700, 0.04912}, {800, 0.05114}}, PlotRange → {{50, 1000}, {0.01, 100}}]